Chapter 2

Kinematics

Student Learning Outcomes

After completing this chapter, students will be able to:

- Differentiate between scalar and vector quantities:
  [A scalar has magnitude (size) only and that a vector quantity has magnitude and direction. Students should be able to represent vectors graphically]
- Justify that distance, speed, time, mass, energy, and temperature are scalar quantities.
- Justify that displacement, force, weight, velocity, acceleration are vector quantities.
- Determine graphically, the resultant of two or more vectors.
- Differentiate between different types of motion [i.e; translatory, {linear, random, and circular); rotatory and vibratory motions and distinguish among them.]
- Differentiate between distance and displacement, speed and velocity.
- Define and calculate average speed [average speed = (total distance travelled)/ (total time taken)]
- Differentiate between average and instantaneous speed (speed shown by speedometer of a vehicle is the speed at any instant.)
- Differentiate between uniform velocity and non -uniform velocity
- Define and calculate acceleration [Includes deriving the units of acceleration as ms^{-2} from the formula $a = \Delta v / \Delta t$ and using the formula to solve problems. This also includes knowing that that deceleration is negative acceleration and using fact in calculations.]
- Differentiate between uniform acceleration and non - uniform acceleration
- Sketch, plot and interpret distance, time and speed-time graphs
  [This includes determining from the shape of a distance -time graph when an object is:
  [(a) at rest, (b) moving with constant speed, (c) accelerating, (d) decelerating. Students are also required to know how to calculate speed from the gradient of a distance - time graph. It also includes determining from the shape of a speed -time graph when an object is:
  (a) at rest, (b) moving with constant speed, (c) moving with constant acceleration.]
- Use the approximate value of g as $10 m/s^2$ for free fall acceleration near Earth to solve problems
- Analyse the distance travelled in speed vs time graphs [By determining the area under the graph for cases of motion with constant speed or constant acceleration]
- Calculate acceleration from the gradient of a speed-time graph
- State that there is a universal speed limit for any object in the universe that is approximately $3 \times 10^8 m s^{-1}$
  [Students should just be aware that this phenomenon is true; they do not need to study relativity in any depth. The purpose is that students appreciate that there is a universal speed limit].

Mechanics is the branch of physics that deals with the motion of objects and the forces that change it.

Generally, mechanics is divided into two branches:

1. Kinematics **2. Dynamics**

Kinematics is the study of motion of objects without referring to forces. On the other hand, dynamics deals with forces and their effect on the motion of objects.

In our everyday life, we observe many objects in motion. For example, cars, buses, bicycles, motorcycles moving on the roads, aeroplanes flying through air, water flowing in canals or some object falling from the table to the ground.

The motion of these objects can be studied with or without considering the force which causes motion in them or changes it.

2.1 Scalars and Vectors

Before we study kinematics in detail, we should know about the nature of various physical quantities. Some quantities are called scalars and the others vectors.

A scalar is that physical quantity which can be described completely by its magnitude only.

Magnitude includes a number and an appropriate unit. When we ask a shopkeeper to give us 5 kilograms of sugar, he can fully understand how much quantity we want. It is the magnitude of mass of sugar. Mass is a scalar quantity. Some other examples of scalar quantities are distance, length, time, speed, energy and temperature. Scalar quantities can be added up like numbers.

For example, 5 metres + 3 metres = 8 metres.

On the other hand,

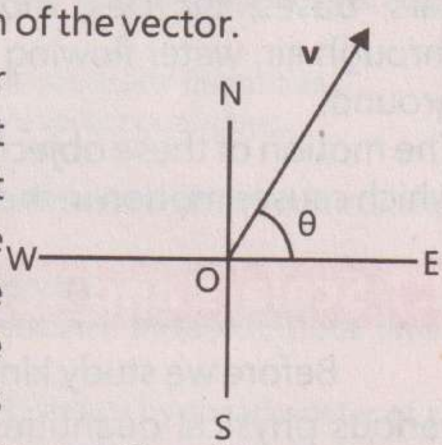
A vector is that physical quantity which needs magnitude as well as direction to describe it completely.

The examples of vector quantities are displacement, velocity, acceleration, weight, force, etc. The velocity of a car moving at 90 kilometre per hour ($25\ m\ s^{-1}$) towards north can be represented by a vector. Velocity is a vector quantity because it has magnitude $25\ m\ s^{-1}$ and direction (towards north). Vectors cannot be added like scalars. There are specific methods to add up vectors. These methods take their directions also into consideration.

Representation of vectors

In the textbooks, symbol used for a vector is a bold face letter such as **A, v, F** and **d** etc. Since we cannot write in bold face script on paper, so a vector is written as the letter with a small arrow over it, i.e. $\vec{A}, \vec{v}, \vec{F}, \vec{d}$. The magnitude of a vector is given by italic letter without arrow head. A vector can be represented graphically by drawing a straight line with an arrow head at one end. The length of line represents the magnitude of the vector quantity according to a suitable scale while the direction of arrow indicates the direction of the vector.

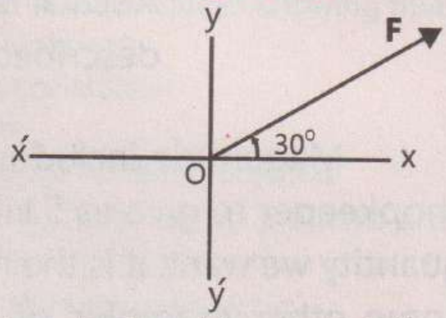
To represent the direction, two mutually perpendicular lines are required. We can draw one line to represent east-west direction and the other line to represent north-south direction as shown in Fig.2.1(a). The direction of a vector can be given with respect to these lines. Mostly, we use any two lines which are perpendicular to each other. Horizontal line (x x́) is called x-axis and vertical line (y ý) is called y-axis (Fig. 2.1-b). The point where these axes meet is known as origin. The origin is usually denoted by O. These axes are also called reference axes.



A vector **v** making an angle θ towards north from east

Fig 2.1 (a)

A vector is drawn starting from the origin of the reference axes towards the given direction. The direction is usually given by an angle θ (theta) with x-axis. The angle with x-axis is always measured from the right side of x-axis in the anti-clockwise direction.



A vector **F** making angle 30° with x-axis

Example 2.1

Draw the velocity vector **v**; a velocity of 300 m s^{-1} at an angle of 60° to the east of north.

For Your Information!

For geographical direction, the reference line is north – south whereas for Cartesian coordinate system +ve *x*-axis is the reference.

Solution

(i) Draw two mutually perpendicular lines indicating N, S, E & W.

(ii) Select a suitable scale. If 100 m s^{-1} = 1 cm, then 300 m s^{-1} are represented by 3 cm line.

(iii) Draw 3 cm line OP at an Angle of 60° starting from N towards E.

(iv) Make an arrow head at the end of line OP. The OP is the vector **v**.

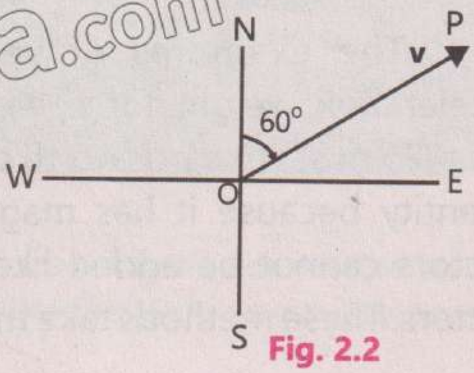


Fig. 2.2

Example 2.2

Draw a force vector **F** having magnitude of 350 N and acting at an angle of 60° with x-axis.

Solution

(i) Draw horizontal and vertical lines to represent x-axis and y-axis as shown in Fig. 2.3.

(ii) Scale: If 100 N = 1 cm, then

$$350 \text{ N} = 3.5 \text{ cm}$$

(iii) Draw 3.5 cm line OQ at an angle of 60° with x-axis.

(iv) Make an arrow head at the end of the line OQ. The OQ is the vector **F**.

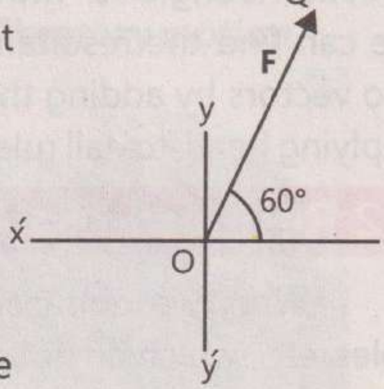


Fig. 2.3

Resultant Vector

We can add two or more vectors to get a single vector. This is called as resultant vector. It has the same effect as the combined effect of all the vectors to be added. We have to determine both magnitude and direction of the resultant vector, therefore, it is quite different from that of scalar addition. One method of addition of vectors is the graphical method.

Addition of Vectors by Graphical Method

Let us add two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ having magnitudes of 300 N and 400 N acting at angles of 30° and 60° with x-axis. By selecting a suitable scale 100 N = 1cm, we can draw the vectors as shown in Fig. 2.4 (a).

To add these vectors, we apply a rule called **head-to-tail rule**, which states that:

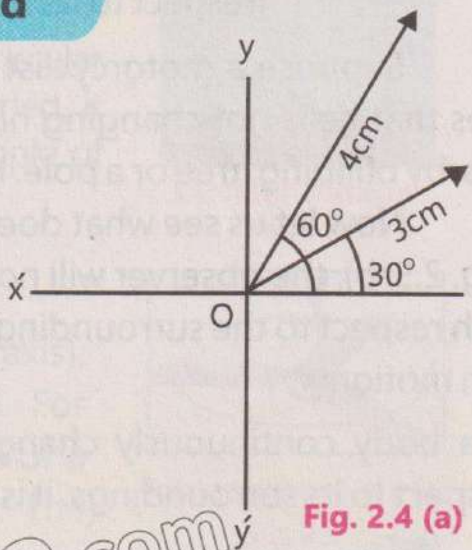


Fig. 2.4 (a)

To add a number of vectors, redraw their representative lines such that the head of one line coincides with the tail of the other. The resultant vector is given by a single vector which is directed from the tail of the first vector to the head of the last vector.

Measured length of resultant vector is 6.8 cm (Fig.2.4-b). According to selected scale, magnitude of the resultant vector **v** is 680 N and direction is angle 49° with x-axis.

We can find the resultant vector of more than two vectors by adding them with the same way applying head-to-tail rule.

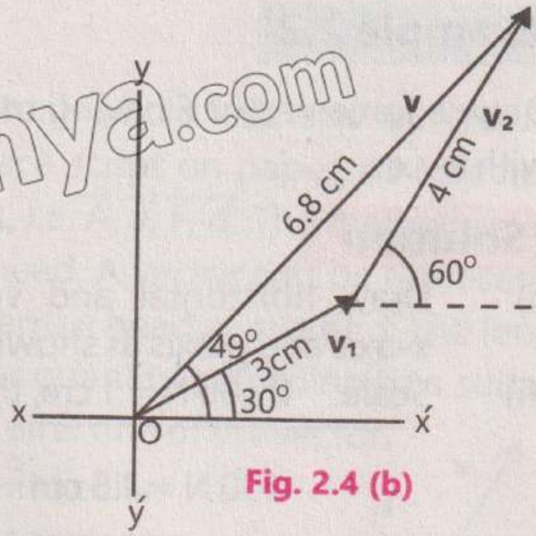


Fig. 2.4 (b)

2.2 Rest and Motion

When we look around us, we see many things like buildings, trees, electric poles, etc. which do not change their positions. We say that they are in a state of rest.


Fig. 2.5 (a)

If a body does not change its position with respect to its surroundings, it is said to be at rest.

Suppose a motorcyclist is standing on the road (Fig. 2.5-a). An observer sees that he is not changing his position with respect to his surroundings i.e., a nearby building, tree or a pole. He will say that the motorcyclist is at rest.

Now let us see what does motion mean? When the motorcyclist is driving (Fig. 2.5-b), the observer will notice that he is continuously changing his position with respect to the surroundings. Then the observer will say that the motorcyclist is in motion.

If a body continuously changes its position with respect to its surroundings, it is said to be in motion.


Fig. 2.5 (b)

The state of rest or motion of a body is always relative. For example, a person standing in the compartment of a moving train is at rest with respect to the other passengers in the compartment but he is in motion with respect to an observer standing on the platform of a railway station.

2.3 Types of Motion

We observe different types of motion in our daily life. A train moves almost along a straight line, the blades of a fan rotate about an axis, a swing vibrates about its mean position. Generally, there are three types of motion of bodies.

1. Translatory motion **2. Rotatory motion** **3. Vibratory motion**

1. Translatory Motion

If the motion of a body is such that every particle of the body moves uniformly in the same direction, it is called tanslatory motion. For example, the motion of a train or a car is tanslatory motion (Fig.2.6). Translatory motion can be of three types:

Fig. 2.6 The motion of a train is translatory motion

(i) Linear Motion

If the body moves along a straight line, it is called linear motion. A freely falling body is the example of linear motion.

(ii) Random Motion

If the body moves along an irregular path (Fig. 2.7), the motion is called random motion.

(a) Irregular path

Fig. 2.7

(b) The motion of bee is random motion

(iii) Circular Motion

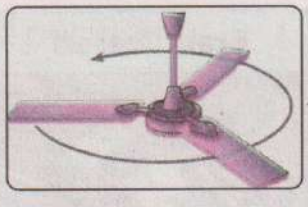
The motion of a body along a circle is called circular motion. If a ball tied to one end of a string is whirled, it moves along a circle. A Ferris wheel is also an example of circular motion (Fig.2.8).

Circular Motion

Fig. 2.8

2. Rotatory Motion

If each point of a body moves around a fixed point (axis), the motion of this body is called rotatory motion. For example, the motion of an electric fan and the drum of a washing machine dryer is rotatory motion (Fig.2.9). The motion of a top is also rotatory motion.



Rotatory motion of a fan

Fig. 2.9

3. Vibratory Motion

When a body repeats its to and fro motion about a fixed position, the motion is called vibratory motion. The motion of a swing in a children park is vibratory motion (Fig. 2.10).

Fig. 2.10 Vibratory motion

2.4 Distance and Displacement

We know that motion is the action of an object going from one place to another or change of position. The length between the original and final positions may be measured in two ways as either distance or displacement.

The distance is the length of actual path of the motion.

Let a person be travelling from Lahore to Multan in a car. On reaching Multan, he reads the speedometer and notices that he has travelled a distance of 320 km. It is the distance travelled by that person. Obviously, it is not the shortest distance from Lahore to Multan, as the car took many turns in the way. He did not travel along a straight line.

The displacement of an object is a vector quantity whose magnitude is the shortest distance between the initial and final positions of the motion and its direction is from the initial position to the final position.

We can also call this as the change in position. Note that displacement is a vector quantity whereas distance is a scalar quantity. Following example will explain the difference between distance and displacement.

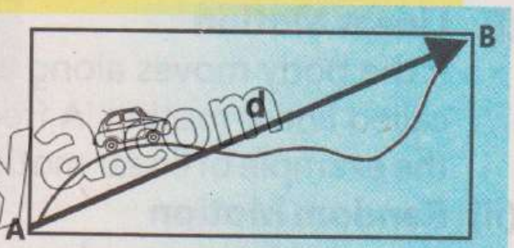


Fig. 2.11

Suppose a car travels from a position A to B. The curved line is the actual path followed by the car (Fig. 2.11). The total distance covered by the car will be equal to the length of the curved line AB. The displacement **d** is the straight line AB directed from A to B as indicated by the arrow head. The SI unit for the displacement is the same as that of distance.

2.5 Speed and Velocity

Brain Teaser!

The car while moving on a circular road may have constant speed, but its velocity is changing at every instant. Why?

We are often interested to know how fast a body is moving. For this purpose, we have to find the distance covered in unit time which is known as speed. If a body covers a distance S in time t, its speed v will be written as:

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} \qquad v = \frac{S}{t}$$

or $S = vt$ ....(2.1)

Do You Know?

The fastest land mammal (cheetah) and the fastest fish (sailfish) have the same highest recorded speed of 110 km h^{-1}.

The speed is a scalar quantity. The SI unit of speed is $m\,s^{-1}$ or $km\,h^{-1}$.

It is obvious that speed of a vehicle does not remain constant throughout the journey. If the reading of the speedometer of the vehicle is observed, it is always changing. The speed of a vehicle that is shown by its speedometer at any instant is called **instantaneous speed**. Practically we make use of the **average speed**. It is defined as:

$$\text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}} \quad \text{or} \quad v_{av} = \frac{S}{t}$$

Example 2.3

An eagle dives to the ground along a 300 m path with an average speed of 60 $m\,s^{-1}$. How long does it take to cover this distance?

For Your Information!

Mount st. Helens erupted in 1980, causing rocks to travel at velocities up to 400 $km\,h^{-1}$

Solution

Total distance covered = S = 300 m

Average speed = v_{av} = 60 $m\,s^{-1}$

Total time taken = t = ?

Using the equation $v_{av} = \frac{S}{t}$

or $t = \frac{S}{v_{av}}$

putting the values t = 300 m / 60 $m\,s^{-1}$ = 5 s

Velocity

The speed of an object does not tell anything about the direction of motion. To take into account the direction, the vector concept is needed. For this, we have to find the displacement **d** between the initial and final positions.

The net displacement of a body in unit time is called velocity.

If a body moves from point A to B along a curved path as shown in Fig.2.11, the displacement **d** is the straight line AB, then

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time}} \quad \text{or} \quad \mathbf{v}_{av} = \frac{\mathbf{d}}{t} \quad \text{............(2.2)}$$

Velocity is a vector quantity. The Equation (2.2) shows that the direction of velocity **v** is the same as that of displacement **d**. The SI unit of velocity is also $m\,s^{-1}$ or $km\,h^{-1}$. Consider the example of a car moving towards north at the rate of 70 $km\,h^{-1}$. To differentiate between speed and velocity, we shall say that the speed of car is 70 km h^{-1} which is a scalar quantity. The velocity of the car is a vector quantity whose magnitude is 70 km h^{-1} and is directed towards north.

For Your Interest!

Time-lapse photo of motorway traffic, the velocity of cars showing straight lines. White lines are the headlights and the red lines are taillights of vehicles moving in opposite directions.

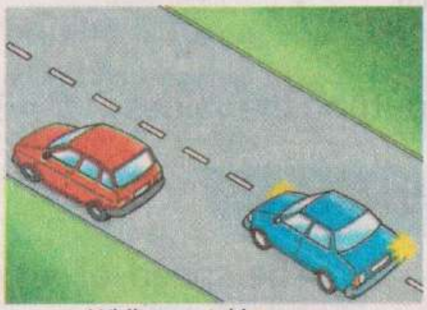
Uniform and Non-uniform Velocity

The velocity is said to be uniform if the speed and direction of a moving body does not change. If the speed or direction or both of them change, it is known as variable velocity or non-uniform velocity.

Practically, a vehicle does not move in a straight line throughout its journey. It changes its speed or its direction frequently. The example of a body moving with uniform velocity is the downward motion of a paratrooper. When a paratrooper jumps from an aeroplane, he falls freely for a few moments. Then the parachute opens. At this stage the force of gravity acting downwards on the paratrooper is balanced by the resistance of air on the parachute that acts upward. Consequently, the paratrooper moves down with uniform velocity.

2.6 Acceleration

Whenever the velocity of an object is increasing, we say that the object is accelerating. For example, when a car overtakes another one, it accelerates to a greater velocity (Fig.2.12). In contrary to that the velocity decreases when brakes are applied to slow down a bicycle or a car. In both the cases, a change in velocity occurs.



While overtaking, a car accelerates to a greater velocity.
Fig. 2.12

Acceleration is defined as the time rate of change of velocity.

The change in velocity can occur in magnitude or direction or both of them. The acceleration is positive if the velocity is increasing and it is negative if the velocity is decreasing. Negative acceleration is also called deceleration or retardation.

Acceleration is a vector quantity like velocity, but the direction of acceleration is that of change of velocity. If a body is moving with an initial velocity $\mathbf{v}_i$ and after some time t its velocity changes to $\mathbf{v}_f$, the change in velocity is $\Delta\mathbf{v} = \mathbf{v}_f - \mathbf{v}_i$ that occurs in time t. In this case, rate of change of velocity i.e., acceleration will be average acceleration.

$$\text{Average acceleration} = \frac{\text{Change in velocity}}{\text{Time taken}}$$

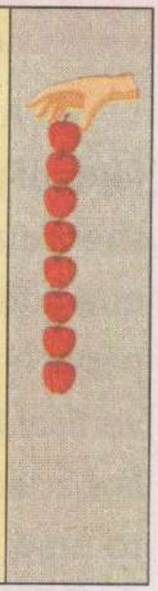
$$\mathbf{a}_{av} = \frac{\mathbf{v}_f - \mathbf{v}_i}{t} \quad \text{............ (2.3)}$$

Equation (2.3) can be written as $\mathbf{a}_{av} = \frac{\Delta \mathbf{v}}{t}$

The SI unit of acceleration is $m\,s^{-2}$.

If acceleration a is constant, then Eq 2.3 can be written as: $v_f = v_i + at$

> **Fascinating Snap:** This is a photograph of a falling apple dropped from som height. The images of apple are captured by the camera at 60 flashes per second. The widening spaces between the images indicate the acceleration of the apple.



Uniform and Non-uniform Acceleration

If time rate of change of velocity is constant, the acceleration is said to be uniform.

If anyone of the magnitude or direction or both of them changes it is called variable or non-uniform acceleration. In this class, we will solve problems only for the motion of the bodies having uniform acceleration and not the variable acceleration.

Example 2.4

A plane starts running from rest on a run-way as shown in the figure below. It accelerates down the run-way and after 20 seconds attains a velocity of 252 km h^{-1}. Determine the average acceleration of the plane.

Solution:

Initial velocity = $v_i = 0$

Final velocity = $v_f = 252$ km h^{-1}

$$= \frac{252 \times 10^3 \text{ m}}{60 \times 60 \text{ s}} = 70 \text{ m s}^{-1}$$

Time taken $= t = 20$ s

Average acceleration $= a_{av} = ?$

Using $a_{av} = \frac{v_f - v_i}{t}$

Putting the values

$$a_{av} = \frac{70 \text{ m s}^{-1} - 0}{20 \text{ s}}$$

$$a_{av} = 3.5 \text{ m s}^{-2}$$

2.7 Graphical Analysis of Motion

A graph is a pictorial diagram in the form of a straight line or a curve which shows the relationship between two physical quantities. Usually, we draw a graph on a paper on which equally spaced horizontal and vertical lines are drawn. Generally, every 10^{th} line is a thick line on the graph paper. In order to draw a graph, two mutually perpendicular thick lines xox́ and yoý are selected as x and y axes as shown in Fig 2.13. The point where the two axes intersect each other is known as origin O. Positive values along x-axis are taken to the right side of the origin and negative values are taken to the left side. Similarly, positive values along y-axis are taken above the origin whereas negative values are taken below the origin. Normally, the independent quantity is taken along x-axis and dependent variable quantity along y-axis. For example, in distance-time graph, *t* is independent and *S* is dependent variable. Therefore, *t* should be along x-axis and *S* along y-axis.

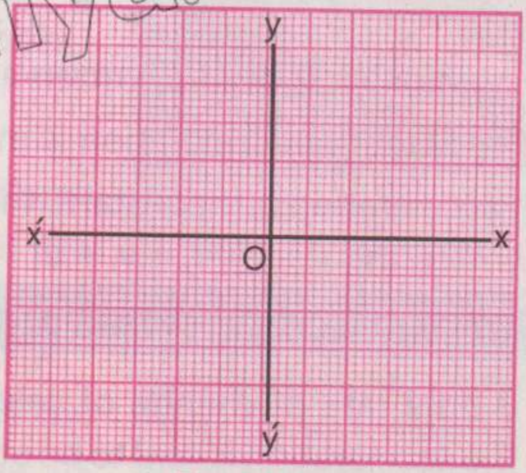


Fig. 2.13

To represent a physical quantity along any axis, a suitable scale is chosen by considering the minimum and maximum values of the quantity.

Distance-Time Graph

Distance-time graph shows the relation between distance *S* and time *t* taken by a moving body.

Let a car be moving in a straight line on a motorway. Suppose that we measure its distance from starting point after every one minute, and record it in the table given below:

Time *t* (min)	0	1	2	3	4	5
Distance *S* (km)	0	1.2	2.4	3.6	4.8	6.0

Follow the steps given below to draw a graph on a centimetre graph paper:

(i) Take time *t* along x-axis and distance *S* along y-axis.

(ii) Select suitable scales (1 minute = 1 cm) along x-axis and (1.2 km =1 cm) along y-axis. The graph paper shown here is not to the scale.

(iii) Mark the values of each big division along x and y axes according to the scale.

(iv) Plot all pairs of values of time and distance by marking points on the graph paper.

(v) Join all the plotted points to obtain a best straight line as shown in Fig. 2.14. From the table, we can observe that car has covered equal distance in equal intervals of time. This shows that the car moves with uniform speed. Therefore, a straight line graph between time and distance represents motion with uniform speed.

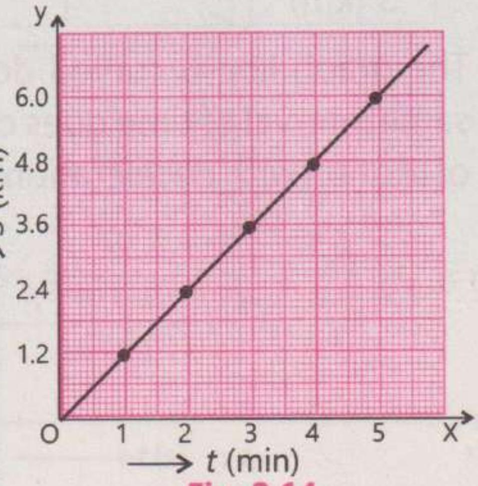


Fig. 2.14

Now consider another journey of the car as recorded in the table given below:

Time *t* (min)	0	1	2	3	4	5
Distance *S* (km)	0	0.240	0.960	2.160	3.840	6.000

Table shows that speed goes on increasing in equal intervals of time. This is very obvious from the graph as shown in Fig. 2.15. The graph line is curved upward. This is the case when the body (car) is moving with certain acceleration.

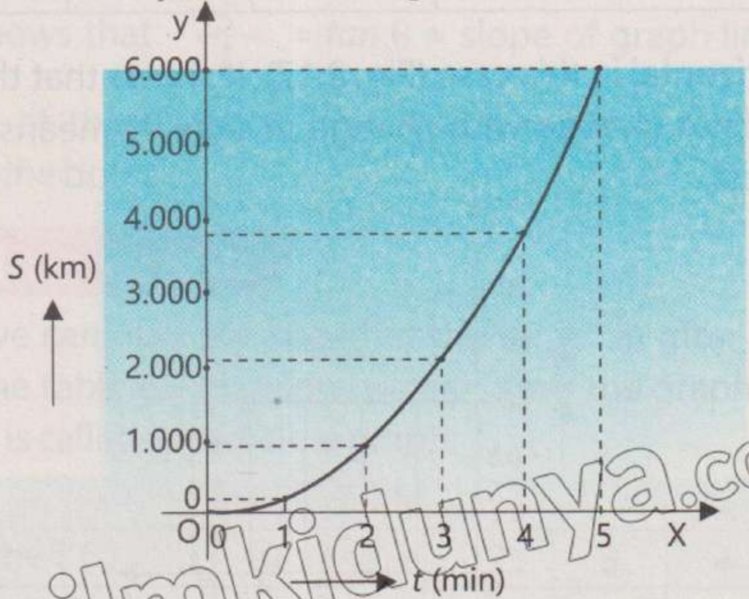


Fig. 2.15

In another case, consider the following table:

Time t (min)	0	1	2	3	4	5
Distance S (km)	0	2.0	3.1	4.0	4.6	5.0

The graph line is curved downwards. This shows that distance travelled in the same interval of time goes on decreasing, so speed is decreasing. This is the case of motion with deceleration or negative acceleration as shown in Fig.2.16.

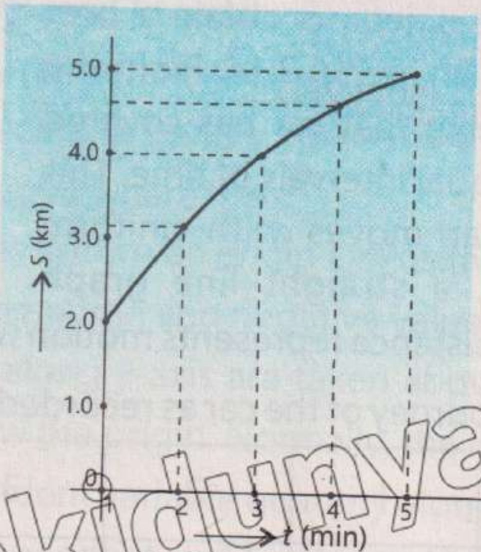


Fig. 2.16

Now consider another case.

Time t (min)	0	1	2	3	4	5
Distance S (km)	1.2	1.2	1.2	1.2	1.2	1.2

Graph line is horizontal in this case (Fig. 2.17). It shows that the distance covered by the car does not change with change in time. It means that the car is not moving; it is at rest.

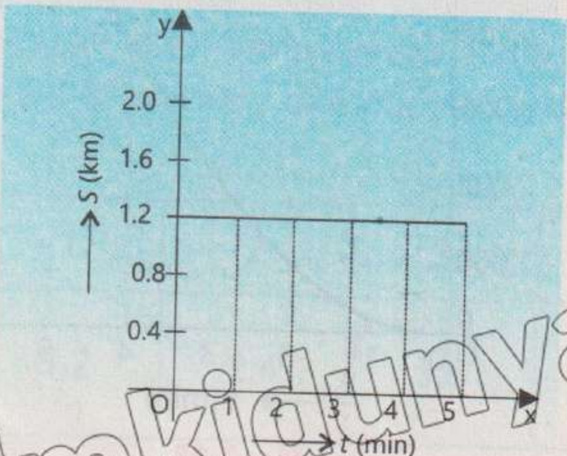


Fig. 2.17

2.8 Gradient of a Distance-Time Graph

The gradient is the measure of slope of a line.

Consider the distance-time graph of uniform speed again. Select any two values of time t_1 and t_2. Draw two vertical dotted lines at t_1 and t_2 on x-axis. These lines meet the graph at points P and Q. From these points draw horizontal lines to meet y-axis at S_1 and S_2 respectively as shown in Fig.2.18.

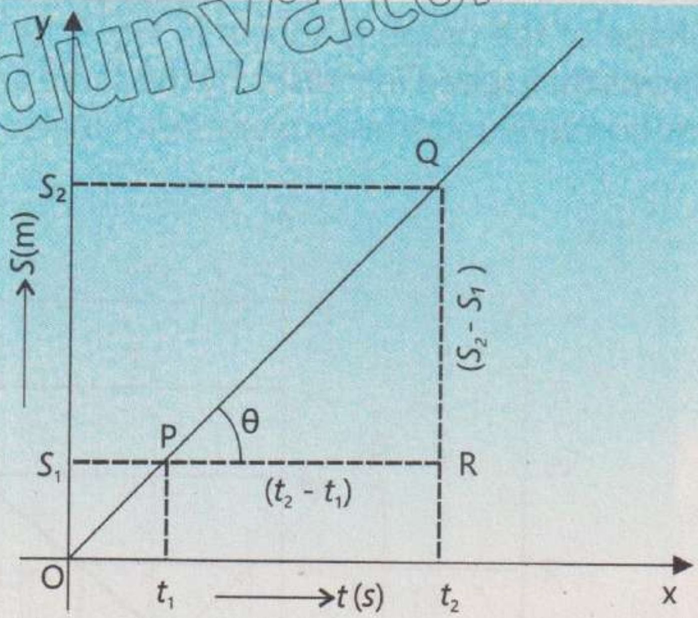


Fig. 2.18

Distance covered in this time interval is $S_2 - S_1 = S$

Time taken $t_2 - t_1 = t$

The slope or gradient of the graph is the measure of tangent θ of the triangle RPQ:

$$\text{Slope} = \frac{RQ}{PR}$$

$$\text{Slope} = \frac{S_2 - S_1}{t_2 - t_1} = \frac{S}{t}$$

From Eq. (2.1), $\frac{S}{t} = v_{av}$, the average speed during the time interval t.

Figure 2.17 shows that $\frac{S}{t} = \tan\theta$ = slope of graph line, therefore,

Gradient of the distance-Time graph is equal to the average speed of the body.

2.9 Speed-Time Graph

Suppose we can note the speed of the same car after every one second and record it in the table given below, we can draw the graph between speed v versus time t. This is called speed-time graph.

Table

Time t (s)	0	1	2	3	4	5
Speed v ($m\ s^{-1}$)	0	8	16	24	32	40

Take t along x-axis and v along y-axis. Scale can be selected as 1 s = 1 cm (x-axis) and speed 10 m s^{-1} = 1 cm along y-axis.

Shape of the graph is shown in Fig. 2.19. It is a straight line rising upward. This shows that speed increases by the same amount after every one second. This is a motion with uniform acceleration. It is also evident from the table.

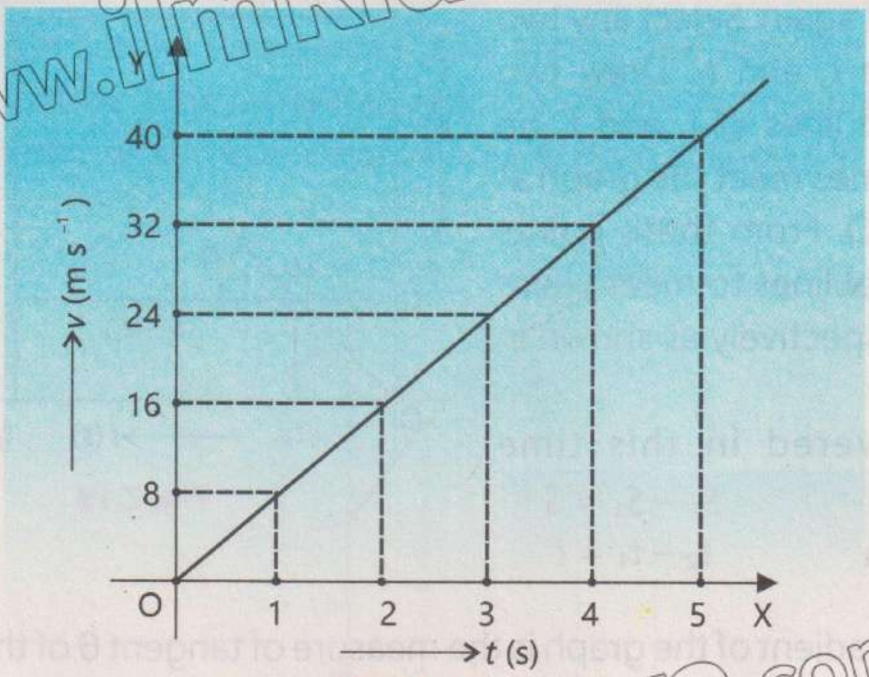


Fig. 2.19

Now consider another case. The observations are recorded in the table given below:

Table

Time t (s)	0	1	2	3	4	5
Speed v (m s^{-1})	20	20	20	20	20	20

In this case, graph line is horizontal (Fig. 2.20) parallel to time x- axis. It shows that speed does not change with change in time. This is a motion with constant speed.

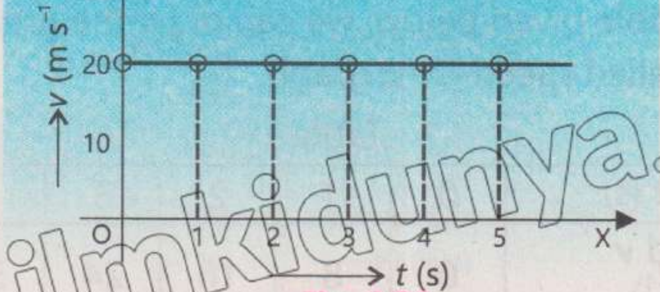


Fig. 2.20

2.10 Gradient of a Speed-Time Graph

Now consider the speed-time graph (Fig. 2.21). The speeds at times t_1 and t_2 are v_1 and v_2 respectively. The change in speed in a time interval $(t_2 - t_1)$ is $(v_2 - v_1)$. Therefore,

$$\text{Slope} = \frac{\text{Change in speed}}{\text{Total time taken}}$$

$$\text{or Slope} = \frac{(v_2 - v_1)}{(t_2 - t_1)}$$

$$\text{Slope} = \frac{\Delta v}{t}$$

But $\frac{\Delta v}{t} = a$, the average acceleration.

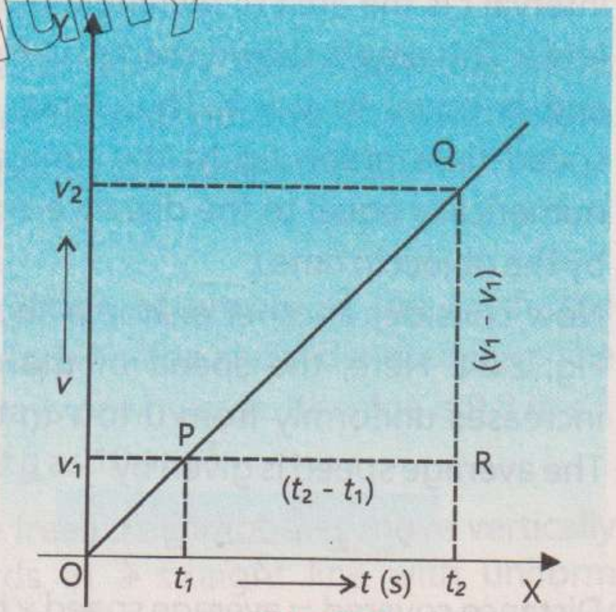


Fig. 2.21

Hence

Gradient of the speed-Time graph is equal to the average acceleration of the body.

This shows that when a car moves with constant acceleration, the velocity-time graph is a straight line which rises through same height for equal intervals of time.

Graph of Fig. 2.19 is redrawn in Fig. 2.22 to find its slope. The speed v_1 at time t_1 is the same as speed v_2 at time t_2, hence, the change in speed is also zero.

$v_2 - v_1 = 0$. Thus, the slope $= \frac{(v_2 - v_1)}{(t_2 - t_1)} = 0$

When the speed of the object is constant, the speed-time graph is a horizontal straight line parallel to time axis.

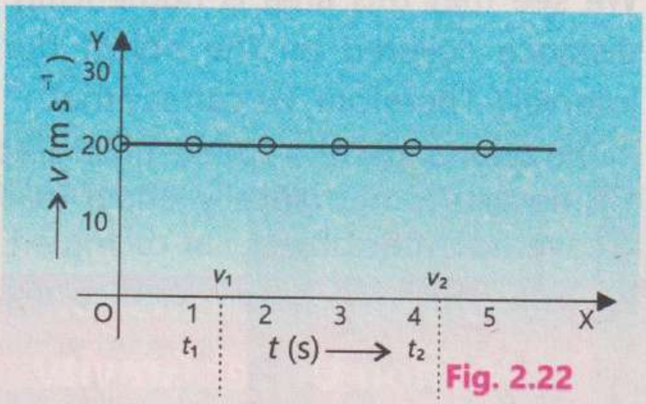


Fig. 2.22

This shows that the acceleration of this motion is zero. It is the motion without the change in speed.

2.11 Area Under Speed-Time Graph

The distance moved by an object can also be determined by using its speed-time graph. For example, figure 2.23 shows that the object is moving with constant speed v. For a time-interval t, the distance covered by the object as given by Eq. 2.1 is $v \times t$.

This distance can also be found by calculating the area under the speed-time graph. The area under the graph for time interval t is the area of rectangle of sides t and v. This area is shown shaded in Fig. 2.23 and is equal to $v \times t$. Thus, area under speed-time graph up to the time axis is numerically equal to the distance covered by the object in time t.

Fig. 2.23

Now consider another example shown in Fig. 2.24. Here, the speed of the object increases uniformly from 0 to v in time t. The average speed is given by

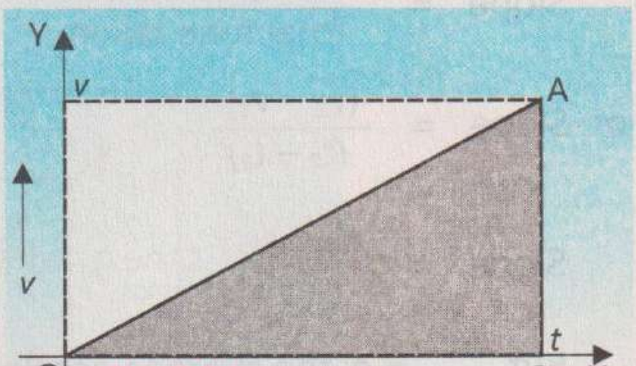


Fig. 2.24

$$v_{av} = \frac{0 + v}{2} = \frac{1}{2} v$$

Distance covered = average speed × time = $1/2\ v \times t$.

If we calculate the area under speed-time graph, it is equal to the area of the right-angled triangle shown shaded in Fig. 2.24. The base of the triangle is equal to t and the perpendicular is equal to v.

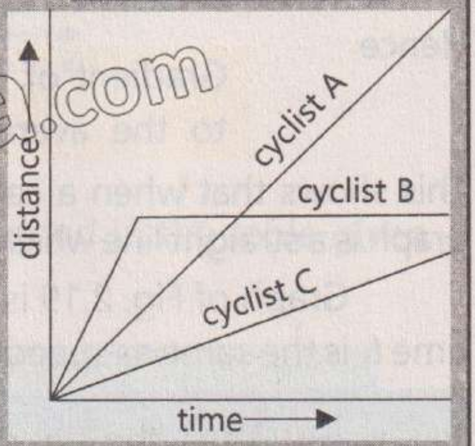
Area of a triangle = 1/2 (perpendicular × base)

= $1/2\ (v \times t)$

We see that this area is numerically equal to the distance covered by the object during the time interval t. Therefore, we can say that:

The area under the speed-time graph up to the time axis is numerically equal to the distance covered by the object.

Mini Exercise



The distance-time graph shows the motion of three cyclists.

(a) What does each line on the graph represent?

(b) Which cyclist travelled the most distance?

(c) Which cyclist travelled at the greatest speed? the lowest speed? at constant speed?

2.12 Solving Problems for Motion Under Gravity

Three equations of motion are used to solve problems for motion of bodies. If v_i is the initial velocity of the body, v_f is the final velocity, t is the time taken, S is the distance covered and a is the acceleration, then:

$v_f = v_i + at$ ------------- (1)

$S = v_i t + \frac{1}{2} at^2$ ------------- (2)

$2aS = v_f^2 - v_i^2$ ------------- (3)

While applying these equations, the following assumptions are made:

(i) Motion is always considered along a straight line

(ii) Only the magnitudes of vector quantities are used.

(iii) Acceleration is assumed to be uniform.

(iv) The direction of initial velocity is taken as positive. Other quantities which are in the direction of initial velocity are taken as positive. The quantities in the direction opposite to the initial velocity are taken as negative.

2.13 Free Fall Acceleration

When a body is falling freely under the action of gravity of the Earth, the acceleration acting on it is the gravitational acceleration and is denoted by g. The direction of gravitational acceleration is always downwards. Its value is 9.8 m s^{-2}, but for convenience we shall use the value of g as 10 m s^{-2}.

For Your Information!

Light and heavier objects when fall through vacuum, move side by side.

Since the freely falling bodies move vertically downwards in a straight line with uniform acceleration g, so the three equations of motion can be applied to the motion of such bodies. While applying equations of motion, the acceleration a is replaced by g. Thus, equations of motion for freely falling bodies can be written as:

$$v_f = v_i + gt \quad \text{------------- (1)}$$

$$S = v_i t + \frac{1}{2} gt^2 \quad \text{------------- (2)}$$

$$2gS = v_f^2 - v_i^2 \quad \text{------------- (3)}$$

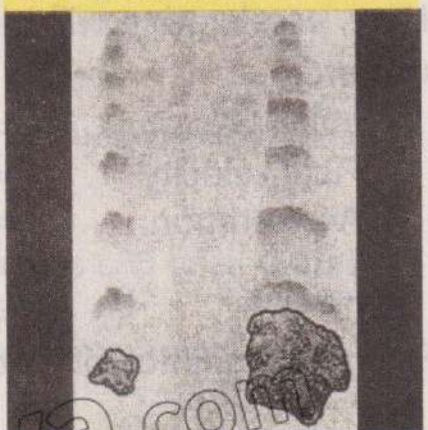
It should be remembered that while using these equations, the following points should be kept in mind:

(i) If a body is released from some height to fall freely, its initial velocity v_i will be taken as zero.

(ii) The gravitational acceleration g will be taken as positive in the downward direction. All other quantities will also be taken as positive in the downward direction. The quantities in the direction opposite to the acceleration will be taken as negative.

(iii) If a body is thrown vertically upward, the value of g will be negative and the final velocity will be zero at the highest point.

For Your Information!



Acceleration of free fall g is 10 m s^{-2} for all objects.

Example 2.5

An iron bob is dropped from the top of a tower. It reaches the ground in 4 seconds. Find: (a) the height of the tower (b) the velocity of the ball as it strikes the ground.

Solution

For freely falling body:

Initial velocity $= v_i = 0$

Acceleration $= g = 10\ \text{m s}^{-2}$

Time $= t = 4\ \text{s}$

Height (distance) $= S = h = ?$

Final velocity $= v_f = ?$

(a) According to second equation of motion,

$$S = v_i t + \frac{1}{2} g t^2$$

Putting the values, $h = 0 \times 4\ \text{s} + \frac{1}{2} \times 10\ \text{m s}^{-2} \times (4)^2\ \text{s}^2$

$$h = 80\ \text{m}$$

(b) From the first equation of motion, we have

$$v_f = v_i + gt$$

Putting the values, $v_f = 0 + 10\ \text{m s}^{-2} \times 4\text{s} = 40\ \text{m s}^{-1}$

Example 2.6

An arrow is thrown vertically upward with the help of a bow. The velocity of the arrow when it leaves the bow is 30 m s^{-1}. Determine time to reach the highest point? Also, find the maximum height attained by the arrow.

Solution

Here, acceleration will be taken as negative, for the arrow is thrown vertically upward.

Initial velocity $= v_i = 30\ \text{m s}^{-1}$

Final velocity $= v_f = 0$

Acceleration $= g = -10\ \text{m s}^{-2}$

Time $= t = ?$

Height $S = h = ?$

From first equation of motion:

$$v_f = v_i + gt$$

or

$$t = \frac{v_f - v_i}{-g}$$

Putting the values $t = \frac{0 - 30\ \text{m s}^{-1}}{-10\ \text{m s}^{-2}} = 3\ \text{s}$

Now from the third equation of motion:

$$2gS = v_f^2 - v_i^2$$

or $$S = \frac{v_f^2 - v_i^2}{2(-g)}$$

Putting the values $$h = \frac{0-(30)^2 \text{ m}^2 \text{ s}^{-2}}{2 \times 10 \text{ m s}^{-2}} = 45 \text{ m}$$

Relativity

In 1905, famous scientist Albert Einstein proposed his revolutionary theory of special relativity which modified many of the basic concepts of physics. According to this theory, speed of light is a universal constant. Its value is approximately 3×10^8 m s^{-1}. Speed of light remains the same for all motions. Any object with mass cannot achieve speeds equal to or greater than that of light. This is known as universal speed limit.

KEY POINTS

- A scalar is that physical quantity which can be described completely by its magnitude only.
- A vector is that physical quantity which needs magnitude as well as direction to describe it completely.
- To add a number of vectors, redraw their representative lines such that the head of one line coincides with the tail of the other. The resultant vector is given by a single vector which is directed from the tail of the first vector to the head of the last vector.
- Translatory motion, rotatory motion and vibratory motions are different types of motion.
- Position of any object is its distance and direction from a fixed point.
- The shortest distance between the initial and final positions of a body is called its displacement.
- Distance covered by a body in a unit time is called its speed.
- Time rate of displacement of a body is called its velocity.
- The velocity is said to be uniform if the speed and direction of a moving body does not change, otherwise it is non-uniform velocity.
- Rate of change of velocity of a body is called its acceleration.
- If change of velocity with time is constant, the acceleration is said to be uniform, otherwise it is non-uniform.
- A graph that shows the relation between distance and time taken by a moving body is called a distance-time graph.
- A graph that shows the relation between the speed and time taken by a moving body is called a speed-time graph.
- Gradient or slope of the distance-time graph is equal to the average speed of the body. Slope of the speed-time graph is equal to the acceleration of the body.

- The area under speed-time graph is numerically equal to the distance covered by the object.
- Following are three equations of motion:

$$v_f = v_i + at$$
$$S = v_i t + \frac{1}{2} at^2$$
$$2aS = v_f^2 - v_i^2$$

- Gravitational acceleration g acts downward on bodies falling freely. The magnitude of g is $10\ m\ s^{-2}$.

EXERCISE

A Multiple Choice Questions

Tick (✓) the correct answer.

2.1 The numerical ratio of displacement to distance is:
(a) always less than one (b) always equal to one
(c) always greater than one (d) equal to or less than one

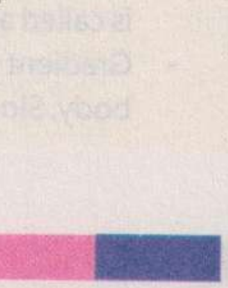
2.2 If a body does not change its position with respect to some fixed point, then it will be in a state of:
(a) rest (b) motion
(c) uniform motion (d) variable motion

2.3 A ball is dropped from the top of a tower, the distance covered by it in the first second is:
(a) 5 m (b) 10 m (c) 50 m (d) 100 m

2.4 A body accelerates from rest to a velocity of $144\ km\ h^{-1}$ in 20 seconds. The distance covered by it is:
(a) 100 m (b) 400 m (c) 1400 m (d) 1440 m

2.5 A body is moving with constant acceleration starting from rest. It covers a distance S in 4 seconds. How much time does it take to cover one-fourth of this distance?
(a) 1 s (b) 2 s (c) 4 s (d) 16 s

2.6 The displacement time graphs of two objects A and B are shown in the figure. Point out the true statement from the following:



(a) The velocity of A is greater than B.
(b) The velocity of A is less than B.
(c) The velocity of A is equal to that of B.
(d) The graph gives no information in this regard.

2.7 The area under the speed-time graph is numerically equal to:
(a) velocity (b) uniform velocity
(c) acceleration (d) distance covered

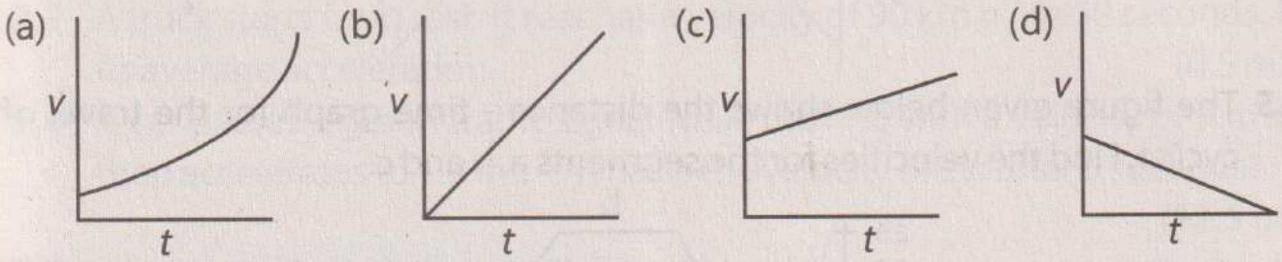
2.8 Gradient of the speed-time graph is equal to:

(a) speed (b) velocity (c) acceleration (d) distance covered

2.9 Gradient of the distance-time graph is equal to the:

(a) speed (b) velocity (c) distance covered (d) acceleration

2.10 A car accelerates uniformly from 80.5 km h^{-1} at $t = 0$ to 113 km h^{-1} at $t = 9$ s. Which graph best describes the motion of the car?



B Short Answer Questions

2.1 Define scalar and vector quantities.

2.2 Give 5 examples each for scalar and vector quantities.

2.3 State head-to-tail rule for addition of vectors.

2.4 What are distance- time graph and speed-time graph?

2.5 Falling objects near the Earth have the same constant acceleration. Does this imply that a heavier object will fall faster than a lighter object?

2.6 The vector quantities are sometimes written in scalar notation (not bold face). How is the direction indicated?

2.7 A body is moving with uniform speed. Will its velocity be uniform? Give reason.

2.8 Is it possible for a body to have acceleration? When moving with:

(i) constant velocity

(ii) constant speed

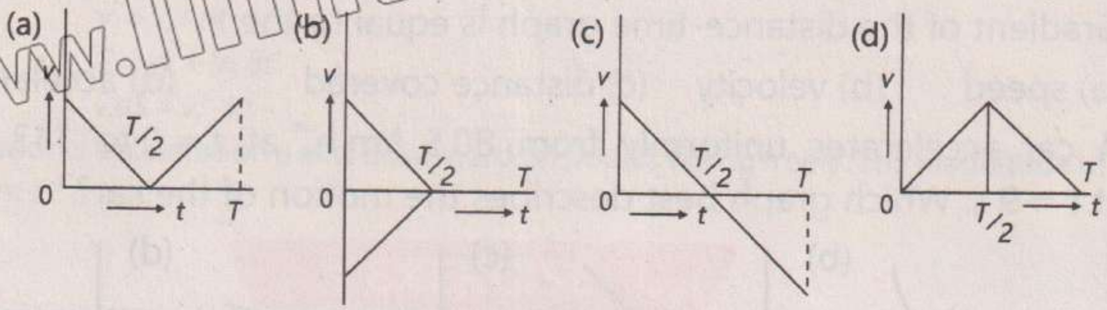
C Constructed Response Questions

2.1 Distance and displacement may or may not be equal in magnitude. Explain this statement.

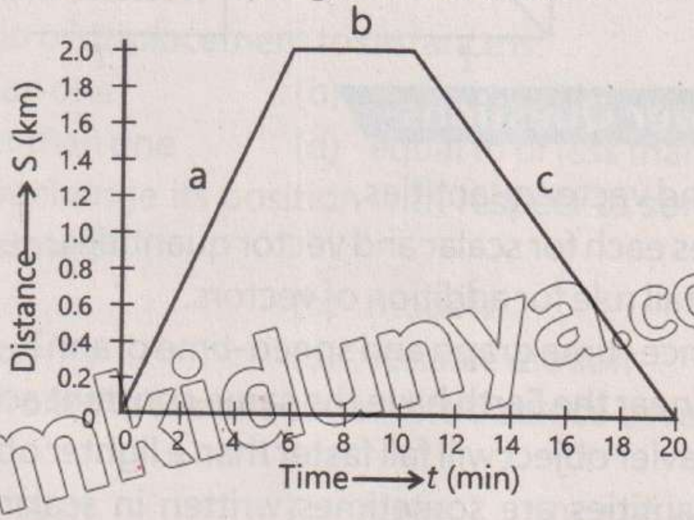
2.2 When a bullet is fired, its velocity with which it leaves the barrel is called the muzzle velocity of the gun. The muzzle velocity of one gun with a longer barrel is lesser than that of another gun with a shorter barrel. In which gun is the acceleration of the bullet larger? Explain your answer.

2.3 For a complete trip, average velocity was calculated. Its value came out to be positive. Is it possible that its instantaneous velocity at any time during the trip had the negative value? Give justification of your answer.

2.4 A ball is thrown vertically upward with velocity *v*. It returns to the ground in time *T*. Which of the following graphs correctly represents the motion? Explain your reasoning.



2.5 The figure given below shows the distance - time graph for the travel of a cyclist. Find the velocities for the segments a, b and c.



2.6 Is it possible that the velocity of an object is zero at an instant of time, but its acceleration is not zero? If yes, give an example of such a case.

D Comprehensive Questions

2.1 How a vector can be represented graphically? Explain.

2.2 Differentiate between:

(i) rest and motion

(ii) speed and velocity

2.3 Describe different types of motion. Also give examples.

2.4 Explain the difference between distance and displacement.

2.5 What do gradients of distance-time graph and speed-time graph represent? Explain it by drawing diagrams.

2.6 Prove that the area under speed-time graph is equal to the distance covered by an object.

2.7 How equations of motion can be applied to the bodies moving under the action of gravity?

E Numerical Problems

2.1 Draw the representative lines of the following vectors:

(a) A velocity of 400 $m\ s^{-1}$ making an angle of 60° with x-axis.

(b) A force of 50 N making an angle of 120° with x-axis.

2.2 A car is moving with an average speed of 72 $km\ h^{-1}$. How much time will it take to cover a distance of 360 km? (5 h)

2.3 A truck starts from rest. It reaches a velocity of 90 $km\ h^{-1}$ in 50 seconds. Find its average acceleration. (0.5 $m\ s^{-2}$)

2.4 A car passes a green traffic signal while moving with a velocity of 5 $m\ s^{-1}$. It then accelerates to 1.5 $m\ s^{-2}$. What is the velocity of car after 5 seconds? (12.5 $m\ s^{-1}$)

2.5 A motorcycle initially travelling at 18 $km\ h^{-1}$ accelerates at constant rate of 2 $m\ s^{-2}$. How far will the motorcycle go in 10 seconds? (150 m)

2.6 A wagon is moving on the road with a velocity of 54 $km\ h^{-1}$. Brakes are applied suddenly. The wagon covers a distance of 25 m before stopping. Determine the acceleration of the wagon. (-4.5 $m\ s^{-2}$)

2.7 A stone is dropped from a height of 45 m. How long will it take to reach the ground? What will be its velocity just before hitting the ground? (3 s, 30 $m\ s^{-1}$)

2.8 A car travels 10 km with an average velocity of 20 $m\ s^{-1}$. Then it travels in the same direction through a diversion at an average velocity of 4 $m\ s^{-1}$ for the next 0.8 km. Determine the average velocity of the car for the total journey. (15.4 $m\ s^{-1}$)

2.9 A ball is dropped from the top of a tower. The ball reaches the ground in 5 seconds. Find the height of the tower and the velocity of the ball with which it strikes the ground. (125 m, 50 $m\ s^{-1}$)

2.10 A cricket ball is hit so that it travels straight up in the air. An observer notes that it took 3 seconds to reach the highest point. What was the initial velocity of the ball? If the ball was hit 1 m above the ground, how high did it rise from the ground? (30 $m\ s^{-1}$, 46 m)