Chapter 4

Turning Effects of Force

Student Learning Outcomes

After completing this chapter, students will be able to:

- Differentiate between like and unlike parallel forces.
- Analyse problems involving turning effects of forces [Student should know that moment of a force = force × perpendicular distance from the pivot and be able to use this in simple problems and give examples and applications of turning effects in real life]
- State what is meant by centre of mass and centre of gravity.
- Describe how to determine the position of the centre of gravity of a plane lamina using a plumb line
- Describe and identify states of equilibrium. [This includes the types, conditions and states of equilibrium and identifying their examples from daily life.]
- Analyse, qualitatively, the effect of the position of the centre of gravity on the stability of simple objects
- Propose how the stability of an object can be improved [by lowering the centre of mass and increasing the base area of the object]
- Illustrate the applications of stability physics in real life [Such as this concept is central to engineering technology such as balancing toys and racing cars
- Predict qualitatively the motion of rotating bodies [Describe qualitatively that, analogous to Newton's 1st law for transnational motion (, an object that is rotating will continue to do so at the same rate unless acted upon by a resultant moment (in which case it would begin to accelerate or decelerate its rotational motion)]
- Describe qualitatively motion in a circular path due to a centripetal force. (Use of the formula $F_c = \frac{mv^2}{r}$)
- Identify the sources of centripetal force in real life examples [e.g., tension in a string for a stone being swirled around, gravity for the Moon orbiting the Earth]

As we know, a force is a vector quantity, so it acts in a particular direction. We observe various effects of forces. Some forces produce acceleration or decelerating in a body, some tend to turn it around a point and some forces balance each other acting in opposite directions.

All those forces which act parallel to one another are known as parallel forces. The points of application of such forces may be different.

4.1 Like and Unlike Parallel Forces

If the parallel forces are acting in the same direction, then they are called like parallel forces and if their directions are opposite to one another, they are called unlike parallel forces. Three forces $\mathbf{F_1}$, $\mathbf{F_2}$ and $\mathbf{F_3}$ are shown in Fig. 4.1 acting on a rigid body at different points. Here, the forces $\mathbf{F_1}$ and $\mathbf{F_2}$ are like parallel forces but $\mathbf{F_2}$ and $\mathbf{F_3}$ are unlike parallel forces.

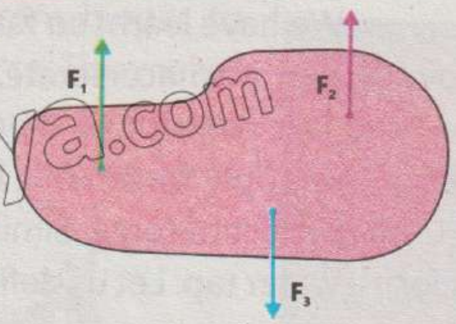


Fig. 4.1

4.2 Addition of Forces

In chapter 2, we have learnt about vectors and their representation. Remember that the resultant is the same for any order of addition of vectors. As forces are vectors, so forces can also be added by head-to-tail rule.

To determine the resultant of two or more forces acting in a plane, the following example will explain its method.

Example 4.1

Let us add three force vectors $\mathbf{F_1}$, $\mathbf{F_2}$ and $\mathbf{F_3}$ having magnitudes of 200 N, 300 N and 250 N acting at angles of 30°, 45°, 60° with x-axis. By selecting a suitable scale 100 N = 1 cm, we can draw the force vectors as shown in Fig. 4.2(a).

To add these vectors, we apply head-to-tail rule as shown in Fig. 4.2(b).

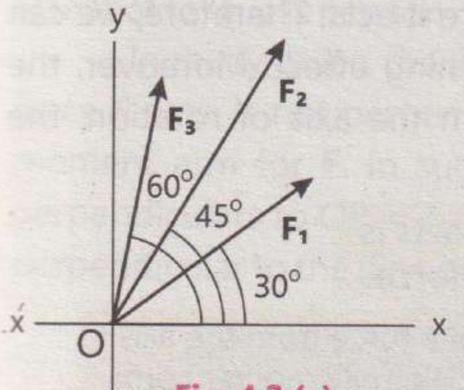


Fig. 4.2 (a)

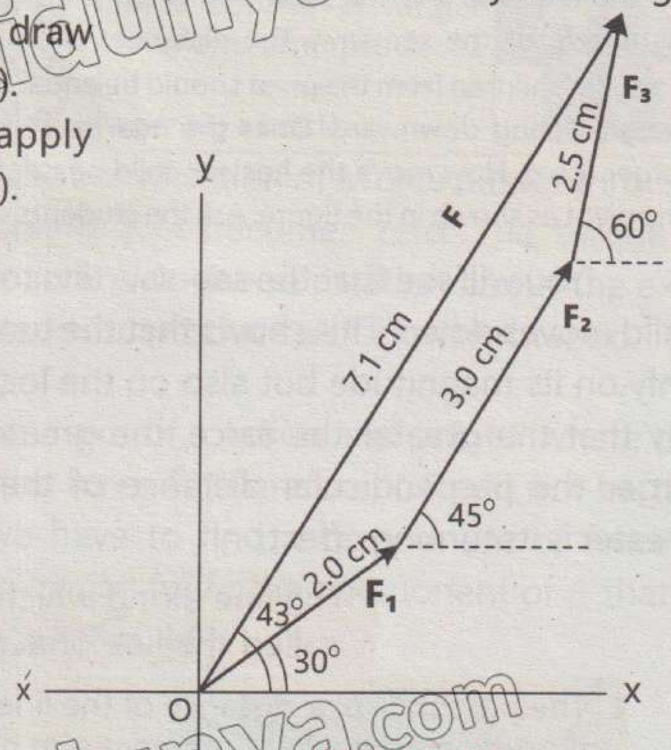


Fig. 4.2 (b)

Measured length of resultant force is 7.1 cm. According to selected scale, magnitude of the resultant force $\mathbf{F}$ is 710 N and direction is at an angle 43° with x-axis.

4.3 Turning Effect of a Force

We have learnt so far that a net force affects the liner motion of an object by causing it to accelerate. Since rigid objects can also rotate, so we need to extend our concept to the turning effect of a force. When we open or close a door, we apply force. This force rotates the door about its hinge. This is called turning effect of force. Similarly, we use turning effect of force when we open or close a water tap. Let us define some terms used in the study of turning effect of a force.

If the distance between two points of the body remains the same under the action of a force, it is called a **rigid body**.

During rotation, all the particles of the rigid body rotate along fixed circles as shown in Fig. 4.3. The straight line joining the centres of these circles is called **the axis of rotation**. In this case, it is OZ. To observe the turning effect of a force, let us perform an activity.

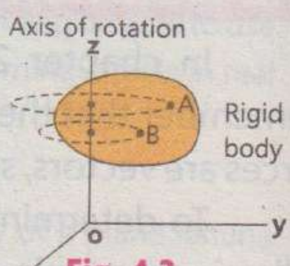


Fig. 4.3

Activity 4.1

Take your class to play ground where a see-saw is available. Let a lighter child sits on the left side and the heavier one on the right side of the see-saw. The distances of both the children from the pivot should be equal. The force exerted by each child is equal to his weight acting downward. Does the heavier child move down? Yes, because he is exerting larger force. Now move the heavier child nearer to the pivot and the lighter child away from the pivot as shown in the figure. Ask the students what do they observe?

You will see that the see-saw tilts to the opposite direction and the lighter child moves down. This shows that the turning effect of a force does not depend only on its magnitude but also on the location where it acts. Therefore, we can say that the greater the force, the greater is its turning effect. Moreover, the larger the perpendicular distance of the force from the axis of rotation, the greater is its turning effect.

The line along which the force acts is called the line of action of the force.

The perpendicular distance of the line of action of a force from the axis of rotation is known as moment arm of the force or simply moment arm.

The moment arms of both the children are shown in the figure of activity 4.1. There are many other examples to observe the turning or rotational effect of a force. It is harder to open a door by pushing it at a point closer to the hinge as compared to push it at the handle (Fig. 4.4). That is why, door or window handles are always installed at larger distances from hinges to produce larger moment of force by applying less force. This makes the doors be opened or closed more easier. Similarly, it requires greater force to open a nut by a spanner if you hold it closer such as point A than point B (Fig. 4.5).

Fig. 4.4

Fig. 4.5

Moment of Force

The turning effect of a force is measured by a quantity known as moment of force or torque.

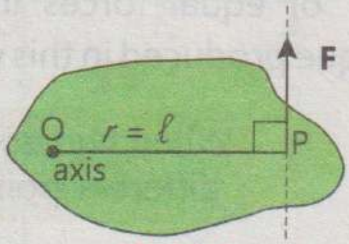


Fig. 4.6

> Moment of a force or torque is defined as the product of the force and the moment arm.

The magnitude of torque is given by

$$\tau = F \times \ell \qquad ....(4.1)$$

Where τ (tau) is the torque and ℓ is the moment arm. In Fig. 4.6, the line of action of a force **F** is perpendicular to r, therefore, moment arm $\ell = r$. Remember

> **Do You Know?**
>
> Moment of force is applicable in the working of bottle opener. A small force applied at longer moment arm produces more torque while opening a bottle.
>
> 

that the torque of a force is zero when the line of action of a force passes through the axis of rotation, because its moment arm becomes zero. The torque is positive if the force tends to produce an anticlockwise rotation about the axis, and it is taken as negative if the force tends to produce a clockwise rotation. The SI unit of torque is newton metre (N m).

In many cases, the line joining the axis of rotation and point P where the force **F** acts, is not perpendicular to the force **F**. Therefore, OP will not be the moment arm for **F**. In such cases, we have to find a component of force **F** perpendiculars to OP = ℓ (Fig. 4.7), or we can find r the component of ℓ that is perpendicular to the (line of action) force **F** (Fig. 4.8).

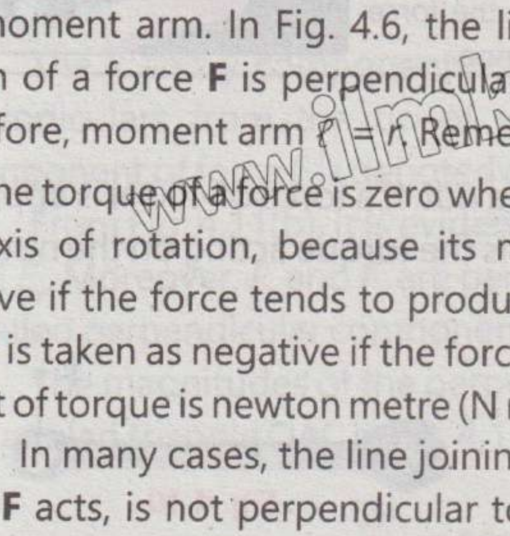


Fig. 4.7

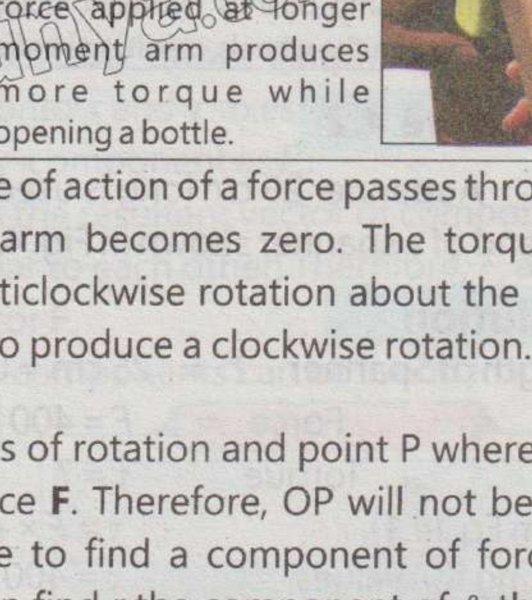


Fig. 4.8

For this, we need to know the method of finding rectangular components of a force or any vector. This is also called as resolution of Forces.

Couple

A couple is a special type of torque. We observe at many situations in our daily life, when two equal and opposite parallel forces produce torque. For example, while opening or closing a water tap, turning key in the lock, opening the lid of a jar and turning steering wheel of a motor car, we apply a pair of equal forces in opposite directions. The torque produced in this way in known as couple.

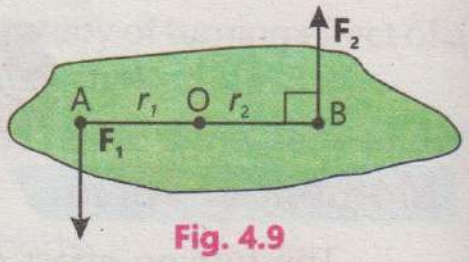


Fig. 4.9

When two equal and opposite parallel forces act at two different points of the same body, they form a couple.

Steering wheel of vehicles

While turning a vehicle, a couple is applied on the steering wheel. It is interesting to know that now-a-days, steering wheels of smaller diameter are installed in vehicles. The reason is that, most of the fvehicles are provided with power steering in which a pump pushes hydraulic fluid to reduce the force needed to turn the wheels, resulting in effortless steering.

Example 4.2

A spanner 25 cm long is used to open a nut. If a force of 400 N is applied at the end of a spanner shown in Fig. 4.10, what is the torque acting on the nut?

Solution

Length of Spanner ℓ = 25 cm = 0.25 m

Force = $F = 400$ N

Torque $\tau = ?$

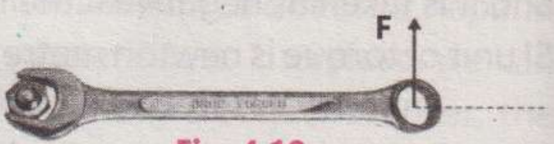


Fig. 4.10

From Eq. (4.1), $\tau = F \times \ell$

Putting the values, $\tau = 400\text{ N} \times 0.25\text{ m} = 100\text{ N m}$

4.4 Resolution of Vectors

By head-to-tail rule, two or more vectors can be added to give a resultant vector. Its reverse process is also possible, i.e., a given vector can be divided into two or more parts. These parts are called as components of the given vector. If these components are added up, their resultant is equal to the given vector. To divide a force into its components is known as **resolution of a force**.

Usually, a force is resolved into two components which are perpendicular to each other. These are called its perpendicular or rectangular components of the force.

Let us resolve a force **F** into its perpendicular components. A force **F** acting on a body at an angle θ with x-axis is shown in Fig. 4.11(a). Imagine a beam of light is placed above the vector **F**. As the light falls perpendicularly to the x-axis, it will cast a shadow OA of vector **F** onto x-axis. We call this shadow as x-component of vector **F**. In the same way, if light is thrown perpendicular to y-axis, the shadow OB of vector **F** on y-axis is the y-component of **F**.

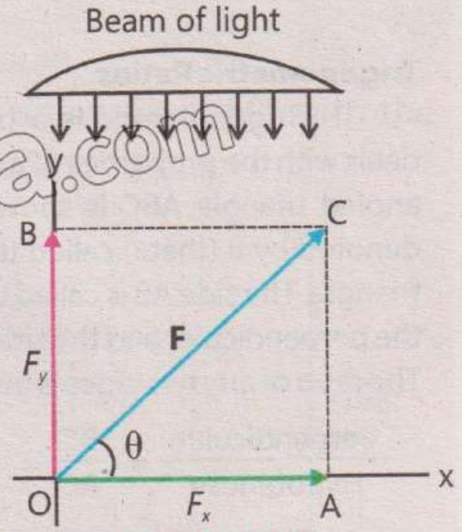


Fig. 4.11(a)

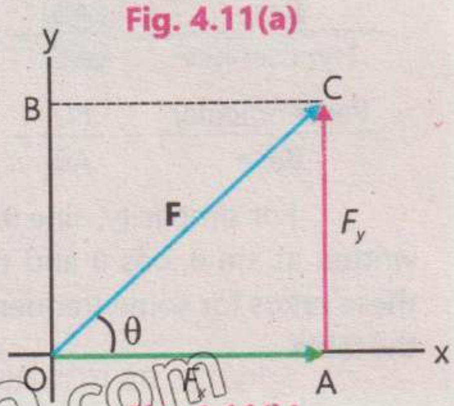


Fig. 4.11(b)

> A component of a vector is its effective value in a given direction.

The x and y components can be practically drawn simply by dropping perpendiculars from the tip of vector **F** onto x and y-axes respectively. The x-component of force **F** is denoted as F_x and y-component as F_y.

From Fig. 4.11(b), it is evident that **F** is the resultant vector of components F_x and F_y. Moreover, F_x and F_y are perpendicular to each other. Therefore, F_x and F_y are called perpendicular components of vector **F**.

The magnitudes of the perpendicular components can be found from the right angled triangle OAC in Fig. 4.11(b).

$$\frac{OA}{OC} = \cos\theta$$

Putting the values, $$\frac{F_x}{F} = \cos\theta$$

or $$F_x = F\cos\theta \qquad \text{...... (4.2)}$$

Similarly, $$\frac{AC}{OC} = \sin\theta$$

$$\frac{F_y}{F} = \sin\theta$$

or $$F_y = F\sin\theta \qquad \text{... .... (4.3)}$$

Do You Know?

A tight rope walker balances himself by holding a bamboo stick. This is an application of principle of moments.

For Your Information!

Trigonometric Ratios

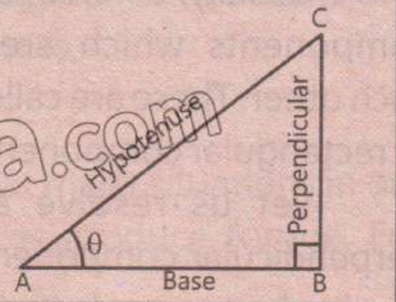
Trigonometric is a branch of mathematics that deals with the properties of a right angled triangle. A right angled triangle ABC is shown in the figure. Angle A is denoted by θ (theta) called the angle of the right angled triangle. The side AB is called the base, the side BC is called the perpendicular and the side AC is called as hypotenuse. The ratio of any two sides is given the names as below:

$$\frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AC} = \text{sine } \theta$$

$$\frac{\text{Base}}{\text{Hypotenuse}} = \frac{AB}{AC} = \text{cosine } \theta$$

$$\frac{\text{Perpendicular}}{\text{Base}} = \frac{BC}{AB} = \text{tangent } \theta$$

For simplicity, sine θ, cosine θ and tangent θ are written as sin θ, cos θ and tan θ respectively. Values of these ratios for some frequently used angles are given in the table.



θ	sinθ	cosθ	tanθ
0°	0	1.0	0
30°	$\frac{1}{2}$ = 0.5	$\frac{\sqrt{3}}{2}$ = 0.866	$\frac{1}{\sqrt{3}}$ = 0.577
45°	$\frac{1}{\sqrt{2}}$ = 0.707	$\frac{1}{\sqrt{2}}$ = 0.707	1.0
60°	$\frac{\sqrt{3}}{2}$ = 0.866	$\frac{1}{2}$ = 0.5	$\sqrt{3}$ = 0.866
90°	1.0	0	∞ Unlimited

4.5 Determination of a Force from its Prependicular Components

The magnitude and direction of a force can be found if its perpendicular components are known. Applying Pythagorean theorem to the right angled triangle OAC (Fig. 4.11-b).

$$(OC)^2 = (OA)^2 + (AC)^2$$

or $$F^2 = F_x^{\,2} + F_y^{\,2}$$

$$F = \sqrt{F_x^{\,2} + F_y^{\,2}} \quad ....(4.4)$$

Hence, using Eq. (4.4) the magnitude F of the required vector **F** can be determined. The direction of **F** is given by

$$\tan\theta = \frac{F_y}{F_x} \quad ....(4.5)$$

or $$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$

By using table of trigonometric ratios or calculator, the value of θ can be determined.

Example 4.3

A force of 160 N is acting on a wooden box at an angle of 60° with the horizontal direction. Determine the values of its x and y components.

Solution

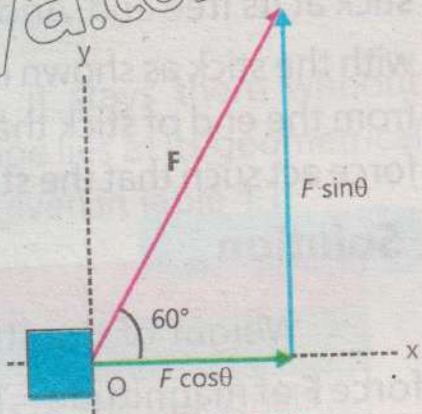


Fig. 4.12

Magnitude of force $F = 160\text{ N}$

Angle $\theta = 60°$

Using calculator, $\sin\theta = \sin 60° = 0.866$

$\cos\theta = \cos 60° = 0.5$

x-component is given by Eq. (4.2)

$$F_x = F\cos\theta$$

Putting the values, $F_x = 160\text{ N} \times 0.5 = 80\text{ N}$

y-component is given by Eq. (4.3)

$$F_y = F\sin\theta$$

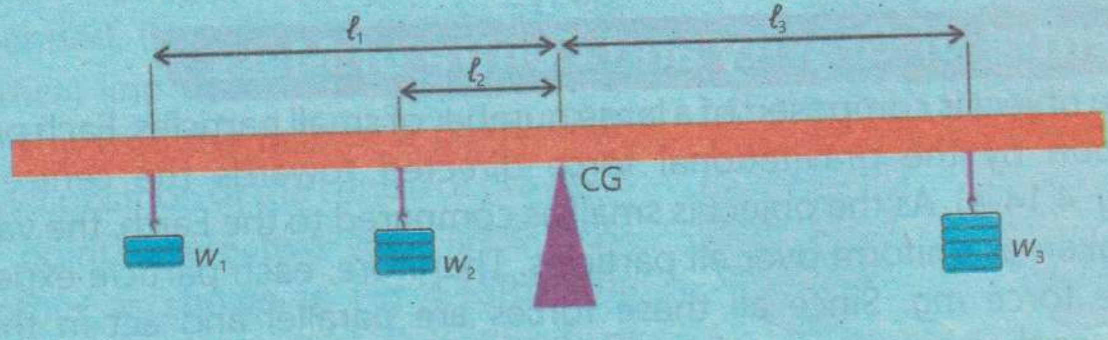
Putting the values, $F_y = 160\text{N} \times 0.866 = 138.6\text{ N}$

4.6 Principle of Moments

To understand the principle of moments, let us perform an activity.

Activity 4.2

Balance a metre rule on a wedge at its centre of gravity such that the metre rule stays horizontal. Suspend two weights w_1 and w_2 on one side of the metre rule at distance ℓ_1 and ℓ_2 from the centre and a third weight w_3 on the other side at distance ℓ_3 until the rule is again balanced.



The weights w_1 and w_2 tend to rotate the rod anticlockwise about CG and the weight w_3 tends to rotate it clockwise. The values of the moments of the weights are $w_1 \times \ell_1$, $w_2 \times \ell_2$ and $w_3 \times \ell_3$. When the metre rule is balanced, then

Total anticlockwise moments = Total clockwise moments

$$w_1 \times \ell_1 + w_2 \ell_2 = w_3 \times \ell_3 \quad \text{.................(4.6)}$$

This is known as principle of moments, which is stated as:

When a body is in balanced position, the sum of clockwise moments about any point equals the sum of anticlockwise moments about that point.

Example 4.4

A metre stick is pinned at its one end O on a table so that it can rotate freely. One force of magnitude 18 N is applied perpendicular to the length of the stick at its free end. Another force of magnitude 60N is acting at an angle of 30° with the stick as shown in Fig. 4.13(a). At what distance from the end of stick that is pinned should the second force act such that the stick does not rotate?

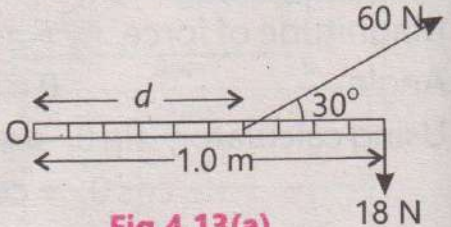


Fig 4.13(a)

Solution

Weight of the stick does not affect in the horizontal plane. Resolving force **F** of magnitude = 60 N into rectangular components that act at distance d from point O:

$F_x = 60\text{ N} \times \cos 30^\circ = 60\text{ N} \times 0.866 = 51.96\text{ N}$

$F_y = 60\text{ N} \times \sin 30^\circ = 60\text{ N} \times 0.5 = 30\text{ N}$

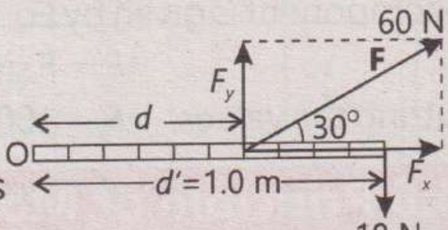


Fig 4.13(b)

As the component F_x passes through the axis of rotation, its torque is zero. Torque τ_1 of 30 N is positive and τ_2 of 18 N force is negative. The stick will not rotate when these two torques balance each others, i.e $\tau_1 = \tau_2$ or $F_y \times d = F_y' \times d'$

$$30\text{ N} \times d = 18\text{ N} \times 1\text{ m}$$

$$d = \frac{18\text{ N} \times 1\text{ m}}{30\text{ N}} = 0.6\text{ m}$$

4.7 Centre of Gravity and Centre of Mass

An object is composed of a large number of small particles. Each particle is acted upon by the gravitational force directed towards the centre of the Earth (Fig. 4.14-a). As the object is small as compared to the Earth, the value of g can be taken as uniform over all particles. Therefore, each particle experiences the same force mg. Since all these forces are parallel and act in the same direction, so their resultant as shown in Fig. 4.14(b) will be equal to the sum of all these forces .i.e,

Resultant force = $\sum mg$ where $\sum$ means the "sum of".

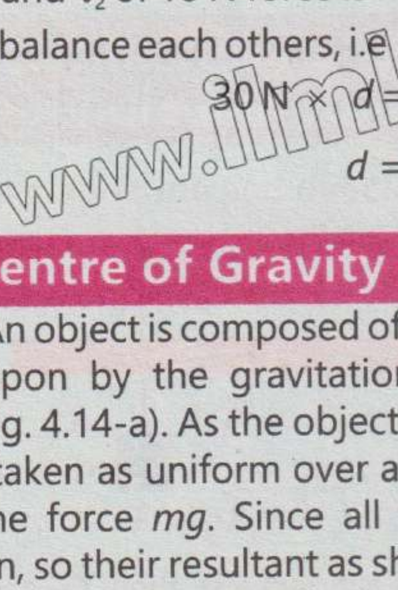


Fig. 4.14(a) Gravitational force acting on various particles

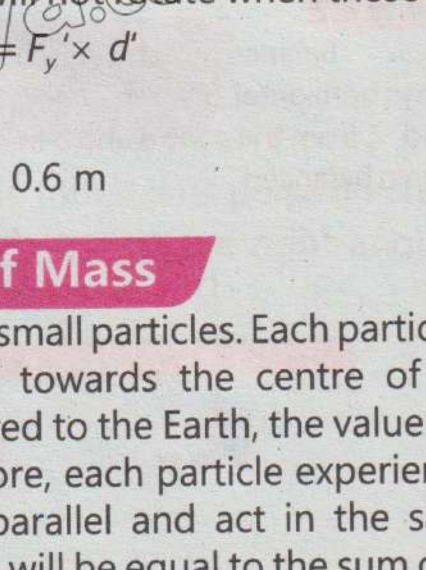


Fig. 4.14(b) Resultant gravitational force

We know that the sum of the gravitational forces acting on all particles is equal to the total weight of the object $w = Mg$ Where $M = \sum m$ = mass of the object.

> Centre of gravity is that point where total weight of the body appears to be acting.

If a body is supported at its centre of gravity, it stays there without rotation. The centre of gravity of an object of regular shape lies at its geometrical centre. Centre of gravity of some geometrical shapes is given in Table 4.1.

Table 4.1

Object	Centre of Gravity
Square, Rectangle	Point of intersection of the diagonals
Triangle	Point of intersection of the medians
Round plate	Centre of the plate
Sphere	Centre of the sphere
Cylinder	Centre of the axis
Metre rule	Centre of the rod

Centre of Gravity of a Plane Lamina

For an irregular shaped plane lamina, the centre of gravity can be found by suspending it freely through different points (Fig. 4.15-a). Each time the object is suspended, its centre of gravity lies on the vertical line drawn from the point of suspension with the help of a plumb line. The exact position of the centre of gravity is at the point where two such lines cross each other as shown in Fig 4.15(b). The centre of gravity can exist inside a body or outside the body as is in case of a cup.

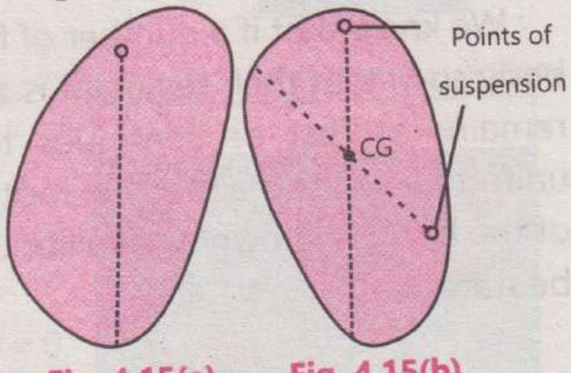


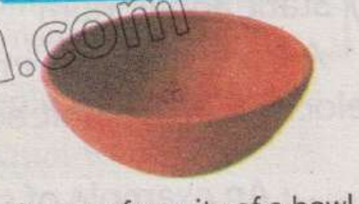
Fig. 4.15(a) Fig. 4.15(b)

Irregular shaped plane lamina

Centre of Mass

Newton's second law of motion is applicable to single particle or system of particles. Even when the parts of a system have different velocities and acceleration, there is still one point in the system whose acceleration could be found by applying second law. This point is called the centre of mass of the system.

For your information!



Centre of gravity of a bowl is outside the material.

The centre of mass of a body is that point where the whole mass of the body is assumed to be concentrated.

Hence, the centre of mass behaves as if all the mass of the body or system is lying at that point. In the Fig. 4.16 given below, a rotating wrench slides along a frictionless floor. There is no resultant force on the wrench. Therefore, its centre of mass, shown by a yellow dot, follows a linear path with constant speed.

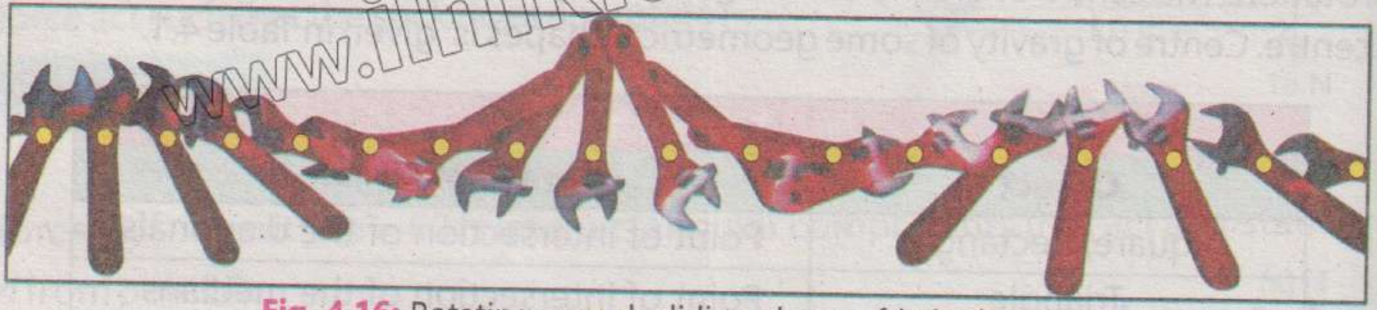


Fig. 4.16: Rotating wrench sliding along a frictionless floor

On the surface of the Earth, where g is almost uniform, the centre of mass of an object coincides with its centre of gravity.

4.8 Equilibrium

We have learnt how translatory and rotational motion can be caused due to the application of external forces. Now, we shall see how external forces can be balanced to produce no translational or rotational effects.

We know that if a number of forces act on a body such that their resultant is zero, the body remains at rest or continues to move with uniform velocity if already in motion. This state of the body is known as equilibrium, which can be stated as:

Do You Know?

This is a fascinating scene of equilibrium.

A body is said to be in equilibrium if it has no acceleration.

There are two types of equilibrium:

(i) Static equilibrium (ii) Dynamic equilibrium

A body at rest is in static equilibrium whereas a body moving with uniform velocity is in dynamic equilibrium.

An example of static equilibrium is a book lying on the table as shown in Fig. 4.17. Only two forces are acting on it. One is its weight $w = mg$ acting downward and the other is $\mathbf{F_n}$ the normal force that the table exerts upward on the book. Since the book is at rest so, it has zero acceleration. Therefore, the sum of all the forces acting on the book should be zero, so that the book is said to be in equilibrium. Hence

Fig. 4.17
Book is in static equilibrium

$$F_n - w = 0$$
$$\text{or} \quad F_n = w$$

This means that forces can act on a body without accelerating it, provided these forces balance each other.

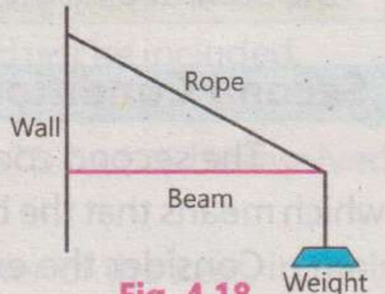


Fig. 4.18
A beam projected from a wall is also in static equilibrium

An electric bulb hanging from the ceiling of a room, a man holding a box, a beam held horizontal against a wall with the help of a rope and a hanging weight (Fig. 4.18), are all examples of static equilibrium.

A good example of dynamic equilibrium is a paratrooper (Fig. 4.19). In a few second after the free fall, the parachute opens and a little later, the paratrooper starts descending with a uniform velocity. In this state, the force of gravity acting vertically downward on the paratrooper is balanced by the resistance of air on the parachute acting upward.


Fig. 4.19
A paratrooper is in dynamic equilibrium

4.9 Conditions of Equilibrium

There are two conditions of equilibrium:

First Condition of Equilibrium

By Newton's second law of motion, $F = ma$
If the body is in translational equilibrium, then $a = 0$, therefore, net force F should be 0 or $\sum F = 0$ ....(4.7)

This is the mathematical form of the first condition of equilibrium which states that:

Fascinating Freefall


A group of paratroopers making in a formation-an example of dynamic equilibrium.

A body is said to be in translational equilibrium only if the vector sum of all the external forces acting on it is equal to zero.

In case a number of coplanar forces $\mathbf{F_1}$, $\mathbf{F_2}$, $\mathbf{F_3}$, ... having their resultant equal to $\mathbf{F}$, are acting on a body, these can be resolved into their rectangular components, and first condition of equilibrium can be then written as:

Along x-direction, $F_{1x} + F_{2x} + F_{3x} + ... = 0$

or $\sum F_x = 0$ ...... (4.8)

Similarly, along y-direction,

$F_{1y} + F_{2y} + F_{3y} + \ ...... = 0$

or $\sum F_y = 0$ ...... (4.9)

Thus, first condition of equilibrium can also be stated as:

The sum of all the components of forces along x-axis should be zero and the sum of all the components of forces along y-axis should also be zero.

Second Condition of Equilibrium

The second condition of equilibrium implies to the rotational equilibrium which means that the body should not rotate under the action of the forces.

Consider the example of a rigid body in Fig.4.20. Two forces $\mathbf{F_1}$ and $\mathbf{F_2}$ of equal magnitude are acting on it. In case (a), both the forces act along the same line of action.

(a) Fig. 4.20 (b)

In case (b), the lines of action of the two forces are different. Since magnitude of $\mathbf{F_1}$ and $\mathbf{F_2}$ are equal, so the resultant force is zero in both the cases. Thus, first condition of equilibrium is satisfied. But you can observe that in case (b), the forces are forming a couple which can apply torque to rotate the body about point O. Therefore, for a body to be completely in equilibrium, a second condition is also required. That is, no net torque should be acting. This is the second condition of equilibrium which can be stated as:

The vector sum of all the torques acting on a body about any point must be zero.

Mathematically, we can write: $\sum \tau = 0$ ......(4.10)

Hence, a body will be in complete equilibrium when,

$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \end{cases}$$

And $\sum \tau = 0$

Solving Problems by Applying Conditions of Equilibrium

The following steps will help to solve problems by applying conditions of equilibrium.

1. First of all, select the objects to which Eqs. (4.8) and (4.9) are to be applied. Each object should be treated separately.
2. Draw a diagram to show the objects and forces acting on them. Only the forces acting on the objects should be included. The forces which the objects exert on their environment should not be included.
3. Choose a set of x, y axes such that as many forces as possible lie directly along x-axis or y-axis, it will minimize the number of forces to be resolved into components.
4. Resolve all the forces which are not parallel to either of the axes, in their rectangular components.
5. Apply Eqs. (4.8) and (4.9) by putting $\sum F_x = 0$ and $\sum F_y = 0$ to get two equations.
6. If needed, apply Eq. (4.10) by putting $\sum \tau = 0$ to get another equation.
7. The equations can be solved simultaneous to find out desired unknown quantities.

Do You Know?

Most balancing toys have a low centre of mass.

Example 4.5

A picture is suspended by means of two vertical strings as shown in Fig 4.21. The weight of the picture is 5 N, and it is acting at its centre of gravity. Find the tension T_1 & T_2 in the two strings.

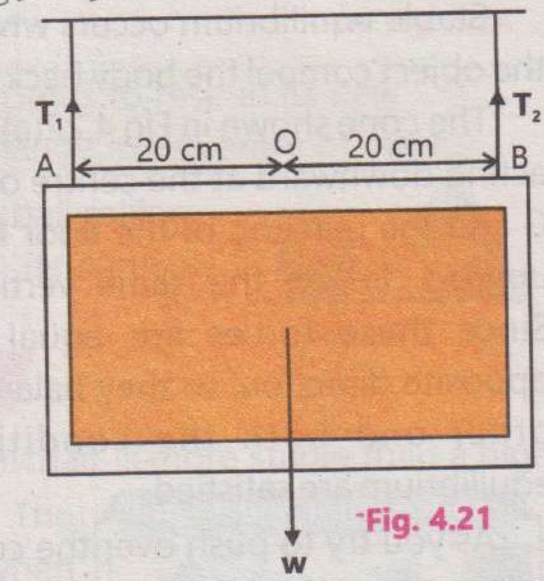


Fig. 4.21

Solution

Total upward force = $T_1 + T_2$

Total downward force = w = 5 N

Tensions in the strings, T_1 = ? and T_2 = ?

Since, there is no horizontal force, so $\sum F_x = 0$

Already $\quad \sum F_x = 0$

Putting $\quad \sum F_y = 0$

$$T_1 + T_2 - w = 0 \quad \text{.......................(i)}$$

Apply $\quad \sum \tau = 0$, selecting point B as point of rotation. Here, torque τ_1 of T_1 is negative whereas torque τ_2 of w is positive about point B. T_2 produces zero torque as it passes through the point of rotation. Hence,

$$\tau_2 - \tau_1 = 0$$

or $$w \times BO - T_1 \times AB = 0$$

putting the values, $$w \times 0.2\text{ m} - T_1 \times 0.4\text{ m} = 0$$

or $$5\text{ N} \times 0.2\text{ m} - T_1 \times 0.4\text{ m} = 0$$

or $$T_1 = \frac{5\text{ N} \times 0.2\text{ m}}{0.4\text{ m}} = 2.5\text{ N}$$

Putting the value of T_1 and w in Eq. (i), we have

$$2.5\text{ N} + T_2 - 5\text{ N} = 0$$

or $$T_2 = 2.5\text{ N}$$

4.10 States of Equilibrium

An object is balanced when its centre of mass and its point of support lie on the same vertical line. Then forces on each side are balanced, and the object is said to be in equilibrium. There are three states of equilibrium in connection with stability of the balanced bodies.

Stable Equilibrium

A body is said to be in a state of stable equilibrium, if after a slight tilt, it comes back to its original position.

Stable equilibrium occurs when the torques arising from the rotation (tilt) of the object compel the body back towards its equilibrium position.

The cone shown in Fig 4.22(a) is in the state of stable equilibrium. Its weight **w** acting downward at the centre of gravity G and the reaction of the floor $\mathbf{F_n}$ acting upward, lie on the same vertical line. Since these forces are equal and in opposite direction, so they balance each other and both the conditions of equilibrium are satisfied.

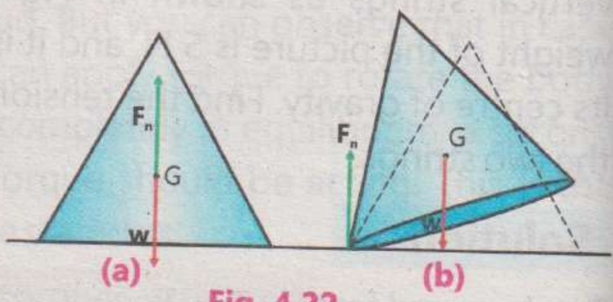


Fig. 4.22

As you try to push over the cone slightly, its centre of gravity is raised but it still remains above the base of the cone. The weight **w** and the normal force $\mathbf{F_n}$ do not remain in the same line but act like two unlike parallel forces. The cone does not remain in equilibrium. Unlike parallel forces produce a clockwise torque which brings the cone back to its original position. It is worth noting that the body remains in equilibrium as long its centre of mass lies within the base.

Unstable Equilibrium

Try to balance the cone on its tip. It is balanced for a moment because **w** and F_n lie along the same line. Even it is slightly tilted, it will not come back to its original position by itself. Rather it will fall downward, because its centre of mass no longer remains above the base. It topples over, because line of action of **w** no longer lies inside the base O (Fig. 4.23). In this case, centre of gravity is lowered on tilting and continues to fall further. It cannot rise up again because the anticlockwise torque produced by **w** moves it further downward.

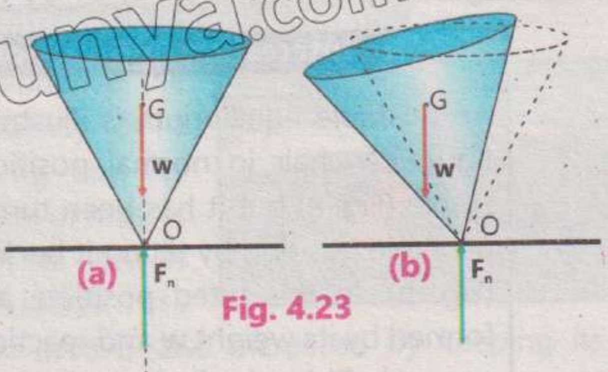


Fig. 4.23

A body is said to be in a state of unstable equilibrium, if after a slight tilt, it tends to move on further away from its original position.

Neutral Equilibrium

A cylinder resting on a horizontal surface (Fig. 4.24) shows the neutral equilibrium. If the cylinder is rotated slightly, there is no force or torque that brings it back to its original position or moves it away. As the cylinder rotates, the height of the centre of mass remains unchanged. In any position of the cylinder, its weight and reaction of the ground lie in the same vertical line.

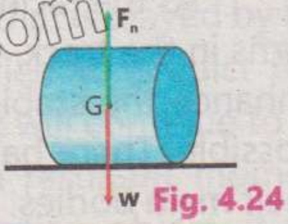


Fig. 4.24

A body is in neutral equilibrium, if it comes to rest in its new position after disturbance without any change in its centre of mass.

Other examples of neutral equilibrium are a ball rolling on a horizontal surfaces, or a cone resting on its curved surface (Fig.4.25).

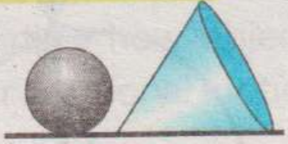


Fig. 4.25

4.11 Improvement of Stability

It is our daily life observation that a low armchair is more stable than a high chair because of its low centre of gravity. The position of centre of gravity is very important when we are talking about stability. A bus can be stable or unstable depending on how it is loaded. If the heavy loads are placed on the floor of the bus, its centre of gravity will be low. Now if it is disturbed slightly, a torque will bring it back to its original position.

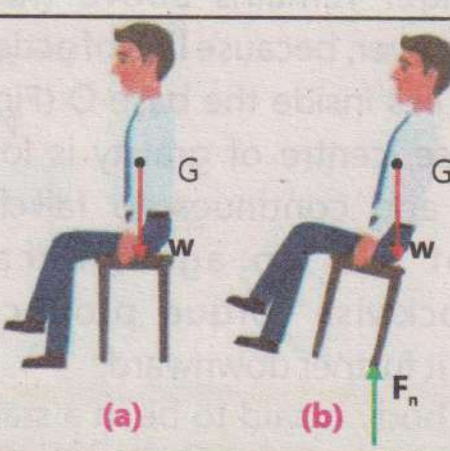
Interesting Information!


A double decker bus is being tilting to test its stability.

In this case, the bus is in stable equilibrium. If the same bus is loaded with steel sheets on the top, the centre of gravity be raised. It is now near to a state of unstable equilibrium. A couple will turn it over if it is slightly tilted. The same is the case of ships and boats. We can improve the stability of a system either by lowering the centre of gravity or by widening the base.

Interesting Information!

An unstable equilibrium is illustrated in this figure. A chair in normal position is quite stable (Fig. a) but it has been turned into an unstable position by tilting it back on its legs (Fig. b). In this tilted position, a couple is formed by its weight **w** and reaction $\mathbf{F_n}$ of the ground. This clockwise couple tends to overturn chair backward.



4.12 Application of Stability in Real Life

The concept of stability is widely applied to engineering technology especially in manufacturing racing cars and balancing toys.

As the racing cars are driven at very high speeds and also there are sharp turns in the track, therefore, the chances of the cars to topple over increase. To enhance the stability of racing cars, their centres of mass are kept as low as possible. There base areas are also increased by keeping the wheel outside of their main bodies. Balancing toys are also very interesting for both children and elders. Look at some balancing toys shown belows.

Fig. 4.26: Balancing toys

The physics behind these types of toys is that stability is built in with balancing toys. These toys are basically in completely stable state and their centres of gravity always remain below the pivot point. If the toys are disturbed in any direction, the centre of gravity is raised and it becomes unstable for a moment. It comes back to its initial stable position by lowering its centre of gravity.

The kids learn from these toys about stable systems and how they return to their state of initial rest position after being disturbed. Educational games on the basis of balancing toys have also been developed for the kids as shown in Fig 4.27.

Fig. 4.27

Interesting Information!

To enhance the stability of a racing car, its centre of mass is kept as low as possible. Its base area is also increased by keeping its wheels outside of its main body.

Rotational Motion Versus Translatinal Motion

Counterparts of velocity, acceleration, force and momentum in translational motion are angular velocity, angular acceleration, moment of force (torque) and angular momentum respectively in rotational motion. It suggests that the torque plays the same role in the rotational motion that is played by the force in the translational motion. Therefore, we are justified to predict that analogous to Newton's first law of motion, a rotating object will continue to do so with constant angular velocity unless acted upon by a resultant moment (torque). However, if a resultant torque is applied to rotating object, it will accelerate depending on the direction of the torque relative to the axis of rotation.

This fundamental principle enhances our understanding how objects move and interact with their environment whether in linear or rotational motion scenarios.

Motion in a Circle

When a body is moving along a circular path, its velocity at any point is directed along the tangent drawn at that point. Figure 4.28 shows that the direction of tangent at each point on a circle is different, therefore, the velocity of an object moving with uniform speed in a circle is changing constantly. Hence, a force perpendicular to the direction of motion is always required to keep the object moving with uniform speed in a circular path.

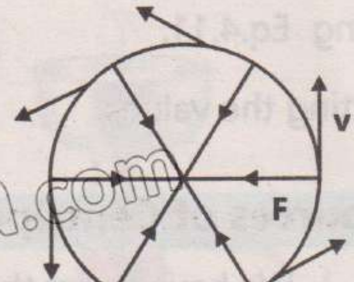


Fig. 4.28

It should be noted that **F** is essentially perpendicular to **v**. For an instance, if it is not perpendicular to **v**, the force **F** will have a component in the direction of **v**. This will change the magnitude of velocity. As the body moves with constant speed, so it is possible only if the component of force along **v** is $F\cos 90° = 0$.

4.13 Centripetal Force

We have studied above that an object can move in a circular path with uniform speed only if a force perpendicular to its velocity is acting constantly on it. This force is always directed towards the centre of the circle. It is called centripetal force and can be defined as:

> The force that causes an object to move in a circle at constant speed is called the centripetal force.

For an object of mass m moving with uniform speed v in circle of radius r, the magnitude of centripetal force F_c acting on it can be calculated by using the relation:

$$F_c = \frac{mv^2}{r} \quad \text{..........(4.11)}$$

Example 4.5

A 150 g stone attached to a string is whirled in a horizontal circle at a constant speed of 8 m s^{-1}. The length of string is 1.2 m. Calculate the centripetal force acting on the stone. Neglect effects of gravity.

Solution

Mass of stone $= m = 150\text{ g} = 0.15\text{ kg}$

Speed of stone $= v = 8\text{ m s}^{-1}$

Radius of circle $= r = 1.2\text{ m}$

Centripetal force $= F_c = ?$

Using Eq.4.11, $F_c = \frac{mv^2}{r}$

Putting the values, $F_c = \frac{0.15\text{ kg} \times (8\text{ m s}^{-1})^2}{1.2\text{ m}} = 8\text{ N}$

Sources of Centripetal Force

We have learnt that centripetal force has to be supplied if the body is to be maintained in its circular path. What could be the sources of centripetal force?

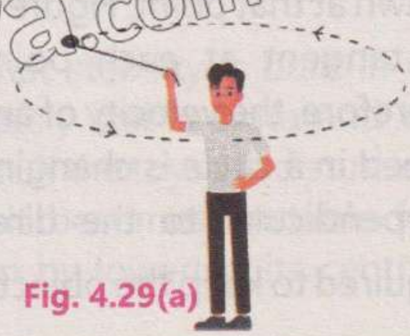


Fig. 4.29(a)
A stone whirled in a circle by a string

If we tie a stone to one end of a string and whirl it from the other end, we will have to exert a force on the stone through the string (Fig 4.29-a). If we release the string when it is at any point P, the stone will fly off along the tangent (PQ) to the circle. Then, it will move along the same straight line with constant velocity unless an unbalanced force acts upon it.

In fact, the tension **T** in the string was providing the stone the necessary centripetal force to keep it along the circular path (Fig 4.29-b). When we release the string we stop applying force on the stone and hence it moves in a straight line.

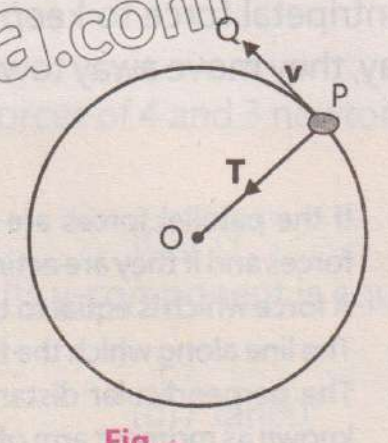


Fig. 4

Now consider the case of the moon which moves around the Earth at constant speed. The gravity of the Earth provides the necessary centripetal force to keep it in its orbit. Same is the case of satellites orbiting the Earth in circular paths with uniform speed. The gravitational pull of the Earth provides centripetal force.


Fig. 4.30
A satellite orbiting the Earth

One of the real life examples is a washing machine dryer. A dryer is a metallic cylinderical drum with many small holes in its walls. Wet clothes are put in it. When the cylinder rotates rapidly, friction between clothes and drum walls provides necessary centripetal force. As the water molecules are free to move, so they cannot get the required centripetal force to move in circular paths and escape from the drum through the holes. This results into quick drying of clothes.

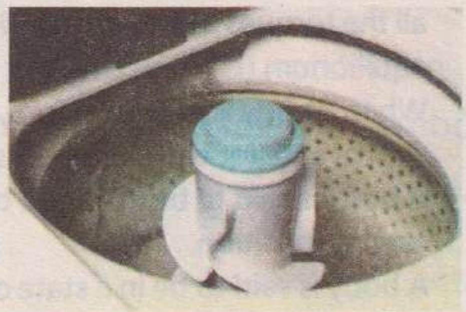


Fig. 4.31
Washing machine

Another interesting example is that of a cream separator. In a cream separator, milk is whirled rapidly.

Cream seperator
Fig. 4.32

The lighter particles of cream experience less centripetal force and gather in the central part of the machine. The heavier particles of milk need greater centripetal force to keep their circular motion in circles of small radius r. In this way, they move away towards the walls.

KEY POINTS

- If the parallel forces are acting in the same direction, then they are called like parallel forces and if they are acting in opposite directions, they are called unlike parallel forces.
- A force which is equal to the sum of all the forces is known as resultant force.
- The line along which the force acts is called the line of action of the force.
- The perpendicular distance of the line of action of a force from the axis of rotation is known as moment arm of the force.
- The torque or moment of a force is defined as the product of the force and the moment arm.
- When two equal and opposite, parallel forces act at two different points of the same body, they form a couple.
- The centre of gravity is a point inside or outside the body at which the whole weight of the body is acting.
- The centre of mass of a body is that point where the whole mass of the body is assumed to be concentrated.
- A body is said to be in equilibrium if it has no acceleration.
- A body will be in translational equilibrium only if the vector sum of all the external forces acting on it is equal to zero. This is called first condition of equilibrium. The vector sum of all the torques acting on a body about any axis should be zero. This is second condition of equilibrium.
- When a body is in equilibrium, the sum of clockwise moments about any point equals the sum of anticlockwise moments about that point.
- A body is said to be in a state of stable equilibrium, if after a slight tilt, it comes back to its original position.
- A body is said to be in a state of unstable equilibrium, if after a slight tilt, it tends to move on further away from its original position.
- A body is in neutral equilibrium, if it comes to rest in its new position after disturbance without any change in its centre of mass.
- Analogous to Newton's first law of motion in a straight line, a rotating object will continue to do so with constant angular velocity unless acted upon by a resultant moment of force.
- The force that causes an object to move in a circle at constant speed is called the centripetal force.

EXERCISE

A Multiple Choice Questions

Tick (✓) the correct answer.

4.1. A particle is simultaneously acted upon by two forces of 4 and 3 newtons. The net force on the particle is:

(a) 1 N (b) between 1 N and 7 N (c) 5 N (d) 7 N

4.2. A force **F** is making an angle of 60° with x-axis. Its y-component is equal to:

(a) F (b) $F\sin 60°$ (c) $F\cos 60°$ (d) $F\tan 60°$

4.3. Moment of force is called:

(a) moment arm (b) couple (c) couple arm (d) torque

4.4. If $\mathbf{F}_1$ and $\mathbf{F}_2$ are the forces acting on a body and τ is the torque produced in it, the body will be completely in equilibrium, when:

(a) $\sum \mathbf{F} = 0$ and $\sum \tau = 0$ (b) $\sum \mathbf{F} = 0$ and $\sum \tau \neq 0$

(c) $\sum \mathbf{F} \neq 0$ and $\sum \tau = 0$ (d) $\sum \mathbf{F} \neq 0$ and $\sum \tau \neq 0$

4.5. A Shopkeeper sells his articles by a balance having unequal arms of the pans. If he puts the weights in the pan having shorter arm, then the customer:

(a) loses (b) gains (c) neither loses nor gains (d) not certain

4.6. A man walks on a tight rope. He balances himself by holding a bamboo stick horizontally. It is an application of:

(a) law of conservation of momentum

(b) Newton's second law of motion

(c) principle of moments

(d) Newton's third law of motion

4.7. In stable equilibrium, the centre of gravity of the body lies:

(a) at the highest position (b) at the lowest position

(c) at any position (d) outside the body

4.8. The centre of mass of a body:

(a) lies always inside the body

(b) lies always outside the body

(c) lies always on the surface of the body

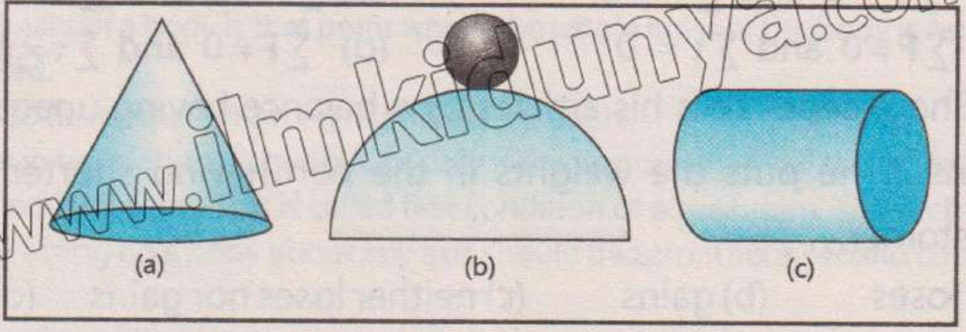
(d) may lie within , outside or on the surface

4.9. A cylinder resting on its circular base is in:
(a) stable equilibrium (b) unstable equilibrium
(c) neutral equilibrium (d) none of these

4.10. Centripetal force is given by:
(a) rF (b) $rF\cos\theta$ (c) $\frac{mv^2}{r}$ (d) $\frac{mv}{r^2}$

B Short Answer Questions

4.1. Define like and unlike parallel forces.
4.2. What are rectangular components of a vector and their values?
4.3. What is the line of action of a force?
4.4. Define moment of a force. Prove that $\tau = rF\sin\theta$, where θ is angle between **r** and **F**.
4.5. With the help of a diagram, show that the resultant force is zero but the resultant torque is not zero.
4.6. Identify the state of equilibrium in each case in the figure given below.



4.7. Give an example of the body which is moving yet in equilibrium.
4.8. Define centre of mass and centre of gravity of a body.
4.9. What are two basic principles of stability in physics which are applied in designing balancing toys and racing cars?
4.10. How can you prove that the centripetal force always acts perpendicular to velocity?

C Constructed Response Questions

4.1. A car travels at the same speed around two curves with different radii. For which radius the car experiences more centripetal force? Prove your answer.
4.2. A ripe mango does not normally fall from the tree. But when the branch of the tree is shaken, the mango falls down easily. Can you tell the reason?
4.3. Discuss the concepts of stability and centre of gravity in relation to objects toppling over. Provide an example where an object's centre of gravity

affects its stability, and explain how altering its base of support can influence stability.

4.4. Why an accelerated body cannot be considered in equilibrium?

4.5. Two boxes of the same weight but different heights are lying on the floor of a truck. If the truck makes a sudden stop, which box is more likely to tumble over? Why?

D Comprehensive Questions

4.1. Explain the principle of moments with an example.

4.2. Describe how could you determine the centre of gravity of an irregular shaped lamina experimentally.

4.3. State and explain two conditions of equilibrium.

4.4. How the stability of an object can be improved? Give a few examples to support your answer.

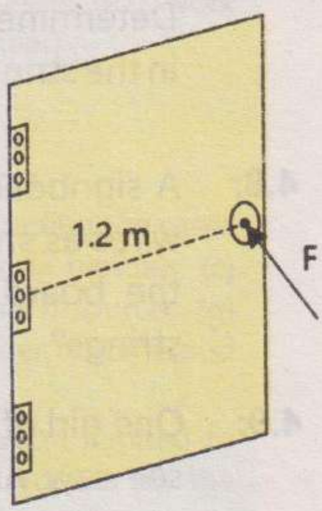
E Numerical Problems

4.1 A force of 200 N is acting on a cart at an angle of 30° with the horizontal direction. Find the x and y-components of the force.

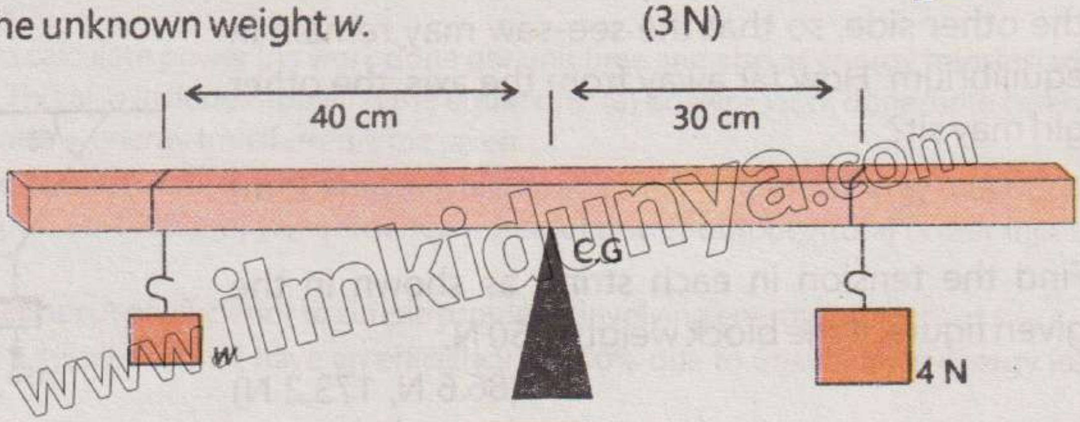
(173.2 N, 100 N)

4.2 A force of 300 N is applied perpendicularly at the knob of a door to open it as shown in the given figure. If the knob is 1.2 m away from the hinge, what is the torque applied? Is it positive or negative torque?

(360 N m, positive)

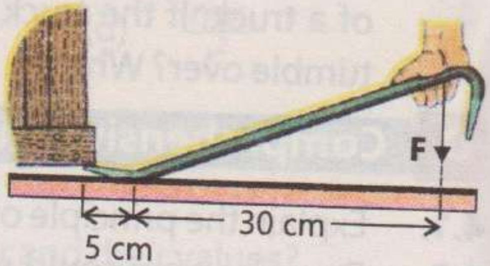


4.3 Two weights are hanging from a metre rule at the positions as shown in the given figure. If the metre rule is balanced at its centre of gravity (C. G), find the unknown weight *w*. (3 N)

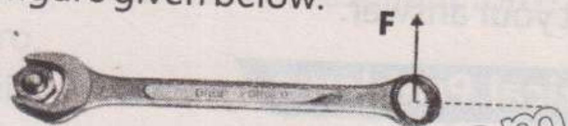


4.4 A see-saw is balanced with two children sitting near either end. Child A weighs 30 kg and sits 2 metres away from the pivot, while child B weighs 40 kg and sits 1.5 metres from the pivot. Calculate the total moment on each side and determine if the sea-saw is in equilibrium. (60 N)

4.5 A crowbar is used to lift a box as shown in the given figure. If the downward force of 250 N is applied at the end of the bar, how much weight does the other end bear? The crowbar itself has negligible weight. (1500 N)

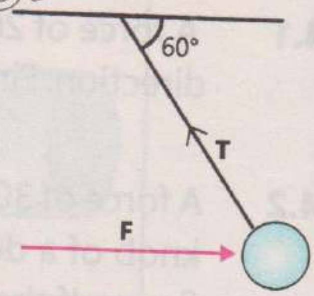


4.6: A 30 cm long spanner is used to open the nut of a car. If the torque required for it is 150 N m, how much force F should be applied on the spanner as shown in the figure given below.



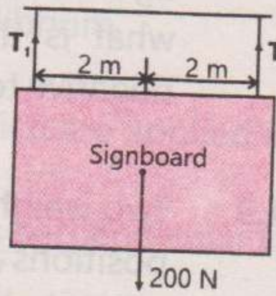
(500 N)

4.7: A 5 N ball hanging from a rope is pulled to the right by a horizontal force **F**. The rope makes an angle of 60° with the ceiling, as shown in the given figure. Determine the magnitude of force **F** and tension **T** in the string.



(2.9 N, 5.8 N)

4.8: A signboard is suspended by means of two steel wires as shown in the given figure. If the weight of the board is 200 N, what is the tension in the strings? (100 N, 100 N)



4.9: One girl of 30 kg mass sits 1.6 m from the axis of a see-saw. Another girl of mass 40 kg wants to sit on the other side, so that the see-saw may remain in equilibrium. How far away from the axis, the other girl may sit?

(1.2 m)

4.10: Find the tension in each string as shown in the given figure, if the block weighs 150 N.

(86.6 N, 173.2 N)