Unit 3 Sets and Functions

Students' Learning Outcomes

At the end of the unit, the students will be able to:

- Recall:
  - Describe mathematics as the study of patterns, structure, and relationships.
  - Identify sets and apply operations on three sets (Subsets, overlapping sets and disjoint sets), using Venn diagrams.
- Solve problems on classification and cataloguing by using Venn diagrams for scenarios involving two sets and three sets. Further application of sets.
- Verify and apply properties/laws of union and intersection of three sets through analytical and Venn diagram methods.
- Apply concepts from set theory to real-world problems (such as in demographic classification, categorizing products in shopping malls)
- Explain product, binary relations and its domain and range.
- Recognize that a relation can be represented by a table, ordered pair and graphs.
- Recognize notation and determine the value of a function and its domain and range.
- Identify types of functions (into, onto, one-to-one, injective, surjective and bijective) by using Venn diagrams.

INTRODUCTION

In this unit, we will revise some basic concepts of set theory and functions, beginning with mathematics as an essential study of patterns, structure, and relationships. Students will learn to identify different types of sets, the laws of union and intersection for two and three sets, and their representation using Venn diagrams. Additionally, they will apply set theory to real-world problems to enhance their understanding of demographic classification and product categorization. Classification develops an understanding of the relationship between various sets. Students will also explore binary relations and functions and their representation in various forms including tables, ordered pairs, and graphs.

3.1 Mathematics as the Study of Patterns, Structures and Relationships

Mathematics is the science of patterns, structures, and relationships, comprising various branches that explore and analyze our world's logical and quantitative aspects. The strength of mathematics is based upon relations that enhance the understanding between the patterns and structure and their generalizations. A mathematical pattern is a predictable arrangement of numbers, shapes, or symbols that follows a specific rule or relationship. Virtually, patterns are the key to learning structural knowledge involving numerical and geometrical relationships. For example, look at the following numerical pattern of the numbers

1st term		2nd term		3rd term		4th term		5th term
1	,	**4**	,	**7**	,	**10**	,	**13**
	+3		+3		+3		+3	

In the above pattern, every term is obtained by adding 3 in the preceding term. This predictable rule or pattern extends continuously, making it a sequence where each term increases at a constant rate.

Consider another example of a famous sequence 0, 1, 1, 2, 3, 5, 8, 13, 21, ..., known as the Fibonacci sequence. This sequence starts with two terms, 0 and 1. Each term of the sequence is obtained by adding the previous two terms. The formula for the Fibonacci sequence is $F_n=F_{n-1}+F_{n-2}$, where $F_0=0$ and $F_1=1$ are the first and second terms respectively. This recursive pattern occurs more frequently in nature.

The study of mathematical structure is essential for mathematical competence. A mathematical structure is typically a rule of a numerical, geometric and logical relationship that holds consistency within a specific domain. A structure is a collection of items or objects, along with particular relationships defined among them. Consider a triangle made up of smaller triangles, as illustrated in Figure (iii).

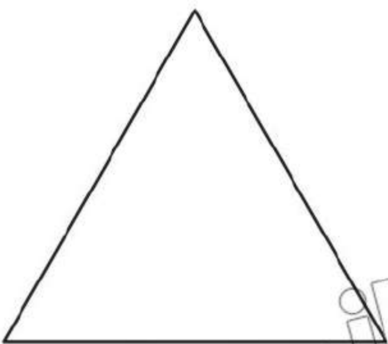


Figure (i)

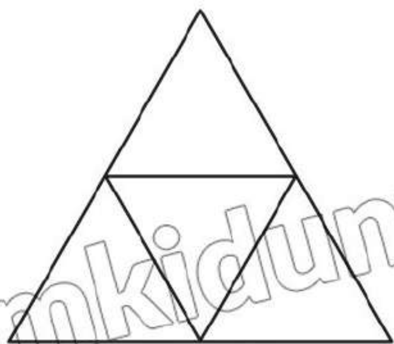


Figure (ii)

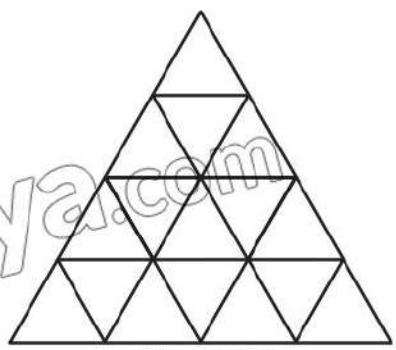


Figure (iii)

The pattern of arranging smaller triangles to form a larger triangle is clear. We can easily recognize the implicit structure: the larger triangle can be seen as consisting of several rows, where each row contains a decreasing number of smaller triangles (e.g., 7 triangles in the first row, 5 in the second, 3 in the third, and 1 at the top).

The repetition of the rows and the spatial relationships between the smaller triangles are critical structural features. The alignment of the smaller triangles creates a sense of congruence as each row is made up of triangles of the same size. At the same time, the arrangement illustrates parallel and perpendicular relationships when viewed in relation to the base of the larger triangle, as shown in Figure (iv). We can develop logical reasoning by understanding these patterns and structures and preparing them for more complex geometric concepts in various fields of mathematics. Similarly, we can establish a relationship between two sets when there is a correspondence between the numbers of these sets.

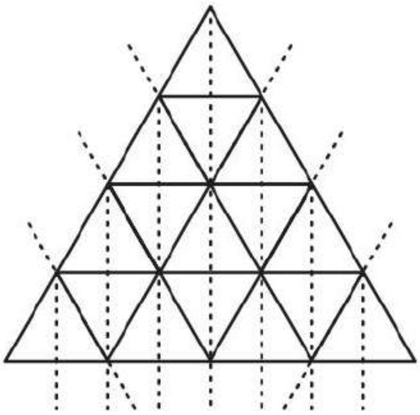


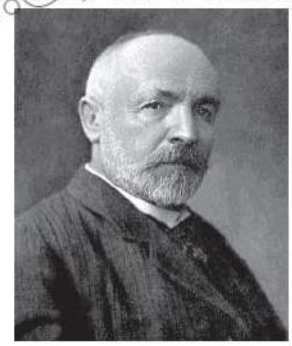
Figure (iv)

3.1.1 Basic Definitions

We are familiar with the notion of a **set** since the word is frequently used in everyday speech, for instance, water set, tea set and sofa set. It is a wonder that mathematicians have developed this ordinary word into a mathematical concept as much as it has become a language that is employed in most branches of modern mathematics. The study of sets helps in understanding the concept of relations, functions and especially in statistics we use sets to understand probability and other important ideas.

A **set** is described as a well-defined collection of distinct objects, numbers or elements, so that we may be able to decide whether the object belongs to the collection or not.

Georg Cantor (1845-1918) was a German mathematician who significantly contributed to the development of set theory, a key area in mathematics. He showed how to compare two sets by matching their members one-to-one. Cantor defined different types of infinite sets and proved that there are more real numbers than natural numbers. His proof revealed that there are many sizes of infinity. Additionally, he introduced the concepts of cardinal and ordinal numbers, along with their arithmetic operations.
https://en.wikipedia.org/wiki/Georg_Cantor



Capital letters A, B, C, X, Y, Z etc., are generally used as names of sets and small letters a, b, c, x, y, z etc., are used as members or elements of sets.

There are three different ways of describing a set.

(i) **The Descriptive form:** A set may be described in words. For instance, the set of all vowels of the English alphabet.

(ii) **The Tabular form:** A set may be described by listing its elements within brackets. If A is the set mentioned above, then we may write:

$$A = \{a, e, i, o, u\}$$

The tabular form is also known as the Roster form.

(iii) **Set-builder method:** It is sometimes more convenient or useful to employ the method of set-builder notation in specifying sets. This is done by using a symbol or letter for an arbitrary set member and stating the property common to all the members. Thus, the above set may be written as:

$$A = \{x \mid x \text{ is a vowel of the English alphabets}\}$$

This is read as A is the set of all x such that x is a vowel of the English alphabets.

The symbol used for membership of a set is $\in$. Thus, $a \in A$ means a is an element of A or a belongs to A. $c \notin A$ means c does not belong *to* A or c is not a member of A. Elements of a set can be anything: people, countries, rivers, objects of our thought. In algebra, we usually deal with sets of numbers. Such sets, along with their names are given below: -

N = The set of natural numbers = $\{1, 2, 3, \ldots\}$
W = The set of whole numbers = $\{0, 1, 2, \ldots\}$
Z = The set of integers = $\{0, \pm 1, \pm 2, \ldots\}$
O = The set of odd integers = $\{\pm 1, \pm 3, \pm 5, \ldots\}$
E = The set of even integers = $\{0, \pm 2, \pm 4, \ldots\}$
P = The set of prime numbers = $\{2, 3, 5, 7, 11, 13, 17, \ldots\}$
Q = The set of all rational numbers = $\left\{x \mid x = \frac{p}{q} \text{ where } p, q \in Z \text{ and } q \neq 0\right\}$
Q' = The set of all irrational numbers = $\left\{x \mid x \neq \frac{p}{q}, \text{ where } p, q \in Z \text{ and } q \neq 0\right\}$
R = The set of all real numbers = $Q \cup Q'$

A set with only one element is called a **singleton set.** For example, {3}, {a}, and {Saturday} are singleton sets. The set with no elements (zero number of elements) is called an **empty set**, **null set**, or **Void set**. The empty set is denoted by the symbol ϕ or { }.

> **Remember!**
> The set {0} is a singleton set having zero as its only element, and not the empty set.

Equal sets: Two sets A and B are equal if they have exactly the same elements or if every element of set A is an element of set B. If two sets A and B are equal, we write $A=B$. Thus, the sets $\{1, 2, 3\}$ and $\{2, 1, 3\}$ are equal.

Equivalent sets: Two sets A and B are equivalent if they have the same number of elements. For example, if $A = \{a, b, c, d, e\}$ and $B = \{1, 2, 3, 4, 5\}$, then A and B are equivalent sets. The symbol $\sim$ is used to represent equivalent sets. Thus, we can write $A \sim B$.

Subset: If every element of a set A is an element of set B, then A is a subset of B. Symbolically this is written as $A \subseteq B$ (A is a subset of B).

In such a case, we say B is a superset of A. Symbolically this is written as:

$$B \supseteq A \text{ (}B\text{ is a superset of }A\text{).}$$

Remember!

The subset of a set can also be stated as follows: $A \subseteq B$ iff $\forall x \in A \Rightarrow x \in B$

Proper subset: If A is a subset of B and B contains at least one element that is not an element of A, then A is said to be a proper subset of B. In such a case, we write:

$$A \subset B \text{ (}A\text{ is a proper subset of }B\text{).}$$

Improper subset: If A is a subset of B and $A = B$, then we say that A is an improper subset of B. From this definition, it also follows that every set A is a subset of itself and is called an improper subset.

Remember!

When we do not want to distinguish between proper and improper subsets, we may use the symbol $\subseteq$ for the relationship. It is easy to see that:

$$N \subset W \subset Z \subset Q \subset R$$

For example, let $A = \{a, b, c\}$, $B = \{c, a, b\}$ and $C = \{a, b, c, d\}$, then clearly

$$A \subset C, \quad B \subset C \quad \text{but} \quad A = B.$$

Notice that each of sets A and B is an improper subset of the other because $A = B$.

Universal set: The set that contains all objects or elements under consideration is called the universal set or the universe of discourse. It is denoted by U.

Power set: The power set of a set S denoted by $P(S)$ is the set containing all the possible subsets of S. For Example:

(i) If $C = \{a, b, c, d\}$, then

$$P(C) = \{\phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, \{a, b, c, d\}\}.$$

(ii) If $D = \{a\}$, then $P(D) = \{\phi, \{a\}\}$

If S is a finite set with $n(S) = m$ representing the number of elements of the set S, then $n\{P(S)\} = 2^m$ is the number of the elements of the power set.

EXERCISE 3.1

1. Write the following sets in set builder notation:
   (i) $\{1, 4, 9, 16, 25, 36, \ldots, 484\}$ (ii) $\{2, 4, 8, 16, 32, 64, \ldots, 150\}$
   (iii) $\{0, \pm 1, \pm 2, \ldots, \pm 1000\}$ (iv) $\{6, 12, 18, \ldots, 120\}$
   (v) $\{100, 102, 104, \ldots, 400\}$ (vi) $\{1, 3, 9, 27, 81, \ldots\}$
   (vii) $\{1, 2, 4, 5, 10, 20, 25, 50, 100\}$ (viii) $\{5, 10, 15, \ldots, 100\}$
   (ix) The set of all integers between – 100 and 1000
2. Write each of the following sets in tabular forms:
   (i) $\{x \mid x \text{ is a multiple of } 3 \wedge x \leq 35\}$ (ii) $\{x \mid x \in R \wedge 2x+1=0\}$
   (iii) $\{x \mid x \in P \wedge x < 12\}$ (iv) $\{x \mid x \text{ is a divisor of } 128\}$
   (v) $\{x \mid x = 2^n, n \in N \wedge n < 8\}$ (vi) $\{x \mid x \in N \wedge x+4=0\}$
   (vii) $\{x \mid x \in N \wedge x = x\}$ (viii) $\{x \mid x \in Z \wedge 3x+1=0\}$
3. Write two proper subsets of each of the following sets:
   (i) $\{a, b, c\}$ (ii) $\{0, 1\}$ (iii) N (iv) Z
   (v) Q (vi) R (vii) $\{x \mid x \in Q \wedge 0 < x \leq 2\}$
4. Is there any set which has no proper subset? If so, name that set.
5. What is the difference between $\{a, b\}$ and $\{\{a, b\}\}$?
6. What is the number of elements of the power set of each of the following sets?
   (i) $\{\ \}$ (ii) $\{0, 1\}$ (iii) $\{1, 2, 3, 4, 5, 6, 7\}$
   (iv) $\{0, 1, 2, 3, 4, 5, 6, 7\}$ (v) $\{a, \{b, c\}\}$
   (vi) $\{\{a, b\}, \{b, c\}, \{d, e\}\}$
7. Write down the power set of each of the following sets:
   (i) $\{9, 11\}$ (ii) $\{+, -, \times, \div\}$ (iii) $\{\phi\}$ (iv) $\{a, \{b, c\}\}$

3.2 Operations on Sets

Just as operations of addition, subtraction etc., are performed on numbers, the operations of union, intersection etc., are performed on sets. We are already familiar with them. A review of the main rules is given below:

Union of Two Sets

The union of two sets A and B, denoted by $A \cup B$, is the set of all elements which belong to A or B. Symbolically;

$$A \cup B = \{x \mid x \in A \vee x \in B\}.$$

Thus if $A = \{1, 2, 3\}$, $B = \{2, 3, 4, 5\}$, then $A \cup B = \{1, 2, 3, 4, 5\}$

Intersection of Two Sets

The intersection of two sets A and B, denoted by $A \cap B$, is the set of all elements that belong to both A and B. Symbolically:

$$A \cap B = \{x \mid x \in A \wedge x \in B\}.$$

Thus, for the above sets A and B, $A \cap B = \{2, 3\}$.

Remember!

The symbol $\vee$ means or.
The symbol $\wedge$ means and.

Disjoint Sets

If the intersection of two sets is the empty set, the sets are said to be disjoint. For example, if

S_1 = The set of odd natural numbers and S_2 = The set of even natural numbers, then S_1 and S_2 are disjoint sets. Similarly, the set of arts students and the set of science students of a college are disjoint sets.

Overlapping Sets

If the intersection of two sets is non-empty but neither is a subset of the other, the sets are called overlapping sets, e.g., if

$L = \{2, 3, 4, 5, 6\}$ and $M = \{5, 6, 7, 8, 9, 10\}$, then L and M are overlapping sets.

Difference of Two Sets

The difference between the sets A and B denoted by $A–B$, consists of all the elements that belong to A but do not belong to B.

Symbolically, $A - B = \{x \mid x \in A \wedge x \notin B\}$ and $B - A = \{x \mid x \in B \wedge x \notin A\}$

For example, if $A = \{1, 2, 3, 4, 5\}$ and

$B = \{4, 5, 6, 7, 8, 9, 10\}$,

then $A - B = \{1, 2, 3\}$ and $B - A = \{6, 7, 8, 9, 10\}$.

Notice that: $A - B \neq B - A$.

Complement of a Set

The complement of a set A, denoted by A' or A^c relative to the universal set U is the set of all elements of U, which do not belong to A. Symbolically:

Note:

In view of the definition of complement and difference sets it is evident that for any set A, $A' = U - A$

$$A' = \{x \mid x \in U \wedge x \notin A\}$$

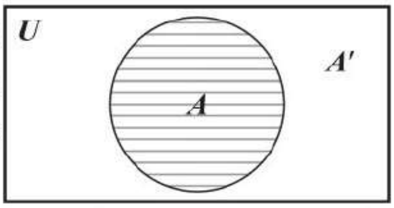
For example, if $U = Z$, then $E' = O$ and $O' = E$

For example, If U = Set of alphabets of English language, C = Set of consonants,

W = Set of vowels, then $C' = W$ and $W' = C$.

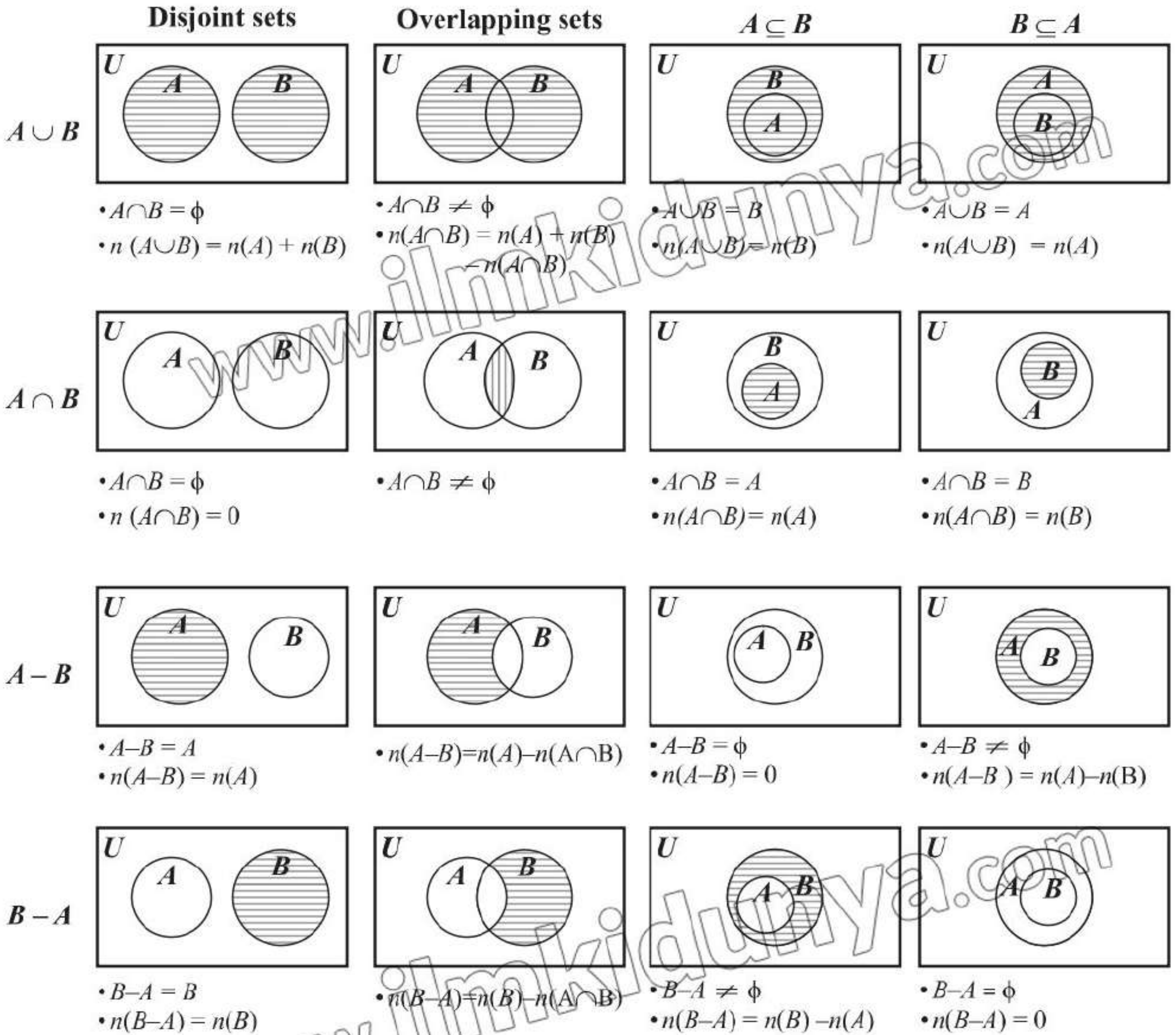
3.2.1 Identification of Sets Using Venn Diagram

Venn diagrams are very useful in depicting visually the basic concepts of sets and relationships between sets. These diagrams were first used by an English logician and mathematician John Venn (1834 to 1883 A.D).



In the adjoining figures, the rectangle represents the universal set U and the shaded circular region represents a set A and the remaining portion of the rectangle represents the A' or $U - A$.

Below are given some more diagrams illustrating basic operations on two sets in different cases (the lined region represents the result of the relevant operation in each case shown below).



3.2.2 Operations on Three Sets

If A, B and C are three given sets, operations of union and intersection can be performed on them in the following ways:

(i) $A \cup (B \cup C)$	(ii) $(A \cup B) \cup C$	(iii) $A \cap (B \cup C)$
(iv) $(A \cap B) \cap C$	(v) $A \cup (B \cap C)$	(vi) $(A \cap C) \cup (B \cap C)$
(vii) $(A \cup B) \cap C$	(viii) $(A \cap B) \cup C$	(ix) $(A \cup C) \cap (B \cup C)$

3.2.2.1 Properties of union and intersection

We now state the fundamental properties of union and intersection of two or three sets.

Properties

(i) $A \cup B = B \cup A$ (Commutative property of Union)

(ii) $A \cap B = B \cap A$ (Commutative property of Intersection)

(iii) $A \cup (B \cup C) = (A \cup B) \cup C$ (Associative property of Union)

(iv) $A \cap (B \cap C) = (A \cap B) \cap C$ (Associative property of Intersection)

(v) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ (Distributivity of Union over intersection)

(vi) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (Distributivity of intersection over Union)

(vii) $(A \cup B)' = A' \cap B'$

(viii) $(A \cap B)' = A' \cup B'$ (De Morgan's Laws)

Verification of the Properties Using Sets

Let $A = \{1, 2, 3\}$, $B = \{2, 3, 4, 5\}$ and $C = \{3, 4, 5, 6, 7, 8\}$

(i) $A \cup B = \{1, 2, 3\} \cup \{2, 3, 4, 5\}$; $B \cup A = \{2, 3, 4, 5\} \cup \{1, 2, 3\}$

$= \{1, 2, 3, 4, 5\}$; $= \{1, 2, 3, 4, 5\}$

$\therefore\ A \cup B = B \cup A$

(ii) $A \cap B = \{1, 2, 3\} \cap \{2, 3, 4, 5\}$; $B \cap A = \{2, 3, 4, 5\} \cap \{1, 2, 3\}$

$= \{2, 3\}$; $= \{2, 3\}$

$\therefore\ A \cap B = B \cap A$

(iii) and (iv) Verify yourself.

(v) $A \cup (B \cap C) = \{1, 2, 3\} \cup [\{2, 3, 4, 5\} \cap \{3, 4, 5, 6, 7, 8\}]$

$= \{1, 2, 3\} \cup \{3, 4, 5\}$

$= \{1, 2, 3, 4, 5\}$ … (i)

$(A \cup B) \cap (A \cup C) = [\{1, 2, 3\} \cup \{2, 3, 4, 5\}] \cap [\{1, 2, 3\} \cup \{3, 4, 5, 6, 7, 8\}]$

$= \{1, 2, 3, 4, 5\} \cap \{1, 2, 3, 4, 5, 6, 7, 8\}$

$= \{1, 2, 3, 4, 5\}$ …(ii)

From (i) and (ii), $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(vi) Verify yourself.

(vii) Let the universal set be $U = \{1,2,3,4,5,6,7,8,9,10\}$

$A \cup B = \{1, 2, 3\} \cup \{2, 3, 4, 5\} = \{1, 2, 3, 4, 5\}$

$(A \cup B)' = U - (A \cup B) = \{6, 7, 8, 9, 10\}$ ...(i)

$A' = U - A = \{4, 5, 6, 7, 8, 9, 10\}$

$B' = U - B = \{1, 6, 7, 8, 9, 10\}$

$A' \cap B' = \{4, 5, 6, 7, 8, 9, 10\} \cap \{1, 6, 7, 8, 9, 10\}$

$= \{6, 7, 8, 9, 10\}$ ...(ii)

From (i) and (ii), $(A \cup B)' = A' \cap B'$

(viii) Verify yourself.

Verification of the properties with the help of Venn diagrams

(i) and (ii): Verification is very simple, therefore, do it by yourself.

(iii): In Fig. (1), set A is represented by a vertically lined region and $B \cup C$ is represented by a horizontally lined region. The set $A \cup (B \cup C)$ is represented by the region lined either in one way or both.

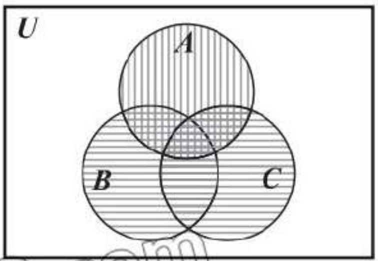


Fig. (1)

In Fig. (2) $A \cup B$ is represented by a horizontally lined region and C by a vertically lined region. $(A \cup B) \cup C$ is represented by the region lined in either one or both ways.

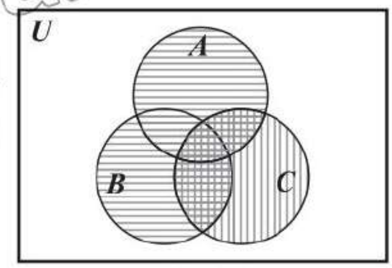


Fig. (2)

From Fig (1) and (2) we can see that

$$A \cup (B \cup C) = (A \cup B) \cup C$$

(iv) In Fig. (3), the doubly lined region represents

$$A \cap (B \cap C)$$

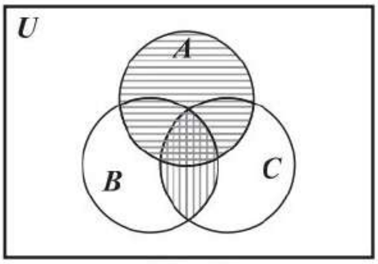


Fig. (3)

In Fig. (4), the doubly lined region represents $(A \cap B) \cap C$.
Since in Fig. (3) and Fig. (4), these regions are the same, therefore, $A \cap (B \cap C) = (A \cap B) \cap C$.

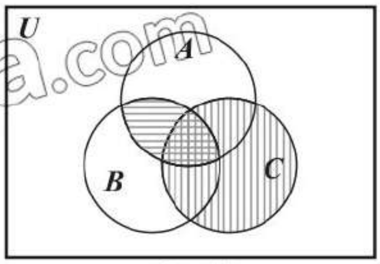


Fig. (4)

(v) In Fig. (5), $A \cup (B \cap C)$ is represented by the region which is lined horizontally or vertically or both ways.

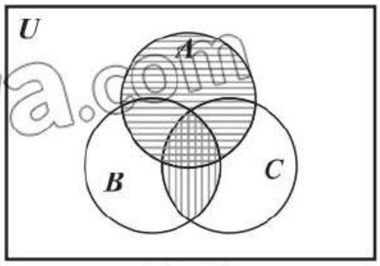


Fig. (5)

In Fig. (6), $(A \cup B) \cap (A \cup C)$ is represented by the doubly lined region.

Since the two regions in Fig (5) and (6) are the same, therefore.

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

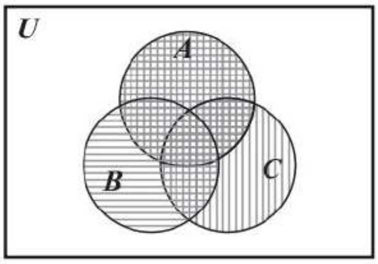


Fig. (6)

(vi) Verify yourselves.

(vii) In Fig. (7), $(A \cup B)'$ is represented by a vertically lined region.

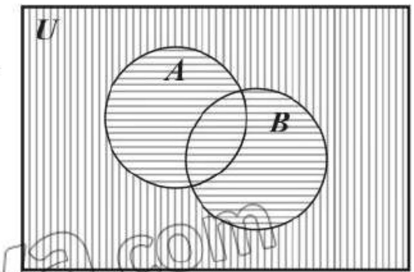


Fig. (7)

In Fig. (8), the doubly lined region represents $A' \cap B'$.

The two regions in Fig. (7) and (8) are the same, therefore, $(A \cup B)' = A' \cap B'$

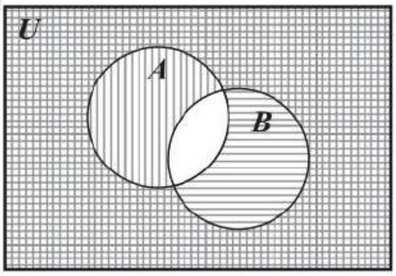
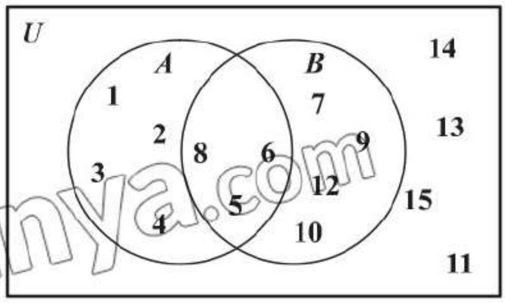


Fig. (8)

(viii) Verify yourselves.

Note:
Only overlapping sets have been considered in the Venn diagrams above. Verification for other cases can be conducted similarly.

Example 1: Consider the adjacent Venn diagram illustrating two non-empty sets, A and B.



(a) Determine the number of elements common to sets A and B.

(b) Identify all the elements exclusively in set B and not in set A.

(c) Calculate the union of sets A and B.

Solution: From the information provided in the Venn diagram, we have:

Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$

$A = \{1, 2, 3, 4, 5, 6, 8\}$

$B = \{5, 6, 7, 8, 9, 10, 12\}$

(a) The elements in both sets A and B are the intersection of the sets:

$A \cap B = \{5, 6, 8\}$

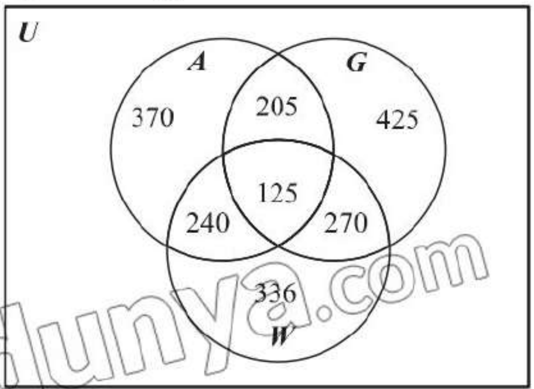
(b) The elements that are only in set B, not in set A, is the sets' differences.

$B - A = \{7, 9, 10, 12\}$

(c) $A \cup B = \{1, 2, 3, 4, 5, 6, 8\} \cup \{5, 6, 7, 8, 9, 10, 12\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12\}$

Example 2: Consider the adjacent Venn diagram representing the students enrolled in different courses in an IT institution.



U = {Students enrolled in IT institutions}

A = {Students enrolled in an Applied Robotics}

G = {Students enrolled in a Game Development}

W = {Students enrolled in a Web Designing}

(a) How many students enrolled in the applied Robotics course?

(b) Determine the total number of Students enrolled in a Game Development.

(c) How many students are enrolled in the Game development and Web designing course?

(d) Identify the students enrolled in Web development but not Applied Robotics.

(e) How many students are enrolled in IT institutions?

(f) How many students enrolled in all three courses?

Solution:

(a) Set A represents the total number of students enrolled in the Applied Robotics program.

$$\text{Total} = 370 + 205 + 125 + 240 = 940$$

So, the total number of students in the Applied Robotics course is 940.

(b) The total number of students enrolled in a Game Development is represented by the set G.

$$\text{Total} = 205 + 125 + 270 + 425 = 1025$$

Thus, the Students enrolled in a Game Development is 1025

(c) Total students are enrolled in both the Game development and Web designing The course is the intersection of G and W.

$$G \cap W = 125 + 270 = 395$$

Therefore, 395 students are enrolled in both the Game development and Web designing Course.

(d) The students who are enrolled in Web development but not in Applied Robotics is the sum of values 336 and 270 in set W.

Total = 336+270 = 606

So, there are 606 students who enrolled in Web development courses but not in Applied Robotics.

(e) The total number of students enrolled in all three courses is represented by all the values inside the circles.

$$\text{Total} = 370 + 205 + 125 + 240 + 425 + 270 + 336 = 1971$$

There are a total of 1971 students enrolled in IT Institutions.

(f) The students who enrolled in all three courses are the intersection of all the circles are represented by the value 125.

3.2.2 Real-World Applications

In this section, we will learn to apply concepts from set theory to real-world problems, such as solving problems on classification and cataloging using Venn diagrams. We will also explore some real-life situations, such as demographic classification and categorizing products in shopping malls.

For this purpose, we use the concept of cardinality of a set. The cardinality of a set is defined as the total number of elements of a set. The cardinality of a set is basically the size of the set. For a non-empty set A, the cardinality of a set is denoted by $n(A)$.

If $A = \{1, 3, 5, 7, 9, 11\}$, then $n(A)=6$. To find the cardinality of a set, we use the following rule called the **inclusion-exclusion principle** for two or three sets.

Principle of Inclusion and Exclusion for Two Sets

Let A and B be finite sets, then

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

and $A \cup B$ and $A \cap B$ are also finite.

Principle of Inclusion and Exclusion for Three Sets

If A, B and C are finite sets, then

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

and $A \cup B \cup C$, $A \cap B$, $A \cap C$, $B \cap C$ and $A \cap B \cap C$ are also finite.

Example 3: There are 98 secondary school students in a sports club. 58 students join the swimming club, and 50 join the tug-of-war club. How many students participated in both games?

Solution: Let U = {total student in a sports club of school}

A = {students who participated in swimming club}

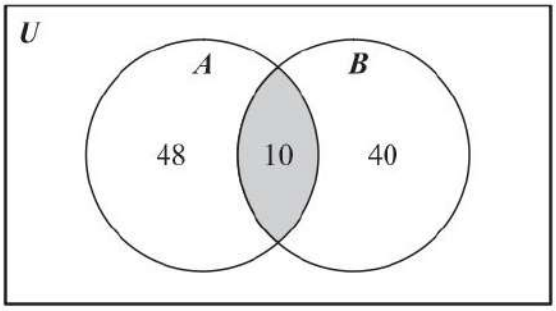
B = {students who participated in tug-of-war club}

From the statement of problems, we have

$n(U) = n(A \cup B) = 98, n(A) = 58, n(B) = 50.$

We want to find the total number of students who participated in both clubs.

$n(A \cap B) = ?$



Using the principles of inclusion and exclusion for two sets:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$\Rightarrow \quad n(A \cap B) = n(A) + n(B) - n(A \cup B)$$

$$= 58 + 50 - 98$$

$$= 10$$

Thus, 10 students participated in both clubs.

The adjacent Venn diagram shows the number of students in each sports club.

Example 4: Mr. Saleem, a school teacher, has a small library in his house containing 150 books. He has two main categories for these books: islamic and science. He categorized 70 books as islamic books and 90 books as science books. There are 15 books that neither belong to the islamic nor science books category. How many books are classified under both the islamic and science categories?

Solution: Let U = {total number of books in library}

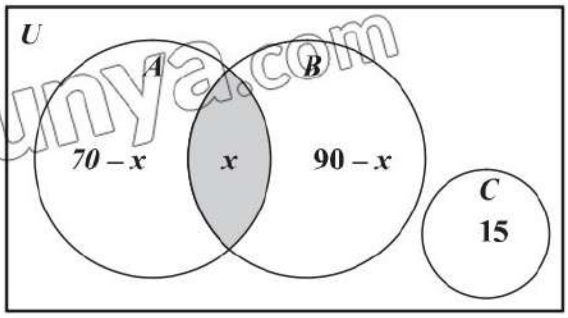
A ={70 books in Islamic category}

B ={90 books in Science category}

C ={15 book that does not belong to any category}

x = number of books that belong to both the categories

The adjacent Venn diagram shows the number of books that are classified under both the islamic and science categories



As, $n(U) = 150$

So, $70 - x + x + 90 - x + 15 = 150$

$\Rightarrow 175 - x = 150$

$\Rightarrow x = 25$

Thus, 25 books are classified under both islamic and science categories.

Example 5: In a college, 45 teachers teach mathematics or physics or chemistry. Here is information about teachers who teach different subjects:

- 18 teach mathematics
- 12 teach physics
- 8 teach chemistry
- 6 teach both mathematics and physics
- 4 teach both physics and chemistry
- 2 teach both mathematics and chemistry.
- How many teachers teach all three subjects?

Solution: Let U = {total number of teachers in the college}

M = {teachers who teach mathematics}

P = {teachers who teach physics}

C = {teachers who teach chemistry}

From the statement of problems, we have

$n(M \cup P \cup C) = 45$, $n(M) = 18$, $n(P) = 12$, $n(C) = 8$, $n(M \cap P) = 6$,

$n(P \cap C) = 4$, $n(M \cap C) = 2$

We want to find the total number of teachers who teach all the subjects.

$n(M \cap P \cap C) = ?$

Using the principle of inclusion and exclusion for three sets:

$$n(M \cup P \cup C) = n(M) + n(P) + n(C) - n(M \cap P) - n(P \cap C) - n(M \cap C) + n(M \cap P \cap C)$$

$$\Rightarrow n(M \cap P \cap C) = n(M \cup P \cup C) - n(M) - n(P) - n(C) + n(M \cap P) + n(P \cap C) + n(M \cap C)$$

$$= 45 - 18 - 12 - 8 + 6 + 4 + 2$$

$$= 19$$

Therefore 19 teachers teach all three subjects.

Example 6: A survey of 130 customers in a shopping mall was conducted in which they were asked about buying preferences.

The survey result showed the following statistics:

- 57 customers bought garments
- 50 customers bought cosmetics
- 46 customers bought electronics
- 31 customers purchased both garments and cosmetics
- 25 customers purchased both garments and electronics
- 21 customers purchased both cosmetics and electronics
- 12 customers purchased all three products i.e. garments, cosmetics, and electronics.

(a) How many of the customers bought at least one of the products: garments, cosmetics or electronics.

(b) How many of the customers bought only one of the products: garments, Cosmetics or electronics?

(c) How many customers did not buy any of the three products?

Solution: Let U = {total number of customers surveyed in the shopping mall}

G = {Customer who bought garments}

C = {Customer who bought cosmetics}

E = {Customer who bought electronics}

From the statement of problems, we have

$$n(U) = 130,\ n(G) = 57,\ n(C) = 50,\ n(E) = 46,\ n(G \cap C) = 31,$$

$$n(G \cap E) = 25,\ n(C \cap E) = 21 \text{ and } n(G \cap C \cap E) = 12.$$

(a) We want to find the total number of customers who have bought at least one of the products: garments, cosmetics, or electronics.

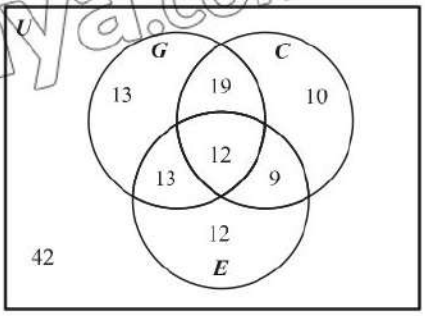
We are to find $n(G \cup C \cup E)$.

Using the principle of inclusion and exclusion for three sets:

$$n(G \cup C \cup E) = n(G) + n(C) + n(E) - n(G \cap C) - n(G \cap E) - n(C \cap E) + n(G \cap C \cap E)$$

$$= 57 + 50 + 46 - 31 - 25 - 21 + 12 = 88$$

Thus, 88 customers bought at least one of the products: garments, cosmetics, or electronics.



(b) Customers who bought only garments

$= n(G) - n(G \cap C) - n(G \cap E) + n(G \cap C \cap E)$

$= 57 - 31 - 25 + 12$

$= 13$

Customers who bought only cosmetics

$= n(C) - n(G \cap C) - n(C \cap E) + n(G \cap C \cap E)$

$= 50 - 31 - 21 + 12$

$= 10$

Customers who bought only electronics

$= n(E) - n(G \cap E) - n(C \cap E) + n(G \cap C \cap E)$

$= 46 - 25 - 21 + 12 = 12$

Therefore, the customers bought only one of the products: garments, cosmetics, or electronics $= 13 + 10 + 12 = 35$

Challenge!

The Venn diagram above illustrates the scenario presented in Example 7. Can you provide a justification for each value within the circles?

(c) Since the total number of Customers surveyed was 130, and 88 customers bought at least one of the products: garments, cosmetics, or electronics. The customers who did not buy any of the three products can be calculated as:

$$n(G \cup C \cup E)^c = n(U) - n(G \cup C \cup E)$$

$$= 130 - 88 = 42$$

So, 42 customers did not buy any of the three products.

Exercise 3.2

1. Consider the universal set $U = \{x: x \text{ is multiple of 2 and } 0 < x \leq 30\}$,

   $A = \{x : x \text{ is a multiple of } 6\}$ and $B = \{x : x \text{ is a multiple of } 8\}$

   (i) List all elements of sets A and B in tabular form

   (ii) Find $A \cap B$ (iii) Draw a Venn diagram

2. Let, $U = \{x : x \text{ is an integer and } 0 < x \leq 150\}$,

   $G = \{x : x = 2^m \text{ for integer } m \text{ and } 0 \leq m \leq 12\}$ and

   $H = \{x : x \text{ is a square}\}$

   (i) List all elements of sets G and H in tabular form

   (ii) Find $G \cup H$ (iii) Find $G \cap H$

3. Consider the sets $P = \{x : x \text{ is a prime number and } 0 < x \leq 20\}$ and

   $Q = \{x : x \text{ is a divisor of 210 and } 0 < x \leq 20\}$

   (i) Find $P \cap Q$ (ii) Find $P \cup Q$

4. Verify the commutative properties of union and intersection for the following pairs of sets:

(i) $A = \{1, 2, 3, 4, 5\}$, $B = \{4, 6, 8, 10\}$ (ii) N, Z

(iii) $A = \{x \mid x \in R \wedge x \geq 0\}$, $B = R$.

5. Let $U = \{a, b, c, d, e, f, g, h, i, j\}$

$A = \{a, b, c, d, g, h\}$, $B = \{c, d, e, f, j\}$,

Verify De Morgan's Laws for these sets. Draw Venn diagram

6. If $U = \{1, 2, 3, \ldots, 20\}$ and $A = \{1, 3, 5, \ldots, 19\}$, verify the following:

(i) $A \cup A' = U$ (ii) $A \cap U = A$ (iii) $A \cap A' = \phi$

7. In a class of 55 students, 34 like to play cricket and 30 like to play hockey. Also each student likes to play at least one of the two games. How many students like to play both games?

8. In a group of 500 employees, 250 can speak Urdu, 150 can speak English, 50 can speak Punjabi, 40 can speak Urdu and English, 30 can speak both English and Punjabi, and 10 can speak Urdu and Punjabi. How many can speak all three languages?

9. In sports events, 19 people wear blue shirts, 15 wear green shirts, 3 wear blue and green shirts, 4 wear a cap and blue shirts, and 2 wear a cap and green shirts. The total number of people with either a blue or green shirt or cap is 25. How many people are wearing caps?

10. In a training session,17 participants have laptops, 11 have tablets, 9 have laptops and tablets, 6 have laptops and books, and 4 have both tablets and books. Eight participants have all three items. The total number of participants with laptops, tablets, or books is 35. How many participants have books?

11. A shopping mall has150 employees labelled 1 to 150, representing the Universal set U. The employees fall into the following categories:

- Set A: 40 employees with a salary range of 30k-45k, labelled from 50 to 89.
- Set B: 50 employees with a salary range of 50k-80k, labelled from 101 to 150.
- Set C: 60 employees with a salary range of 100k-150k, labelled from 1 to 49 and 90 to 100.

(a) Find $(A' \cup B') \cap C$ (a) Find $n\{A \cap (B^c \cap C^C)\}$

12. In a secondary school with 125 students participate in at least one of the following sports: cricket, football, or hockey.
- 60 students play cricket.
- 70 students play football.
- 40 students play hockey.
- 25 students play both cricket and football.
- 15 students play both football and hockey.
- 10 students play both cricket and hockey.
  - (a) How many students play all three sports?
  - (b) Draw a Venn diagram showing the distribution of sports participation in all the games.

13. A survey was conducted in which 130 people were asked about their favourite foods. The survey results showed the following information:
- 40 people said they liked nihari
- 65 people said they liked biryani
- 50 people said they liked korma
- 20 people said they liked nihari and biryani
- 35 people said they liked biryani and korma
- 27 people said they liked nihari and korma
- 12 people said they liked all three foods nihari, biryani, and korma
  - (a) At least how many people like nihari, biryani or korma?
  - (b) How many people did not like nihari, biryani, or korma?
  - (c) How many people like only one of the following foods: nihari, biryani, or korma?
  - (d) Draw a Venn diagram.

3.3 Binary Relations

In everyday use, relation means an abstract type of connection between two persons or objects, for instance, (teacher, pupil), (mother, son), (husband, wife), (brother, sister), (friend, friend), (house, owner). In mathematics also some operations determine the relationship between two numbers, for example:

$>$: (5, 4) ; square: (25, 5) ; Square root: (2,4) ; Equal: $(2 \times 2, 4)$.

In the above examples >, square, square root and equal are examples of relations.

Mathematically, a relation is any set of ordered pairs. The relationship between the components of an ordered pair may or may not be mentioned.

(i) Let A and B be two non-empty sets, then the Cartesian product is the set of all ordered pairs (x, y) such that $x \in A$ and $y \in B$ and is denoted by $A \times B$. Symbolically we can write it as $A \times B = \{(x, y) \mid x \in A \text{ and } y \in B\}$.

(ii) Any subset of $A \times B$ is called a binary relation, or simply a relation, from A to B. Ordinarily a relation will be denoted by the letter r.

(iii) The set of the first elements of the ordered pairs forming a relation is called its domain. The domain of any relation r is denoted as Dom r.

(iv) The set of the second elements of the ordered pairs forming a relation is called its range. The range of any relation r is denoted as Ran r.

(v) If A is a non-empty set, any subset of $A \times A$ is called a relation in A.

Example 7: Let c_1, c_2, c_3 be three children and m_1, m_2 be two men such that the father of both c_1, c_2 is m_1 and father of c_3 is m_2. Find the relation {(child, father)}

Solution: C = Set of children = $\{c_1, c_2, c_3\}$ and F = set of fathers = $\{m_1, m_2\}$

The Cartesian product of C and F:

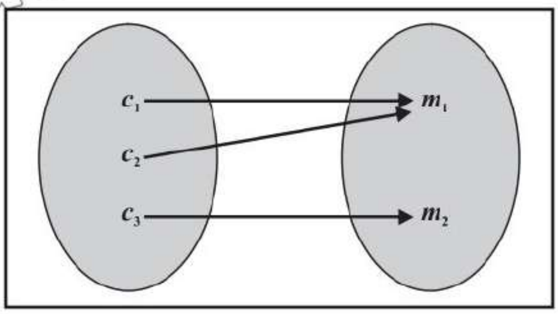
$C \times F = \{(c_1, m_1), (c_1, m_2), (c_2, m_1), (c_2, m_2), (c_3, m_1), (c_3, m_2)\}$

r = set of ordered pairs (child, father).

$= \{(c_1, m_1), (c_2, m_1), (c_3, m_2)\}$

Dom $r = \{c_1, c_2, c_3\}$, Range $r = \{m_1, m_2\}$

The relation is shown diagrammatically in adjacent figure.



Example 8: Let $A = \{1, 2, 3\}$. Determine the relation r such that $x\, r\, y$ iff $x < y$.

Solution: $A \times A = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$

Clearly, required relation is:

$r = \{(1, 2), (1, 3), (2, 3)\}$, Dom $r = \{1, 2\}$, Range $r = \{2, 3\}$

3.3.1 Relation as Table, Ordered Pair and Graphs

We have learned that a relation in mathematics is any subset of the Cartesian product, which contains all ordered pairs. Each ordered pair consists of two coordinates, x and y. The x coordinate is called abscissa, and the y coordinate is ordinate, often representing an input and an output. Now, we describe the relation in three different ways.

Ordered Pairs: A relation can be represented by a set of ordered pairs. For example, consider a water tank that starts with 1 litre of water already inside. Each minute, 1 additional litre of water is added to the tank. The situation can be represented by the relation $r = \{ (x, y) | y = x + 1 \}$, where x is the number of minutes (time) that have passed since the filling started and y is the total amount of water (in litres) in the tank.

When $x = 0, y = 1$ and $x = 1, y = 2$

In order pair this relation is represented as:

$$\{(0, 1), (1, 2), (2, 3), (3, 4), (4, 5), (5, 6)\}$$

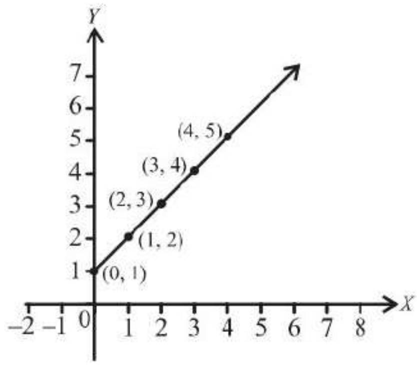
The above relation in table form can be represented as given below:

Table

x (time in minutes)	$y = x + 1$ (water in litres)
0	$y = 0 + 1 = 1$
1	$y = 1 + 1 = 2$
2	$y = 2 + 1 = 3$
3	$y = 3 + 1 = 4$
4	$y = 4 + 1 = 5$
5	$y = 5 + 1 = 6$

Graph: We can also represent the relations visually by drawing a graph. To draw the diagram, we use ordered pairs. Each ordered pair (x, y) is plotted as a point in the coordinate plane, where x is the first element and y is the second element of the ordered pair.

The relation is represented graphically by the line passing through the points,



$\{ (0,1), (1,2), (2,3), (3,4), (4,5), (5,6) \}$ as shown in the adjacent Figure.

3.3.2 Function and its Domain and Range

Functions

A very important particular type of relation is a function defined as below:

Let A and B be two non-empty sets such that:

(i) f is a relation from A to B, that is, f is a subset of $A \times B$

(ii) Domain $f = A$

(iii) First element of no two pairs of f are equal, then f is said to be a function from A to B.

The function f is also written as:

$$f : A \to B$$

Which is read as f is a function from A to B. The set of all first elements of each ordered pair represents the domain of f, and all second elements represent the range of f. Here, the domain of f is A, and the range of f is B.

If (x, y) is an element of f when regarded as a set of ordered pairs.

We write $y = f(x)$. y is called the value of f for x or the image of x under f.

Example 9: If $A = \{0, 1, 2, 3, 4\}$ and $B = \{3, 5, 7, 9, 11\}$, define a function $f : A \to B$, $f = \{(x, y) \mid y = 2x + 3, x \in A \text{ and } y \in B\}$, Find the value of function f, its domain, co-domain and range.

Solution: Given: $y = 2x + 3$; $x \in A$ and $y \in B$, then value of function,

$$f = \{(0, 3), (1, 5), (2, 7), (3, 9), (4, 11)\}$$

$$\text{Dom } f = \{0, 1, 2, 3, 4\} = A$$

$\Rightarrow$ Co-domain $f = B$ and

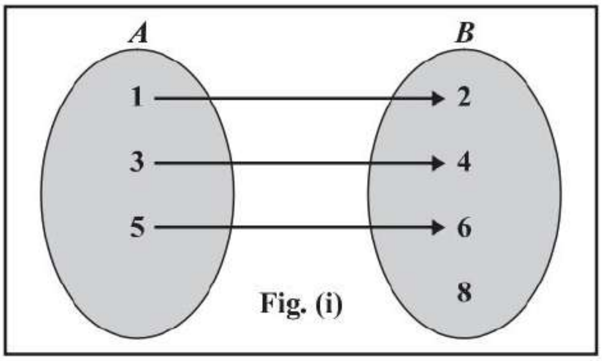
$\Rightarrow$ Range $f = \{3, 5, 7, 9, 11\} \subseteq B$

Types of functions

In this section we discuss different types of functions:

(i) Into Function

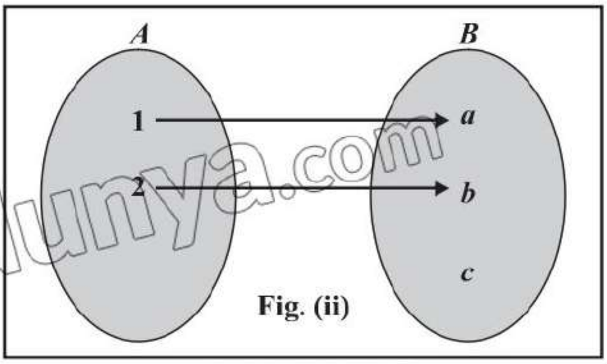
If a function $f : A \to B$ is such that Range $f \subset B$ i.e., Range $f \neq B$, then f is said to be a function from A into B. In Fig. (i), f is clearly a function. But Range $f \neq B$. Therefore, f is a function from A into B.



$f = \{(1, 2), (3, 4), (5, 6)\}$

(ii) (One - One) Function (or Injective Function)

If a function f from A into B is such that second elements of no two of its ordered pairs are same, then it is



$f = \{(1, a), (2, b)\}$

called an injective function; the function shown in Fig. (iii) is such a function.

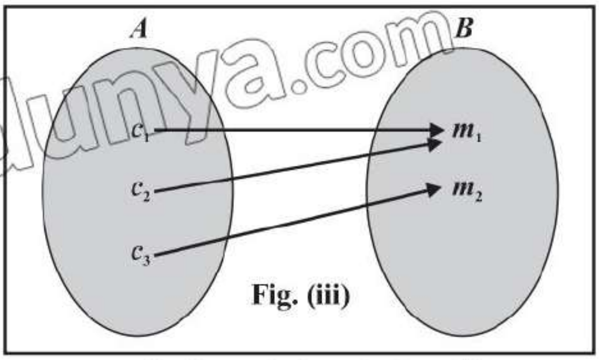


Fig. (iii)

$f = \{(c_1, m_1), (c_2, m_1), (c_3, m_2)\}$

(iii) Onto Function (or Surjective function)

If a function $f: A \to B$ is such that Range $f = B$ i.e., every element of B is the image of some element of A, then f is called an **onto** function or a surjective function.

(iv) (One – One) and onto Function (or Bijective Function)

A function f from A to B is said to be a Bijective function if it is both one-one and onto. Such a function is also called (1 – 1) correspondence between the sets A and B.

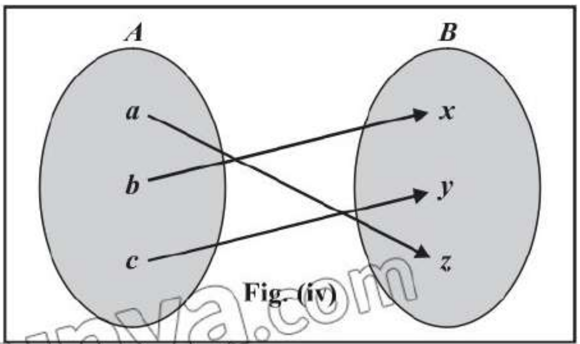


Fig. (iv)

$f = \{(a, z), (b, x), (c, y)\}$

(a, z), (b, x) and (c, y) are the pairs of corresponding elements i.e., in this case $f = \{(a, z), (b, x), (c, y)\}$ which is a bijective function or (1 – 1) correspondence between the sets A and B.

3.3.3 Notation of Function

We know that set-builder notation is more suitable for infinite sets. So is the case with respect to a function comprising an infinite number of ordered pairs. Consider for instance, the function

$$f = \{(-1, 1), (0, 0), (1, 1), (2, 4), (3, 9), (4, 16), \ldots\}$$

Dom $f = \{-1, 0, 1, 2, 3, 4, \ldots\}$ and

Range $f = \{0, 1, 4, 9, 16, \ldots\}$

This function may be written as:

$$f = \{(x, y) \mid y = x^2, x \in N\}$$

The mapping diagram for the function is shown in the Fig.(v).

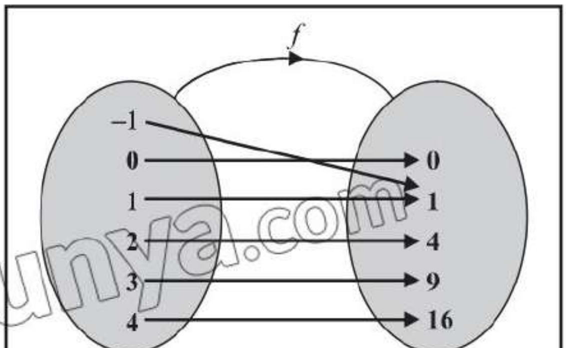


Fig. (v)

3.3.4 Linear and Quadratic Functions

The function $\{(x, y) \mid y = m\,x + c\}$ is called a linear function because its graph (geometric representation) is a straight line. We know that an equation of the form $y = mx + c$ represents a straight line. The function $\{(x, y) \mid y = ax^2 + bx + c\}$ is called a quadratic function. We will study their geometric representation in the next chapter.

Example 10: If $f(x) = 2x - 1$ and $g(x) = x^2 - 3$, then find:

(i) $f(1)$ (ii) $f(-3)$ (iii) $f(7)$

(iv) $g(1)$ (v) $g(-3)$ (vi) $g(4)$

Solution: (i) $f(1) = 2 \times 1 - 1 = 1$ (ii) $f(-3) = 2 \times (-3) - 1 = -7$

(iii) $f(7) = 2 \times 7 - 1 = 13$ (iv) $g(1) = (1)^2 - 3 = -3$

(v) $g(-3) = (-3)^2 - 3 = 6$ (vi) $g(4) = (4)^2 - 3 = 13$

Example 11: Consider $f(x) = ax + b + 3$, where a and b are constant numbers. If $f(1) = 4$ and $f(5) = 9$, then find the value of a and b.

Solution: Given function $f(x) = ax + b + 3$

If $f(1) = 4$

Then $a \times 1 + b + 3 = 4$

$\Rightarrow \quad a + b = 1 \quad \text{...(i)}$

Similarly, $f(5) = 9$

$\Rightarrow \quad a \times 1 + b + 3 = 4$

$\Rightarrow \quad 5a + b = 6 \quad \text{...(ii)}$

Subtract equation (i) from equation (ii), we get.

$$(5a + b) - (a + b) = 6 - 1$$

$$5a + b - a - b = 5$$

$$4a = 5 \Rightarrow a = \frac{5}{4}$$

Substitute $a = \frac{5}{4}$ in the equation (i)

$$\frac{5}{4} + b = 1$$

$$b = 1 - \frac{5}{4}$$

$$\Rightarrow b = -\frac{1}{4}$$

Thus, $a = \frac{5}{4}$ and $b = -\frac{1}{4}$

EXERCISE 3.3

1. For $A = \{1, 2, 3, 4\}$, find the following relations in A. State the domain and range of each relation.

   (i) $\{(x, y) \mid y = x\}$ (ii) $\{(x, y) \mid y + x = 5\}$

   (iii) $\{(x, y) \mid x + y < 5\}$ (iv) $\{(x, y) \mid x + y > 5\}$

2. Which of the following diagrams represent functions and of which type?

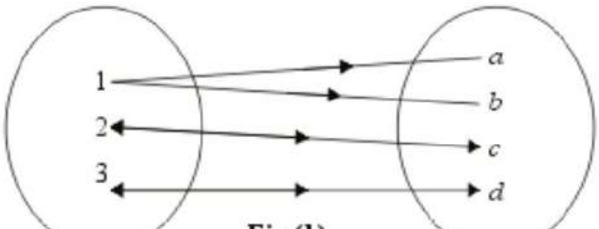


Fig (1)

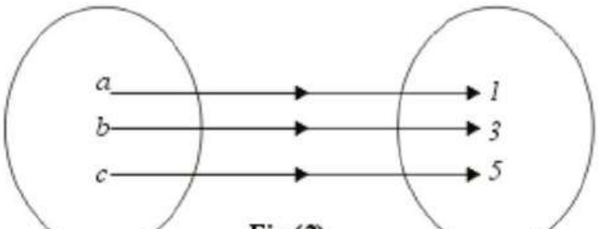


Fig (2)

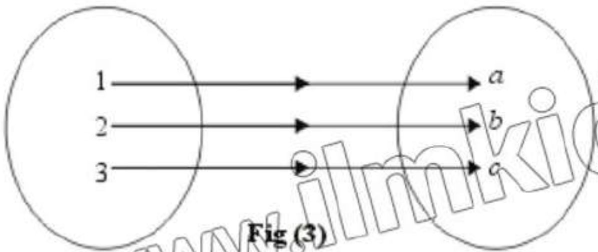


Fig (3)

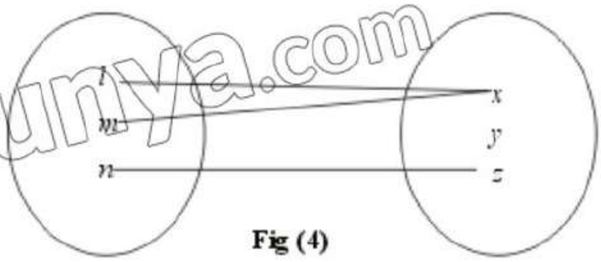


Fig (4)

3. If $g(x) = 3x + 2$ and $h(x) = x^2 + 1$, then find:

   (i) $g(0)$ (ii) $g(-3)$ (iii) $g\left(\frac{2}{3}\right)$

   (iv) $h(1)$ (v) $h(-4)$ (vi) $h\left(-\frac{1}{2}\right)$

4. Given that $f(x) = ax + b + 1$, where a and b are constant numbers. If $f(3) = 8$ and $f(6) = 14$, then find the values of a and b.

5. Given that $g(x) = ax + b + 5$, where a and b are constant numbers. If $g(-1) = 0$ and $g(2) = 10$, find the values of a and b.

6. Consider the function defined by $f(x) = 5x + 1$. If $f(x) = 32$, find the x value.

7. Consider the function $f(x) = cx^2 + d$, where c and d are constant numbers. If $f(1) = 6$ and $f(-2) = 10$, then find the values of c and d.

REVIEW EXERCISE 3

1. Four options are given against each statement. Encircle the correct option.

(i) The set builder form of the set $\left\{1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, ...\right\}$ is:

(a) $\left\{x \mid x = \frac{1}{n}, n \in W\right\}$ (b) $\left\{x \mid x = \frac{1}{2n+1}, n \in W\right\}$

(c) $\left\{x \mid x = \frac{1}{n+1}, n \in W\right\}$ (d) $\{x \mid x = 2n+1, n \in W\}$

(ii) If $A = \{\ \}$, then $P(A)$ is:

(a) $\{\ \}$ (b) $\{1\}$ (c) $\{\{\ \}\}$ (d) ϕ

(iii) If $U = \{1, 2, 3, 4, 5\}$, $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$, then $U - (A \cap B)$ is:

(a) $\{1, 2, 4, 5\}$ (b) $\{2, 3\}$ (c) $\{1, 3, 4, 5\}$ (d) $\{1, 2, 3\}$

(iv) If A and B are overlapping sets, then $n(A - B)$ is equal to

(a) $n(A)$ (b) $n(B)$ (c) $A \cap B$ (d) $n(A) - n(A \cap B)$

(v) If $A \subseteq B$ and $B - A \neq \phi$, then $n(B - A)$ is equal to

(a) 0 (b) $n(B)$ (c) $n(A)$ (d) $n(B) - n(A)$

(vi) If $n(A \cup B) = 50$, $n(A) = 30$ and $n(B) = 35$, then $n(A \cap B) =$:

(a) 23 (b) 15 (c) 9 (d) 40

(vii) If $A = \{1, 2, 3, 4\}$ and $B = \{x, y, z\}$, then cartesian product of A and B contains exactly _______ elements.

(a) 13 (b) 12 (c) 10 (d) 6

(viii) If $f(x) = x^2 - 3x + 2$, then the value of $f(a + 1)$ is equal to:

(a) $a + 1$ (b) $a^2 + 1$ (c) $a^2 + 2a + 1$ (d) $a^2 - a$

(ix) Given that $f(x) = 3x+1$, if $f(x)=28$, then the value of x is:

(a) 9 (b) 27 (c) 3 (d) 18

(x) Let $A = \{1, 2, 3\}$ and $B = \{a, b\}$ two non-empty sets and $f: A \rightarrow B$ be a a function defined as $f = \{(1, a), (2, b), (3, b)\}$, then which of the following statement is true?

(a) f is injective (b) f is surjective (c) f is bijective (d) f is into only

2. Write each of the following sets in tabular forms:

(i) $\{x \mid x = 2n, n \in N\}$ (ii) $\{x \mid x = 2m+1, m \in N\}$

(iii) $\{x \mid x = 11n, n \in W \wedge n < 11\}$ (iv) $\{x \mid x \in E \wedge 4 < x < 6\}$

(v) $\{x \mid x \in O \wedge 5 \leq x < 7\}$ (vi) $\{x \mid x \in Q \wedge x^2 = 2\}$

(vii) $\{x \mid x \in Q \wedge x = -x\}$ (viii) $\{x \mid x \in R \wedge x \notin Q'\}$

3. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{2, 4, 6, 8, 10\}$, $B = \{1, 2, 3, 4, 5\}$ and $C = \{1, 3, 5, 7, 9\}$
   List the members of each of the following sets:
   (i) A' (ii) B' (iii) $A \cup B$ (iv) $A - B$
   (v) $A \cap C$ (vi) $A' \cup C'$ (vii) $A' \cup C$ (viii) U'
4. Using the Venn diagrams, if necessary, find the single sets equal to the following:
   (i) A' (ii) $A \cap U$ (iii) $A \cup U$
   (iv) $A \cup \phi$ (v) $\phi \cap \phi$
5. Use Venn diagrams to verify the following:
   (i) $A - B = A \cup B'$ (ii) $(A - B)' \cap B = B$
6. Verify the properties for the sets A, B and C given below:
   (i) Associativity of Union (ii) Associativity of intersection.
   (iii) Distributivity of Union over intersection.
   (iv) Distributivity of intersection over union.
   (a) $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6, 7, 8\}$, $C = \{5, 6, 7, 9, 10\}$
   (b) $A = \phi$, $B = \{0\}$, $C = \{0, 1, 2\}$
   (c) $A = N, B = Z, C = Q$
7. Verify De Morgan's Laws for the following sets:
   $U = \{1, 2, 3, \ldots, 20\}$, $A = \{2, 4, 6, \ldots, 20\}$ and $B = \{1, 3, 5, \ldots, 19\}$.
8. Consider the set $P = \{x \mid x = 5m, m \in N\}$ and $Q = \{x \mid x = 2m, m \in N\}$. Find $P \cap Q$
9. From suitable properties of union and intersection, deduce the following results:
   (i) $A \cap (A \cup B) = A \cup (A \cap B)$ (ii) $A \cup (A \cap B) = A \cap (A \cup B)$
10. If $g(x) = 7x - 2$ and $s(x) = 8x^2 - 3$ find:
    (i) $g(0)$ (ii) $g(-1)$ (iii) $g\left(-\frac{5}{3}\right)$ (iv) $s(1)$ (v) $s(-9)$ (vi) $s\left(\frac{7}{2}\right)$
11. Given that $f(x) = ax + b$, where a and b are constant numbers. If $f(-2) = 3$ and $f(4) = 10$, then find the values of a and b.
12. Consider the function defined by $k(x) = 7x - 5$. If $k(x) = 100$, find the value of x.
13. Consider the function $g(x) = mx^2 + n$, where m and n are constant numbers. If

$g(4) = 20$ and $g(0) = 5$, find the values of m and n.

14. A shopping mall has 100 products from various categories labeled 1 to 100, representing the universal set U. The products are categorized as follows:
    - Set A: Electronics, consisting of 30 products labeled from 1 to 30.
    - Set B: Clothing comprises 25 products labeled from 31 to 55.
    - Set C: Beauty Products, comprising 25 products labeled from 76 to 100.

    Write each set in tabular form, and find the union of all three sets.

15. Out of the 180 students who appeared in the annual examination, 120 passed the math test, 90 passed the science test, and 60 passed both the math and science tests.
    (a) How many passed either the math or science test?
    (b) How many did not pass either of the two tests?
    (c) How many passed the science test but not the math test?
    (d) How many failed the science test?

16. In a software house of a city with 300 software developers, a survey was conducted to determine which programming languages are liked more. The survey revealed the following statistics:
    - 150 developers like Python.
    - 130 developers like Java.
    - 120 developers like PHP.
    - 70 developers like both Python and Java.
    - 60 developers like both Python and PHP.
    - 50 developers like both Java and PHP.
    - 40 developers like all three languages: Python, Java and PHP.

    (a) How many developers use at least one of these languages?
    (b) How many developers use only one of these languages?
    (c) How many developers do not use any of these languages?
    (d) How many developers use only PHP?