Unit -1 PERCENTAGE, RATIO AND PROPORTION

PERCENTAGE

The word "percent" is a short form of the Latin word "percentum". Percent means out of hundred or per hundred. The symbol for percentage is "%".

71% of the Earth's surface is water.
Percentage of land is 29%.

Ratio

Ratio is a comparison of like quantities measured in like units. The symbol for ratio is ":"

ANTECEDENT AND CONSEQUENT

In a ratio $a : b$, 'a' is called the antecedent and 'b' is called the consequent, e.g. in ratio $2 : 5$, antecedent is 2 and consequent is 5.

THE ORDER OF RATIO

If the magnitudes of the two quantities are denoted by 'a' and 'b' then ratio from 'a' to 'b' is $a : b$.

We cannot write this ratio as $b : a$, because, $a : b \neq b : a$ since $\frac{a}{b} \neq \frac{b}{a}$.

PROPORTION

The equality of two ratios is known as proportion. The symbol for proportion is "::" or "=".

EXTREMES AND MEANS

If $a : b = c : d$, then the proportion is $a : b :: c : d$. We read it as ratio a is to b is as ratio c is to d. a, b, c and d are called the terms of the proportion.

The first and fourth terms, i.e. "a" and "d" are called the extremes, while the second and third terms "b" and "c" are called the means of the proportion.

The product of means is equal to the product of extremes, i.e.

$$ad = bc$$

DIRECT PROPORTION

The relationship between two ratios in which increase in one quantity causes a proportional increase in the other quantity or decrease in one quantity causes a decrease in the other quantity is called "direct proportion".

INVERSE PROPORTION

The relationship between two ratios in which increase in one quantity causes a proportional decrease in the other quantity or a decrease in the one quantity cause a proportional increase in the other quantity is an inverse proportion.

COMPOUND PROPORTION

The relationship between two or more proportions is known as compound proportion.

Exercise 1.1

1. ***Express the following percentages as fractions in their lowest form.***

(i) 95% (ii) 65% (iii) 75% (iv) 25% (v) 56% (vi) 48%

(vii) 8% (viii) $33\frac{1}{2}\%$ (ix) $37\frac{1}{2}\%$ (x) $87\frac{1}{2}\%$

(xi) $5\frac{1}{4}\%$ (xii) $42\frac{1}{2}\%$

Solutions:

(i) $95\% = \frac{95}{100} = \frac{19}{20}$

(ii) $65\% = \frac{65}{100} = \frac{13}{20}$

(iii) $75\% = \frac{75}{100} = \frac{3}{4}$

(iv) $25\% = \frac{25}{100} = \frac{1}{4}$

(v) $56\% = \frac{56}{100} = \frac{14}{25}$

(vi) $48\% = \frac{48}{100} = \frac{12}{25}$

(vii) $8\% = \frac{8}{100} = \frac{2}{25}$

(viii) $33\frac{1}{2}\% = \frac{67}{2}\% = \frac{67}{2\times100} = \frac{67}{200}$

(ix) $37\frac{1}{2}\% = \frac{75}{2}\%$

$= \frac{75}{2\times100}$

$= \frac{3}{8}$

(x) $87\frac{1}{2}\% = \frac{175}{2}\%$

$= \frac{175}{2\times100}$

$= \frac{7}{8}$

(xi) $5\frac{1}{4}\% = \frac{21}{4}\%$

$= \frac{21}{4\times100}$

$= \frac{21}{400}$

(xii) $42\frac{1}{2}\% = \frac{85}{2}\%$

$= \frac{85}{2\times100}$

$= \frac{17}{40}$

2. ***Express the following fractions as percentages, giving your answer correct to 1 decimal place, where necessary.***

(i) $\frac{3}{4}$ (ii) $\frac{3}{5}$ (iii) $\frac{4}{25}$ (iv) $\frac{13}{20}$

(v) $\frac{31}{25}$ (vi) $\frac{21}{40}$ (vii) $\frac{23}{60}$ (viii) $\frac{8}{3}$

(ix) $\frac{8}{5}$ (x) $\frac{7}{8}$ (xi) $\frac{5}{8}$ (xii) $\frac{3}{8}$

Solutions:

(i) $\frac{3}{4} = \frac{3}{4}\times\frac{100}{100}$

$= \frac{3}{4}\times100\%$

$=3\times25\%$

$=75\%$

(ii) $\frac{3}{5} = \frac{3}{5} \times \frac{100}{100}$

$= \frac{3}{5} \times 100\%$

$= 3 \times 20\%$

$= 60\%$

(iii) $\frac{4}{25} = \frac{4}{25} \times \frac{100}{100}$

$= \frac{4}{25} \times 100\%$

$= 16\%$

(iv) $\frac{13}{20} = \frac{13}{20} \times \frac{100}{100}$

$= \frac{13}{20} \times 100\%$

$= 65\%$

(v) $\frac{31}{25} = \frac{31}{25} \times \frac{100}{100}$

$= \frac{31}{25} \times 100\%$

$= 124\%$

(vi) $\frac{21}{40} = \frac{21}{40} \times \frac{100}{100}$

$= \frac{21}{40} \times 100\%$

$= 52.5\%$

(vii) $\frac{23}{60} = \frac{23}{60} \times \frac{100}{100}$

$= \frac{23}{60} \times 100\%$

$= \frac{23}{3} \times 5\%$

$= \frac{115}{3}\%$

$=38.33\%$

$=38.30\%$

(viii) $\frac{8}{3} = \frac{8}{3} \times \frac{100}{100}$

$= \frac{8}{3} \times 100\%$

$= \frac{800}{3}\%$

$=266.666\%$

$=266.67\%$

(ix) $\frac{8}{5} = \frac{8}{5} \times \frac{100}{100}$

$= \frac{8}{5} \times 100\%$

$=160\%$

(x) $\frac{7}{8} = \frac{7}{8} \times \frac{100}{100}$

$= \frac{7}{8} \times 100\%$

$= \frac{700}{8}\%$

$=87.5\%$

(xi) $\frac{5}{8} = \frac{5}{8} \times \frac{100}{100}$

$= \frac{5}{8} \times 100\%$

$= \frac{125}{2}\%$

$=62.5\%$

(xii) $\frac{3}{8} = \frac{3}{8} \times \frac{100}{100}$

$= \frac{3}{8} \times 100\%$

$= \frac{75}{2}\%$

$=37.5\%$

3. Express the following percentages as decimals, giving your answer correct to 3 places of decimal.

(i) 47% (ii) 58% (iii) 92% (iv) 8%

(v) 12% (vi) 120% (vii) 180% (viii) 145%

(ix) $5\frac{1}{2}\%$ (x) $5\frac{1}{3}\%$ (xi) $48\frac{2}{3}\%$ (xii) $58\frac{1}{3}\%$

Solutions:

(i) $47\% = \frac{47}{100} = .47$

(ii) $58\% = \frac{58}{100} = .58$

(iii) $92\% = \frac{92}{100} = 0.92$

(iv) $8\% = \frac{8}{100} = 0.08$

(v) $12\% = \frac{12}{100} = 0.12$

(vi) $180\% = \frac{120}{100} = 1.8$

(vii) $120\% = \frac{180}{100} = 1.2$

(viii) $145\% = \frac{145}{100} = 1.45$

(ix) $5\frac{1}{2}\% = \frac{11}{2}\% = 5.5\% = \frac{5.5}{100} = 0.055$

(x) $5\frac{1}{3}\% = \frac{16}{3}\% = 5.33\% = \frac{5.33}{100} = 0.0533$

(xi) $48\frac{2}{3}\% = \frac{146}{3}\% = 48.66\% = \frac{48.7}{100} = 0.487$

(xii) $58\frac{1}{3}\% = \frac{175}{3}\% = 58.3\% = \frac{58.3}{100} = 0.583$

4. Express the following decimals as percentages.

(i)	0.5	(ii)	0.9	(iii)	1.25	(iv)	1.39
(v)	1.72	(vi)	0.22	(vii)	2.64	(viii)	3.41
(ix)	0.845	(x)	1.78	(xi)	1.58	(xii)	0.065

Solutions:

(i) $0.5 = 0.5 \times \frac{100}{100}$

$= 0.5 \times 100\%$

$= \frac{5}{10} \times 100\%$

$= 5 \times 10\%$

$= 50\%$

(ii) $0.9 = 0.9 \times \frac{100}{100}$

$= 0.9 \times 100\%$

$= \frac{9}{10} \times 100\%$

$= 9 \times 10\%$

$= 90\%$

(iii) $1.25 = 1.25 \times \frac{100}{100}$

$= 1.25 \times 100\%$

$= \frac{125}{100} \times 100\%$

$= 125\%$

(iv) $1.39 = 1.39 \times \frac{100}{100}$

$= 1.39 \times 100\%$

$= \frac{139}{100} \times 100\%$

$= 139\%$

(v) $1.72 = 1.72 \times \frac{100}{100}$

$= 1.72 \times 100\%$

$= \frac{172}{100} \times 100\%$

$= 172\%$

(vi) $0.22 = 0.22 \times \frac{100}{100}$

$= 0.22 \times 100\%$

$= \frac{22}{100} \times 100\%$

$= 22\%$

(vii) $2.64 = 2.64 \times \frac{100}{100}$

$= 2.64 \times 100\%$

$= \frac{264}{100} \times 100\%$

$= 264\%$

(viii) $3.41 = 3.41 \times \frac{100}{100}$

$= 3.41 \times 100\%$

$= \frac{341}{100} \times 100\%$

$= 341\%$

(ix) $0.845 = 0.845 \times \frac{100}{100}$

$= 0.845 \times 100\%$

$= 84.5\%$

(x) $1.78 = 1.78 \times \frac{100}{100}$

$= 1.78 \times 100\%$

$= \frac{178}{100} \times 100\%$

$= 178\%$

(xi) $1.58 = 1.58 \times \frac{100}{100}$

$= 1.58 \times 100\%$

$= 158\%$

(xii) $0.065 = 0.065 \times \frac{100}{100}$

$= 0.065 \times 100\%$

$= 6.5\%$

5. ***Complete the following table.***

Fraction	Percentage	Decimal
i. $\frac{3}{4}$	75%	0.75
ii. $\frac{4}{5}$	80%	0.80
iii. $\frac{2}{5}$	40%	0.4
iv. $\frac{31}{50}$	62%	0.62
v. $\frac{11}{25}$	44%	0.44

Exercise 1.2

1. ***If 45% of the students in a school are girls. What percentage are boys?***

Solution:

Number of Girls = 45%

Number of boys = (100 - 45)%

= 55%

2. ***If 82% of the house have a television, what percentage do not have?***

Solution:

Percentage of home which have Television = 82%

Percentage of home which have not television = (100 - 82)%

= 18%

3. ***A hockey team won 62% of their matches and 26% of them were ended in a draw. What percentage of the matches they lost?***

Solution:

won % matches of hockey team = 62%

percentage of matches which were ended = 26%

Percentage of both matches = (62+26)%

= 88%

Percentage of those matches which were lost = (100 - 88)%

= 12%

4. ***An aeroplane carries 400 passengers, 52% of the passengers were Pakistani, 17% were Chinese, 12% were from Iran and the rest were from British?***

(i) ***How many people each nationality were on the plane.***

(ii) ***What percentage were British?***

Solution:

Total number of passengers in plane = 400

Percentage of Pakistani passengers = 52

Numbers of Pakistani passengers = $\frac{52}{100} \times 400$

(i) = 208

Percentage of Chinese passengers = 17%

Thus

Number of Chinese passengers = $\frac{17}{100} \times 400$

= 68 (ii)

Percentage of Irani passengers = 12%

Number of Irani passerngers = $\frac{12}{100} \times 400$

= 48

Number of British passengers = 400 - 208 - 68 -48

= 76

Note:

Number of British passengers are subtracted from the total number of Pakistani, Chinese and Irani passengers.

5. ***Amna scored 46 out of 50 in a Maths test, 64 out of 75 in Chemistry test and 72 out of 80 in a Physics test. In which sybject did she perform best?***

Solution:

Out of 50 marks in maths = 46

Percentage scored in math = $\frac{46}{50} \times 100\%$

(i) = 92%

Out of 75 marks in Chemistry = 64

Percentage marks in Chemistry = $\frac{64}{75_3} \times \frac{100^4}{}$

$$= \frac{64 \times 4}{3}$$

$$= \frac{256}{3}$$

(ii) $= 85.3\%$

Out of 80 marks in Physics $= 72$

Percentage marks in Physics $= \frac{72}{80} \times 100\%$

(iii) $= 90\%$

Amna got the best marks in maths.

6. ***A table costs a carpenter Rs. 720 to make. He sells it for Rs. 920. What percentage of profit did he earn?***

Solution:

Total cost of a table = Rs. 720

Sale price of a table = Rs. 920

Percentage of Profit on Rs. 720 = (920 - 720)

= Rs. 200

$$\text{Percentage Profit} = \frac{200}{720} \times 100$$

$$= \frac{20000}{720}$$

$$= 27.78\%$$

7. ***If 8.4% of a book consists of 42 pages. Find total number of pages in the book.***

Solution:

Number of pages in 8.4% part of a book = 42

$$\text{Total number of pages in the book} = \frac{42}{84} \times 100 \times 10$$

$$= \frac{42000}{84}$$

= 500 Pages

8. ***Out of his total income Hamza spends 20% on house rent and 70% of the rest on household expenditure. If he saves Rs. 1800, What is his total income?***

Solution:

Amount spend on house rent = 20%

Rest value = (100 - 20)%

= 80%

Amount spend on house hold expenditure = $\frac{70}{100} \times 80$

Total value = 56%

Share of % income on house rent and house hold expenditure = (20 + 56)%

= 76%

= (100 - 76)%

Saving % of income = 24%

Total income if saving will be = Rs. 100

Total income if saving will be Rs.24 = $\frac{100}{24} \times 1800$

Total income if saving will be Rs. 1800 = Rs. 7500

9. ***Raheel's income is 25% more than that of Rauf. What percent is Rauf's income less than Raheel's?***

Solution:

Let Income of Rauf = Rs. 100

Raheel's income is more than 25% extra of Rauf's = Rs. 125

If Raheel's income Rs.125 than Rauf's income = Rs. 100

If Raheel's income Rs.100 less than Rauf's income = $\frac{100}{125} \times 100$

= Rs. 80

= (100 - 80)%

Less percentage of Rauf's income than Raheel's. = 20%

Exercise 1.3

1. ***Find the ratio of first quantity to the second in its lowest term.***

(i) Rs. 24, Rs.6 (ii) 20 Kg, 5 Kg (iii) 20cm, 80cm
(iv) 5m, 5m (v) 1500km, 1200km (vi) Rs.150,Rs.275

Solution:

Required Ratio = 24:6
= 4:1 (Dividing by 6)

(ii) 20kg, 5kg

Required Ratio = 20:5
= 4:1 (Dividing by 5)

(iii) 20cm, 80cm

Requierd Ratio = 20:80
= 1:4 (Dividing by 20)

(iv) 5m, 5m

Required Ratio = 5:5
= 1:1 (Dividing by 5)

(v) 1500km, 1200km

Required Ratio = 1500:1200
= 15:12 (Dividing by 100)
= 5:4 (Dividing by 3)

(vi) Rs. 150, Rs.275

Required Ratio = 150: 275
= 30:55 Dividing by 5
= 6:11 Dividing 5

2. ***Express each of the following ratios in its simplest form.***

(i) $\frac{2}{3}:\frac{3}{5}$ (ii) $\frac{4}{5}:\frac{3}{4}$ (iii) $\frac{5}{6}:\frac{7}{10}$

(iv) $\frac{13}{40}:\frac{3}{20}$ (v) $\frac{2}{3}:\frac{1}{6}$ (vi) $\frac{4}{10}:20$

(vii) $\frac{15}{10}:2$ (viii) $\frac{12}{10}:\frac{28}{10}$ (ix) $\frac{2}{5}:\frac{1}{3}$

Solutions:

(i) $\frac{2}{3}:\frac{3}{5}$

$\frac{2}{3}\div\frac{3}{5} = \frac{2}{3}\times\frac{5}{3}$

$= \frac{10}{9} = 10:9$

(ii) $\frac{4}{5}:\frac{3}{4}$

$\frac{4}{5}:\frac{3}{4} = \frac{4}{5}\div\frac{3}{4}$

$= \frac{4}{5}\times\frac{4}{3}$

$= \frac{16}{15}$

$= 16:15$

(iii) $\frac{5}{6}:\frac{7}{10}$

$\frac{5}{6}:\frac{7}{10} = \frac{5}{6}\times\frac{10}{7}$

$= \frac{5}{3}\times\frac{5}{7} = \frac{25}{21}$

$=25:21$

(iv) $\frac{13}{40}:\frac{3}{20}$

$\frac{13}{40}:\frac{3}{20} = \frac{13}{40}\div\frac{3}{20}$

$$= \frac{13}{40} \times \frac{20}{3}$$

$$= \frac{13}{6}$$

$$= 13:6$$

(v) $\frac{2}{3} : \frac{1}{6}$

$$\frac{2}{3} : \frac{1}{6} = \frac{2}{3} \div \frac{1}{6}$$

$$= \frac{2}{3} \times \frac{6}{1}$$

$$= \frac{2}{1} \times \frac{2}{1}$$

$$= \frac{4}{1}$$

$$= 4:1$$

(vi) $\frac{4}{10} : 20$

$$\frac{4}{10} : 20 = \frac{4}{10} \div 20$$

$$= \frac{4}{10} \times \frac{1}{20}$$

$$= \frac{1}{10} \times \frac{1}{5}$$

$$= \frac{1}{50}$$

$$= 1:50$$

(vii) $\frac{15}{10} : 2$

$$\frac{15}{10} : 2 = \frac{15}{10} \div 2$$

$$= \frac{15}{10} \times \frac{1}{2}$$

$$= \frac{15}{20}$$

$$= \frac{3}{4}$$

$$= 3:4$$

(viii) $\frac{12}{10} : \frac{28}{10}$

$$\frac{12}{10} : \frac{28}{10} = \frac{12}{10} \div \frac{28}{10}$$

$$= \frac{12}{10} \times \frac{10}{28}$$

$$= \frac{12}{28}$$

$$= \frac{3}{7}$$

$$= 3:7$$

(ix) $\frac{2}{5} : \frac{1}{3}$

$$\frac{2}{5} : \frac{1}{3} = \frac{2}{5} \div \frac{1}{3}$$

$$= \frac{2}{5} \times \frac{3}{1}$$

$$= \frac{6}{5}$$

$$= \mathbf{6:5}$$

3. ***In a city 126 medical students traveled by:***

Rikshaw	Taxi	Bus	Car
14	9	75	28

Find ratio of the students who used.

(i) Rikshaw to taxi

(ii) Taxi to bus

(iii) Taxi to car.

Solution:

Number of students who travelled by car = 28

Number of students who travelled by bus = 75

Number of students who travelled by Taxi = 9

Number of students who travelled by Rikshaw = 14

(i) Rikshaw to Taxi

14 : 9

= 14 : 9

(ii) by Taxi by bus

9 : 75

$9 \div 75$

$$= \frac{9}{75}$$

$$= \frac{3}{25}$$

$$= 3 : 25$$

(iii) By Taxi : By car

9 : 28

4. ***In a school library, there are 75 books on Mathematics, 115 on English, 85 on Chemistry and 60 on Physics. Find the ratio of the following:***

(i) Mathematics books to English books

(ii) English books to Chemistry books

(iii) English books to Physics books

(iv) Physics books to Chemistry books

(v) Physics books to Mathematics books

(vi) Chemistry books to Mathematics books

Solution:

Number of Math's books = 75

Number of English books = 115

Number of Chemistry books = 85

Number of Physics books = 60

No. of Math books : No. of English books

75 : 115

75 : 115

$$= \frac{75}{115}$$

or **15 : 23** $$= \frac{15}{23}$$

No. of English books : No. of Chem books

115 : 85

115 : 85

$$= \frac{115}{85}$$

or 23 : 17 $$= \frac{23}{17}$$

No. of English books : No. of Physics books

115 : 60

115 : 60

$$= \frac{115}{60}$$

or 23 : 12 $$= \frac{23}{12}$$

No. of Physics book : No. of Chemistry book

60 : 85

60 : 85

$$= \frac{60}{85}$$

or 12 : 17 $$= \frac{12}{17}$$

No. of Physics books : No. of Math

60 : books

60 : 75

$= \frac{75}{60}$

$= \frac{12}{15}$

or $4 : 5 = \frac{4}{5}$

No. of Chemistry book : No. of Math books

85 : 75

85 : 75

$= \frac{85}{75}$

or $17 : 15 = \frac{17}{15}$

Exercise 1.4

1. Find the ratio of 6 rupees each to 72 rupees per dozen.

Solution:

$$72 \text{ rupees Perdozen mean} = \frac{72}{12}$$

$$= 6$$

Therefore there is same ratio between each Rs. 6 and Rs. 7256 thats mean Rs. 6 and every Rs. 6.

$$6 : 6 = \frac{6}{6}$$

$$= \frac{1}{1}$$

$$= 1 : 1$$

2. Find the ratio of Rs. 160 per meter to Rs. 150 per meter.

Solution:

Rs.150 per meter : Rs.160 per meter

$$160 : 150$$

$$= \frac{160}{150}$$

$$\text{or} \quad 16 : 15 = \frac{16}{15}$$

3. Find the ratio of Rs. 72 for 24 to rupees 4 each.

Solution:

$$\text{Rs.72 for 24 mean} = \frac{72}{24}$$

For each one Rs. 3 = 3

Rs. 72 for 24 : For each 4

Thus 3 : 4

4. ***A square 'A' has side 2cm and a square 'B' has side 6 cm. Find the ratio of:***

(i) The length of the side of the square A to the length of the side of the square B.

(ii) The perimeter of the square A to the perimeter of the square 'B'.

(iii) The area of the square 'A' to the area of the square 'B'.

Solution:

Length of a square A side = 2 cm

Therefore Area of A side = 2×2

$= 4 \text{ cm}^2$

$= 4 \times 2$

Perimeter of square A = 8cm

Length of the square B side = 6cm

Area of B side = 6×6

$= 36 \text{ cm}^2$

Perimeter of square B = 4×6

$= 24$ cm

Length of square A side : Length of square B side

Now $2 : 6 = \frac{2}{6}$

$2 : 6 = \frac{1}{3}$

$1 : 3$

Perimeter of square A : Perimeter of square B

(ii) $8 : 24 = \frac{8}{24}$

Now $8 : 24$

$= \frac{1}{3}$

$= 1:3$

Area of square A : Area of square B

$$4 : 36 = \frac{4}{36}$$

$$4 : 36 = \frac{1}{9}$$

$$= 1 : 9$$

5. If a : b = 2 : 3, find the ratio 6a : 2b.

Solution:

$$a:b = 2:3$$

$$\text{i.e. } a = 2$$

$$b = 3$$

$$\text{then } 6a = 6\times2 = 12$$

$$\text{and } 2b = 2\times3 = 6$$

$$\text{Hence } 6a:2b = 12:6$$

$$= \frac{12}{6}$$

$$= \frac{2}{1}$$

$$= 2:1$$

6. A triangle has sides of lengths 3cm, 4cm, and 6cm. Find the ratio of the lengths of the sides to one another.

Solution:

First side of length = 3 cm

Second side of length = 4 cm

Third side of length = 6 cm

(i) First side of length : Second side of length

3 : 4

(ii) Second side of length : Third side of length

$$4 : 6$$

$$\text{and} \quad 4 : 6 = \frac{4}{6}$$

$$= \frac{2}{3}$$

$$= 2 : 3$$

(iii) First side of length : Third side of length

$$3 : 6$$

$$\text{and} \quad 3 : 6 = \frac{3}{6}$$

$$= \frac{1}{2}$$

$$= 1:2$$

7. ***Two angles in a triangle are 54° and 72°. Find the ratio of the third angle to the sum of the first two.***

Solution:

$$\text{First angle} = 54^\circ$$

$$\text{Second angle} = 72^\circ$$

Therefore $\quad \text{Sum of two angles} = 54 + 72$

$$= 126^\circ$$

$$\text{Amount of 3rd side} = 180^\circ - 126^\circ$$

$$= 54^\circ$$

Required Ratio

Sum of remaining amount of two angles : amount of 3rd angle

$$54 : 126$$

$$\text{and} \quad 54 : 126 = \frac{54}{126}$$

$$= \frac{18\times 3}{18\times 7}$$

$$= \frac{3}{7}$$

$$= 3 : 7$$

8. ***Ali's father earns a salary of Rs. 40,000 in a month, while his father's monthly expenditures are Rs. 35,000. Find the ratio of his father's:***

(i) Income to expenditure

(ii) Expenditure to savings

(iii) Income to saving

Solution:

Salary = Rs. 40,000

Expenditure = Rs. 35,000

Therefore, Saving = 40,000−35,000

= 5000 Rupees

Income Expensis required ratios

40,000 : 35,000

Now $40{,}000 : 35{,}000 = \frac{40{,}000}{35{,}000}$

$$= \frac{8}{7}$$

$$= 8 : 7$$

Expenditure : Saving

35,000 : 5000

and $35{,}000 : 5000 = \frac{35000}{5000}$

$$= \frac{7}{1}$$

$= 7:1$

Income : Saving

40,000 : 5000

and $40{,}000 : 5000 = \frac{40{,}000}{5{,}000}$

$= \frac{8}{1}$

$= 8 : 1$

9. ***A square A has side 6cm and square B has side 8cm. Find the ratio of:***

(i) The length of the side of a square A to the length of the side of the square B.

(ii) The area of square A to the area of square B.

Solution:

Length of square A side = 6 cm

Area of A square = 6×6

$= 36\text{cm}^2$

Length of square B side = 8cm

Area of B square = 8×8

$= 64\ \text{cm}^2$

(i) Required ratios

Length of square side A : Length of square side B

6 : 8

$6 : 8 = \frac{6}{8}$

and $= \frac{3}{4}$

Area of square A side : Area of square B side

$$36 : 64$$

$$\text{and } 36 : 64 = \frac{36}{64}$$

$$= \frac{9}{16}$$

$$= 9 : 16$$

10. A family has 12 pets c which 6 are cats, 2 are dogs and rest are birds. Find the ratio of hte number of:

(i) birds to dogs

(ii) birds to pets

Solution:

$$\text{Total birds} = 12$$

$$\text{Number of Parrots} = 6$$

$$\text{Number of Sparrows} = 2$$

$$\text{Number of rest birds} = 12 - 6 - 2$$

$$= 4$$

$$\text{Number of Pigeons} : \text{Number of Sparrows}$$

$$4 : 2$$

$$\text{and } 4 : 2 = \frac{4}{2}$$

$$= \frac{2}{1}$$

$$= 2 : 1$$

$$\text{Number of Pigeons} : \text{Number of total birds}$$

$$4 : 12$$

$$\text{and } 4 : 12 = \frac{4}{12}$$

$$= \frac{1}{3}$$

$$= 1 : 3$$

Exercise 1.5

1. Find the value of x in the proportion 20 : 50 :: 8 : x?

Solution:

Product of extremes = $20x$

Product of means = 50×8

Product of extremes = Product of means

Thus $20x = 50 \times 8$

Therefore, $$x = \frac{50 \times 8}{20}$$

$$= 20$$

2. The price of 15 suits is Rs. 6750. How many such suits can be purchased by an amount of Rs. 4050?

Solution:

No. of suits purchased by = 15

Rs. 6750

No. of suits purchased by = x

Rs. 4050

Suits	Price
15 ↑	6750 ↑
x ↑	4050 ↑

Therefore,

Direct Proportion $x : 15 :: 4050 : 6750$

Product of extremes = $6750x$

Product of means = 15×4050

Product of extremes = Product of means

$6750x = 15 \times 4050$

Therefore, $$x = \frac{15 \times 4050}{6750} = 9$$

Thus 9 suits can be purchased in Rs. 4050

3. ***A motorcycle covers 90km in 2 liters of petrol. In how many liters of petrol will it cover 225km?***

Solution:

Let 1st condition

Quantity of petrol = 2 liter

Distance covered = 90 km

Second condition

Distance covered = 225km

Let Quantity of petrol = x (l)

Now

	Petrol	distance covers
Direct Proportion	2 ↓	90 ↓
	x ↓	225 ↓

Therefore $x : 2 :: 225 : 90$

Product of extremes = $90x$

Product of means = 2×225

Product of extremes = Product of means

Thus $90x = 2 \times 225$

Therefore $$x = \frac{{}^{1}\cancel{2} \times \cancel{225}^{5}}{\cancel{90}_{45_1}}$$

$$= 5$$

Thus the motorcycle will cover 225 km distance in 5 liters.

4. ***A certain journey by train takes 5 hours at the speed of 45 km/h. What will be the speed of the train to complete the same journey in 3 hours?***

Solution:

1st condition

Train speed per hour = 45km

Time = 5 hours

2nd condition = 3 hours

Time

Train speed = x hours

Now time Train speed time Time

Inverse Proportion $\uparrow \begin{matrix} 45 & 5 \downarrow \\ x & 3 \downarrow \end{matrix}$

$x : 45 :: 5 : 3$

Product of extremes = $3x$

Product of mean = 45×5

Product of extremes = Product of mean

$$3x = 45 \times 5$$

$$x = \frac{45 \times 5}{3}$$

Hence $x = 75$

5. ***Six men can paint a house in four days. How long it would take to paint the house if three men are employed?***

Solution:

1st condition

Number of men = 6 men

2nd condition

Number of days for work = 4 days

Number of mens = 3 men

Let

Number of days for work = x days

Number of days Number of men

Inverse Proportion $\uparrow \begin{matrix} 4 & 6 \downarrow \\ x & 3 \downarrow \end{matrix}$

$x : 4 :: 6 : 3$

Product of extremes = $3x$

Product of means = 4×6

Product of extremes = Product of means

$$3x = 4 \times 6$$

Thus $$x = \frac{4 \times 6}{3}$$

8 days

6. ***A manager plans to produce 100 bicycles with the help of 25 persons working 4 hours daily. How many bicycles can be made by 40 persons if they work 3 hours daily?***

Solution:

1st Condition

Number of men = 25men

daily woking hours = 4 hours

Number of ready cycles = 100cycles

2nd Condition

Number of men = 40men

Daily working hours = 3 hours

Let number of ready cycle = x

Number of cycles	Number of men	Daily time
100	25	4
x	40	3

There is direct proportion between number of cycles and men.

And there is direct proportion between cycles and time

$$x : 100 = \begin{Bmatrix} 40 : 25 \\ 3 : 4 \end{Bmatrix}$$

here.

Product of extremes = $25 \times 4 \times x$

and Product of means = $100 \times 40 \times 3$

Product of extremes = Product of means

$$25 \times 4x = 100 \times 40 \times 3$$

Hence $$x = \frac{100 \times 40 \times 3}{25 \times 4}$$

$$= 120 \text{ cycles}$$

7. ***A factory makes 560 fans in 7 days with the help of 20 machines. How many fans can be made in 12 days with the help of 18 machines?***

Solution:

1st condition

Number of machines = 20

Number of days = 7

Number of fans = 560

2nd condition

Number of machines = 18

Number of days = 12

Let Number of fans = x

No. of fans	No. of machines	No. of days
↑ 560	↑ 20	↑ 7
x	18	12

There is direct proportion between numbers of fans and machines.

$$x : 560 :: \begin{Bmatrix} 18 : 20 \\ 12 : 7 \end{Bmatrix}$$

Product of extremes = $20 \times 7 \times x$

Product of means = $560 \times 18 \times 12$

Thus $20 \times 7 \times x = 560 \times 18 \times 12$

$$x = \frac{560 \times 18 \times 12}{20 \times 7}$$

$= 864$ fans

8. ***A factory makes 600 soaps in 9 days with the help of 20 machines. How many soaps can be made in 12 days with the help of 18 machines?***

Solution:

1st condition

Number of machines = 20

Number of days = 9

Numbre of suits = 600

2nd condition

Number of machines = 18

Number of days = 12

Number of suits = x

No. of suits	No. of days	No. of machines
600 ↑	↑ 9	↑ 20
x ↑	↑ 12	↑ 18

$$x : 600 :: \left\{\begin{matrix} 12 : 9 \\ 18 : 20 \end{matrix}\right\}$$

There is direct proportion between number of suits and No. days.

Product of extremes = Product of means

$$9 \times 20 \times x = 600 \times 12 \times 18$$

Hence $$x = \frac{600 \times 12 \times 18}{9 \times 20}$$

= 720 Suits

9. ***If the stay of 12 men for 28 days in a hotel costs Rs. 6720. Find the cost for the stay of 8 men for 14 days in the hotel.***

Solution:

1st condition

Number of men = 12 men

Number of days = 28days

Cost = Rs. 6720

2nd condition

Number of men = 8 men

Number of days = 14days

Let cost = Rs. x

Cost	No. of men	No. of days
6720 ↑	↑ 12	↑ 28
x	8	14

There is also direct proportion between cost and men.

$$x : 6720 :: \left\{ \begin{matrix} 8 : 12 \\ 14 : 28 \end{matrix} \right\}$$

There is also direct proportion between cost and days.

Here Product of extremes = $12 \times 28 \times x$

Product of means = $6720 \times 8 \times 14$

and Product of extremes = Product of means

$12 \times 28x = 6720 \times 8 \times 14$

Hence $$x = \frac{6720 \times 8 \times 14}{12 \times 28}$$

= Rs. 2240

2nd condition

Quantity of hay = 770

Number of days = 28

Let Number of Cows = x

No. of Cows	No. of days	No. of hay
↑ 14	↓ 18	↑ 63
x	28	770

$$x : 14 :: \begin{Bmatrix} 18 : 28 \\ 770 : 63 \end{Bmatrix}$$

Remember That:

We have some quantity of hay.

Now this quantity is enough for 14 Cows for 18 days.

Now same quantity have to finish in 20 days, then number of Cows less than before. Thus there is direct proportion between Cows and days.

Now number of days are fixed. And 63 kg hay finished 14 Cows. For these fix days for 770 kg hay, number of Cows more than before. This is direct proportion.

Now Product of extremes = Product of means

$$28 \times 63x = 14 \times 18 \times 770$$

$$x = \frac{14 \times 18 \times 770}{28 \times 63}$$

$$= 110 \quad \text{(Cows)}$$

12. ***Juice manufacturer produces 3000 bottles in a day employing 15 workers working 8 hours. Find the***

10. If the stay of 14 men for 8 days in a hotel costs Rs. 22, 400. Find the cost for the stay of 7 men for 13 days.

Solution:

1st condition

Number of men = 14

Number of days = 8

Cost = 22400

2nd condition

Number of men = 7

Number of days = 13

Let Cost = x

There is also direct proportion between cost and days.

Cost No. of men No. of days

22400 ↑ 14 ↑ 8

7 13

There is ... ...en cost and men.

Similarly $x : 22400 :: \begin{Bmatrix} 7 : 14 \\ 13 : 8 \end{Bmatrix}$

Here Prodcut of extremes = $14 \times 8 \times x$

and Product of means = $22400 \times 7 \times 13$

Product of extremes = Product of means

$$14 \times 8\, x = 13 \times 7 \times 22400$$

Hence $$x = \frac{22400 \times 7 \times 13}{14 \times 8}$$

$$= \text{Rs. } 18200$$

11. 14 cows consume 63 kg of hay in 18 days. How many cows will eat 770 kg of hay in 28 days at same rate?

Solution:

1st condition

Number of Cows = 14 Cows

Quantity of Hay = 63 kg

Number of days = 18 days

number of bottles manufactured when he employs 18 workers working 6 hours.

Solution:

1st condition

Number of employs = 15

Daily time duration = 8 hours

Number of produced bottles = 3000

2nd condition

Number of employs = 18

daily time duration = 6 hours

Let

Number of produced bottles = Rs. x

No. of bottles	Daily time duration	No. of employs
3000 ↑	↑ 8	↑ 15
x	6	18

$$x : 3000 :: \left\{ \begin{matrix} 6 : 8 \\ 18 : 15 \end{matrix} \right\}$$

Product of extremes = Product of means

$$8 \times 15x = 3000 \times 6 \times 18$$

$$x = \frac{3000 \times 6 \times 18}{8 \times 15}$$

$$= 2700 \quad \text{Bottles}$$

1. Payment of funeral expenses.
2. Payment of his/her debts.
3. Execution of his/her will.

Exercise 2.1

1. Calculate Zakat on gold amounting to Rs. 11,10,000.

Solution:

Amount of Gold = Rs. 11,10,000

Rate of Zakat = 2.5%

$$= \frac{25}{10}\%$$

$$= \frac{25}{10} \times \frac{1}{100}$$

$$= \frac{25}{1000}$$

$$\text{Amount of zakat} = 11,10,000 \times \frac{25}{1000}$$

$$= \text{Rs. } 27750$$

2. Calculate Zakat on silver amounting to Rs. 3,00,000.

Solution:

Worth of silver = 3.00,000

Rate of zakat = 2.5%

$$= \frac{25}{10}\%$$

$$= \frac{25}{10} \times \frac{1}{100}$$

$$= \frac{25}{1000}$$

$$\text{Amount of zakat} = 3,00,000 \times \frac{25}{1000}$$

$$= \text{Rs. } 7500$$

3. ***Calculate the amount of Zakat on 10 tola gold and 40 tola silver, if the rate of gold in Rs. 40,000 per tola and the rate of silver is Rs. 5000 per tola.***

Solution:

Weight of Gold = 10 tola

Rate of Gold per tola = Rs. 40,000

Rate of Gold 10 tola = 40,000 × 10

= Rs. 4,00,000 (i)

Weight of silver = 40 tola

Rate of silver per tola = Rs. 5,000

Rate of silver 40 tola = 5,000 × 40

= Rs. 2,00,000 (ii)

Total amount for Zakat = 4,00,000 + 2,00,000 (i) + (ii)

= 6,00,000

Now Rate of Zakat = 2.5%

$$= \frac{25}{10} \times \frac{1}{100}$$

$$= \frac{25}{1000}$$

$$\text{zakat on } 6,00,000 = \frac{25}{1000} \times 6,00,000$$

$$= \text{Rs. } 15,000$$

4. ***Calculate Zakat on gold of worth Rs. 8,00,000, cash of amount Rs. 4,00,000 and silver of weight 50 tola (Rs. 5000 per tola).***

Solution:

Worth of gold = Rs. 80,00,000 (i)

cash amount = Rs. 4,00,000 (ii)

weight of silver = 50 tola

Rate of silver per tola = Rs. 5,000

Rate of silver 50 tola = 5,000 × 50

= Rs. 2,50,000 (iii)

Total amount for zakat = 8,00,000+4,00,000+2,50,000

(i)+(ii)+(iii)

= Rs. 14,50,000

Now Rate of Zakat = 2.5%

$$= \frac{25}{10}\%$$

$$= \frac{25}{10} \times \frac{1}{100}$$

$$= \frac{25}{1000}$$

$$\text{Zakat on Rs. } 14,50,000 = \frac{25}{1000} \times 14,50,000$$

$$= \text{Rs. } 36,250$$

5. ***Calculate Ushr on a rice crop produced by natural resources amounting to Rs. 6,00,000.***

Solution:

Amount produced by natural resources = Rs. 6,00,000

Rate of Usher = 10%

$$= \frac{10}{100}$$

$$\text{Amount of Usher} = \frac{10}{100} \times 6,00,000$$

= Rs. 60,000

6. ***Calculate Ushr on a wheat crop amounting to Rs. 3,50,000 produced by artificial resources.***

Solution:

Amounting produceed by artificial resources = 3,50,000

Amount of Usher = 3,50,000

Here Rate of Usher = 5%

$$= \frac{5}{100}$$

Thus $$\text{Amount of Usher} = \frac{5}{100} \times 3,50,000$$

= Rs. 17500

7. ***Work out the share of each, if the inherited property amounting to Rs. 7,50,000 is left by a deceased, who also left a widow, two son's and one daughter.***

Solution:

Total amounting on property = Rs. 7,50,000

$$\text{Share of widow} = \frac{1}{8} \times 7,50,000$$

= Rs. 93,750

Remaining amounting = 7,50,000 - 93,750

= Rs. 6,56,250

Ratio between daughters and sons = 2 : 2 : 1

sum of the ratio = 2 + 2 + 1

= 5

$$\text{Share of sons} = \frac{2}{5} \times 6,56,250$$

= Rs. 2,62,500

$$\text{Share of daughter} = \frac{1}{5} \times 6,56,250$$

= Rs. 1,31,250

= Rs. 93,750

Share of each son = Rs. 2,62,500

Share of daughter = Rs. 1,31,250

8. ***An amount of Rs. 4,00,000 left as an inheritance is to be distributed among a widow and four daughters. Work out the share of each.***

Solution:

Left inheritance = Rs. 4,00,000

$$\text{Share of widow} = \frac{1}{8} \times 4,00,000$$

= Rs. 50,000

Remaining inherited amount = 4,00,000 - 50,000

= Rs. 3,50,000

Number of daughters = 4

$$\text{Share of each daughter} = \frac{1}{4} \times 3,50,000$$

$$= 87500$$

9. ***If a deceased left a property of worth Rs. 15,00,000 workout the property, if he left behind a widow.***

Solution:

$$\text{Property of worth} = 15,00,000$$

$$\text{share of widow} = \frac{1}{4} \times 15,00,000$$

$$= \text{Rs. } 375000$$

10. ***The inherited property amounting to Rs. 20,00,000 is left by a deceased. He left behind a widow and two son's. Workout the share of each.***

Solution:

$$\text{Inherited property} = \text{Rs. } 20,00,000$$

$$\text{share of widow} = \frac{1}{8} \times 20,00,000$$

$$= \text{Rs. } 2,50,000$$

$$\text{Remaining behind share} = 20,00,000 - 2,50,000$$

$$= \text{Rs. } 17,50,000$$

$$\text{Number of sons} = 2$$

$$\text{share of each son} = \frac{17,50,000}{2}$$

$$= \text{Rs. } 8,75,000$$

11. ***Asghar left a property of worth Rs. 4,80,000. He left behind a widow, three sons and four daughters. Calculate the share of each one.***

Solution:

$$\text{Property of worth} = \text{Rs. } 4,80,000$$

share of widow $= \frac{1}{8}(4,80,000)$

Remaining behind share = Rs. 60,000

= 4,80,000−60,000

= Rs. 4,20,000

Number of sons = 3

Number of daughters = 4

Ratio between sons and daughters share = 2 : 2 : 2 : 1 : 1 : 1 : 1

Sum of the ratio = 2 + 2 + 2 + 1 + 1+ 1 + 1

= 10

Share of each son $= \frac{2}{10} \times 420,000$

= Rs. 84,000

$= \frac{1}{10} \times 420,000$

share of each daughter = Rs. 42,000

12. Najeeb left a wealth amounting to Rs. 4,00,000. H left behind a widow, while they did not have an child. Find the share of Najeeb's widow.

Solution:

wealth amounting = Rs. 4,00,000

share of widow $= \frac{1}{4} \times 4,00,000$

= Rs. 1,00,000

Excercise 3.1

1. Find the SP, when

(i)	CP = Rs. 950	Profit	=	10%
(ii)	CP = Rs. 1540	Loss	=	5%
(iii)	CP = Rs. 9600	Profit	=	10%
(iv)	CP = Rs. 1,26,000	Loss	=	5%
(v)	CP = Rs. 480	Profit	=	3%
(vi)	CP = Rs. 760	Loss	=	4%

Solution 1(i)

Cost price = Rs. 950

Profit = 10%

Let Cost price = Rs. 100

Profit = 10%

Sale price = Cost price + Profit

Cost price = 100 + 10

Cost price = 110

If cost price is Rs.100 then S.P = Rs. 110

If C.P is Rs.1 then S.P = $\frac{110}{100}$

$= \text{Rs. } \frac{11}{10}$

Now If C.P is Rs.950 then S.P = $\frac{11}{10} \times 950$

= Rs. 1045

Solution: 1 (ii)

C.P = Rs. 1540

Loss = 5%

Loss = 5%

S.P = C.P - Loss

= 100 - 5

= Rs. 95

If C.P is Rs.100 then S.P is = Rs. 95

If C.P is Re.1 then S.P is = Rs. $\frac{95}{100} = \frac{19}{20}$

If C.P is Rs.1540 then S.P is = $\frac{19}{20} \times 1540$

= Rs. 1463

Solution: 1 (iii)

C.P = Rs. 9600

Profit = 10%

Let C.P = Rs. 100

Profit = 10%

S.P = Cost price + Profit

= 100 + 10

= Rs. 110

Now If C.P is Rs,100 then S.P = Rs. 110

If C.P is Re. 1 then S.P = $\frac{110}{100}$

= Rs. $\frac{11}{10}$

If C.P is Rs 9600 then S.P = $\frac{11}{10} \times 9600$

= Rs. 10560

Solution: 1 (iv)

C.P = Rs. 126000

Loss = 5%

Let C.P = Rs. 100

Loss = 5%

S.P = 100 - 5

= Rs. 95

Now

If C.P is Re. 1 then S.P = $\frac{95}{100}$

= Rs. $\frac{19}{20}$

If C.P is Rs.126000 then S.P = $\frac{19}{20} \times 126000$

= Rs. 119700

Solution: 1 (v)

C.P = Rs. 480

Profit = 3%

Let C.P = Rs. 100

Profit = 3%

S.P = 100 + 3

= Rs. 103

If C.P is Rs.100 then S.P = Rs. $\frac{103}{100}$

If C.P is Rs.480 then S.P = $\frac{103}{100} \times 480$

= Rs. 494

Solution: 1 (vi)

C.P = Rs. 760

Loss = 4%

Let C.P = Rs. 100

$$S.P = 100 - 4$$

$$= 96$$

Now

If C.P is Rs.100 then S.P = Rs. 96

If C.P is Re.1 then S.P = Rs. $\frac{96}{100}$

If C.P is Rs.760 then S.P = $\frac{96}{100} \times 760$

$$S.P = Rs.\ 729$$

2. ***Haris purchased a car for Rs. 248000 and spent Rs. 12000 on its denting and painting. He sold that at a profit of 5%. What did the customer pay to Haris?***

Solution:

C.P of a car = Rs. 2,48,000

Rs. spent on denting painting of a car = Rs. 12000

Total cost on car = 2,48,000+12000

= Rs. 2,60,000

Profit = 5%

Let C.P = Rs. 100

Profit = 5%

S.P = 100 + 5

If cost is 100 then S.P = Rs. 105

If cost is Re. 1 then S.P = $\frac{105}{100}$

= Rs. $\frac{21}{}$

If cost is Rs. 2,60,000 then S.P $= \frac{21}{20} \times 2,60,000$

$= \text{Rs. } 2,73,000$

Hence . S.P = Rs. 2,73,000

3. ***Find the CP, when***

(i)	S.P = Rs. 672	Profit	= 5%
(ii)	S.P = Rs. 851	Loss	= 8%
(iii)	S.P = Rs. 1755	Profit	$= 12\frac{1}{2}\%$
(iv)	S.P = Rs. 2640	Loss	= 12%
(v)	S.P = Rs. 100	Profit	$= 33\frac{1}{2}\%$

Solution: 3 (i)

S.P = Rs. 672

Profit = 5%

C.P = ?

Let C.P = Rs. 100

Profit = 5%

Therefore S.P = 100 + 5

Now = Rs. 105

If S.P is Rs.105 then C.P = Rs. 100

If S.P is Re.1 then C.P = $\text{Rs. } \frac{100}{105}$

If S.P is Rs.672 then C.P $= \frac{100}{105} \times 672$

C.P = Rs. 640

Solution: 3 (ii)

S.P = Rs. 851

Loss = 8%

C.P = ?

Let C.P = Rs. 100

Loss = 8%

S.P = 100 - 8

= Rs. 92

Now If S.P is Rs. 92 then C.P = Rs. 100

If S.P is Re.1 then C.P = Rs. $\frac{100}{92}$

If S.P is Rs.851 then C.P = $\frac{100}{92} \times 851$

Hence = Rs. 925

C.P = Rs. 925

3 (iii)

S.P = Rs. 1755

Profit = $12\frac{1}{2}\%$

C.P = ?

Let C. P = Rs. 100

Profit = Rs. $12\frac{1}{2}$

S.P = $100 + 12\frac{1}{2}$

= $112\frac{1}{2}$

= Rs. $\frac{225}{2}$

If S.P is Rs. $\frac{225}{2}$ then C.P = Rs. 100

If S.P is Re.1 then C.P = Rs. $\frac{100}{225} \times 2 = \frac{8}{9}$.

If S.P is Rs.1755 then C.P $= \frac{8}{9} \times 1755$

$= Rs.\ 1560$

C.P = Rs. 1560

Solution: 3 (iv)

S.P = Rs. 2640

Loss = 12%

C.P = ?

Let C.P = Rs. 100

Loss = 12%

Therefore S.P = 100 - 12

= Rs. 88

If S.P is Rs.88 then C.P = Rs. 100

If S.P is Re.1 then C.P = Rs. $\frac{100}{88}$

If S.P is Rs.2640 then C.P $= \frac{100}{88} \times 2640$

$= Rs.\ 3000$

Hence C.P = Rs. 3000

Solution: 3 (v)

S.P = Rs. 100

Profit $= 33\frac{1}{3}\%$

C.P = ?

Let C.P = Rs. 100

Profit $= 33\frac{1}{3}\%$

Therefore S.P $= 100 + 33\frac{1}{3}$

$$= 133\frac{1}{3}$$

$$= \text{Rs. } \frac{400}{3}$$

If S.P is Rs. $\frac{400}{3}$ then C.P = Rs. 100

If S.P is Re. 1 then C.P = $\frac{100}{400} \times 3$

$$= \text{Rs. } \frac{3}{4}$$

If S.P is Rs.100 then C.P = $\frac{3}{4} \times 100$

$$= \text{Rs. } 75$$

Hence C.P = Rs. 75

4. ***A shop-keeper gains a profit of 7% by selling a dinner set for Rs. 3852. If he sells it for Rs. 4050, find his profit percentage.***

Solution:

1st condition

S.P = Rs. 3852

Profit = 7%

C.P = ?

Let C.P = Rs. 100

Profit = 7%

Hence S.P = 100 + 7

= Rs. 107

If S.P is Rs.107 then C.P = Rs. 100

If S.P is Re.1 then C.P = $\text{Rs. } \frac{100}{107}$

$$\text{If S.P is Rs.3852 then C.P} = \frac{100}{107} \times 3852$$

$$\text{C.P} = \text{Rs. } 3600$$

2nd condition

$$\text{C.P is} = \text{Rs. } 3600$$

$$\text{S.P} = \text{Rs. } 4050$$

$$\text{Cost price of a profit on Rs.3600} = 4050 - 3600$$

$$= \text{Rs. } 450$$

$$\text{Profit on Re. } 1 = \text{Rs. } \frac{450}{3600}$$

$$\text{Profit on Rs. } 100 = \frac{450}{3600} \times 100$$

$$= \frac{25}{2}\%$$

$$\text{Profit \%} = 12\frac{1}{2}\%$$

5. The selling price of 12 articles is equal to the cost price of 15 articles. Find profit percentage.

Solution:

Let

$$\text{C.P of 15 articles} = \text{Rs. } 100$$

$$\text{S.P of 12 articles} = \text{Rs. } 100$$

Now we find the S.P of 15 articles.

$$\text{S.P of 12 articles} = \text{Rs. } 100$$

$$\text{S.P of 1 articles} = \text{Rs. } \frac{100}{12}$$

$$\text{S.P of 15 articles} = \frac{100}{12} \times 15$$

$$= \text{Rs. } 125$$

$$\text{Profit} = \text{S.P} - \text{C.P}$$

$= 125 - 100$

$= \text{Rs. } 25$

Therefore Profit % $= 25\%$

6. ***Find the cost price, if a fan is sold for Rs. 1470, to get a profit*** $\frac{1}{6}th$ ***of its cost price.***

Solution:

Let

$$C.P = \text{Rs. } 100$$

$$\text{Profit} = \frac{1}{6} \times 100$$

$$= \frac{100}{6}$$

Therefore $$S.P = 100 + \frac{100}{6}$$

$$= \text{Rs. } \frac{700}{6}$$

Now

If S.P is Rs. $\frac{700}{6}$ then C.P $= \text{Rs. } 100$

If S.P is Re. 1 then $C.P = \frac{100}{700} \times 6$

$$= \text{Rs. } \frac{6}{7}$$

If S.P is Rs. 1470 then $C.P = \frac{6}{7} \times 1470$

$$= 6 \times 210$$

C.P = Rs. 1260

7. ***A man sold an almirah at a profit of*** $7\frac{1}{2}\%$ ***, had he sold it for Rs. 209, he would have lost 2%. For how much the man purchased it?***

Solution:

$$\text{S.P} = \text{Rs. } 209$$
$$\text{Loss} = 2\%$$

Now Let S.P = Rs. 100

Therefore S.P = 100 - 2

= Rs. 98

If S.P is Rs. 98 then C.P = Rs. 100

If S.P is Re. 1 then C.P = Rs. $\frac{100}{98}$

If S.P is Rs. 209 then C.P = $\frac{100}{98} \times 209$

= Rs. 213.3

C.P = Rs. 213

8. ***Three chairs are purchased at Rs. 450 each. One of these is sold at a loss of 10%. At what price should the other two be sold so as to gain 20% on the whole transaction?***

Solution:

Cost price of 1 chair = Rs. 450

Cost price of 3 chairs = 450×3

1st condition = Rs. 1350

Total cost price = Rs. 1350

C.P of 1 chair = Rs. 450

Loss = 10%

Now S.P = 100-10=Rs. 90

If C.P is Rs.100 then S.P = Rs. 90

If C.P is Re.1 then S.P = $\frac{90}{100} = \frac{9}{10}$

If C.P is Rs.450 then S.P

$$= \frac{9}{\cancel{10}} \times \cancel{450}$$

$$= \text{Rs. } 405$$

2nd condition

S.P of 1st (one) chair = Rs. 405

C.P of three chairs = Rs. 1350

Let C.P = Rs. 100

Profit = 20%

S.P = 100 + 20

Now = Rs. 120

If C.P is Rs.100 then S.P = Rs. 120

If C.P is Re.1 then S.P = $\frac{120}{100}$

If C.P is Rs.1350 then S.P = $\frac{120}{100} \times 1350$

= Rs. 1620

Required S.P = Rs. 1620

Price of 1st chair = Rs. 405

Now required S.P = 1620 - 405

Required S.P = Rs. 1215

Exercise 3.2

1. ***Find the selling price, when,***

(i) MP = Rs. 728, Discount = 6%

(ii) MP = Rs. 2760, Discount = 5%

(iii) MP = Rs. 395.75, Discount = 8%

Solution: 1 (i)

MP = Rs. 728

Discount = 6%

Let M.P = Rs. 100

Discount = 6%

S.P = 100 - 6

= Rs. 94

Now if MP is Rs.100 then S.P = Rs. 94

If MP is Re.1 then S.P = Rs. $\frac{94}{100}$

If MP is Rs.728 then S.P = $\frac{94}{100} \times 728$

Hence S.P = Rs. 684.32

= Rs. 684.32

Solution: 1 (ii)

MP = Rs. 2760

Discount = 5%

Let M.P = Rs. 100

= 5%

Discount = 100 - 5

If MP is Rs.100 then S.P = Rs. 95

If MP is Re.1 then S.P = $\frac{95}{100}$

If MP is Rs.2760 then S.P = $\frac{95}{100} \times 2760$

= Rs. 2622

Hence S.P = Rs. 2622

Solution: 1 (iii)

M.P = Rs. 395.75

Discount = 8%

Let MP = Rs. 100

Discount = 8%

S.P = 100 - 8

If MP is Rs.100 then S.P = Rs. 92

If MP is Re.1 then S.P = Rs. $\frac{92}{100}$

If MP is Rs.395.75 then S.P = $\frac{92}{100} \times 395.75$

$= \frac{92}{100} \times \frac{39575}{100}$

$= \frac{3640900}{10000}$

Hence S.P = Rs. 364.09

= Rs. 364.09

2. Find the marked price when,

(i) SP = Rs. 515.20, Discount = 8%

(ii) SP = Rs. 858, Discount = 12%

(iii) SP = Rs. 2400, Discount = 4%

Solution: 2 (i)

S.P = Rs. 515.20

Discount = 8%

MP = ?

Let MP = Rs. 100

$$= \frac{8}{100} \times 100$$

$$= \text{Rs. } 8$$

Therefore $S.P = 100 - 8$

Now $= \text{Rs. } 92$

If S.P is Rs.92 then MP $= \text{Rs. } 100$

If S.P is Re.1 then MP $= \text{Rs. } \frac{100}{92}$

If S.P is Rs.515.20 then MP $= \frac{100}{92} \times 515.20$

$$= \frac{100}{92} \times \frac{51520}{100}$$

$$= \frac{51520}{92}$$

$$= \text{Rs. } 560$$

Hence $MP = \text{Rs. } 560$

Solution: 2 (ii)

$S.P = \text{Rs. } 858$

$\text{Discount} = 12\%$

$MP = ?$

Let $MP \text{ is} = \text{Rs. } 100$

$\text{Discount} = 12\%$

$$= \frac{12}{100} \times 100$$

$$= \text{Rs. } 12$$

$S.P = 100 - 12$

$= \text{Rs. } 88$

If S.P is Rs. 88 then MP $= \text{Rs. } 100$

If S.P is Re. 1 then MP $= \text{Rs. } \frac{100}{88}$

If S.P is Rs. 858 then MP $= \frac{100}{88} \times 858$

$= Rs. 975$

Hence MP is $= Rs. 975$

Solution: 2 (iii)

S.P $= Rs. 2400$

Discount $= 4\%$

MP $= ?$

Let MP is $= Rs. 100$

Discount $= 4\%$

$= \frac{4}{100} \times 100$

$= Rs. 4$

S.P $= 100 - 4$

$= Rs. 96$

If S.P is Rs.96 then MP $= Rs. 100$

If S.P is Rs.1 then MP $= Rs. \frac{100}{96}$

If S.P is Rs.2400 then MP $= \frac{100}{96} \times 2400$

$= Rs. 2500$

Hence MP $= Rs. 2500$

3. The marked price of a ceiling fan is Rs. 720. It is sold for Rs. 684. What percentage discount is being allowed?

Solution:

MP $= Rs. 720$

S.P $= Rs. 684$

Discount % $= ?$

Discount of Rs. 720 $= 720 - 684$

$= Rs.36$

$$\text{Discount of Re.1} = \text{Rs.}\ \frac{36}{720}$$

$$\text{Discount \%} = \frac{36}{720} \times 100$$

$$= 5$$

Hence $= 5\%$

4. ***The marked price of washing machine is Rs. 3640. During sale season it is sold for Rs. 3367. What percent sale discount is being given?***

Solution:

$$\text{MP} = \text{Rs. } 3640$$

$$\text{SP} = \text{Rs. } 3367$$

$$\text{Discount \%} = ?$$

$$\text{Discount of Rs. } 3640 = 3640 - 3367$$

$$= \text{Rs.}273$$

$$\text{Discount of Re.1} = \text{Rs.}\ \frac{273}{3640}$$

$$\text{Discount of Rs. } 100 = \frac{273}{3640} \times 100$$

$$= \frac{15}{2}$$

$$= 7.5$$

Hence Discount percent $= 7.5\%$

5. ***The marked price of a book is Rs. 480. The shopkeeper offers a discount of 10% and still gains 8%. Find the price at which the shopkeeper purchased it.***

Solution:

$$\text{MP} = \text{Rs. } 480$$

$$\text{Discount percent} = 10\%$$

Let Marked price = Rs. 100

Discount = 10%

$$= \frac{10}{100} \times 100$$

= Rs. 10

S.P = 100 - 10

= Rs. 90

Now

If MP is Rs.100 its S.P = Rs. 90

If MP is Re.1 its S.P = Rs. $\frac{90}{100}$

If MP is Rs.480 its S.P = $\frac{90}{100} \times 480$

= Rs. 432

S.P = Rs. 432

2nd condition

S.P = Rs. 432

Profit = 8%

C.P = ?

Let C.P = Rs. 100

Profit = 8%

Profit = $\frac{8}{100} \times 100$

= Rs. 8

S.P = 100 + 8

= Rs. 108

Now

If S.P is Rs.108 its C.P = Rs. 100

If S.P is Re.1 its C.P = Rs. $\frac{100}{108}$

If S.P is Rs.432 its C.P = $\frac{100}{108} \times 432$

$= Rs. 400$

Hence C.P of a book $= Rs. 400$

6. ***A trader marks his goods in such a way that after allowing a discount of 10%, he gains 15%. If an article costs him Rs. 720, what is its marked price?***

Let MP of goods = Rs. 100

Discount = 10%

S.P = M.P - Discount

= Rs. (100 - 10)

= Rs. 90

If S.P is Rs.90 its MP = Rs. 100

If S.P is Re.1 its MP = Rs. $\frac{100}{90}$

If S.P is Rs.720 its MP = $\frac{100}{\cancel{90}} \times \frac{8}{\cancel{720}}$

$= Rs. 800$

Profit percentage = $\frac{15}{100} \times 800$

$= 15 \times 8$

$= Rs. 120$

Marked price = Rs. 800+120=920

7. ***The list price of a T.V is Rs. 12600. A discount of 5% is allowed on it. Further for cash payment a second discount of 2% is given. How much cash payment is to be made for buying it?***

Solution:

1st condition

MP = Rs. 12600

Discount percent = 5%

S.P = ?

Let MP = Rs. 100

Discount percent = 5%

$$\text{Discount} = \frac{5}{100} \times 100$$

$$= \text{Rs. } 5$$

$$\text{S.P} = 100 - 5$$

$$= \text{Rs. } 95$$

Now

If M.P is Rs. 100 its S.P = Rs. 95

$$\text{If M.P is Re. 1 its S.P} = \text{Rs. } \frac{95}{100}$$

$$\text{If M.P is Rs. 12600 its S.P} = \frac{95}{100} \times 12600$$

$$\text{S.P} = \text{Rs. } 11970$$

Now in condition of cash payment S.P is Rs. 11970 and extra two percent discount is given. In this condition Rs.11970 is consider marked price, find sale price OR customer's cost price.

2nd Condition:

Sale price = 11970

Discount = 2%

S.P Or Costumer's C.P = ?

Let M.P is = Rs. 100

Discount = 2%

$$= \frac{2}{100} \times 100$$

$$= \text{Rs. } 2$$

(2nd condition) S.P = 100 - 2

$$= \text{Rs. } 98$$

If MP is Rs.100 its S.P = Rs. 98

$$\text{If MP is Rs.1 its S.P} = \text{Rs. } \frac{98}{100}$$

$$\text{If MP is Rs.11970 its S.P} = \frac{98}{100} \times 11970$$

$$= \text{Rs. } 11730.6$$

$$\text{Customer's requirement} = \text{Rs. } 11730.6$$

8. ***If 15% discount on MP of a heater is allowed and still makes a profit of 2%. If it is sold on MP, what is profit percentage?***

Solution:

$$\text{MP} = \text{Rs. } 100$$

$$\text{Discount} = 15\%$$

$$= \frac{15}{100} \times 100$$

$$= \text{Rs. } 15$$

$$\text{S.P} = 100 - 15$$

$$= \text{Rs. } 85$$

$$= 15 + 2$$

$$\text{Total profit on Rs.85} = 17\%$$

$$\text{Total profit on Rs.100} = \frac{17}{100} \times 100$$

$$= \text{Rs. } 17$$

$$\text{Total profit in S.P of Rs. } 100 = \frac{17}{85} \times 100$$

$$= \text{Rs. } 20$$

$$\text{Profit percentage} = 20\%$$

Exercise 3.3

1. ***Distribute Rs. 200,000 as a profit in a business regarding three persons, if their shares are in the ratio 3:2:5.***

Solution:

Total amount in Profit = 2,00,000

Ratio b/t business men = 3 : 2 : 5

sum of ratio = 3 + 2 + 5

= 10

$$\text{Profit of one person} = \frac{3}{10} \times 2,00,000$$

= Rs. 60,000 (i)

$$\text{Profit of 2nd person} = \frac{2}{10} \times 2,00,000$$

= Rs. 40,000 (ii)

$$\text{Profit of 3rd person} = \frac{5}{10} \times 2,00,000$$

= Rs. 10,00,000 (iii)

2. ***If Ali, Daniyal and Abdullah earned 15% profit against an investment of Rs. 750,000. Find the profit of each if their shares are in the ratio 2:3:5.***

Solution:

Amount in investment = Rs. 7,50,000

Profit = 15%

Now profit in investment of Rs. 100 = Rs. 15

$$\text{Profit in investment of Re.1} = \text{Rs. } \frac{15}{100}$$

Profit of investment on Rs.7,50,000 $= \frac{15}{100} \times 7,50,000$

$= Rs.\ 112500$

Ratio between Ali's Daniyal and Abdullah's share $= 2 : 3 : 5$

Sum of the ratio $= 2 + 3 + 5$

$= 10$

Ali's share in profit $= \frac{2}{10} \times 112500$

$= Rs.\ 22,500$

Daniyal's share in profit $= \frac{3}{10} \times 112500$

$= Rs.\ 33,750$

Abdullah's share in profit $= \frac{5}{10} \times 112500$

$= Rs.\ 56,250$

3. ***Distribute Rs. 720 as profit amongst three people, so that their shares are in the ratio 3:4:5.***

Solution:

Amount of profit $= Rs.\ 720$

sum of ratio $= 3 : 4 : 5$

$= 3 + 4 + 5$

$= 12$

1st person's share in profit $= \frac{3}{12} \times 720$

$= Rs.\ 180$ (i)

2nd person's share in profit $= \frac{4}{12} \times 720$

$= Rs.\ 240$ (ii)

$$= \frac{5}{12} \times 720$$

$$= \text{Rs. } 300 \qquad \text{(iii)}$$

4. ***Three persons invested an amount of Rs. 30,00,000 in a business with shares ratio 2:3:7. They earned a profit of Rs. 600,000. If they are interested to wind up their business, what amount every share holder would get?***

Solution:

1st part

amount of investment = 30,00,000

Ratio = 2 : 3 : 7

sum of ratio = 2 + 3 + 7

= 12

$$\text{1st investment} = \frac{2}{12} \times 30,00,000$$

$$= \text{Rs. } 5,00,000$$

$$\text{2nd investment} = \frac{3}{12} \times 30,00,000$$

$$= \text{Rs. } 7,50,000$$

$$\text{3rd investment} = \frac{7}{12} \times 30,00,000$$

$$= \text{Rs. } 17,50,000$$

2nd part

total amount of profit = Rs. 6,00,000

Ratio = 2 : 3 : 7

sum of the ratio = 2 + 3 + 7

= 12

$$\text{profit of the 1st share holder} = \frac{2}{12} \times 6,00,000$$

$$= \text{Rs. } 1,00,000$$

profit of the 2nd share holder $= \frac{3}{12} \times 6,00,000$

$= \text{Rs. } 1,50,000$

Profit of the 3rd share holder $= \frac{7}{12} \times 6,00,000$

$= \text{Rs. } 3,50,000$

	1st	*2nd*	*3rd*
Cost in Profit	1,00,000	1,50,000	3,50,000
Cost in investment	+ 5,00,000	+ 750,000	+ 17.50,000
Total amount =	6,00,000	9,00,000	21,00,000

5. ***Three members of a firm divide the profit Rs. 67,200 among themselves in the ratio 2:3:7. What is the biggest share of the profit?***

Solution:

Profit = Rs. 67,200

Ratio in the shares = 2 : 3 : 7

sum of the ratio = 2 + 3 + 7

=

Share of 1st person $= \frac{2}{12} \times 67,200$

= Rs. 11,200

Share of 2nd person $= \frac{3}{12} \times 67,200$

= Rs. 16,800

Share of 3rd person $= \frac{7}{12} \times 67,200$

= Rs. 39,200

6. ***A sum of money is divided among three persons, A,B and C in the ratio 10:7:5. If "B" gets Rs. 14 more than "C". How much will "A" get and what is the total sum of money?***

Solution:

Let full amount = Rs. x

Ratio among A,B,C share = 10 : 7 : 5

Sum of ratio = 10 + 7 + 5

= 22

$$\text{Share of A} = \frac{10}{22}x$$

$$\text{Share of B} = \frac{7}{22}x$$

$$\text{Share of C} = \frac{5}{22}x$$

Now Share of B - Share of C $= \frac{7}{22}x - \frac{5}{22}x$

$$= \frac{2x}{22}$$

Now according to given condition

$$\frac{2x}{22} = 14$$

$$x = 14 \times \frac{22}{2}$$

Hence full amount = 154

$$\text{Share of A} = \frac{10}{22}x$$

$$\text{Share of A} = \frac{10}{22} \times 154$$

Exercise 4.1

1. Convert 250 US Dollar into Sterling Pound.

Solution:

Rate of US Dollar = Rs. 83.800

250 dollar = 250×83.800

= Rs. 20,950

Rate of pound sterling = Rs. 129.7968

Pounds of sterling in exchange of 250 dollars $= \frac{20950}{129.7968}$

= 161.406 Pounds

2. Convert 5000 Riyals into Pak Rupee.

Solution:

Rate of Saudi Riyal = Rs. 22.3449

Amount of Saudi Riyal = 5000

5000 Saudi Riyals in exchange of Pakistani rupees = 5000×22.3449

= Rs. 111724.5

3. An importer inports a car from Japan for 5000Yen delivery was to be made after three months. At the time of contract Re1 = 0.895236Yen. At the time of delivery Re.1=0892236 Yen. Payment was made at the time of contract. Determinie the profit or loss of the importer.

Solution:

Rate of import car in yen = 5000 yen

The value of Re. 1 atcontract time = 0.895236 yen

0.895236 Yen = 1 rupee

5000 Yen = 5000×0.895236

= Rs. 4476.18

The value of Re. 1 at delivery time = 0.892236

5000 Yen = 5000 × 0.892236

= Rs. 4461.18

Loss = 4476.18-4461.18

= Rs. 15

4. ***A customer wants to convert 150 American dollers into rupees. He goes; to an authorised dealer. He offers him conversion at the rate of 1 doller = Rs. 84.100. If it is converted with a money changer, the rate is 1 dollers = Rs. 83.4495, determine the amount into rupees if it is converted with:***

(i) Authorised dealer

(ii) Money Changer

(iii) The loss due to conversion with the money changer.

Solution:-

Number of American dollar = 150

Money exchanger with an authorised dealer

1 US dollar = Rs. 84.100

150 US dollar = 150×84.100

Rs. 12615

Rate of exchange with money changer

1 US dollar = Rs. 83.4495

Therefore 150 US dollar = 150×83.4495

= Rs. 12517.43

The loss due to conversion with the money changer = 12615-12517.43

= Rs. 97.57

5. ***Rate of tea in Pakistan is Rs. 2.1 per pound. Distribute the rate per kilogram.***

(i) 1 pound = 2.2 kilogram

(ii) What will be the rate in Saudi Arbia if Saudi Riyal = Rs. 22.400.

Solution:

One pound = .45 kg

Rate of 1 pound tea = Rs. 19.089

Rate of .45 kg tea = Rs. 19.089

$$\text{Rate of 1 kg tea} = \frac{19.089}{.45}$$

= Rs. 42.42

1 Saudi Riyal = Rs. 22.40

$$\frac{1}{22.40} \times 42.42 = \text{Rs. } 42.42$$

= 1.90 Riyal per pound

6. ***An exporter of carpets exports to England Carpets amounting to 40000 Sterling Pound. The spot buying rate exchange at that time was Rs. 129.4542 to 1 Sterling. He receives the amount at the time when rate is Rs. 129.0599 to 1 Sterling. How much he looses?***

Solution:

Export price of a carpet = 4000 pound sterling

On contract time exchange of Rate.

1 pound sterling = Rs. 129.4542

4000 pound sterling = 4000×129.4542

= Rs. 517816.8

Exchange of Rate after time.

1 pound sterling = Rs. 129.0599

4000 pound sterling = 4000×129.0599

= Rs. 516239.6

Loss = 517816.8 - 516239.6

= Rs. 1577.2

7. ***A Pakistani living in Saudi Arabia earns 4370 Riyals a month. His monthly expenses comes to 3450 Riyals. He remits his saving monthly to Pakistan. How much he saved in a year if rate of exchange is Rs. 22.400 = 1 Saudi Riyals. After a year Rate of exchange is Rs. 22.3004. Determine the loss due to***

monthly remitance.

Rate of exchange

(1 Saudi Riyal) = Rs. 22.400

After one year

(1 Saudi Riyal) = Rs. 22.3004

Rate of exchange.

Solution:

Monthly income = 4370 Riyals

Monthly expenses = 3450 Riyals

Monthly saving = 4370 - 3450

= 920 Riyals

Annually saving = 12×920

Annually saving = 11040 Riyals

Rate of exhange

1 Riyal = Rs. 22.400

Therefore 11040 Riyals = 11040×22.400

= Rs. 247296

Rate of exchange after one year.

1 Riyal = Rs. 22.3004

Therefore 11040 Riyals = 11040×22.3004

= 246196.416

Loss = 247296 - 246196.42

= Rs. 1099.58

Monthly Less = 920 (22.400 - 22.3004)

= $920 \times .0996$

= 91.632

= Rs. 92

8. ***Rizwan purchases a car in Saudi Arabia for 15000 Riyals. Delivery was to be made after three months and payment is also to be made at the time of delivery. At the time of contract, the rate was 1 Riyal = Rs. 22.400, while at the time of delivery the rate was 1 Riyal Rs. 22.0827. Determine the loss in rupees due to change in the rate.***

• 1 Saudi Riyal = Rs. 22.400

Time of delivery rate

1 Saudi Riyal = Rs. 22.0827

Determine the loss in rupees due to change in the rate.

CP of a car = 15000 Riyals

On contract time rate of Riyal

1 Riyal = Rs. 22.400

Rate of 15000 Riyal = 15000×22.400

= Rs. 336000

After three months rate of Riyal

1 Riyal = Rs. 22.0827

Rate of 15000 Riyal = 15000×22.0827

= Rs. 331240.5

Loss = 336000 - 331240.5

= 4759.5

= Rs. 4759

9. ***A friend of Ali living in Saudi Arabia remits Ali 450 Riyals. The bank offers two conversions rate. T.T. Buying Rs. 22.3449 = 1 Riyal T/C Buying Rate: Rs. 22.2146 = 1 Riyal***

Which one of the rate will be applicable and also calculate the amount in rupees.

Solution:

1 Saudi Riyal = Rs. 22.3449 (By T.T)

1 Saudi Riyal = Rs. 22.2146 (By T/C)

By T.T the rate will be applicable and best.

1 Saudi Riyal = Rs. 22.3449

450 Saudi Riyal = 450×22.3449

= Rs.10055

Exercise 4.2

1. ***A financial institution charges Rs. 55 simple profit on a sum of money which is borrowed for five months. Given that the rate of profit is 12% per annum, find the sum of money.***

Solution:

$$\text{Profit} = \text{Rs. } 55$$

$$\text{Time/period} = 5 \text{ months} = \frac{5}{12} \text{ years}$$

$$\text{Rate} = 12\%$$

$$\text{Principal} = \frac{\text{Amount of Rate} \times 100}{\text{Rate} \times \text{Time}} \qquad \text{Formula}$$

$$= \frac{55 \times 100}{12 \times \frac{5}{12}}$$

$$= \frac{55 \times 100}{5}$$

$$= 11 \times 100$$

$$= \text{Rs. } 1100$$

2. ***Mrs. Javed invests in Savings Scheme Rs. 800 at 6% per annum and Rs. 1,200 at 7% per annum. What is her total amount of profit on these two investments?***

Solution:

1st condition

$$\text{Principal amount} = \text{Rs. } 800$$

$$\text{Rate} = 6\%$$

$$\text{Time/period} = 1 \text{ year}$$

$$\text{Profit (simple)} = \frac{Principal \times Time \times Rate}{100}$$ **Formula**

$$= \frac{800 \times 1 \times 6}{100}$$

2nd condition = Rs. 48

Principal amount = Rs. 1200

Rate = 7%

Time/period = 1 year

$$\text{Profit} = \frac{1200 \times 1 \times 7}{100}$$

= Rs. 84

Total profit = 48 + 84

Total profit = Rs. 132

3. How long would Rs. 1250 have to be deposited at 6% per year simple profit to gain Rs. 750 simple profit?

Solution:

Principal amount = Rs. 1250

Rate = 6%

Profit = Rs. 750

Time/period = ?

$$\text{Period/Time} = \frac{Amount\ of\ Profit \times 100}{Rate \times Principal}$$ Formula

$$= \frac{750 \times 100}{6 \times 1250}$$

= 10 years

4. Ali lent to Abid Rs. 4,800 for 7 months. At the end of this period Abid had to pay Ali profit of Rs. 119. What was the rate of simple profit per annum?

Solution:

Principal amount = Rs. 4800

Time/period = 7 years

$$= \frac{7}{12} \text{ years}$$

Amount of profit = Rs. 119

Rate = ?

$$\text{Rate} = \frac{\textit{Amount of Profit} \times 100}{\textit{Time} \times \textit{Principal}} \quad \text{Formula}$$

$$= \frac{119 \times 100}{\frac{7}{12} \times 4800}$$

$$= \frac{119 \times 100}{7 \times 400}$$

$$= \frac{17}{4}\%$$

$$= 4\frac{1}{4}\%$$

5. ***In a certain year, Javed puts Rs. 600 in a private bank at the end of March and Rs. 400 in the same bank at the end of June. The bank offers 3% per annum simple profit rate. Find the total amount Javed receives from the bank at the end of December in that year?***

Solution:

1st condition

Principal amount = Rs. 600

Rate = 3%

Time/period = 3 months

$$= \frac{3}{12} = \frac{1}{4} \text{ years}$$

$$\text{Profit (simple)} = \frac{\textit{Principal} \times \textit{Time} \times \textit{Rate}}{100} \quad \text{Formula}$$

$$= \frac{600 \times 3 \times 1}{100 \times 4}$$

(i) $$= \text{Rs. } \frac{9}{2} = 4.50$$

2nd Condition

Principal amount = Rs. 600+400 = 1000

Rate = 3%

Time/period = 6 months

$$= \frac{6}{12} = \frac{1}{2} \text{ years}$$

$$\text{Profit (simple)} = \frac{Principal \times Time \times Rate}{100}$$ Formula

$$= \frac{1000 \times 3 \times 1}{100 \times 2}$$

= Rs. 15 (ii)

Total profit amount = 4.50 + 15 (i) + (ii)

= Rs. 19.50

Javed received at the end of Dec. = 1000 + 19.50

= Rs. 1019.50

6. ***At what annual rate of profit would a sum of Rs. 680 will increase to Rs. 850 in 3 years and 4 months?***

Solution:

Principal amount = Rs. 680

Amount = Rs. 850

Amount of profit = 850 - 680

= Rs. 170

$$\text{Time/period} = 3\frac{4}{12}$$

$$= 3\frac{1}{3} = \frac{10}{3} \text{ years}$$

Rate = ?

$$\text{Rate} = \frac{\text{Amount of profit} \times 100}{\text{Time} \times \text{Principal}} \quad \text{Formula}$$

$$= \frac{170 \times 100}{680 \times \frac{10}{3}}$$

$$= \frac{170 \times 100 \times 3}{680 \times 10}$$

$$= \frac{15}{2}$$

$$\text{Rate} = 7\frac{1}{2}\%$$

7. Copy and complete the following table with the help of formula given in this unit?

	Principal	Profit rate	Time	Simple profit	Amount
a	Rs. 12.000	8%	7 years		
b	Rs.500	11%		Rs.220	
c		9%	4 years	Rs.108	
d	Rs.3000		10 years	Rs.1200	
e			2 years	Rs.360	Rs.3960
f	Rs.1800		18 years	Rs.189	
g	Rs.4500		2 years		Rs.5040
h		5%		Rs.90	Rs.1290

Solution:

7(a) ***1st Part***

Principal amount = Rs. 12,000

Rate = 8%

Time = 7 years

Profit (simple) = ?

Amount = ?

$$\text{Profit (simple)} = \frac{Principal \times Time \times Rate}{100}$$

$$= \frac{12000 \times 7 \times 8}{100}$$

$$= 120 \times 7 \times 8$$

$$\text{Profit} = \text{Rs. } 6720 \qquad \text{(i)}$$

2nd Part

$$\text{Amount} = \text{Principal} + \text{Profit}$$

$$= 12000 + 6720$$

$$= \text{Rs. } 18720 \qquad \text{(ii)}$$

Solution:

7(b) $\text{Principal amount} = \text{Rs. } 500$

$\text{Rate} = 11\%$

$\text{Profit (simple)} = \text{Rs. } 220$

$\text{Rate} = ?$

$\text{Amount} = ?$

$$\text{Time} = \frac{Amount\ of\ Profit \times 100}{Rate \times Principal}$$

$$= \frac{220 \times 100}{500 \times 11}$$

$$= 4 \text{ years} \qquad \text{(i)}$$

$$=$$

$$= 500 + 220$$

$$= \text{Rs. } 720 \qquad \text{(ii)}$$

Solution:

7(c) $\text{Rate} = 9\%$

$\text{Time} = 4 \text{ years}$

$\text{Profit} = \text{Rs. } 108$

$\text{Principal amount} = ?$

$\text{Amount} = ?$

$$\text{Principal} = \frac{Amount\ of\ Profit \times 100}{Rate \times Time}$$

$$= \frac{108 \times 100}{9 \times 4}$$

$$= \text{Rs. } 300$$

$$\text{Amount} = \text{Profit} + \text{principal}$$

$$= 108 + 300$$

$$= \text{Rs. } 408$$

Solution:

7(d) $\text{Principal amount} = \text{Rs. } 3000$

$\text{Time} = 10 \text{ years}$

$\text{Profit (simple)} = \text{Rs. } 1200$

$\text{Profit rate} = ?$

$\text{Amount} = ?$

$$\text{Rate} = \frac{\textit{Amount of Profit} \times 100}{\textit{Time} \times \textit{Principal}}$$

$$= \frac{1200 \times 100}{10 \times 3000}$$

$$= 4\%$$

$$\text{Amount} = \text{Profit} + \text{principal}$$

$$= 1200 + 3000$$

$$= \text{Rs. } 4200$$

Solution:

7(e) $\text{Time} = 2 \text{ years}$

$\text{Profit} = \text{Rs. } 360$

$\text{Amount} = \text{Rs. } 3960$

$\text{Principal} = ?$

$\text{Rate} = ?$

$$\text{Principal} = \text{Amount} - \text{profit}$$

$$= 3960 - 360$$

$$\text{Principal} = \text{Rs. } 3600$$

$$\text{Rate} = \frac{\textit{Amount of Profit} \times 100}{\textit{Time} \times \textit{Principal}}$$

$$= \frac{360 \times 100}{2 \times 3600}$$

$$= 5\%$$

Solution:

7(f)

Principal = Rs. 1800

$$\text{Time} = {}_{18\text{months}} = \frac{18}{12} = \frac{3}{2} \text{ years}$$

Profit = Rs. 189

Rate = ?

Amount = ?

$$\text{Rate} = \frac{\text{Amount of Profit} \times 100}{\text{Time} \times \text{Principal}}$$

$$= \frac{189 \times 100}{\frac{3}{2} \times 1800}$$

$$= \frac{189 \times 100}{3 \times 900}$$

Rate = 7%

Amount = Principal + Profit

= 1800 + 189

Amount = Rs. 1989

Solution:

7(g)

Principal = Rs. 4500

Time = 2 years

Amount = 5040

Profit = ?

Rate = ?

Profit = Amount - Principal

= 5040 - 4500

Profit = Rs. 540

$$\text{Rate} = \frac{\text{Amount of Profit} \times 100}{\text{Time} \times \text{Principal}}$$

$$= \frac{540 \times 100}{2 \times 4500}$$

$$= 6\%$$

Solution:

7(h)

Rate = 5%

Profit = Rs. 90

Amount = Rs. 1290

Principal = ?

Time = ?

Principal = Amount - Profit

= 1290 - 90

Principal = Rs.1200

$$\text{Time} = \frac{\text{Amount of Profit} \times 100}{\text{Rate} \times \text{Principal}}$$

$$= \frac{90 \times 100}{5 \times 1200}$$

$$\text{Time} = \frac{3}{2} \text{ years}$$

$$\text{Time} = 1\frac{1}{2} \text{ years}$$

8. *A **bank increased the rate of profit from 3.5% to 4% per annum. Find how much more profit Saeed would receive if he deposited Rs. 6400 in the bank for 6 months at the new profit rate.***

Solution:

1st condition

Principal = Rs. 6400

Time = 6 months

$$= \frac{6}{12} = \frac{1}{2} \text{ year}$$

1st Rate = 3.5%

$$= \frac{35}{10}\%$$

$$\text{Profit amount} = \frac{Principal \times Rate \times Time}{100}$$

$$= \frac{6400 \times 35 \times 1}{100 \times 10 \times 2}$$

Profit amount = Rs. 112

2nd condition

Principal = 6400

Time = $\frac{1}{2}$ year

Rate = 4%

$$\text{Profit} = \frac{Principal \times Rate \times Time}{100}$$

$$= \frac{6400 \times 4 \times 1}{100 \times 2}$$

Profit = Rs. 128

Additional amount = 2nd profit - 1st profit

= 128 - 112

Hence Additional amount = Rs.16

9. ***Mrs. Jamshed invested Rs. 4000 in X... Bank Limited which paid simple profit at a rate $7\frac{1}{2}\%$ per annum to its investors. After 2 years, the rate was increased to 8% per annum, Find the amount she had at the end of 7 years.***

Solution:

1st condition

Principal = Rs. 4000

$$\text{Rate} = 7\frac{1}{4}\%$$

$$= \frac{29}{4}\%$$

$$\text{Time} = 2 \text{ years}$$

$$\text{Profit} = \frac{Principal \times Rate \times Time}{100}$$

$$= \frac{4000 \times 29 \times 2}{100 \times 4}$$

2nd Condition

$$= \text{Rs. } 580 \qquad \text{(i)}$$

$$\text{Principal} = \text{Rs. } 4000$$

$$\text{Rate} = 8\%$$

$$\text{Time} = 7 - 2 = 5 \text{ years}$$

$$\text{Profit} = \frac{4000 \times 8 \times 5}{100}$$

$$= \text{Rs. } 1600 \qquad \text{(ii)}$$

$$\text{Hence Total profit} = 580 + 1600 \qquad \text{(ii),(i)}$$

$$\text{Total profit} = \text{Rs. } 2180$$

$$\text{After seven years Jamshed} = 4000 + 2180$$

$$= \text{Rs. } 6180$$

$$\text{Wife will get amount} = \text{Rs. } 6180$$

10. Mr. Dawood deposits a certain sum of money in ABC Limited. If the profit rate of the bank decreases from $3\frac{3}{4}\%$ per annum to $3\frac{1}{2}\%$ per annum, Mr. Dawood's profit will decrease by Rs. 50 in a year. Find the sum of money he deposits.

Solution:

1st condition

Deposit amount = Rs. x

Time = 1 year

$$\text{Rate} = 3\frac{3}{4}\% = \frac{15}{4}\%$$

$$\text{Profit} = \frac{Principal \times Rate \times time}{100}$$

$$= \frac{x \times 15 \times 1}{4 \times 100}$$

$$= \text{Rs. } \frac{15x}{400} \quad \text{(i)}$$

2nd condition

Principle = Rs. x

$$\text{Rate} = 3\frac{1}{2}\% = \frac{7}{2}\%$$

$$\text{Profit} = \frac{Principal \times Rate \times time}{100}$$

$$= \frac{x \times 7 \times 1}{2 \times 100}$$

$$= \text{Rs. } \frac{7x}{200} \quad \text{(ii)}$$

Difference in profit = Rs. 50

According to given condition

$$\frac{15x}{400} - \frac{7x}{200} = 50$$

$$400 \times \frac{15x}{400} - 400 \times \frac{7x}{200} = 400 \times 50 \text{ Multiplying by } 400$$

OR $15x - 14x = 20000$

$x = \text{Rs. } 20{,}000$

Principle amount = Rs. 20,000

11. Find the compound prfit on:

(i) Rs. 450 for 2 years at 10% per annum compounded yearly;

(ii) Rs. 700 for 3 years at 11% per annum compounded yearly;

(iii) Rs. 5000 for 2 years at $11\frac{3}{4}\%$ per annum compounded yearly;

(iv) Rs. 1200 for 3 years at 4% per annum compounded yearly;

(v) Rs. 10000 for 3 years at $7\frac{1}{2}\%$ per annum compounded yearly;

Solution:

11(i) Principal = Rs. 450

Rate = 10%

Time = 2 years

Compound profit = ?

$$\text{Compound profit} = Principal\left[1+\frac{Rate}{100}\right]^{time} - Principl$$

$$= 450\times\left[1+\frac{10}{100}\right]^{2} - 450$$

$$= 450\times\left[1+\frac{1}{10}\right]^2-450$$

$$= 450\times\left(\frac{11}{10}\right)^2-450$$

$$= 450\times\frac{11}{10}\times\frac{11}{10}-450$$

$$= 45\times11\times\frac{11}{10}-450$$

$$= \frac{5445}{10}-450$$

$$= \frac{5445-4500}{10}$$

$$= \frac{945}{10}$$

$$= \text{Rs. } 94.50$$

Solution:

11(ii) **Principal** = Rs. 750

Time = 3 years

Rate = 11%

Compound profit = ?

$$\text{Compound profit} = Principal\left[1+\frac{Rate}{100}\right]^{time}-Principle$$

$$= 750\times\left[1+\frac{11}{100}\right]^3-750$$

$$= 750\times\left[\frac{111}{100}\right]^3 - 750$$

$$= 750\times\frac{111\times111\times111}{100\times100\times100} - 750$$

$$= \frac{1025723250 - 750000000}{1000000}$$

$$= \frac{275723250}{1000000}$$

$$= \text{Rs. } 275.72$$

Solution:

11(iii) Principal = Rs. 5000

$$\text{Rate} = 11\frac{3}{4}\% = \frac{47}{4}\%$$

Time = 2 years

Compound profit = ?

$$\text{Compound Profit} = Principal\left[1+\frac{Rate}{100}\right]^{time} - Principle$$

$$= 5000\times\left[1+\frac{47}{4\times100}\right]^2 - 5000$$

$$= 5000\times\left[1+\frac{47}{400}\right]^2 - 5000$$

$$= 5000\times\left[\frac{400+47}{400}\right]^2 - 5000$$

$$= 5000\times\left(\frac{447}{400}\right)^2 - 5000$$

$$= 5000 \times \frac{447 \times 447}{400 \times 400} - 5000$$

$$= 5 \times \frac{447 \times 447}{4 \times 40} - 5000$$

$$= \frac{447 \times 447}{4 \times 8} - 5000$$

$$= \frac{199809}{32} - 5000$$

$$= \frac{199809 - 160000}{32}$$

$$\text{Compound profit} = \frac{39809}{32}$$

$$= \text{Rs. } 1244.03$$

$$\text{Compound profit} = \text{Rs. } 1244$$

Solution:

11(iv)

$$\text{Principal} = \text{Rs. } 1200$$

$$\text{time} = 3 \text{ years}$$

$$\text{Rate} = 4\%$$

$$\text{Compound profit} = ?$$

$$\text{Compound profit} = \left[1 + \frac{Rate}{100}\right]^{time} - Principle$$

$$= 1200 \times \left[1 + \frac{4}{100}\right]^3 - 1200$$

$$= 1200 \times \left[1 + \frac{1}{25}\right]^3 - 1200$$

$$= 1200 \times \left(\frac{26}{25}\right)^3 - 1200$$

$$= 1200 \times \frac{26 \times 26 \times 26}{25 \times 25 \times 25} - 120($$

$$= 48 \times 26 \times \frac{26 \times 26}{25 \times 25} - 1200$$

$$= \frac{843648}{625} - 1200$$

$$= \frac{843648 - 750000}{625}$$

$$= \frac{93648}{625}$$

$$= \text{Rs. } 149.84$$

Solution:

11(v) Principal = Rs. 10,000

time = 3 years

$$\text{Rate} = 7\frac{1}{2}\% = \frac{15}{2}\%$$

Compound profit = ?

$$\text{Compound profit} = \left[1 + \frac{Rate}{100}\right]^{time} - Principle$$

$$= 10,000 \times \left[1 + \frac{15}{2 \times 100}\right]^3 - 10,0($$

$$= 10,000 \times \left[1 + \frac{3}{40}\right]^3 - 10,000$$

$$= 10,000 \times \left[\frac{40+3}{40}\right]^3 - 10,000$$

$$= 10,000 \times \left[\frac{43}{40}\right]^3 - 10,000$$

$$= 10,000 \times \frac{43 \times 43 \times 43}{40 \times 40 \times 40} - 10,000$$

$$= 10,000 \times \frac{43 \times 43 \times 43}{40 \times 40 \times 40} - 10,000$$

$$= 2,000 \times \frac{43 \times 43 \times 43}{8 \times 40 \times 40} - 10,000$$

$$= 400 \times \frac{43 \times 43 \times 43}{8 \times 8 \times 40} - 10,000$$

$$= 50 \times \frac{43 \times 43 \times 43}{1 \times 8 \times 40} - 10,000$$

$$= 10 \times \frac{43 \times 43 \times 43}{1 \times 8 \times 8} - 10,000$$

$$= \frac{10 \times 43 \times 43 \times 43}{64} - 10,000$$

$$= \frac{795070}{64} - 10,000$$

$$= \frac{795070 - 64,000}{64}$$

$$= \frac{731070}{64}$$

$$= 11422.96$$

$$= \frac{155068}{64}$$

$$= \text{Rs. } 2422.94$$

12. ***Waseem invests Rs. 5000 at $5\frac{1}{4}\%$ per annum profit compounded annually. Find the amount at the end of the third year.***

Solution:

$$\text{Principal} = \text{Rs. } 5000$$

$$\text{Rate} = 5\frac{1}{4}\% = \frac{21}{4}\%$$

$$\text{Time} = 3 \text{ years}$$

$$\text{Principal amount} = Principal \times \left[1+\frac{Rate}{100}\right]^{time}$$

$$= 5000\times\left[1+\frac{21}{4\times 100}\right]^3$$

$$= 5000\times\left[1+\frac{21}{400}\right]^3$$

$$= 5000\times\left[\frac{400+21}{400}\right]^3$$

$$= 5000\times\left(\frac{421}{400}\right)^3$$

$$= 5000\times\frac{421}{400}\times\frac{421}{400}\times\frac{42}{40}$$

$$= 25 \times \frac{421}{2} \times \frac{421}{400} \times \frac{421}{400}$$

$$= \frac{421}{2} \times \frac{421}{16} \times \frac{421}{400}$$

$$= \frac{74618461}{12800}$$

$$= \text{Rs. } 5829.56$$

13. Javed invests Rs. 800 at $12\frac{1}{2}\%$ per annum compound profit compounded half-yearly. What is the amount at the end of the first year?

Solution:

$$\text{Principal} = \text{Rs. } 800$$

$$\text{Yearly Rate} = 12\frac{1}{2}\%$$

$$\text{half yearly Rate} = \frac{25}{2}$$

$$= \frac{1}{2} \times \frac{25}{2}$$

$$= \frac{25}{4}\%$$

$$\text{time} = 1 \text{ year}$$

$$\text{time} = 2 \text{ half year}$$

$$\text{Total amount} = Principal \times \left[1 + \frac{Rate}{100}\right]^{time}$$

$$= 800 \times \left[1 + \frac{25}{4 \times 100}\right]^{2}$$

$$= 800 \times \left[1 + \frac{1}{4 \times 4}\right]^2$$

$$= 800 \times \left[1 + \frac{1}{16}\right]^2$$

$$= 800 \times \left(\frac{17}{16}\right)^2$$

$$= 800 \times \frac{17}{16} \times \frac{17}{16}$$

$$= 50 \times 17 \times \frac{17}{16}$$

$$= 25 \times 17 \times \frac{17}{8}$$

$$= \frac{7225}{8}$$

Total amount = Rs.903.13

Compound profit = 903.13 - 800

= Rs.103.13

14. ***Mr. Saleem invests Rs. 9000 at 2% per annum compound profit compounded daily. What is his amount at the end of the third day.***

Solution:

Principal = Rs. 9000

Yearly Rate = 2%

$$= \frac{2}{365}\% \text{ daily}$$

time = 3 days

$$\text{Final amount} = Principal \times \left[1 + \frac{Rate}{100}\right]^{time}$$

$$= 9000 \times \left[1 + \frac{2}{365 \times 100}\right]^3$$

$$= 9000 \times \left[1 + \frac{1}{365 \times 50}\right]^3$$

$$= 9000 \times \left[1 + \frac{1}{18250}\right]^3$$

$$= 9000 \times \left[\frac{18250+1}{18250}\right]^3$$

$$= 9000 \times \left[\frac{182501}{18250}\right]^3$$

$$= 9000 \times 1.00005$$

$$= 9000 \times 1.00015$$

Final amount = Rs. 9001.35

Exercise 4.3

1. *A man borrowed Rs. 1460 from ABC Bank on the 3rd of March at* $12\frac{1}{2}\%$ *what should he pay on the 1st of July to pay off the debt.*

fortime period /Principle amount = Rs. 1460

March = 28 Rate $= 12\frac{1}{2}\%$

April = 30

May = 31

June = 30

July = 1

120 days

$$= \frac{25}{2}\%$$

time = 120 days

$$\text{mark up} = \frac{1460 \times 25 \times 120}{100 \times 2 \times 365}$$

mark up = Rs. 60

2. *A shopkeeper borrowed Rs. 3540 from ABC Bank at* $10\frac{3}{4}\%$ *and lent the whole amount at* $11\frac{1}{2}\%$ *on the same day, what would be gianed from this after 3*

Solution:

1st condition

Principal amount = Rs. 3540

$$\text{Rate} = 10\frac{3}{4}\% = \frac{43}{4}\%$$

$$\text{time} = 3\frac{4}{12} = 3\frac{1}{3} = \frac{10}{3} \text{ years}$$

$$\text{mark up } \frac{Principal \times Rate \times time}{100}$$

$$\text{mark up} = \frac{3540 \times 43 \times 10}{100 \times 4 \times 3}$$

mark up = Rs. 1268.50 (i)

= Rs. 3540

2nd condition

$$\text{Rate} = 11\frac{1}{2} = \frac{23}{2}\%$$

$$\text{time} = \frac{10}{3} \text{ years}$$

$$\text{mark up (Profit) } \frac{Principal \times Rate \times time}{100}$$

$$= \frac{3540 \times 23 \times 10}{100 \times 2 \times 3}$$

= Rs. 1357 (ii)

Shopkeeper's profit = 1357 – 1268.50 (ii),(i)

= Rs. 88.50

3. ***XYZ Bank gained Rs. 3834 on its loan at 6% compound markup in 2 years. What amount did it lend?***

Solution:

Profit amount = Rs. 8034

time = 2 years

Rate = 6%

Principal amount = ?

Let amount = Rs. 100

Rate = 6%

time = 2 years

Compound profit on Rs.100 $= Principal \times \left[1+\frac{Rate}{100}\right]^{time} - Principal$

$$= 100 \times \left[1+\frac{6}{100}\right]^{2} - 100$$

$$= 100 \times \left(\frac{106}{100}\right)^{2} - 100$$

$$= 100 \times \left(\frac{53}{50}\right)^{2} - 100$$

$$= 100 \times \frac{53 \times 53}{50 \times 50} - 100$$

$$= \frac{53 \times 53}{25} - 100$$

$$= \frac{2809}{25} - 100$$

Compound profit on Rs.100 $= \frac{2809 - 2500}{25} - 100$

Now

$$\text{Rs.} = \frac{309}{25}$$

If $\frac{309}{25}$ is profit its loan $= 100$

If Re.1 is profit its loan $= 100 \times \frac{25}{309}$

If 8034 is profit its loan $= \frac{100 \times 25}{309} \times 8034$

$= 100 \times 25 \times 26$

Amount which lent = Rs. 65,000

4. ***A company borrowed Rs. 6.600 from ABC Bank Ltd at 8% simple markup per annum. How much did the company owe to the bank at the end of 11 months?***

Solution:

Principal amount = Rs. 6000

Rate = 8%

time = 11 months

Amount of profit $= \frac{11}{12}$ years

Profit (simple) $\frac{Principal \times Rate \times time}{100}$

$= \frac{6000 \times 8 \times 11}{100 \times 12}$

Profit = Rs. 440

5. ***XYZ Bank charges 2.25% per month simple markup on personal loans. If Ali borrows Rs. 6.400 for a period of 2 years 1 month, find the total markup he has to pay to XYZ Bank.***

Solution:

Principal amount = Rs. 6400

$$\text{Monthly rate} = 2.25\% = \frac{225}{100}\%$$

time = 2 years 1 month

= 25 months

$$\text{Profit (simple)} = \frac{Principal \times Rate \times time}{100}$$

$$= \frac{6400 \times 225 \times 25}{100 \times 100}$$

$$= \frac{64 \times 225 \times 25}{100}$$

$$= \frac{\overset{16}{\cancel{64}} \times 225 \times \overset{1}{\cancel{25}}}{\underset{4}{\cancel{100}}}$$

$$= 16 \times 225$$

Profit (simple) mark up = 3600

6. ***Find out the compound markup on Rs. 250,000 for one year @ 14% compounded annually.***

Solution:

Principal amount = Rs. 2,50,000

time = 1 year

Yearly Rate = 14%

$$\text{compound mark up} = Principal \times \left[1 + \frac{Rate}{100}\right]^{time} - Principal$$

$$= 2,50,000 \times \left[1 + \frac{14}{100}\right]^{1} - 2,50,000$$

$$= 2,50,000 \times \left[1 + \frac{7}{50}\right] - 2,50,000$$

$$= 2,50,000 \times \frac{57}{50} - 2,50,000$$

$= 5000 \times 57 - 2,50,000$

$= 285000 - 2,50,000$

Compound mark up = Rs. 35, 000

7. Find compound profit on Rs. 600 for 4 years at 6 percent per annum.

Solution:

Principal amount = Rs. 600

Rate = 6%

= 4 years

Compound profit $= Principal \times \left[1 + \frac{Rate}{100}\right]^{time} - Principal$

$= 600 \times \left[1 + \frac{6}{100}\right]^4 - 600$

$= 600 \times \left[1 + \frac{3}{50}\right]^4 - 600$

$= 600 \times \left(\frac{53}{50}\right)^4 - 600$

$= 600 \times \frac{53 \times 53 \times 53 \times 53}{50 \times 50 \times 50 \times 50} - 600$

$= \frac{6 \times 53 \times 53 \times 53 \times 53}{5 \times 5 \times 50 \times 50} - 600$

$= \frac{47342886}{62500} - 600$

$= 757.48 - 600$

Compound profit = Rs. 157.48

8. ***Find the compound profit of Rs. 50000 at 4% for*** $1\frac{1}{2}$ ***years.***

Solution:

Principal amount = Rs. 50,000

Yeraly rate = 4%

time $= 1\frac{1}{2}$ years

time = One year and half year

$$= \frac{4}{2} = 2\%$$

Compound profit $= Principal \times \left[1 + \frac{Rate}{100}\right]^{time} - Principal$

Note this step $= 50,000 \times \left[1 + \frac{4}{100}\right]^{1} \times \left[1 + \frac{2}{100}\right]^{1} - 50,000$

$$= 50,000 \times \left[1 + \frac{2}{50}\right] \times \left[1 + \frac{1}{50}\right] - 50,000$$

$$= 50,000 \times \frac{52}{50} \times \frac{51}{50} - 50,000$$

$$= 1000 \times 52 \times \frac{51}{50} - 50,000$$

$$= 20 \times 52 \times 51 - 50,000$$

$$= 53040 - 50,000$$

Hence

Compound profit = 3040

پس

9. ***Find the compound profit on Rs. 54000 for one year at 12% per annum.***

Solution:

Principal amount = Rs. 54000

time = 1 year

Rate of % 1 year = 12%

Compound profit = ?

$$\text{Compound profit} = Principal \times \left[1 + \frac{Rate}{100}\right]^{time} - Principal$$

$$= 54000 \times \left[1 + \frac{12}{100}\right]^{1} - 54000$$

$$= 54000 \times \frac{112}{100} - 54000$$

$$= 540 \times 112 - 54000$$

$$= 60480 - 54000$$

$$= \text{Rs. } 6480$$

Exercise 4.4

1. *If the amount of premium is calculated as. Yearly premium @ 4.5% of the policy income + policy fee @ 0.25% of the policy amount or at themost Rs. 200. Half yearly premium @ 52% of yearly premium. Quarterly premium @ 27% of yearly premium. Monthly premium @ 9% of yearly premium. Then complete the table below for calculation of the premiums. Aslo find the total amount he pays to the company.*

Amount of policy	Yearly premium	Half yearly premium	Quarterly premium	Monthly premium
(i) 50,000 *(ii)* 100,000 *(iii)* 150,000 *(iv)* 200,000				

Solution: (i)

Amount of policy = 50,000 Rs.

$$\text{1st Premium @ of 4.5\%} = \frac{4.5}{100} \times 50{,}000$$

$$= \text{Rs. } 2250$$

$$\text{Amount of policy @ 0.25} = \frac{0.25}{100} \times 50{,}000$$

$$= \text{Rs. } 125$$

Therefore

Annual Premium = 1st Premium + policy fee

= 2250+125

= 2375 Rs.

$$\text{Half yearly premium} = \frac{52}{100} \times \text{Annual premium}$$

$$\text{Half yearly premium} = \frac{52}{100} \times 2375$$

$$= \text{Rs. } 1235 .$$

$$\text{Quarterly premium} = \frac{27}{100} \times 2375$$

$$= \text{Rs. } 641.25.$$

$$\text{Monthly premium} = 9\% \text{ Annual premium}$$

$$= \frac{9}{100} \times 2375$$

$$= \text{Rs. } 213.75$$

Solution (ii)

$$\text{Total amount of policy fee} = \text{Rs. } 100{,}000$$

$$\text{1st premium @ 4.5\%} = \frac{4.5}{100} \times 100{,}000$$

$$\text{policy fee} = \text{Rs. } 4500$$

$$= \frac{0.25}{100} \times 100{,}000 = 250$$

which is more than 200

$$\text{Therefore Policy fee} = \text{Rs. } 200$$

$$\text{Yearly premium} = \text{1st premium} + \text{policy fee}$$

$$= \text{Rs. } 4500 + 200 = 4700$$

$$\text{Half yeraly} = \frac{52}{100} \times \text{yearly premium}$$

$$= \frac{52}{100} \times 4700$$

$$= \text{Rs. } 2444$$

$$\text{Quarterly premium} = \frac{27}{100} \times 4700$$

$$= \text{Rs. } 1269$$

$$\text{Monthly premium} = 9\% \text{ of yearly premium}$$

$$= \frac{9}{100} \times 4700$$

= Rs.423.

Solution (iii)

Total policy fee = Rs. 1,50,000

1st premium@ 4.5% $= \frac{4.5}{100} \times 1,50,000$

= Rs. 6750

Policy fee $= \frac{0.25}{100} \times 1,50,000$

= Rs. 375

Policy fee is more than 200

= Rs. 200

Yearly premium = first premium + policy fee

= 6750 + 200

= Rs. 6950

Half yearly premium $= \frac{52}{100} \times$ yearly premium

$$= \frac{52}{100} \times 6950$$

= 3614 Rs.

$$= \frac{27}{100} \times$$

$$= \frac{27}{100} \times 6950$$

= Rs. 1876.50

Monthly premium = 9% of yearly premium

$$= \frac{9}{100} \times 6950$$

= Rs. 625.50

Solution: (iv)

Policy fee = Rs. 2,00,000

$$\text{1st premium @4.5\%} = \frac{4.5}{100} \times 2,00,000$$

$$= \text{Rs. } 9000$$

$$\text{Policy fee} = \frac{.25}{100} \times 2,00,000$$

$$= \text{Rs. } 500$$

500 policy fee is more than 200.

Therefore

Policy fee = Rs. 200

Yearly premium = 9000 + 200

= Rs. 9200

$$\text{Half yearly premium} = \frac{52}{100} \times \text{yearly premium}$$

$$= \frac{52}{100} \times 9200$$

$$= \text{Rs. } 4784$$

$$\text{Quarterly premium} = \frac{27}{100} \times \text{yearly premium}$$

$$= \frac{27}{100} \times 9200$$

$$= \text{Rs. } 2484$$

Monthly premium = 9% of yearly premium

$$= \frac{9}{100} \times 9200$$

$$= \text{Rs. } 828$$

No	Policy fee	yearly premmium Rs	Half yearly premium Rs	Quarterly premium	Monthly premium Rs
i	50,000	2250+125 ≐ 2375	1235	641.50	213.75

ii	1,00,000	4500+200 = 4700	2444	1269	423
iii	1,50,000	6750+200 = 6950	3614	1876.50	625.30
iv	2,00,000	9000+200 = 9200	4784	2484	828

2. ***Calculate the amount to be received by the heirs of an insured if he died 2 years after buying the policy while. The amount of policy = Rs. 50,000Premium is fixed @ 4.2% yearly Policy fee @ 0.3% Family income contract @ 0.6% Maturity period = 22 years Bonus @ 4.5% and Rs. 6000 yearly income is promised by the company.***

Solution: (i)

Policy fee = Rs. 50,000 (i)

Rs. 6000 for 20 years = 6000 × 20

= Rs.1,20,000 . (ii)

for 2 years amount of Bonus @ 4.5% $= \frac{4.5}{100} \times 50,000 \times 2$

Rs = 4500 (iii)

Amount to be received by the heirs = 50,000+1,20,000+4500 (i) + (ii) + (iii)

Rs = 1,74,500

3. ***Mr. Ahmed Ali insured his house worth Rs. 75,00,000 @ 2% for 4 years, calculate the amount paid in 4 years. While the rate of depreciation is 10% yearly.***

Solution:

Worth of house = Rs. 75,00,000

Rate of insurance = 2%

Time = 4 years

1st year amount of insurance $= \frac{2}{100} \times 75,00,000$

= Rs. 1,50,000

Depreciation $= \frac{10}{100} \times 75,00,000$

= Rs. 750000

Depreciation price = 75,00,000 – 750000

= Rs. 6750000

2nd year amount of insurance $= \frac{2}{100} \times 6750000$

= Rs. 1,35,000

Depriciation $= \frac{10}{100} \times 6750000$

= Rs. 675000

Depriciated price = 6750000 – 675000

= Rs. 6075000

3rd year amount of insurance $= \frac{2}{100} \times 6075000$

= Rs. 121500

4th year amount of insurance = Rs. 0

Total amount of insurance

1st Year = Rs. 1, 50. 000

2nd Year = Rs. 1.35.000

3rd Year = Rs. 1,21,500

4th Year = 0

Full paid amount =1,50,000+1,35,000+1.21,500

= Rs. 406500

4. ***Mr. Nadeem insured his shop @ 3% for 3 years, the depreciation rate is 5% yearly. If he paid an amount of Rs. 21000 as the 1st premium, what is the worth of his shop. If he got a claim of Rs. 200,000 after two years, how much benefits did he get?***

Solution:

Let worth of a shop = Rs. x

yearly rate of insurance = 3%

$$\text{1st premium} = \frac{3}{100}x \text{ Rs.}$$

$$= \text{Rs. } 21000 \qquad \text{(i)}$$

$$\frac{3x}{100} = \text{Rs. } 21000$$

And $$x = 21000 \times \frac{100}{3}$$

$$x = \text{Rs. } 7{,}00{,}000$$

worth of shop = Rs. 7,00,000

after one year depriciated price of shop

= 5%

$$\text{Rate of depriciation} = \frac{5}{100} \times 7{,}00{,}000$$

Worth of shop = 35,000

Depriciated price = 7,00,000 – 35000

2nd yearly premium = Rs. 6,65,000

$$= \frac{3}{100} \times 6{,}65{,}000$$

= Rs. 19950 (ii)

full amount paid by premium = 21000 + 19950 (i)+(ii)

claim amount = Rs.40,950 (iii)

= 2,00,000

Benifit = 2,00,000 - 40,950 (iii)

= Rs. 1,59,050

5. ***Mr. Adil bought a running business worth Rs. 10,00,000 and got it insured @ 2.5% as yearly premium for 4 years. After 3 years he got a claim of Rs. 500,000 for actual damages. How much loss had he recovered through insurance?***

Solution:

Policy fee = Rs. 10,00,000 (i)

1st premium @2.5% $= \frac{2.5}{100} \times 10,00,000$

= Rs. 25,000

After one year worth of business (10% less) $= \frac{10}{100} \times 10,00,000$

= Rs. 100000 (ii)

After one yera worth of business = 10,00,000 – 1,00,000

= Rs. 9,00,000

2nd year premium $= \frac{2.5}{100} \times 9,00,000$ (iii)

= Rs. 22500

After two years depriciation in worth $= \frac{10}{100} \times 9,00,000$

= Rs. 90,000

= 9,00,000 – 90,000

= Rs. 8,10,000

3rd year premium $= \frac{2.5}{100} \times 8,10,000$

= Rs. 20250

Total three years paid premium = 25000 + 22500 + 20250

= Rs. 67,750

Worth of claim = Rs. 5,00,000

Loss = 5,00,000 – 67,750

= Rs. 4,32,250

6. ***Mr. Javeed bought an insurance policy against his car worth Rs. 8,50,000, @ 4.25% for 3 years. What total amount will he pay as premium, if he had not claimed and damages during the period? Where depreciation is 10%?***

Solution:

Policy fee = 8,50,000

1st premium @ 4.25% $= \frac{4.25}{100} \times 8,50,000$

$= \frac{425 \times 8,50,000}{100 \times 100}$

= Rs. 36125 (i)

Depriciation $= \frac{10}{100} \times 8,50,000$

= Rs. 85000

Depriciated price = 8,50,000 - 85000

= Rs. 7,65,000

2nd year premium $= \frac{4.25}{100} \times 7,65,000$

$= \frac{425 \times 7,65000}{100 \times 100}$

= Rs. 32512.50 (ii)

3rd yearly premium = Rs. 0

Total premium = 36125 + 32512.50 (i)+(ii)

= Rs. 68637.50

7. ***Mr. Rehman bought a vehicle worth Rs. 7,50,000. He***

got it insured @ 3.5% for 5 years. How much he paid in total for covering the risks, if he had got a claim of damages worth Rs. 100,000 during the period? Where depreciation is 10%.

Solution:

Policy fee = Rs. 7,50,000

Rate of policy = 3.5%

Time of policy = 5 years

Rate of less price = 10%

claimed amount = Rs. 1,00,000

$$\text{1st year premium} = \frac{3.5}{100} \times 7,50,000$$

$$= \frac{35 \times 7,50,000}{10 \times 100}$$

= Rs. 26,250

$$\text{Depriciation} = \frac{10}{100} \times 7,50,000$$

= Rs. 7,50,000

Depriciated price = 7,50,000 - 75000

= Rs. 6,75,000

$$\text{2nd year premium} = \frac{3.5}{100} \times 6,75,000$$

$$= \frac{35 \times 6,75,000}{10 \times 100}$$

= Rs. 23625

$$\text{2nd depriciated price} = \frac{10}{100} \times 6,75,000$$

= Rs. 67500

depriciated price = 6,75,000 - 67500

= Rs. 6,07,500

$$= \frac{3.5}{100} \times 6,07,500$$

$$= \frac{35 \times 607500}{10 \times 100}$$

= Rs. 21262.50

$$\text{Depriciation} = \frac{10}{100} \times 6,07,500$$

= Rs. 60750

Depriciated price = 607500 - 60750

= Rs. 5,46,750

$$\text{4th year premium} = \frac{3.5}{100} \times 546750$$

$$= \frac{35 \times 546750}{10 \times 100}$$

= Rs. 19136.25

5th year premium = Rs. 0

Total paid premium =26250+23625+21262.50+19136.25

= Rs. 90273.75

Claim amount = Rs. 1,00,000

Benifit = 1,00,000 - 90273.75

Profit = Rs. 9726.25

8. ***Ms. Maria bought an insurance policy @ 3.25% for her car for 3 years. Her 1st premium is Rs. 26000. Tell the price of her car. Also calculate the amounts of her 2nd and 3rd premium.***

Rate of policy = 3.25%

Time = 3 years

1st premium = Rs. 26000

Worth of car = ?

Let worth of a car = Rs. x

$$\text{1st Premium} = \frac{3.25(x)}{100} \text{ First premium}$$

$$= \frac{325(x)}{100 \times 100}$$

According to condition

$$\frac{325(x)}{100 \times 100} = \text{Rs. } 26000$$

Therefore $x = \dfrac{26000 \times 100 \times 100}{325}$

$= \text{Rs. } 8,00,000$

Depriciation $= \dfrac{10}{100} \times 8,00,000$

$= \text{Rs. } 80000$

Depriciated price $= 8,00,000 - 80000$

$= \text{Rs. } 7,20,000$

Premium of second year $= \dfrac{3.25}{100} \times 7,20,000$

$$= \frac{325 \times 7,20,000}{100 \times 100}$$

Premium of second year = Rs. 23400

Premium of third year = Rs. 0

8,00,000 ; 23400, zero

Exercise 4.5

1. For each of the following.

(i) find the additional amount you have to pay by financing and

(ii) express the additional amount obtained in as a percentage of the cash price:

		Financing Term		
	Cash (Rs.)	Down (Rs.)	Monthly installment (Rs.)	Number of installments
(a)	Rs. 360	Rs. 50	Rs. 40	10
(b)	Rs. 900	Rs. 150	Rs. 75	12
(c)	Rs. 25000	Rs. 10000	Rs. 500	36

Solution: 1 (a)

1st condition:

Cash payment = Rs. 360

2nd condition

down payment = Rs. 50

Monthly installment = Rs. 40

Number of installment = 10

By installment paid = 40 × 10

= Rs. 400

By installment paid and down payment = 50 + 400

additional amount = Rs. 450

= 450 - 360

= Rs. 90

$$\text{additional amount with rate} = \frac{90 \times 100}{360}$$

= 25%

Solution: 1 (b)

Cash payment = Rs. 900

Down payment = Rs. 150

Monthly premium = Rs. 75

Number of installment = 12

Payment by installment = 75 × 12

= Rs. 900

Payment by installment and down payment = 150 + 900

= Rs. 1050

Additional amount = 1050 - 900

= Rs. 150

$$\text{Additional amount with rate} = \frac{150}{900} \times 100 = 16\frac{2}{3}\%$$

or = 16.67%

Solution: 1 (c)

Cash payment = Rs. 25000

Down payment = Rs. 10000

Monthly installment = Rs. 500

Number of installment = 36

Payment by installment = 500 × 36

= Rs. 18000

Payment by installment and down payment = 18000 + 10000

= Rs. 28000

= 28000 - 25000

Additional amount = Rs. 3000

$$\text{amount which is taken as cash rate} = \frac{3000}{25000} \times 100$$

= 12%

2. ***Pervaiz buys a window air-conditioner at Rs. 900. He pays 20% deposit and the outstanding balance plus markup in 48 months. Markup on the balance is charged at 10% Find***

(i) the cost of his monthly installment;

(ii) the amount he saves by paying cash.

Solution:

Cash payment of air-condition Rs. 900

$$20\% \text{ Cash payment } \frac{20}{100} \times 900$$

$$= \text{Rs. } 180$$

$$\text{Balance amount } = 900 - 180$$

$$= \text{Rs. } 720$$

$$\text{Rate of mark up} = 10\%$$

$$= \frac{720 \times 10 \times 4}{100}$$

$$= \text{Rs. } 288$$

$$\text{Payment with mark up} = 720 - 288$$

$$= \text{Rs. } 1008$$

$$\text{Monthly installment} = \frac{1008}{48}$$

$$= \text{Rs. } 21$$

3. ***On each of the following***

(i) find the financial price of the goods and

(ii) express the amount saved by paying cash as a percentage of the cash price

	Item	Cash Rs.	Deposit	Number of Installments	Monthly Installments Rs.
(a)	Computer	Rs. 200	10%	24	Rs. 9
(b)	Printer	Rs. 450	15%	18	Rs. 25
(c)	Scanner	Rs. 1600	25%	30	Rs. 52

Solution 3 (a)

Cash amount of computer = Rs. 200

10% cash amount paid = $\frac{10}{100} \times 200$

= Rs. 20

Saving amount = 200 - 20 = Rs. 180

Monthly installment = Rs. 9

Number of installment = Rs. 24

Cash payment with installment = 9 × 24

= Rs. 216

Additional payment = 216 - 180

= Rs. 36

Financial price of computer = 200 + 36

= Rs.236 *(i)*

$= \left[\frac{Additional\ amount \times 100}{Cash\ payment}\right]$

Amount saved by paying cash as a percentage = $\frac{36 \times 100}{200}$

= 18%

Solution 3 (b)

Cash amount = Rs. 450

Cash payment 15% = $\frac{15}{100} \times 450$

= Rs. 67.50

Balance amount = 450 - 67.50 = Rs. 382.50

Monthly installment = Rs. 25

Number of installment = 18

nstallmentCash payment withi = 25 × 18

= Rs. 450

Additional payment = 450 - 382.50

Financial price of printer = Rs. 67.50

Amount saved by paying cash

as a percentage = 450 + 67.50

$$\left[\frac{Additional\ amount \times 100}{Cash\ payment}\right]$$

= Rs. 517.50 *(i)*

$$= \frac{67.50 \times 100}{450}$$

= 15% *(ii)*

Solution 3 (c)

Cash amount of scanner = Rs. 1600

25% Cash payment $= \frac{25}{100} \times 1600$

= Rs. 400

Balance = 1600 - 400 = Rs. 1200

Monthly installment = Rs. 52

Number of installment = 30

Paid amount by installment = 52 × 30

= Rs. 1560

Additional amount = 1560 - 1200

= Rs. 360

Financial price of scanner = 1600 + 360

= Rs. 1960 *(i)*

$$= \frac{360 \times 100}{1600}$$

Paying cash as a percentage $= 22\frac{1}{2}\%$

= 22.50% *(ii)*

4. For each of the following, find

(i) the monthly installment and

(ii) the difference in the hire purchase price and the cash price as a percentage of the cash price:

	Cash	Hire-purchase terms
(a)	Rs. 800	Rs. 100 deposit; balance 8%; 1 year
(b)	Rs. 8000	Rs. 200 deposit; balance 10%; $2\frac{1}{2}$ year
(c)	Rs. 1200	Rs. 200 deposit; balance 15%; $1\frac{1}{3}$ year

Solution 4 (a)

$$= \text{Rs. } 800$$

$$\text{Cash payment} = \text{Rs. } 100$$

$$\text{Balance} = 800 - 100 = \text{Rs. } 700$$

$$\text{Rate of mark up} = 8\%$$

$$\text{Time} = 1 \text{ year}$$

$$\text{Mark up} = \frac{Principal \times Rate \times time}{100}$$

$$= \frac{700 \times 8 \times 1}{100} = 56$$

$$\text{Amount paid by instalment} = 700 + 56$$

$$= \text{Rs. } 756$$

$$\text{Time} = 1 \text{ year} = 12 \text{ months}$$

$$\text{Monthly installment} = \frac{756}{12} = 63 \qquad (i)$$

$$\text{Amount saved by paying cash as a percentage} = \frac{additional\ amount \times 100}{Cash\ amount}$$

$$= \frac{56 \times 100}{800}$$

$$= 7\%$$

Solution 4 (b)

$$\text{Cash price} = \text{Rs. } 8000$$

$$\text{Cash payment} = \text{Rs. } 3200$$

$$\text{Balance} = 8000 - 3200$$
$$= \text{Rs. } 4800$$
$$\text{Rate of mark up} = 10\%$$
$$\text{Time} = 2\frac{1}{2} = \frac{5}{2} \text{ years}$$
$$\text{Mark up} = \frac{Principal \times Rate \times time}{100}$$
$$= \frac{4800 \times 10 \times 5}{100 \times 2}$$
$$\text{Mark up} = \text{Rs. } 1200$$
$$\text{Payment by instalment} = 4800 + 1200$$
$$= \text{Rs. } 6000$$
$$\text{Time} = \frac{5}{2} \text{ years} = 30 \text{ months}$$
$$\text{Monthly installment} = \frac{6000}{30}$$
$$= \text{Rs. } 200 \qquad (i)$$
$$\text{Amount saved by paying cash as a percentage} = \frac{Mark\ up \times 100}{Cash\ amount}$$
$$= \frac{1200 \times 100}{8000}$$
$$= 15\%$$

Solution 4 (c)

$$\text{Cash price} = \text{Rs. } 1200$$
$$\text{Cash payment} = \text{Rs. } 200$$
$$\text{Balance} = 1200 - 200$$
$$= \text{Rs. } 1000$$
$$\text{Rate of mark up} = 15\%$$
$$\text{Time} = 1\frac{1}{3} = \frac{4}{3} \text{ years}$$

$$\text{Mark up} = \frac{Principal \times Rate \times time}{100}$$

$$= \frac{1000 \times 15 \times 4}{100 \times 3}$$

$$= \text{Rs. } 200$$

$$\text{Amount paid in instalment} = 1000 + 200$$

$$= \text{Rs. } 1200$$

$$\text{Time} = \frac{4}{3} \text{ years} = 16 \text{ months}$$

$$\text{Monthly instalment} = \frac{1200}{16}$$

$$= \text{Rs. } 75$$

$$\text{Amount saved by paying cash as a percentage} = \frac{Mark\ up \times 100}{Cash\ amount}$$

$$= \frac{200 \times 100}{1200}$$

$$= \frac{50}{3}$$

$$= 16\frac{2}{3}\%$$

5. ***The cash price of a computer package deal was Rs. 3200. Markup paid @ 15% down payment and the outstanding balance plus markup over 24 months. Markup on the balance was charged at 9.5%.***

(i) ***Find the cost of the package deal if it is bought on hire-purchase.***

(ii) ***Find the difference between the hire-purchase price and the cash price.***

(iii) ***Express the difference obtained in (ii) as a percentage of the cash price.***

Solution:

Cash price of computer package = Rs. 3200

$$\text{Down payment} = \frac{15}{100} \times 3200$$

$$\text{Cash payment} = \text{Rs. } 480$$

$$\text{Balance} = 3200 - 480$$

$$= \text{Rs. } 2720$$

$$\text{Rate of mark up} = 9.5\% = 9\frac{1}{2} = \frac{19}{2}\%$$

$$\text{Time} = 24 \text{ months}$$

$$\text{Time} = 2 \text{ years}$$

$$\text{Mark up} =$$

$$= \frac{2720 \times 19 \times 2}{100 \times 2}$$

$$\text{Mark up} = \text{Rs. } 516.80 \qquad (i)$$

$$= \text{Principle} + \text{Mark up}$$

$$= 3200 + 516.80$$

$$= 3716.80 \text{ روپے} \qquad (ii)$$

$$\text{Amount saved by paying cash as a percentage} = \frac{\textit{Mark up} \times 100}{\textit{Cash price}}$$

$$= \frac{516.80 \times 100}{3200}$$

$$= 16.15\%$$

Exercise 5.1

1. ***The price of a bicycle is Rs. 3500. If 16% sales tax is charged, then calculate the amount of sale tax on 50 such bicycles.***

Solution:-

Price of one bicycle = Rs. 3500

Price of 50 bicycles = 3500×50

= 175000

Rate of sales tax = 16%

$$= \frac{16}{100} \times 175000$$

= Rs. 28000

2. ***If the price of an air conditioner is Rs. 40,000, then work out the amount of sale tax on it at the rate of 16%. Also calculate the price of air conditioner without sales tax.***

Solution:-

(Price of an air conditioner with tax) = Rs. 40,000

Rate of sales tax = 16%

amount of sales tax = ?

without sales tax, price of air conditioner = ?

Let Price = Rs. 100

Rate of sales tax = 16%

Price including sales tax = 100 + 16 = Rs. 116

If price with sales tax Rs. 116

then without sales tax cost = Rs. 100

If price with sales tax Rs. 40,000 $= \frac{100}{116} \times 40,000$

then without sales tax cost

= Rs. 34,482.76

= 40,000 - 34482.76

= Rs. 5517.24

3. ***The price of two cars of 1300 cc and 1600 cc without excise duty are 6,00,000 and Rs. 8,00,000 respectively. If the excise duty on these two are 200% and 250% respectively. Find the prices of the two cars inclusive duties.***

Solution:-

Price of 13 cc car = 6,00,000

Let cost = Rs. 100

Excise duty = 200%

price with exise duty = 100 + 200

If price is Rs. 100 then price

with excise duty = Rs. 300

If price is 6,00,000 then price $= \frac{300}{100} \times 6,00,000$

with excise duty

= Rs. 18,00,000

Cost of 1600 cc = Rs. 8,00,000

Let price = Rs. 100

Excise duty = 250%

Price with excise duty = 100 + 250

If price is Rs. 100 then price

with excise duty = Rs. 350

If price is 8,00,000 then $= \frac{350}{100} \times 8,00,000$

price including with excise duty = Rs. 28,00,000

4. ***The annual price of a house and price of land is Rs. 15,00,000 and Rs. 20,00,000 respectively. Find the property tax on each of these two at the rate of 16%.***

Solution:-

Annual price of a house = Rs. 15,00,000

Rate of property tax = 16%

Let annual income = Rs. 100

Property tax = Rs. 16

Annual income tax on Rs. 100 = Rs. 16

$$\text{Annual income tax on Rs. 15,00,000} = \frac{16}{100} \times 15,00,000$$

$$= \text{Rs. } 2,40,000 \quad (i)$$

Annual income on land = Rs. 20,00,000

Rate of property tax = 16%

Let annual income = Rs. 100

Property tax = Rs. 16

Annual income tax on Rs. 100 = Rs. 16

$$\text{Annual income tax on Rs. 20,00,000} = \frac{16}{100} \times 20,00,000$$

$$= \text{Rs. } 3,20,000 \quad (ii)$$

5. ***The total taxable income of two persons is Rs. 2,50,000 and Rs. 3,10,000 respectively. Work out the income tax for each of them @ 4.5%.***

Solution:-

1st person taxable income = Rs. 2,50,000

Rate of tax = 4.5%

Let taxable income = Rs. 100

$$\text{If Rs. 100 is taxable income then income tax} = \frac{4.5}{100} \times 2,50,000$$

$$\text{If Rs. 2,50,000 is taxable income then income tax} = \frac{45}{10 \times 100} \times 2,50,000$$

Taxable income of 2nd person = Rs. 3,10,000

Rate of income tax = Rs. 4.5%

Let taxable income = 100

Tax = 4.5

If taxable income is Rs. 100

then incometax = Rs. 4.5

If taxable income is Rs. 3,10,000 then incometax $= \frac{4.5}{100} \times 3,10,000$

$$= \frac{45}{10 \times 100} \times 3,10,000$$

= Rs. 13950 *(ii)*

6. ***The total taxable income of a person is Rs. 4,30,000. If he is given rebate Rs. 3000 on the tax chargeable, then work out the amount he has to pay as an income tax @ 4.5%.***

Solution:-

Annual income = Rs. 4,30,000

Rebate on tax = Rs. 3000

Rate of income tax = 4.5%

Income tax on Rs. 100 = Rs. 4.5

Income tax on Rs. 4,30,000 $= \frac{4.5}{100} \times 4,30,000$

$$= \frac{45}{10 \times 100} \times 4,30,000$$

Amount of income tax = Rs. 19350

Rebate on income tax = Rs. 3000

Paid income tax = 19350 - 3000

= Rs. 16350

7. *If the total annual income of a person is Rs. 6,25,000 with exemption of amount of Rs. 1,50,000, then find the tax chargeable @ 4.5%.*

Solution:-

Annual income = Rs. 6,25,000
Amount of rebate = Rs. 1,50,000
Rate of income tax = 4.5%
Taxable income = Income - Rebate
= 6,25,000 - 1,50,000
= Rs. 4,75,000

Income tax on Rs. 100 = Rs. 4.5

$$\text{Income tax on Rs. } 4,75,000 = \frac{4.5}{100} \times 4,75,000$$

$$= \frac{45}{10 \times 100} \times 4,75,000$$

= Rs. 21375

8. *The total income of a person is Rs. 5,25,000. Whereas the exemption is Rs. 1,50,000. Work out the tax payable @ 4.5% along with the income tax rate, if Rs. 10,000 has already been deducted at source as income tax.*

Solution:-

Total annual amount = Rs. 5,25,000
Exempted amount = Rs. 1,50,000
Taxable income = 5,25,000 - 1,50,000
= Rs. 375000

Tax rate = 4.5%

$$\text{Total tax payable} = \frac{4.5}{100} \times 375000$$

$$= \frac{45 \times 375000}{10 \times 100}$$

= Rs. 16875

Tax deducted at source = Rs. 10,000
Tax = 16875 - 10,000
= Rs. 6875

Exercise 5.2

1. ***In the following the gas meter reading has been given. Complete the gas bills with the help of the slabs given in the unit. Also include the meter rent and GST.***

Solution:-the table available in text book

(i) $3.0756\ Hm^3$ **(ii)** $4.285\ Hm^3$ **(iii)** $2.796\ Hm^3$
(iv) $1.378\ Hm^3$ **(v)** $5.235\ Hm^3$ **(vi)** $4.665\ Hm^3$

1 (i)

Amount of Gas $= 3.0756\ Hm^3$

Gas slab rates for domestic purpose are

Gas charges $3Hm^3$ = Rs. 325.48 *(i)*

Gas charges $0.756Hm^3$ = Rs. 80.65 *(ii)*

Meter Rent = Rs. 120.00 *(iii)*

Total amount (i)+ (ii)+(iii) = Rs. 526.13

$$\text{GST @ } 16\% = \frac{16}{100} \times 526.13$$

= Rs. 84.18

Current bill = 526.13 + 84.18

= Rs. 610.31

1 (ii)

Amount of Gas $= 4.285\ Hm^3$

from the given table

Gas charges $4Hm^3$ = Rs. 423.42 *(i)*

Gas charges $.285\ Hm^3$ = Rs. 80.65 *(ii)*

Meter Rent = Rs. 120.00 *(iii)*

Total amount *(i)+ (ii)+(iii)* = Rs. 624.07

$$\text{GST @ } 16\% = \frac{16}{100} \times 624.07$$

= 99.85

Current bill = 624.07 + 99.85

= Rs. 723.92

1 (iii) Amount of Gas = 2.796 Hm^3

from the given table

Gas charges 2 Hm^3 = Rs. 153.73 *(i)*

Gas charges .796 Hm^3 = Rs. 84.45 *(ii)*

Meter Rent = Rs. 120.00 *(iii)*

Total amount *(i)+(ii)+(iii)* = Rs. 358.18

$$\text{GST @ 16\%} = \frac{16}{100} \times 358.18$$

= Rs. 57.31

Current bill = 358.18 + 57.31

= Rs. 415.49

1 (iv) Amount of Gas = 1.378 Hm^3

from the given table

Gas charges 1Hm^3 = Rs. 84.45 *(i)*

Gas charges .378 Hm^3 = Rs. 80.65 *(ii)*

Meter Rent = Rs. 120.00 *(iii)*

Total amount *(i)+(ii)+(iii)* = Rs. 285.10

$$\text{GST @ 16\%} = \frac{16}{100} \times 285.10$$

= Rs. 45.62

Current bill = 285.10 + 45.62

= Rs. 330.72

1 (v) Amount of Gas = 5.235 Hm^3

from the given table

Gas charges in 5 Hm^3 = Rs. 550.44 *(i)*

Gas charges in .235 Hm^3 = Rs. 80.65 *(ii)*

Meter Rent = Rs. 120.00 *(iii)*

Total amount *(i)+(ii)+(iii)* = Rs. 751.09

$$\text{GST @ 16\%} = \frac{16}{100} \times 751.09$$

= Rs. 120.17

Current Bill = 751.09 + 120.17

= Rs. 871.26

1 (vi) Amount of Gas = 4.665 Hm^3

from the given table

Gas charges in 4 Hm^3 = Rs. 423.42 *(i)*

Gas charges in .665 Hm^3 = Rs. 84.45 *(ii)*

Meter Rent = Rs. 120.00 *(iii)*

Total amount *(i)+ (ii)+(iii)* = Rs. 627.87

$$\text{GST @ 16\%} = \frac{16}{100} \times 627.87$$

= Rs. 100.46

Current Bill = 627.87 + 100.46

= Rs. 728.33

2. ***In the following the number of units consumed while using electricity are given. Complete the Electricity bills, including the items as well as shown in the example of electricity bill.***

Solution:-

(i) 315 Units (ii) 210 Units (iii) 375 Units (iv) 290 Units

2 (i) Number of units consumed = 315

= 100×2.65 *(i)*

Cost of 100 units @ Rs. 2.65 is cost of per unit = Rs. 265

Cost of next 200 units @ Rs. 3.64 is = 200×3.64

= Rs. 728 *(ii)*

Cost of next 15 units @ Rs. 6.15 is = 15×6.15

= Rs. 92.25 *(iii)*

Total cost of 315 units from *(i) + (ii) + (iii)* = Rs. *1085.25* *(iv)*

Excise duty @ 1.5% is = Rs. 16.28 *(v)*

Electricity duty = Rs. 65.52 *(vi)*

PTV fee = Rs. 25.00 *(vii)*

Income tax @ 1.6% = 17.36

Total amount = Rs. 1206.41

(iv)+ (v) + (vi) + (vii)

Cost of 315 units = 1085.25 (A)

Total amount = 1206.41 (B)

Number of units consumed = 210

2 (ii) Cost of 100 units @ Rs. 2.65 is = 100×2.65

Cost of Next 110 units @ 3.64 per unit = Rs. 265 *(i)*

= 100×3.64

= Rs. 400.40 *(ii)*

Total amount 210 units (i)+(ii) = Rs. *665.40*

Excise duty @ 1.5% is = 9.98

Electricity duty = 62.52

PTV fee = 25.00

Income tax @ 1.6% = 10.65

Total amount = Rs. 1827.9

2 (iii) Number of units consumed = 375

Cost of 1st 100 units @ 2.65 is = 100×2.65

= Rs. 265 *(i)*

Cost of next 200 units @ 3.64 per unit is = 200×3.64

= Rs. 728 *(ii)*

Cost of next 75 units @ 6.15 per unit is = 75×6.15

= Rs. 461.25 *(iii)*

Cost of 375 units from *(i) + (ii) + (iii)* = Rs. *1454.25* *(iv)*

Excise duty @ 1.5% is = Rs. 21.81 *(v)*

Electricity duty = Rs. 62.52 *(vi)*

PTV fee = Rs. 25.00 *(vii)*

Income tax @ 1.6% from Rs. 23.27 *(viii)*

(iv)+(v)+(vi)+(vii)+(viii) Total amount Rs. 1586.85

2 (iv) Number of units consumed = 290

Cost of 1st 100 units @

2.65 per unit is = 100×2.65

= Rs. 265 *(i)*

Cost of next 190 units @

3.64 per unit is = 190×3.64

= Rs. 691.60 *(ii)*

Cost of 290 units *(i)* + *(ii)* = Rs. 956.60 *(iii)*

Excise duty @ 1.5% is = Rs. 14.35 *(iv)*

Electricity duty = Rs. 62.52 *(v)*

PTV fee = Rs. 25.00 *(vi)*

Income tax @ 1.6% is = Rs. 15.31

Total amount (iii)+(iv)+(v)+(vi) = Rs. 1073.78

3. In the following the number of calls made are given. Complete the telephone bill including the items.

(i)	530	(ii)	640	(iii)	750
(iv)	270	(v)	480	(vi)	315

3 (i) Number of calls = 530

Call charges @ Rs. 5 per call = 530×5

= Rs. 2650 *(i)*

$$\text{CED @ 15\%} = \frac{15}{100} \times 2650$$

= Rs. 397.50 *(ii)*

$$\text{W.H tax @ 4\%} = \frac{4}{100} \times 2650$$

= Rs. 106 *(iii)*

Total amount payable

(i) + *(ii)* + *(iii)* = Rs. 3153.50

3 (ii) Solution:

Number of calls = 640

Charges of 640 calls @

Rs. 5 per call = 640×5

= Rs. 3200 *(i)*

$$\text{C.E.D @ 15\% is} = \frac{15}{100} \times 3200$$

$$= \text{Rs. } 480 \qquad (ii)$$

$$\text{W.H tax @ 4\% is} = \frac{4}{100} \times 3200$$

$$\text{Total payable amount} = \text{Rs. } 128 \qquad (iii)$$

$$= \text{Rs. } 3808$$

3 (iii)

$$\text{Number of calls} = 750$$

Charges of 750 calls @

$$\text{Rs. 5 per call} = 750 \times 5$$

$$= \text{Rs. } 3750 \qquad (i)$$

$$\text{C.E.D @ 15\%} = \frac{15}{100} \times 3750$$

$$= \text{Rs. } 562.50 \qquad (ii)$$

$$\text{W.H tax @ 4\% is} = \frac{4}{100} \times 3750$$

$$= \text{Rs. } 150 \qquad (iii)$$

$$\text{Total payable amount} = \text{Rs. } 4462.50$$

3 (iv)

$$\text{Number of calls} = 270$$

Charges of 750 calls @

$$\text{Rs. 5 per call} = 270 \times 5$$

$$= \text{Rs. } 1350 \qquad (i)$$

$$\text{C.E.D @ 15\%} = \frac{15}{100} \times 1350$$

$$= \text{Rs. } 202.50 \qquad (ii)$$

$$\text{W.H Tax @ 4\% is} = \frac{4}{100} \times 1350$$

$$= \text{Rs. } 54 \qquad (iii)$$

$$\text{Total payable amount} = \text{Rs. } 1606.5$$

3 (v) Number of calls $=480$

Charges of 480 calls @

Rs. 5 per call $=480\times5$

$=$Rs. 2400 *(i)*

C.E.D @ 15% is $=\frac{15}{100}\times2400$

$=$Rs. 360 *(ii)*

W.H. Tax @ 4% is $=\frac{4}{100}\times2400$

$=$Rs. 96 *(iii)*

Total payable amount $=$Rs. 2856

3 (vi) Number of calls $=315$

Charges of 315 calls @

Rs. 5 per call $=315\times5$

$=$Rs. 1575 *(i)*

C.E.D @ 15% is $=\frac{15}{100}\times1575$

$=$Rs. 236.25 *(ii)*

W.H. Tax @ 4% is $=\frac{4}{100}\times1575$

$=$Rs. 63 *(iii)*

Total payabel amount $=$Rs. 1874.25

Exercise 5.3

1. ***A lady worker works a six-day week. She starts work at 7.00 am and finishes at 4pm. She has 15 minutes break in the morning and 45 minutes break in the afternoon. How long does she actually work in a week and how much she is paid, if the rate of payment is Rs. 40 per hour?***

Solution:

Numbers of hours from 7am to 4 pm = 9 hours

Break = 15 + 45

= 60 mints

= 1 hour

= 9 - 1

therefore, actually daily working hours = 8 hours

Number of hours works in 6 days = 8 × 6

= **48 hours**

She works 48 hours in a week

Rate of payment per hour = Rs. 40

Rate of payment 48 hour = 40 × 48

= **Rs. 1920**

2. ***Khalid works 6 day-week. Find his gross monthly wage, if his rate of pay is Rs. 200 per day.***

Solution:

Number of weeks in a month = 6

Number of days in a week = 4

Khalid's total working days of a month = 6 × 4

= 24 days

Daily wages = Rs. 200

24 days wages = 200 × 24

= **Rs. 4800**

3. ***Aslam gets paid Rs. 70 per hour for his normal working 8 hours daily (6 day week). The rate of overtime is 1.5 of Rs. 70 per hour. If he works 40 hours as overtime, then work out his gross monthly pay.***

Solution:

Aslam's working hours = 8 hours
Number of days Aslam works in a week = 6 days
Total weeks in a month = 4
Total number of days = 6 × 4
Aslam works in a month = 24 days
Number of hours = 24 × 8
Aslam works in a month = 192 hours
Payment of 1 hour working = Rs. 70
Payment of 192 hours working = 70 × 192
= Rs. 13440 *(i)*
extra wages per hour = 1.5 × 70
= Rs. 105
Number of hours Aslam works as overtime = 40 hours
Over time payment = 105 × 40
= Rs. 4200 *(ii)*
Gross payment = 13440 + 4200
= **Rs. 17640**

4. ***Calculate the gross monthly pay of a person, if his basic pay is Rs. 18000, house rent allowances is Rs, 3500, dearness allowances is Rs. 3000, conveyance allowance is Rs. 1500 and medical allowance is Rs. 500.***

Solution:

Basic salary = Rs. 18000 *(i)*
house allowance = Rs. 3500 *(ii)*
dearness allowance = Rs. 3000 *(iii)*
conveyance allowance = Rs. 1500 *(iv)*
Medical allowance = Rs. 500 *(v)*
= 18000 + 3500 + 3000 + 1500 + 500

Gross monthly income = **Rs. 26500**

5. ***If gross pay of a person is Rs. 45,000, then calculate his net take home salary, after deductions of Rs. 400 as income tax, Rs. 1200 as benevolent fund, Rs. 1500 as G.P found and Rs. 400 as group insurance.***

Solution:

Gross income = Rs. 4500 *(i)*
Deduction = Rs. 400 *(i)*
Benevolent funds = Rs. 1200 *(ii)*
G.P funds = Rs. 1500 *(iii)*
amount of group insurance = Rs. 400 *(iv)*
Total payment of deduction = Rs. 3500
Net income = Gross Salary - total dedcution
= 45000 - 3500
= **Rs. 41,500**

6. ***Noman works in a factory where the basic hourly rate is Rs. 50 for a 35 hour week. An over time is paid at time and - a - half. How much will he earn in a week when he works for:***
(i) 38 hours (ii) 48 hours (iii) 50 hours

Solution:

Working time in a week = 35 hours
Wages of 1 hour work = Rs. 50
Wages of 35 hours work = 35 × 50
= Rs. 1750 (A)
Wages of over time work = 1.5 × 50
= Rs. 75

(1st condition)

over time = 38 - 35
= 3 hours
Total wages of over time = 75 × 3
= Rs. 225 (B)
Total wages A + B = 1750 + 225

2nd condition

= 1975

= **13 hours**

wages of extra time = 75 × 13

= Rs. 975 (C)

Total wages = 1750 + 975

3rd condition = **Rs. 2725**

over time = 50 - 35

= 15 hours

wages of over time = 75 × 15

= Rs. 1125

Total wages = 1750 + 1125

= **Rs. 2875**

7. ***Abdullah's pay slip showed that he had worked 6 hours over time in addition to his basic 36 hours week. If his basic rate to pay is Rs. 60 and overtime is paid at time and a - half. Find his gross pay for the month.***

Solution:

Time works in a week = 36 hours

wages for 1 hour = Rs. 60

wages of 1 week = 36 × 60

= Rs. 2160

Total weeks in a month = 4

Monthly Salary = 2160 × 4

= **Rs. 8640** **(A)**

Over time for 1 hour = 6 hours

Wages of 1 over time hour = 1.5 × 60

= Rs. 90

Wages of 1 hour = 90 × 6

= 540

Total weeks in a month = 4

over time wages = 540 × 4

= **Rs. 2160** **(B)**

= 8640 + 2160

Total gross ... = Rs. 10800

Exercise 7.1

1- ***Determine the radicals and the radicands from the following.***

(i) $\sqrt{3}$ (ii) $4+3\sqrt{a}$

(iii) $\sqrt{11}$ (iv) $8-2\sqrt{6}$

(v) $\frac{\sqrt{5}}{7}$ (vi) $\frac{9}{\sqrt{13}}$

Solution:-

	Radical	Radicands
(i) $\sqrt{3}$	$\sqrt{3}$	3
(iii) $\sqrt{11}$	$\sqrt{11}$	11
(v) $\frac{\sqrt{5}}{7}$	$\sqrt{5}$	5

	Radical	Radicands
(ii) $4+3\sqrt{a}$	$\sqrt{a}$	a
(iv) $8-2\sqrt{6}$	$\sqrt{6}$	6
(vi) $\frac{9}{\sqrt{13}}$	$\sqrt{13}$	13

2. ***Express the following in exponential form:***

(i) $\sqrt{a^3}$ (ii) $\sqrt[5]{a^3}$

(iii) $\frac{1}{\sqrt[p]{a^k}}$ (iv) $\frac{1}{\sqrt[b]{a^k}}$

Solution:-

(i) $\sqrt{a^3} = (a^3)^{\frac{1}{2}}$

$= a^{3\times\frac{1}{2}}$

$= a^{\frac{3}{2}}$

(ii) $\sqrt[5]{a^3} = (a^3)^{\frac{1}{5}}$

$= a^{3\times\frac{1}{5}}$

$= a^{\frac{3}{5}}$

(iii) $\frac{1}{\sqrt[p]{a^k}} = \frac{1}{(a^k)^{\frac{1}{p}}}$

$= \frac{1}{a^{k \times \frac{1}{p}}}$

$= a^{\frac{k}{p}}$

(vi) $\frac{1}{\sqrt[b]{a^k}} = \frac{1}{(a^k)^{\frac{1}{b}}}$

$= \frac{1}{a^{k \times \frac{1}{b}}}$

$= \frac{1}{a^{\frac{k}{b}}}$

$= a^{\frac{k}{b}}$

3. *Write in the radical form and evaluate the result.*

(i) $(25)^{1/2}$ (ii) $(64)^{1/3}$

(iii) $(81)^{1/4}$ (iv) $(27)^{1/3}$

(v) $(27)^{2/3}$ (vi) $8^{-1/3}$

(vii) $(1000)^{2/3}$ (viii) $(64)^{1/2}$

Solution:-(i)

$(25)^{1/2} = \sqrt{25}$

$= \sqrt{5^2}$

$= (5^2)^{\frac{1}{2}}$

$= 5^{2 \times \frac{1}{2}} = 5$

$\sqrt{25} = 5$ Ans.

Solution:-(ii)

$(64)^{1/3} = \sqrt[3]{54}$

$$= \sqrt[3]{4^3}$$

$$= (4^3)^{\frac{1}{3}}$$

$$= 4^{3 \times \frac{1}{3}} = 4$$

$$\sqrt[3]{64} = 4 \text{ Ans.}$$

Solution:-(iii)

$$(81)^{1/4} = \sqrt[4]{81}$$

$$= \sqrt[4]{3^4}$$

$$= (3^4)^{\frac{1}{4}}$$

$$= 3^{4 \times \frac{1}{4}} = 3$$

$$\sqrt[4]{81} = 3 \text{ Ans.}$$

Solution:-(iv)

$$(27)^{1/3} = \sqrt[3]{(27)}$$

$$= \sqrt[3]{(3^3)}$$

$$= 3^{3 \times \frac{1}{3}} = 3$$

$$\sqrt[3]{27} = 3$$

Solution:-(v)

$$(27)^{2/3} = \sqrt[3]{(27)^2}$$

$$= \sqrt[3]{(3^3)^2}$$

$$= \sqrt[3]{3^{3 \times 2}}$$

$$= 3^{3\times 2\times \frac{1}{3}}$$

$$= 3^2 = 9$$

$$\sqrt[3]{(27)^2} = 9 \text{ Ans.}$$

Solution:-(vi)

$$8^{-1/3} = \frac{1}{8^{\frac{1}{3}}}$$

$$= \frac{1}{\sqrt[3]{8}}$$

$$= \frac{1}{\sqrt[3]{2^3}}$$

$$= \frac{1}{2^{3\times\frac{1}{3}}}$$

$$= \frac{1}{2}$$

$$\frac{1}{\sqrt[3]{8}} = \frac{1}{2} \text{ Ans.}$$

Solution:-(vii)

$$(1000)^{2/3} = \sqrt[3]{(1000)^2}$$

$$= \sqrt[3]{(10^3)^2}$$

$$= \sqrt[3]{(10)^{3\times 2}}$$

$$= (10)^{3\times 2\times\frac{1}{3}}$$

$$= (10)^2 = 100$$

$$\sqrt[3]{(1000)^2} = 100 \text{ Ans.}$$

$$(64)^{1/2} = \sqrt{64}$$

$$= \sqrt{8^2}$$

$$= 8^{2\times\frac{1}{2}} = 8$$

$$\sqrt{64} = 8 \text{ Ans.}$$

4. ***Simplify and give answer in exponential form.***

(i) $\sqrt{a^{16}}$ **(ii)** $\sqrt[3]{a^{15}}$

(iii) $\sqrt[3]{27a^9}$ **(iv)** $\sqrt[3]{8a^9}$

(v) $\sqrt[4]{x^{32}}$ **(vi)** $\sqrt[4]{81x^{20}}$

(vii) $\sqrt[3]{125x^9y^{15}}$ **(viii)** $\sqrt{(8+y)^7}$

(ix) $\sqrt[4]{16x^2y^6}$ **(x)** $\sqrt[4]{\dfrac{x^5y^6}{z^2}}$

(xi) $\sqrt[3]{\dfrac{8x}{x+y}}$ **(xii)** $\sqrt[p]{\dfrac{y^n}{a^m}}$

Solution:-(i)

$$\sqrt{a^{16}} = (a^{16})^{\frac{1}{2}}$$

$$= a^{16\times\frac{1}{2}}$$

$$= a^8 \text{ Ans.}$$

Solution:-(ii)

$$\sqrt[3]{a^{15}} = (a^{15})^{\frac{1}{3}}$$

$$= a^{15\times\frac{1}{3}}$$

$$= a^5 \text{ Ans.}$$

Solution:-(iii)

$$\sqrt[3]{27a^9} = \sqrt[3]{3^3 a^9}$$

$$= (3^3 a^9)^{\frac{1}{3}}$$

$$= 3^{3\times\frac{1}{3}} a^{9\times\frac{1}{3}}$$

$$= 3a^3 \quad \text{Ans.}$$

Solution:-(iv)

$$\sqrt[3]{8a^9} = \sqrt[3]{2^3 a^9}$$

$$= (2^3 a^9)^{\frac{1}{3}}$$

$$= 2^{3\times\frac{1}{3}} a^{9\times\frac{1}{3}}$$

$$= 2a^3 \quad \text{Ans.}$$

Solution:-(v)

$$\sqrt[4]{x^{32}} = (x^{32})^{\frac{1}{4}}$$

$$= x^{32\times\frac{1}{4}}$$

$$= x^8 \quad \text{Ans.}$$

Solution:-(vi)

$$\sqrt[4]{81x^{20}} = \sqrt[4]{3^4 x^{20}}$$

$$= (3^4 x^{20})^{\frac{1}{4}}$$

$$= 3^{4\times\frac{1}{4}} x^{20\times\frac{1}{4}} \quad \text{Ans.}$$

$$= 3x^5$$

Solution:-(vii)

$$\sqrt[3]{125x^9y^{15}} = \sqrt[3]{5^3x^9y^{15}}$$

$$= (5^3x^9y^{15})^{\frac{1}{3}}$$

$$= 5^{3\times\frac{1}{3}}x^{9\times\frac{1}{3}}y^{15\times\frac{1}{3}}$$

$$= 5x^3y^5 \quad \text{Ans.}$$

Solution:-(viii)

$$\sqrt{(8+y)^7} = \left\{(8+y)^7\right\}^{\frac{1}{2}}$$

$$= (8+y)^{7\times\frac{1}{2}}$$

$$= (8+y)^{\frac{7}{2}}$$

Solution:-(ix)

$$\sqrt[4]{16x^2y^6} = \sqrt[4]{2^4x^2y^6} \quad \text{Ans.}$$

$$= (2^4x^2y^6)^{\frac{1}{4}}$$

$$= 2^{4\times\frac{1}{4}}x^{2\times\frac{1}{4}}y^{6\times\frac{1}{4}}$$

$$= 2x^{\frac{1}{2}}y^{\frac{3}{2}} \quad \text{Ans.}$$

Solution:-(x)

$$\sqrt[4]{\frac{x^5y^6}{z^2}} = \left(\frac{x^5y^6}{z^2}\right)^{\frac{1}{4}}$$

$$= \frac{x^{5\times\frac{1}{4}}y^{6\times\frac{1}{4}}}{z^{2\times\frac{1}{4}}}$$

$$= \frac{x^{\frac{5}{4}}y^{\frac{3}{2}}}{\quad} \quad \text{Ans.}$$

Solution:-(xi)

$$\sqrt[3]{\frac{8x}{x+y}} = \sqrt[3]{\frac{2^3 x}{x+y}}$$

$$= \left(\frac{2^3 x}{x+y}\right)^{\frac{1}{3}}$$

$$= \frac{2^{3\times\frac{1}{3}} x^{\frac{1}{3}}}{(x+y)^{\frac{1}{3}}}$$

$$= \frac{2x^{\frac{1}{3}}}{(x+y)^{\frac{1}{3}}} \text{ Ans.}$$

Solution:-(xii)

$$\sqrt[p]{\frac{y^m}{a^m}} = \left(\frac{y^m}{a^m}\right)^{\frac{1}{p}}$$

$$= \frac{y^{m\times\frac{1}{p}}}{a^{m\times\frac{1}{p}}}$$

$$= \frac{y^{\frac{m}{p}}}{a^{\frac{m}{p}}} \text{ Ans.}$$

5. Simplify:

(i) $\sqrt{3}\times\sqrt{7}$

(ii) $\sqrt[5]{4}\times\sqrt[5]{128}$

(iii) $\sqrt[5]{81}\times\sqrt[5]{27}$

(iv) $\sqrt{2}\div\sqrt[9]{32}$

(v) $\sqrt[5]{118}\div\sqrt[5]{2}$

(vi) $\sqrt{27}\div\sqrt{81}$

(vii) $a^{\frac{1}{4}}\times a^{\frac{2}{3}}$

(viii) $\frac{6}{\quad}$ $\frac{1}{\quad}$

(ix) $\left(x^{\frac{3}{4}} \times y^{\frac{1}{6}}\right)^{6}$ (x) $\left(x^3y^2\right)^{\frac{1}{2}} \times \left(y^3x^4\right)^{\frac{-1}{3}}$

(xi) $\left(x^3y^2\right)^{\frac{1}{4}} \times \left(x^{\frac{1}{3}}y\right)^{\frac{3}{4}}$ (xii) $\left(a^{\frac{1}{4}}b^{\frac{1}{3}}\right)^{\frac{-1}{2}} \div \left(a^{\frac{1}{3}}b^{\frac{1}{4}}\right)^{-5}$

(xiii) $\left(x^2y^3\right)^{\frac{1}{5}} \times \left(x^{\frac{1}{3}}y^2\right)^{\frac{1}{4}}$

Solution:-(i)

$$\sqrt{3} \times \sqrt{7} = \sqrt{3 \times 7}$$

$$= \sqrt{21} \text{ Ans.}$$

Solution:-(ii)

$$\sqrt[5]{4} \times \sqrt[5]{128} = \sqrt[5]{4 \times 128}$$

$$= \sqrt[5]{512} \text{ Ans.}$$

Solution:-(iii)

$$\sqrt[5]{81} \times \sqrt[5]{27} = \sqrt[5]{81 \times 27}$$

$$= \sqrt[5]{2187} \text{ Ans.}$$

Solution:-(iv)

$$\sqrt{2} \div \sqrt[9]{32} = \sqrt{2} \div \sqrt[9]{2^5} \text{ Ans.}$$

$$= \sqrt{2} \div \left(2^5\right)^{\frac{1}{9}}$$

$$= (2)^{\frac{1}{2}} \div \left(2^{5 \times \frac{1}{9}}\right)$$

$$= \frac{2^{\frac{1}{2}}}{2^{\frac{5}{9}}}$$

$$= 2^{\frac{1}{2}-\frac{5}{9}}$$

$$= 2^{\frac{-1}{18}}$$

$$= \frac{1}{2^{\frac{1}{18}}}$$

$$= \frac{1}{2^{\frac{1}{9}\times\frac{1}{2}}}$$

Solution:-(v)

$$= \sqrt{2^{9}}^{P} \text{ Ans.}$$

$$\sqrt[5]{118} \div \sqrt[5]{2} = \frac{\sqrt[5]{118}}{\sqrt[5]{2}}$$

$$= \sqrt[5]{\frac{118}{2}}$$

Solution:-(vi)

$$\sqrt[5]{59} \text{ Ans.}$$

$$\sqrt{27} \div \sqrt{81} = \frac{\sqrt{27}}{\sqrt{81}}$$

$$= \sqrt{\frac{27}{81}}$$

$$= \sqrt{\frac{3}{9}}$$

$$= \frac{\sqrt{3}}{3} \text{ Ans.}$$

Solution:-(vii)

$$a^{\frac{1}{4}} \times a^{\frac{2}{3}} = a^{\frac{1}{4}+\frac{2}{3}}$$

$$= a^{\frac{3+8}{12}}$$

$$= a^{\frac{11}{12}}$$

$$= a^{11\times\frac{1}{12}}$$

$$= \sqrt[12]{a^{11}} \quad \text{Ans.}$$

Solution:-(viii)

$$x^{\frac{6}{7}} y^{\frac{1}{4}} = x^{\frac{6\times4}{7\times4}} \times y^{\frac{1}{4}}$$

$$= \sqrt[4]{x^{\frac{24}{7}} y} \quad \text{Ans.}$$

Solution:-(ix)

$$\left(x^{\frac{3}{4}} y^{\frac{1}{6}}\right)^6 = \left(x^{\frac{3}{4}} y^{\frac{1}{6}}\right)^6$$

$$= x^{\frac{3}{4}\times6}\, x^{\frac{1}{6}\times6}$$

$$= x^{\frac{9}{2}} y$$

$$= y\sqrt[2]{x^9}$$

$$= y\sqrt{x^3} \quad \text{Ans.}$$

Solution:-(x)

$$\left(x^3 y^2\right)^{\frac{1}{2}} \times \left(y^3 x^4\right)^{-\frac{1}{3}} = x^{3\times\frac{1}{2}} y^{2\times\frac{1}{2}} \times y^{3\left(-\frac{1}{3}\right)} x^{4\left(-\frac{1}{3}\right)}$$

$$= x^{\frac{3}{2}} y^{-1} x^{-\frac{4}{3}}$$

$$= x^{\frac{3}{2}-\frac{4}{3}} y^{-1} \times y^1$$

$$= x^{\frac{9-8}{6}} \times y^{-1} \quad y^0 = 1$$

$$= x^{\frac{1}{6}}$$

$$= \sqrt[6]{x} \quad \text{Ans.}$$

Solution:-(xi)

$$\left(x^3y^2\right)^{\frac{1}{4}} \times \left(x^{\frac{1}{3}}y\right)^{\frac{3}{4}} = \left(x^{3\times\frac{1}{4}}y^{2\times\frac{1}{4}}\right) \times \left(x^{\frac{1}{3}\times\frac{3}{4}}y^{1\times\frac{3}{4}}\right)$$

$$= x^{\frac{3}{4}}y^{\frac{1}{2}} \times x^{\frac{1}{4}}y^{\frac{4}{3}}$$

$$= x^{\frac{3+1}{4}}y^{\frac{2+3}{4}}$$

$$= x^{\frac{4}{4}}y^{\frac{5}{4}}$$

$$= \sqrt[4]{x^4y^5} \quad \text{Ans.}$$

Solution:-(xii)

$$)\left(a^{\frac{1}{4}}b^{\frac{1}{3}}\right)^{-\frac{1}{2}} \div \left(a^{\frac{1}{3}}b^{\frac{1}{4}}\right)^{-5} = \left(a^{\frac{1}{4}\left(-\frac{1}{2}\right)}b^{\frac{1}{3}\left(-\frac{1}{2}\right)}\right) \div \left(a^{\frac{1}{3}(-5)}b^{\frac{1}{4}(-5)}\right)$$

$$= \left(a^{-\frac{1}{8}}b^{-\frac{1}{6}}\right) \div \left(a^{-\frac{5}{3}}b^{-\frac{5}{4}}\right)$$

$$= \frac{a^{-\frac{1}{8}}b^{-\frac{1}{6}}}{a^{-\frac{5}{3}}b^{-\frac{5}{4}}}$$

$$= a^{-\frac{1}{8}+\frac{5}{3}} \times b^{-\frac{1}{6}+\frac{5}{4}}$$

$$= a^{\frac{-3+40}{24}} \times b^{\frac{-2+15}{12}}$$

$$= a^{\frac{37}{24}} b^{\frac{13}{12}}$$

$$= a^{\frac{37}{2\times 12}} b^{\frac{13}{12}}$$

$$= \sqrt[12]{a^{\frac{37}{2}} b^{13}} \quad \text{Ans.}$$

Solution:-(xiii)

$$\left(x^2 y^3\right)^{\frac{1}{5}} \times \left(x^{\frac{1}{3}} y^2\right)^{\frac{1}{4}} = \left(x^{2\times\frac{1}{5}} y^{3\times\frac{1}{5}}\right) \times \left(x^{\frac{1}{3}\times\frac{1}{4}} y^{2\times\frac{1}{4}}\right)$$

$$= x^{\frac{2}{5}} y^{\frac{3}{5}} \times x^{\frac{1}{12}} y^{\frac{1}{2}}$$

$$= x^{\frac{2}{5}} \times x^{\frac{1}{12}} \times y^{\frac{3}{5}} \times y^{\frac{1}{2}}$$

$$= x^{\frac{2}{5}+\frac{1}{12}} \times y^{\frac{3}{5}+\frac{1}{2}}$$

$$= x^{\frac{24+5}{60}} \times y^{\frac{6+5}{10}}$$

$$= x^{\frac{29}{60}} \times y^{\frac{11}{10}} \quad \text{Ans.}$$

Exercise 6.2

1- ***Write the base and exponent in the following.***

(i) $16x^3$ **(ii)** x^9 **(iii)** $(4y)^3$

(iv) $(x-2)^3$ **(v)** $18x^5$ **(vi)** $5x^{\frac{3}{2}} \times x^{\frac{1}{2}}$

Solution:-

(i) $16x^3$

Base = x ; = exponent 3

(ii) x^9

Base = x ; = exponent 9

(iii) $(4y)^3$

Base = $4y$; = exponent 3

(iv) $(x-2)^3$

Base = $(x-2)$; = exponent 3

(v) $18x^5$

Base = x ; = exponent 5

(vi) $5x^{\frac{3}{2}} \times x^{\frac{1}{2}}$

$= 5x^{\frac{3}{2}+\frac{1}{2}} = 5x^2$

Base = x ; = exponent 2

Simplify and express with positive indices

2- $\sqrt{(a^2b^3)^6}$ **3-** $\sqrt[9]{(x^{-4}y^3)^{-3}}$

4- $(x^a y^{-b})^3 \times (x^3 y^2)^{-a}$ **5-** $\left(\frac{16x^2}{y^{-2}}\right)^{\frac{-1}{4}}$

6- $\left(\frac{27x^3}{8a^{-3}}\right)^{\frac{-2}{3}}$

7- $\left(\frac{a^{\frac{-1}{2}}}{4c^2}\right)^{-2}$

8- $\sqrt{a^{-2}b}\times 3\sqrt{ab^{-3}}$

9- $\left(\frac{a^{-3}}{b^{\frac{-2}{3}}c}\right)^{\frac{-3}{2}} \div \frac{ab^2c}{a^2c}$

10- $\frac{(a^4)^3(a^{-1}b)^{10}}{a^2b^7}$

11- $\frac{(x^3y)^3(2xy)^{-2}}{4x^{-4}y^{-5}}$

12- $\frac{(a^{-5})^3(ab)^{15}}{a^{-1}b^2}$

13- $a^5b^4c^2 \div abc$

14- $(2ab^2)^2(3abc^2)^{-2} \div (ab)^{-4}(bca)^5$

15- $\frac{2^3\times 6^5}{3^{-3}\times 4^{-4}}$

16- $\frac{2^5\times 9^{-1}}{27^{-3}\times 8^{-3}}$

17- $(2^{-3}a^4b)^{-1}\times(4^{-2}b^{-5})$

18- $(3^2)^5 \div 9^3 \times 27^{-1}$

19- $\left(\frac{3}{4}\right)^{-2} \div \left(\frac{4}{9}\right)^3 \times \left(\frac{27}{16}\right)^{-1}$

20- $\left(\frac{2}{3}\right)^{-1} \div \left(\frac{4}{9}\right)^{-2} \times 27$

21- $\frac{5^4}{3^7}\times\left(\frac{9}{15}\right)^3 \div \frac{27}{25}$

22- $a^{\frac{1}{2}}b^{\frac{2}{3}} \times a^{\frac{2}{3}}b^{\frac{1}{4}}$

23- $a^{\frac{2}{3}}b^{\frac{5}{6}} \times a^{\frac{1}{2}}b \div (ab)^{\frac{1}{3}}$

24- $(a^{\frac{1}{2}}b^{\frac{1}{3}}c^{\frac{1}{4}})^6$

25- $(a^{\frac{1}{2}}b^{\frac{1}{3}})^{\frac{4}{3}} \div (a^{\frac{1}{3}}b^{\frac{1}{4}})^{\frac{1}{2}}$

26- $a^{\frac{2}{3}} \times a^{\frac{1}{2}} \div a^{\frac{1}{4}}$

2. $\sqrt{(a^3b^3)^6}$

Solution:-

$$= \sqrt{a^{2\times6} b^{3\times6}}$$

$$= \sqrt{a^{12} b18}$$

$$= (a^{12} b^{18})^{\frac{1}{2}} = a^{12\times\frac{1}{2}}\ b^{18\times\frac{1}{2}}$$

$$= a^6 b^9$$

3. $\sqrt[9]{(x^{-4} y^3)^{-3}}$ Ans.

Solution:-

$$= \sqrt[9]{x^{(-4)(-3)} \times y^{3(-3)}}$$

$$= (x^{12} \times y^{-9})^{\frac{1}{9}}$$

$$= x^{12(\frac{1}{9})} \times y^{-9(\frac{1}{9})}$$

$$= x^{\frac{4}{3}} \times y^{-1}$$

$$= \frac{x^{\frac{4}{3}}}{y}$$ Ans.

4. $(x^a y^{-b})^3 \times (x^3 y^2)^{-a}$

Solution:-

$$= (x^{(a)(3)} y^{(-b)(3)}) \times (x^{(3)(-a)} y^{(2)(-a)})$$

$$= (x^{3a} y^{-3b}) \times (x^{-3a} y^{-2a})$$

$$= x^{3a} \times y^{-3b} \times x^{-3a} \times y^{-2a}$$

$$= x^{3a} \times x^{-3a} \times y^{-3b} \times y^{-2a}$$

$$= x^{3a-3b} \times y^{-3b-2a}$$

$$= x^0 \times y^{\frac{1}{3b+2a}}$$

$$= 1 \times y^{\frac{1}{3b+2a}} = y^{\frac{1}{2a+3b}} \text{ Ans.}$$

5. $\left(\frac{16x^2}{y^{-2}}\right)^{-\frac{1}{4}}$

Solution:-

$$= \left(\frac{2^4 x^2}{y^{-2}}\right)^{-\frac{1}{4}}$$

$$= \frac{2^{4(-\frac{1}{4})} x^{2(-\frac{1}{4})}}{y^{-2(-\frac{1}{4})}}$$

$$= \frac{2^{-1} x^{\frac{1}{2}}}{y^{\frac{1}{2}}} = \frac{1}{2x^{\frac{1}{2}} y^{\frac{1}{2}}} \text{ Ans.}$$

6. $= \left(\frac{27x^3}{8a^{-3}}\right)^{-\frac{2}{3}}$

Solution:-

$$= \left(\frac{3^3 x^3}{2^3 a^{-3}}\right)^{-\frac{2}{3}}$$

$$= \frac{3^{3(-\frac{2}{3})} x^{3(-\frac{2}{3})}}{2^{3(-\frac{2}{3})} a^{-3(-\frac{2}{3})}}$$

$$= \frac{3^{-2} x^{-2}}{2^{-2} a^2}$$

$$= \frac{2^2}{3^2 \times a^2 \times x^2}$$

$$= \frac{4}{9a^2x^2} \text{ Ans.}$$

7. $$\left(\frac{a^{-\frac{1}{2}}}{4c^2}\right)^{-2}$$

Solution:-

$$= \left(\frac{a^{-\frac{1}{2}}}{2^2 c^2}\right)^{-2}$$

$$= \frac{a^{(-\frac{1}{2})(-2)}}{2^{(2)(-2)} c^{(2)(-2)}}$$

$$= \frac{a}{2^{-4} c^{-4}} = 2^4 ac^4$$

$$= 16ac^4 \text{ Ans.}$$

8. $$\sqrt{a^{-2}b} \times \sqrt[3]{ab^{-3}}$$

Solution:-

$$= (a^{-2}b)^{\frac{1}{2}} \times 3 \times (ab^{-3})^{\frac{1}{2}}$$

$$= 3 \times (a^{(-2)(\frac{1}{2})} \times b^{\frac{1}{2}}) \times (a^{\frac{1}{2}} b^{-3(\frac{1}{2})})$$

$$= 3a^{-1} \times b^{\frac{1}{2}} \times a^{\frac{1}{2}} \times b^{\frac{-3}{2}}$$

$$= 3a^{-1} \times a^{\frac{1}{2}} \times b^{\frac{1}{2}} \times b^{\frac{-3}{2}}$$

$$= 3a^{-1+\frac{1}{2}} \times b^{\frac{1}{2}-\frac{3}{2}}$$

$$= 3a^{-\frac{1}{2}} \times b^{-\frac{2}{2}}$$

$$= 3a^{-\frac{1}{2}} \times b^{-1} = \frac{3}{a^{\frac{1}{2}}b}$$ Ans.

9. $$\left(\frac{a^{-3}}{b^{-\frac{2}{3}}c}\right)^{-\frac{3}{2}} \div \frac{ab^2c}{a^2c}$$

Solution:-

$$= \frac{a^{(-3)(-\frac{3}{2})}}{b^{(-\frac{2}{3})(-\frac{3}{2})} \times c^{-\frac{3}{2}}} \times \frac{a^2c}{ab^2c}$$

$$= \frac{a^{\frac{9}{2}}}{b \times c^{-\frac{3}{2}}} \times \frac{a^2}{a^1b^2}$$

$$= \frac{a^{\frac{9}{2}+2-1}c^{\frac{3}{2}}}{b^{1+2}}$$

$$= \frac{a^{\frac{11}{2}}c^{\frac{3}{2}}}{b^3}$$ Ans.

10. $$\frac{(a^4)^3(a^{-1}b)^{10}}{a^2b^7}$$

Solution:-

$$= \frac{(a^{4\times3})(a^{(-1)(10)}b^{(1)(10)})}{a^2b^7}$$

$$= \frac{a^{12} \times a^{-10} \times b^{10}}{a^2 b^7}$$

$$= a^{12-10-2} \times b^{10-7} = a^0 b^3$$

$$= 1 \times b^3 = b^3 \text{ Ans.}$$

11. $\dfrac{(x^3 y)^3 (2xy)^{-2}}{4x^{-4} y^{-5}}$

Solution:-

$$= \frac{(x^{3\times3} y^{1\times3})(2^{-2} x^{-2} y^{-2})}{4x^{-4} y^{-5}}$$

$$= \frac{(x^9 y^3)(^{-2} x^{-2} y^{-2})}{2^2 \times 4x^{-4} y^{-5}}$$

$$= \frac{x^{9-2+4} \times y^{3-2+5}}{4\times4} = \frac{x^{11} y^6}{16} \text{ Ans.}$$

12. $\dfrac{(a^{-5})^3 \times (ab)^{15}}{a^{-1} b^2}$

Solution:-

$$= \frac{a^{(-5)(3)} \times a^{1\times15} b^{1\times15}}{a^{-1} \times b^2}$$

$$= \frac{a^{-15} \times a^{15} b^{15}}{a^{-1} \times b^2}$$

$$= a^{-15+15+1} \times b^{15-2} = ab^{13} \text{ Ans.}$$

13. $a^5 b^4 c^2 \div abc$

Solution:-

$$= a^5 b^4 c^2 \times \frac{1}{abc}$$

$= a^{5-1}b^{4-1}c^{2-1} = a^4b^3c$ Ans.

14. $(2ab^2)^2(3abc^2)^{-2} \div (ab)^{-4}(bca)^5$

Solution:-

$= (2^2a^2b^{2\times2})(3^{-2}a^{-2}b^{-2}c^{2(-2)}) \div (a^{-4}b^{-4})(b^5c^5a^5$

$$= \frac{(4a^2b^4)(\frac{1}{3^2}a^{-2}b^{-2}c^{-4})}{(a^{-4}b^{-4})(b^5c^5a^5)}$$

$$= \frac{4a^2b^4a^{-2}b^{-2}c^{-4}}{9a^{-4}b^{-4}b^5c^5a^5}$$

$$= \frac{4a^2a^{-2}b^4b^{-2}c^{-4}}{9a^{-4}a^5b^{-4}b^5c^5}$$

$$= \frac{4a^0b^2c^{-4}}{9abc^5}$$

$$= \frac{4a^0b^2c^{-4}}{9abc^5} = \frac{4}{9}a^{0-1}b^{2-1}c^{-4-5} = \frac{4}{9}a^{-1}bc^{-9}$$

$$= \frac{4b}{9ac^9}$$ Ans.

15. $\dfrac{2^3 \times 6^5}{3^{-3} \times 4^{-4}}$

Solution:-

$$= \frac{2^3 \times (2\times3)^5}{3^{-3} \times (2\times2)^{-4}}$$

$$= \frac{2^3 \times 2^5 \times 3^5}{3^{-3} \times 2^{-4} \times 2^{-4}}$$

$$= \frac{2^3 \times 2^5 \times 3^5}{\quad}$$

$$= 2^{3+5+4+4} \times 3^{5+3} = 2^{16} 3^8$$

$$= 65536 \times 6561 = 429981696 \text{ Ans.}$$

also $= \dfrac{2^3 \times 6^5}{3^{-3} \times 4^{-4}} = 2^3 \times 6^5 \times 3^3 \times 4^4$

$$= 2^3 \times 4^4 \times 3^3 \times 6^5 \text{ Ans.}$$

16. $\dfrac{2^5 \times 9^{-1}}{(27)^{-3} \times (8)^{-3}}$

Solution:-

$$= \frac{2^5 \times (3^2)^{-1}}{(3^3)^{-3} \times (2^3)^{-3}} = \frac{2^5 \times 3^{-2}}{3^{-9} \times 2^{-9}}$$

$$= 2^{5+9} \times 3^{-2+9} = 2^{14} \times 3^7 \text{ Ans.}$$

17. $(2^{-3} a^4 b)^{-1} \times (4^{-2} b^{-5})$

Solution:-

$$= 2^{(-3)(-1)} \times a^{4(-1)} \times b^{1(-1)} \times 4^{-2} b^{-5}$$

$$= 2^3 \times a^{-4} \times b^{-1} \times \frac{1}{4^2} \times b^{-5}$$

$$= 8a^{-4} b^{-1-5} \times \frac{1}{16}$$

$$= \frac{1}{2} \times \frac{1}{a^4} \times \frac{1}{b^6} \qquad \text{as } -1-5=-6$$

$$= \frac{1}{2a^4 b^6} \text{ Ans.}$$

18. $(3^2)^5 \div 9^3 \times (27)^{-1}$

Solution:-

$$= 3^{2\times5} \times \frac{1}{9^3 \times (27)^{-1}} = 3^{10} \times \frac{27}{9^3}$$

$$= \frac{3^{10} \times 3^3}{(3^2)^3} = \frac{3^{10} \times 3^3}{3^6} = 3^{10-6} \times 3^3 = 3^{4+3} = 3^7$$

$$= 3\times3\times3\times3\times3\times3\times3 = 2187 \text{ Ans.}$$

19. $\left(\frac{3}{4}\right)^{-2} \div \left(\frac{4}{9}\right)^3 \times \left(\frac{27}{16}\right)^{-1}$

Solution:-

$$= \frac{3^{-2}}{4^{-2}} \div \left(\frac{2^2}{3^2}\right)^3 \times \left(\frac{3^3}{2^4}\right)^{-1}$$

$$= \frac{4^2}{3^2} \div \left(\frac{2^{2\times3}}{3^{2\times3}}\right) \times \left(\frac{3^{3(-1)}}{2^{4(-1)}}\right)$$

$$= \frac{(2^2)^2}{3^2} \div \left(\frac{2^6}{3^6}\right) \times \left(\frac{3^{-3}}{2^{-4}}\right)$$

$$= \frac{2^{2(2)}}{3^2} \div (2^{6+4} \times 3^{-3-6})$$

$$= \frac{2^4}{3^2} \div (2^{10} \times 3^{-9})$$

$$= \frac{2^4}{3^2} \times \frac{1}{2^{10} \times 3^{-9}} = 2^{4-10} \times 3^{-2+9}$$

$$= 2^{-6} \times 3^7 = \frac{3^7}{2^6} = \frac{2187}{64} \text{ Ans.}$$

20. $\left(\frac{2}{3}\right)^{-1} \div \left(\frac{4}{9}\right)^{-2} \times 27$

Solution:-

$$= \frac{2^{-1}}{3^{-1}} \div \left(\frac{2^2}{3^2}\right)^{-2} \times 3^3$$

$$= \frac{3}{2} \div \frac{2^{2(-2)}}{3^{2(-2)}} \times 3^3 = \frac{3}{2} \div \frac{2^{-4}}{3^{-4}} \times 3^3$$

$$= \frac{3}{2} \div \frac{1}{2^4} \times 3^{3+4} = \frac{3}{2} \div \frac{3^7}{2^4}$$

$$= \frac{3}{2} \times \frac{2^4}{3^7} = \frac{2^{4-1}}{3^{7-1}}$$

$$= \frac{2^3}{3^6} = \frac{8}{729} \text{ Ans.}$$

21. $\frac{5^4}{3^4} \times \left(\frac{9}{15}\right)^3 \div \frac{27}{25}$

Solution:-

$$= \frac{5^4}{3^4} \times \left(\frac{3}{5}\right)^3 \div \frac{3^3}{5^2} = \frac{5^4}{3^4} \times \frac{3^3}{5^3} \times \frac{5^2}{3^3}$$

$$= \frac{5^{4+2-3}}{3^{4+3-3}} = \frac{5^3}{3^4} = \frac{125}{81} \text{ Ans.}$$

22. $a^{\frac{1}{3}} b^{\frac{2}{3}} \times a^{\frac{2}{3}} b^{\frac{1}{4}}$

Solution:-

$$= a^{\frac{1}{3}} \times a^{\frac{2}{3}} \times b^{\frac{2}{3}} \times b^{\frac{1}{4}}$$

$$= a^{\frac{1}{3}+\frac{2}{3}} \times b^{\frac{2}{3}+\frac{1}{4}} = a^{\frac{1+2}{3}} \times b^{\frac{8+3}{12}}$$

$$= a^{\frac{3}{3}} \times b^{\frac{11}{12}} = ab^{\frac{11}{12}} \text{ Ans.}$$

23. $a^{\frac{2}{3}} b^{\frac{5}{6}} \times a^{\frac{1}{2}} b \div (ab)^{\frac{1}{3}}$

Solution:-

$$= a^{\frac{2}{3}} b^{\frac{5}{6}} \times a^{\frac{1}{2}} b \div (a^{\frac{1}{3}} b^{\frac{1}{3}})$$

$$= \frac{a^{\frac{2}{3}} b^{\frac{5}{6}} \times a^{\frac{1}{2}} b}{a^{\frac{1}{3}} b^{\frac{1}{3}}}$$

$$= a^{\frac{2}{3}+\frac{1}{2}-\frac{1}{3}} \times b^{\frac{5}{6}+1-\frac{1}{3}}$$

$$= a^{\frac{4+3-2}{6}} \times b^{\frac{5+6-2}{6}}$$

$$= a^{\frac{5}{6}} \times b^{\frac{9}{6}} = a^{\frac{5}{6}} \times b^{\frac{3}{2}} \text{ Ans.}$$

24. $(a^{\frac{1}{2}} b^{\frac{1}{3}} c^{\frac{1}{4}})^6$

Solution:-

$$= a^{\frac{1}{2}(6)} b^{\frac{1}{3}(6)} c^{\frac{1}{4}(6)}$$

$$= a^3 b^2 c^{\frac{3}{2}} \text{ Ans.}$$

25. $(a^{\frac{1}{8}} b^{\frac{1}{3}})^{\frac{4}{3}} \div (a^{\frac{1}{3}} b^{\frac{1}{4}})^{\frac{1}{2}}$

Solution:-

$$= \left(a^{(\frac{1}{8})(\frac{4}{3})} b^{(\frac{1}{3})(\frac{4}{3})} \right) \div \left(a^{(\frac{1}{3})(\frac{1}{2})} b^{(\frac{1}{4})(\frac{1}{2})} \right)$$

$$= (a^{\frac{1}{6}} b^{\frac{4}{9}}) \div (a^{\frac{1}{6}} b^{\frac{1}{8}})$$

$$= \frac{a^{\frac{1}{6}} b^{\frac{4}{9}}}{a^{\frac{1}{6}} b^{\frac{1}{8}}} = a^{\frac{1}{6}-\frac{1}{6}} \times b^{\frac{4}{9}-\frac{1}{8}}$$

$$= a^{0} \times b^{\frac{32-9}{72}} = 1 \times b^{\frac{32-9}{72}} = b^{\frac{23}{72}} \text{ Ans.}$$

26. $a^{\frac{2}{3}} \times a^{\frac{1}{2}} \div a^{\frac{1}{4}}$

Solution:-

$$= \frac{a^{\frac{2}{3}} \times a^{\frac{1}{2}}}{a^{\frac{1}{4}}} = a^{\frac{2}{3}+\frac{1}{2}-\frac{1}{4}} \text{ Ans.}$$

$$= a^{\frac{8+6-3}{12}} = a^{\frac{11}{12}}$$

27- Simplify each of the following.

(i) $4^{\frac{3}{5}} \times 4^{\frac{1}{5}}$ (ii) $2^{\frac{1}{8}} \times 2^{\frac{3}{8}}$

(iii) $5x^{\frac{1}{3}} \times 2x^{\frac{1}{5}}$ (iv) $x^{\frac{3}{4}} \times x^{\frac{2}{5}}$

(v) $\frac{1}{2} y^{\frac{3}{7}} \times 4y^{\frac{2}{7}}$ (vi) $5x^{\frac{3}{2}} \times x^{\frac{1}{2}}$

27 (i) $4^{\frac{3}{5}} \times 4^{\frac{1}{5}}$

Solution:-

$$= 4^{\frac{3}{5}+\frac{1}{5}} = 4^{\frac{3+1}{5}} = 4^{\frac{4}{5}} \text{ Ans.}$$

27 (ii) $2^{\frac{1}{8}} \times 2^{\frac{3}{8}}$

Solution:-

$$= 2^{\frac{1}{8}+\frac{3}{8}} = 2^{\frac{1+3}{8}} = 2^{\frac{4}{8}} = 2^{\frac{1}{2}} \text{ Ans.}$$

27 (iii) $5x^{\frac{1}{3}} \times 2x^{\frac{1}{5}}$

Solution:-

$$= x^{\frac{3}{4}+\frac{2}{5}} = x^{\frac{15+8}{20}} = x^{\frac{23}{20}} \text{ Ans.}$$

27 (iv) $x^{\frac{3}{4}} \times x^{\frac{2}{5}}$

Solution:-

$$= 5 \times 2 \times x^{\frac{1}{3}} \times x^{\frac{1}{5}} = 10x^{\frac{1}{3}+\frac{1}{5}}$$

$$= 10x^{\frac{5+3}{15}} = 10x^{\frac{8}{15}} \text{ Ans.}$$

27 (v) $\frac{1}{2}y^{\frac{3}{7}} \times 4y^{\frac{2}{7}}$

Solution:-

$$= \frac{1}{2} \times 4 \times y^{\frac{3}{7}} \times y^{\frac{2}{7}}$$

$$= 2y^{\frac{3}{7}+\frac{2}{7}} = 2y^{\frac{3+2}{7}} = 2y^{\frac{5}{7}} \text{ Ans.}$$

27 (vi) $5x^{\frac{3}{2}} \times x^{\frac{1}{2}}$

Solution:-

$$= 5x^{\frac{3}{2}+\frac{1}{2}} = 5x^{\frac{3+1}{2}}$$

$$= 5x^{\frac{4}{2}} = 5x^2 \text{ Ans.}$$

28- *Simplify each of the following.*

(i) $a^{\frac{2}{3}}b^{\frac{3}{4}} \times a^{\frac{1}{3}}b^{\frac{3}{4}}$ **(ii)** $x^{\frac{3}{5}}y^{\frac{2}{9}} \times x^{\frac{1}{5}}y^{\frac{1}{3}}$

(iii) $2ab^{\frac{1}{3}} \times 3a^{\frac{3}{5}}b^{\frac{4}{5}}$ **(iv)** $6x^{\frac{3}{7}} \times \frac{1}{3}x^{\frac{1}{4}}y^{\frac{2}{5}}$

(v) $x^3y^{\frac{1}{2}}z^{\frac{1}{3}} \times x^{\frac{1}{6}}y^{\frac{1}{3}}z^{\frac{1}{2}}$

28 (i) $a^{\frac{2}{3}}b^{\frac{3}{4}} \times a^{\frac{1}{3}}b^{\frac{3}{4}}$

Solution:-

$$= a^{\frac{2}{3}} \times a^{\frac{1}{3}} \times b^{\frac{3}{4}} \times b^{\frac{3}{4}}$$

$$= a^{\frac{2}{3}+\frac{1}{3}} \times b^{\frac{3}{4}+\frac{3}{4}} = a^{\frac{2+1}{3}} \times b^{\frac{3+3}{4}}$$

$$= a^{\frac{3}{3}} \times b^{\frac{6}{4}} = a \times b^{\frac{3}{2}} \text{ Ans.}$$

28 (ii) $x^{\frac{3}{5}}y^{\frac{2}{9}} \times x^{\frac{1}{5}}y^{\frac{1}{3}}$

Solution:-

$$= x^{\frac{3}{5}} \times x^{\frac{1}{5}} \times y^{\frac{2}{9}} \times y^{\frac{1}{3}}$$

$$= x^{\frac{3}{5}+\frac{1}{5}} y^{\frac{2}{9}+\frac{1}{3}} = x^{\frac{3+1}{5}} \times y^{\frac{2+3}{9}}$$

$$= x^{\frac{4}{5}} \times y^{\frac{5}{9}} = x^{\frac{4}{5}}y^{\frac{5}{9}} \text{ Ans.}$$

28 (iii) $2ab^{\frac{1}{3}} \times 3a^{\frac{3}{5}}b^{\frac{4}{5}}$

Solution:-

$$= 2 \times 3a \times a^{\frac{3}{5}} \times b^{\frac{1}{3}} \times b^{\frac{4}{5}}$$

$$= 6a^{1+\frac{3}{5}} \times b^{\frac{1}{3}+\frac{4}{5}} = 6a^{\frac{5+3}{5}} \times b^{\frac{5+12}{15}}$$

$$= 6a^{\frac{8}{5}} \times b^{\frac{17}{15}} = 6a^{\frac{8}{5}} b^{\frac{17}{15}} \text{ Ans.}$$

28 (iv) $6x^{\frac{3}{7}} \times \frac{1}{3} x^{\frac{1}{4}} y^{\frac{2}{5}}$

Solution:-

$$= 6 \times \frac{1}{3} \times x^{\frac{3}{7}} \times x^{\frac{1}{4}} y^{\frac{2}{5}}$$

$$= 2x^{\frac{3}{7}+\frac{1}{4}} y^{\frac{2}{5}} = 2x^{\frac{12+7}{28}} \times y^{\frac{2}{5}}$$

$$= 2x^{\frac{19}{28}} y^{\frac{2}{5}} \text{ Ans.}$$

28 (v) $x^3 y^{\frac{1}{2}} z^{\frac{1}{3}} \times x^{\frac{1}{6}} y^{\frac{1}{3}} z^{\frac{1}{2}}$

Solution:-

$$= x^3 \times x^{\frac{1}{6}} \times y^{\frac{1}{2}} \times y^{\frac{1}{3}} \times z^{\frac{1}{3}} \times z^{\frac{1}{2}}$$

$$= x^{3+\frac{1}{6}} \times y^{\frac{1}{2}+\frac{1}{3}} z^{\frac{1}{3}+\frac{1}{2}}$$

$$= x^{\frac{18+1}{6}} \times y^{\frac{3+2}{6}} \times z^{\frac{2+3}{6}}$$

$$= x^{\frac{19}{6}} y^{\frac{5}{6}} z^{\frac{5}{6}} \text{ Ans.}$$

29- Simplify each of the following.

(i) $3^{\frac{1}{2}} \div 3^{\frac{1}{3}}$

(ii) $\dfrac{x^{\frac{4}{4}}}{x^{\frac{5}{9}}}$

(iii) $\dfrac{2x^{\frac{3}{}}}{4x^{\frac{3}{5}}}$

(iv) $\dfrac{25y^{\frac{3}{5}}}{20y^{\frac{1}{4}}}$

(v) $x^3y^2 \div x^{\frac{4}{3}}y^{\frac{3}{5}}$ (vi) $a^{\frac{5}{9}}b^{\frac{2}{3}} \div a^{\frac{2}{5}}b^{\frac{2}{5}}$

(vii) $10x^{\frac{4}{5}}y \div 5x^{\frac{2}{3}}y^{\frac{1}{4}}$ (viii) $\dfrac{5a^{\frac{3}{4}}b^{\frac{3}{5}}}{20a^{\frac{1}{5}}b^{\frac{1}{4}}}$

Solution:-29 (i)

$$3^{\frac{1}{2}} \div 3^{\frac{1}{3}} = \frac{3^{\frac{1}{2}}}{3^{\frac{1}{3}}} = 3^{\frac{1}{2}-\frac{1}{3}}$$

$$= 3^{\frac{3-2}{6}} = 3^{\frac{1}{6}} \text{ Ans.}$$

Solution:-29 (ii)

$$\frac{x^{\frac{4}{4}}}{x^{\frac{5}{9}}}$$

$$= x^{\frac{4}{5}-\frac{5}{9}} = x^{\frac{36-25}{45}} = x^{\frac{11}{45}} \text{ Ans.}$$

Solution:-29 (iii)

$$\frac{2x^{\frac{3}{4}}}{4x^{\frac{3}{5}}}$$

$$= \frac{2}{4}x^{\frac{3}{4}-\frac{3}{5}}$$

$$= \frac{1}{2}x^{\frac{15-12}{20}} = \frac{1}{2}x^{\frac{3}{20}} \text{ Ans.}$$

Solution:-29 (iv)

$$\frac{25y^{\frac{3}{5}}}{20y^{\frac{1}{4}}}$$

$$= \frac{5}{4} \times y^{\frac{3}{5}-\frac{1}{4}} = \frac{5}{4} \times y^{\frac{12-5}{20}}$$

$$= \frac{5}{4} y^{\frac{7}{20}} \text{ Ans.}$$

Solution:-29(v)

$$x^3 y^2 \div x^{\frac{4}{3}} y^{\frac{3}{5}}$$

$$= \frac{x^3 y^2}{x^{\frac{4}{3}} y^{\frac{3}{5}}}$$

$$= x^{3-\frac{4}{3}} \times y^{2-\frac{3}{5}}$$

$$= x^{\frac{9-4}{3}} \times y^{\frac{10-3}{5}} = x^{\frac{5}{3}} y^{\frac{7}{5}} \text{ Ans.}$$

Solution:- 29(vi)

$$a^{\frac{5}{9}} b^{\frac{2}{3}} \div a^{\frac{2}{5}} b^{\frac{2}{5}}$$

$$= \frac{a^{\frac{5}{9}} b^{\frac{2}{3}}}{a^{\frac{2}{5}} b^{\frac{2}{5}}}$$

$$= a^{\frac{5}{9}-\frac{2}{5}} \times b^{\frac{2}{3}-\frac{2}{5}}$$

$$= a^{\frac{25-18}{45}} \times b^{\frac{10-6}{15}}$$

$$= a^{\frac{7}{45}} \times b^{\frac{4}{15}} = a^{\frac{7}{45}} b^{\frac{4}{15}} \text{ Ans.}$$

Solution:-29 (vii)

$$10x^{\frac{4}{5}} y \div 5x^{\frac{2}{3}} y^{\frac{1}{4}}$$

$$= \frac{10x^{\frac{4}{5}} y}{5x^{\frac{2}{3}} y^{\frac{1}{4}}}$$

$$= 2x^{\frac{4}{5}-\frac{2}{3}} \times y^{1-\frac{1}{4}} = 2x^{\frac{12-10}{15}} \times y^{\frac{4-1}{4}}$$

$$= 2x^{\frac{2}{15}} \times y^{\frac{3}{4}} = 2x^{\frac{2}{15}} y^{\frac{3}{4}} \text{ Ans.}$$

Solution:-29 (viii)

$$\frac{5a^{\frac{3}{4}} b^{\frac{3}{5}}}{20a^{\frac{1}{5}} b^{\frac{1}{4}}}$$

$$= \frac{1}{4} a^{\frac{3}{4}-\frac{1}{5}} \times b^{\frac{3}{5}-\frac{1}{4}}$$

$$= \frac{1}{4} a^{\frac{15-4}{20}} \times b^{\frac{12-5}{20}}$$

$$= \frac{1}{4} a^{\frac{11}{20}} \times b^{\frac{7}{20}} = \frac{1}{4} a^{\frac{11}{20}} b^{\frac{7}{20}} \text{ Ans.}$$

Exercise 6.3

Write the following in scientific notation:

1- *0.051*

2- *89.99*

3- *0.424*

4- *2566324*

5- *0.00000075*

Write the following in the decimal form:

6- 0.86×10^{4}

7- 1.345×10^{-5}

8- 5.1×10^{-9}

9- 0.525×10^{-7}

10- 636.5×10^{-6}

Simplify and write your answer in scientific notation:

11- $\dfrac{0.96\times10^{7}}{2\times10^{4}}$

12- $\dfrac{2.61\times4\times10^{8}}{10^{3}}$

13- $\dfrac{521\times10^{3}\times12}{2\times10^{2}}$

Solution:-1.

$$0.051$$

$$=\frac{51}{1000}=\frac{51}{10^{3}}$$

$$=51\times10^{-3}=5.1\times10\times10^{-3}$$

$$=51\times10^{-3+1}=5.1\times10^{-2}$$

Solution:-2.

$$89.99$$

$$= \frac{8999}{100} = \frac{8999}{10^2}$$

$$= 8999 \times 10^{-2}$$

$$= (8.999 \times 1000) \times 10^{-2}$$

$$= 8.999 \times 10^3 \times 10^{-2}$$

$$= 8.999 \times 10^{3-2}$$

$$= 8.999 \times 10 \text{ Ans.}$$

Solution:-3.

0.424

$$= \frac{424}{1000} = \frac{424}{10^3}$$

$$= 424 \times 10^{-3} = (4.24 \times 100) \times 10^{-3}$$

$$= 4.24 \times 10^2 \times 10^{-3} = 4.24 \times 10^{2-3}$$

$$= 4.24 \times 10^{-1} \text{ Ans.}$$

Solution:-4.

2566324

$$= 2.566324 \times 10^6 \text{ Ans.}$$

Solution:5.-

0.00000075

$$= \frac{75}{100000000} = \frac{75}{10^8}$$

$$= 75 \times 10^{-8} = (7.5 \times 10) \times 10^{-8}$$

$$= 7.5 \times 10 \times 10^{-8} = 7.5 \times 10^{1-8}$$

$= 7.5 \times 10^{-7}$ Ans.

Solution:-6.

$$0.86 \times 10^{4}$$

$= 0.86 \times 100000 = 8600$ Ans.

Solution:-7.

$$1.345 \times 10^{-5}$$

$$= 1.345 \times \frac{1}{10^{5}} = \frac{1.345}{100000} = .00001345$$ Ans.

Solution:-8.

$$5.1 \times 10^{-9}$$

$$= 5.1 \times 10^{\frac{1}{9}} = 5.1 \times \frac{1}{1000000000}$$

$= .0000000051$ Ans.

Solution:-9.

$$0.525 \times 10^{-7}$$

$$= 0.525 \times \frac{1}{10^{7}} = 0.525 \times \frac{1}{10000000}$$

$= .0000000525$ Ans.

Solution:-10.

$$636.5 \times 10^{-6}$$

$$= 636.5 \times \frac{1}{10^{6}} = 636.5 \times \frac{1}{1000000}$$

$= .0006365$ Ans.

Solution:-11.

$$\frac{0.96\times10^{7}}{2\times10^{4}}$$

$$=\left(\frac{0.96}{2}\right)\times\frac{10^{7}}{10^{4}}$$

$$=0.48\times10^{7-4}=0.48\times10^{3}\ \text{Ans.}$$

Solution:-12.

$$\frac{2.61\times4\times10^{8}}{10^{3}}$$

$$=2.61\times4\times\frac{10^{8}}{10^{3}}=10.44\times10^{8-3}$$

$$=(1.044\times10)\times10^{5}=1.044\times10^{5+1}$$

$$=1.044\times10^{6}\ \text{Ans.}$$

Solution:13.-

$$\frac{521\times10^{3}\times12}{2\times10^{2}}$$

$$=\frac{521\times12}{2}\times\frac{10^{3}}{10^{2}}=3126\times10$$

$$=3.126\times1000\times10=3.126\times10^{3}\times10$$

$$=3.126\times10^{3+1}=3.126\times10^{4}\ \text{Ans.}$$

14- Convert 4.5 x 10^{5} cm into meters write the soluti in decimal form.

Solution:-14. $=4.5\times10^{5}$ cm

$$= \frac{4.5\times10^5}{100} = 4.5\times\frac{10^5}{10^2}\ \text{m}$$

$$= 4.5\times10^{5-2}\ \text{m}\ 4.5\times10^3 \quad \text{Ans.}$$

15- The radius of earth is 6400 km. Convert it into meters and write the solution in scientific notation.

Solution:-15.

Radius of earth $= 6400$

$$= 6400 \times 1000\ \text{m}$$

$$= 64 \times 100 \times 1000$$

$$= (6.4\times10)\times100\times1000$$

$$= 6.4\times10^1\times10^2\times10^3$$

$$= 6.4\times10^{1+2+3} = 6.4\times10^6 \quad \text{Ans.}$$

Exercise 6.4

1- ***Write down the characteristic of the logarithms of following numbers.***

(i) 6350 *(ii) 2035.6*

(iii) 2.057 *(iv) 0.8657*

(v) 0.0732 *(vi) 0.000721*

Solution:-

(i) The characteristic of *log 6350* is *3*, because $6350 = 6.350 \times 10^3$

(ii) The characteristic of *log 2035.6* is *3*, because $2035.6 = 2.0356 \times 10^3$

(iii) The characteristic of *log 2.057* is *0*, because $2.057 = 2.057 \times 10^0$

(iv) The characteristic of *log 0.8657* is *-1*, because $0.8657 = 8.757 \times 10^{-1}$

(v) The characteristic of *log 0.732* is *-2*, because $0.0732 = 7.32 \times 10^{-2}$

(vi) *The characteristics of log 0.000721 is -4 because* $0.000721 = 7.21 \times 10^{-4}$

2- ***Write down the values of:***

(i) log 52.13 *(ii) log 6.304*

(iii) log 0.6127 *(iv) log 0.0057*

(v) log 0.00003

Solution:-

log 52.13 = ?

(i) The characteristic of *log 52.13* is *1*, because

$52.13 = 5.213 \times 10^1$

(ii) To find mantissa we search 52 in log table. The next digit of 52 in log table is '1' we search it on the top. We draw vertical line from '1' and horizontal lines from 52 these two lines meet at digit 7168.

	0	1	2	3	4	5	6	7	8	9	1 2 3 4 5 6 7 8 9
52		7168									2

(iii) In number 5213 last digit is 3, we can see differenc for 3 in the extreme right side of the table 3 t vertically and front of 52, lines meets with number 2 which we add in 7168, therefore the mantissa is

7168 + 2 = 7170.

Thus log 52.13 = 1.7170

2(ii) *log 6.304 = ?*

Solution:-

The characteristics of log 6.304 is 0 because

$6.304 = 6.304 \times 10^0$

	0	1	2	3	4	5	6	7	8	9	1 2 3 4 5 6 7 8 9
63	7993										3

The next digit of 63 is '0' the number 7993 is situate on the vertical side of '0' and horizontal side of 63.

The last digit of 6304 is '4'.

In the difference column the vertical side of 4 and horizontal side of 63 digit is 3.

Therefore mantissa is 7993 + 3 = 7996.

Hence log 6.304 = 0.7996.

2(iii) *log 0.6127*

Solution:-

The characteristics of log 0.6127 is -1 because

$0.6127 = 6.127 \times 10^{-1}$

	0	1	2	3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
61			7868		5

In 6127 the next digit of 61 is '2'. In log table vertical lines of 2 and horizontal lines of 61 are cut on 7868 Now in 6127 the last digit is 7. In difference column vertical of 7 and horizontal lines of 61 are cut on its digit '5'.

Therefore mantissa is 7868 + 5 = 7873

Hence log 0.6127 = $\bar{1}.7873$

2(iv) *log 0.0057*

Solution:-

The characteristics of log 0.0057 is -3 because

$.0057 = 5.7 \times 10^{-3}$

	0	1 2 3 4 5 6 7 8 9	2 3 4 5 6 7 8 9	
57	7559			

2 (iv) Now we draw horizontal line from 57 and vertical line from '0' which cut on the number 7559. Therefore the mantissa is 7559

Thus $\log 0.0057 = \bar{3}.7559$

2(v) *log 0.00003*

In log 0.00003 the characteristic is -5 because

$0.00003 = 3\times10^{-5}$.

	0	1 2 3 4 5 6 7 8 9	1,2,3,4,5,6 7 8 9
30	4771		

Now instead of 3, we take horizontal line. from 30 and vertical line from 0, which cut with each other at the number 4771.

Hence log 0.00003 = $\bar{5}$.4771

3- ***If log 6374 = 3.8044, write down the values of:***

(i) log 6.374 *(ii) log 0.6374*

(iii) log 0.00637

3(i) *log 6374 = 3.8044*

Then log 6.374 = ?

Solution:- *You can observe that in log 6374 and log 6.374 the mantissa will be one, which will be 8044.*

Now in log 6.374 the characteristic is 0 because

$6.374 = 6.374 \times 10^0$

Thus log 6.374 = 0.8044

3:(ii) *log 6374 = 3.8044*

Then *log 0.6374 = ?*

Solution:- *log 6374 and log 0.6374 the mantissa is same which is 8044. Now the characteristic in log 0.6374 is -1 because .6374 = 6.374 × 10^{-1}*

Hence log 0.6374 = $\bar{1}$.8044

3(iii) ***log 0.00637***

Solution:

Log 6374 and log 0.00637. The mantissa is same. That is 8044.

Now characteristic in log 0.00637 is -3 because. 0.00637 = 6.37 × 10^{-3}. Hence log 0.00637 = $\bar{3}$.8044.

4- *(i)* If *log x = $\bar{2}$.0374, find x*

(ii) If *log x = 0.1597, find x*

(iii) If *log x = 4.4236, find x*

Solution:(i)

logx = $\bar{2}$.0374

Taking antilog

Antilog (log *x*) = Antilog $\bar{2}$.0374

Now see antilog table

	1 2 3 4 5 6	7	8 9	12 3	4	5 6 7 8 9
.03		1089			1	

Characteristic -2 helpful for putting point.

With the help of mantissa 0374 we find required digits - and .0374 the next digit from .03 is '7'. We draw horizontal line from .03 and vertical line from '7' these are cut at 1089.

Thus in .0374 next digit of the 7 is '4'.

Now in different column draw vertical line from 4 and horizontal line from 0.3 these cut on '1'. So required digit is 1089 + 1 = 1090. The characteristic -2 shows the right side of point is 0. The required digit is .01090.

Thus Antilog (log) = Antilog $\bar{2}.0374$.

Hence x = 0.01090

Solution:(ii)

If *log x = 0.1597, find x*

log $x = 0.1597$

Taking Antilog (logx) = Antilog 0.1597

Antilog Table

	1 2 3 4 5 6 7 8	*9*	*1 2 3 4 5 6 7 8 9*
.15		*1442*	*2*

The characteristic '0' is helpful for putting point

In .1597 the next digit from .15 is 9.

From .15 we draw horizontal line from 9 draw vertical line these lines cut digit 1442. In .1597 the next digit of 9 is '7'. Now in difference column vertical lines

from 7 and horizontal line from .15 which cut with each other at '2'.

Hence required digit is 1442 + 2 = 1444

Thus Antilog (logx) = Antilog .1597

$$x = 1.444$$

Characteristic 0, show there is '1' digit at left side of point.

Solution: (iii)

If *log x = 4.4236, find x*

Antilog (logx) = Antilog 4.4236

	12	3	4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
.42		2649		4

Characteristic = 4

(i) In number .4236 next digit of .42 is 3 search .42 in the log table vertical line from 3 and horizontal line from .42 meet each other at 2649.

(ii) For the difference of 6 .42 search in 6 column that is 2649 + 4 = 2653. Characteristic is 4, therefore required digit will be 5 digit number.

Hence 26530

$x = 26530$ (0 will be added at the right side.)

Laws of Logarithms

(i) $\log_a^{mn} = \log_a^m + \log_a^n$

(ii) $\log_a^{(\frac{m}{n})} = \log_a^m - \log_a^n$

(iii) $\log_a^{mn} = n\log_a^m$

1st Law:

Proof: Let m and *n* are positive integers and '*a*' is any adimissible base, then taking

$x = \log a\, m$.....(*i*)

$y = \log a\, n$.....(*ii*)

Then $a^x = m$ $a^y = n$

Thus $mn = a^x \times a^y = a^{x+y}$

$\log_a mn = x + y$

$= x + y$

Putting valules of x and y from (i) and (ii)

$$\log_a mn = \log_a m + \log_a n$$

2nd Law

Proof: ii. Let *m* and *n* are positive integers and '*a*' is any admissible base, (i.e. $a > 1$), then

$x = \log_a m$.....(*i*)

$y = \log_a n$.....(*ii*)

Therefore $a^x = m$, $a^y = n$

$$\frac{m}{n} = \frac{a^x}{a^y}$$

$$= a^x . a^{-y}$$

$$= a^{x-y}$$

$$\log_a \left(\frac{m}{n} \right) = x - y$$

Putting valules of x and y from (i) and (ii)

$$\boxed{\log_a \left(\frac{m}{n} \right) = \log_a m - \log_a n}$$

3rd Law

Proof: iii. Let $x = \log_a m$, then as before

$$x = \log_a m$$

$$a^x = m$$

$$(a^x)^n = m^n$$

$$m^n = (a^x)^n$$

$$= a^{nx}$$

Therefore $\log_a m^n = nx$

Putting value of x

$$\boxed{\log_a m^n = n \log_a m}$$

Exercise 6.5

1- Solve

$$(i)\ \frac{\log 81}{\log 9} \quad (ii)\ \frac{\log 36}{\log 6} \quad (iii)\ \frac{\log 243}{\log 9}$$

Solution:-(i)

$$\frac{log\,81}{log\,9} = \frac{log\,9^2}{log\,9}$$

$$= \frac{2log^{9}}{log^{9}} = 2 \quad Ans.$$

Solution:- (ii)

$$\frac{log\,36}{log\,6} = \frac{log\,6^2}{log\,6}$$

$$= \frac{2log^{6}}{log^{6}} = 2 \quad Ans.$$

Solution:-(iii)

$$\frac{log\,243}{log\,9} = \frac{log\,3^5}{log\,3^2}$$

$$= \frac{5log\,3}{2log\,3} = \frac{5}{2} \quad Ans.$$

2- Evaluate

(i) *log 5+ log 4+ log3- log 6*

(ii) $\log 5 + \log 20 + \log 24 + \log 25 - \log 60$

(iii) $2\log 3 + 3\log 4 + 4\log 5 - 2\log 6$

(iv) $2\log 5 + \log 8 - \frac{1}{2}\log 4$

(v) $\log 200 + \log 5$

Hint $\left(\log 5 = \log\left(\frac{10}{2}\right) = \log 10 - \log 2 = 1 - \log 2\right)$

Solution:-2(i)

$$\log 5 + \log 4 + \log 3 - \log 6$$

$$= \log \frac{5 \times 4 \times 3}{6}$$

$$= \log 10 = 1.000$$

Solution:-2(ii)

$$\log 5 + \log + 20 + \log 24 + \log 25 - \log 60$$

$$= \log \frac{\cancel{5} \times 20 \times \cancel{24} \times 25}{\cancel{60}}$$

$$= \log 5 \times 8 \times 25 = \log 5 \times 2^3 \times 5^2$$

$$= \log 5^3 \times 2^3 = \log 5^3 + \log 2^3$$

$$= 3\log 5 + 3\log 2 = 3[\log 5 + \log 2]$$

$$= 3 \log 5 \times 2$$

$$= 3\log 10 = 3[1.0000] = 3.000 \text{ Ans.}$$

Solution:-2(iii)

$$2\log 3 + 3\log 4 + 4\log 5 - 2\log 6$$

$$= \log 3^2 + \log 4^3 + \log 5^4 - \log 6^2$$

$$= \log \frac{3^2 \times 4^3 \times 5^4}{6^2}$$

$$= \log \frac{\cancel{3} \times \cancel{3} \times \cancel{4} \times 4 \times 4 \times 5^4}{\cancel{6}_1 \times \cancel{6}_1}$$

$$= \log 4 \times 4 \times 5^4 = \log 2^2 \times 2^2 \times 5^4$$

$$= \log 2^4 \times 5^4 = 4\log 2 + 4\log 5$$

$$= 4[\log 2 + \log 5] = 4[\log 2 \times 5]$$

$$= 4\log 10 = 4(1.0000) = 4.0000 \text{ Ans.}$$

Solution:-2(iv)

$$2\log 5 + \log 8 - \frac{1}{2}\log 4$$

$$= \log 5^2 + \log 2^3 - \frac{1}{2}\log 2^2$$

$$= \log 5^2 + \log 2^3 - \frac{1}{2} \times 2\log 2$$

$$= \log 5^2 + \log 2^3 - \log 2$$

$$= \log \frac{5^2 \times 2^3}{2} = \log 5^2 \times \log 2^2$$

$$= \log(5 \times 2)^2 = 2\log 10$$

$$= 2(1.0000) = 2.0000 \text{ Ans.}$$

Solution:-2(v)

$$\log 200 + \log 5$$

$$= \log 200 \times 5$$

$$= \log 5^2 \times 2^3 \times 5 = \log 5^3 \times 2^3$$

$$= \log(5 \times 2)^3 = 3 \log 10$$

$$= 3 \times 1.0000 = 3.000 \quad Ans.$$

3- *Simplify without using logarithm table.*

(i) $\log 1.3472 + \log 22.79 - \log 5$

(ii) $\log 22.13 + \log 0.354 + \log 7 - \log 3$

(iii) $\log 57.86 + \log 4.385 - \log 2.391 - \log 3.072$

Solution:-3(i)

$$\log 1.3472 + \log 22.79 - \log 5$$

$$= \log \frac{1.3472 \times 22.79}{5} \quad Ans.$$

Solution:-3(ii)

$$\log 22.13 + \log 0.354 + \log 7 - \log 3$$

$$= \log \frac{22.13 \times 0.354 \times 7}{3} \quad Ans.$$

Solution:-3(iii)

$$\log 57.86 + \log 4.385 - \log 2.391 - \log 3.072$$

$$= \log \frac{57.86 \times 4.385}{2.391 \times 3.072} \quad Ans.$$

4- *Solve with the help of logarithm table.*

$$(i)\ \frac{2.38\times3.901}{4.83} \qquad (ii)\ \frac{8.67\times3.94}{1.78}$$

$$(iii)\ \frac{25.36\times3.4569}{9.87\times8.93}$$

Solution:-4(i)

$$x=\frac{2.38\times3.901}{4.83}$$

taking log

$$\log x=\log\frac{2.38\times3.901}{4.83}$$

$$=\log 2.38+\log 3.901-\log 4.83$$

$$=0.3766+0.5911-0.6839$$

$$\log x=0.2838$$

0

$$\text{Antilog}(\log x)=\text{Antilog}\,0.2838$$

$$x=1.923 \text{ Ans.}$$

Solution:-4(ii)

$$\text{Let } x=\frac{8.67\times3.94}{1.78}$$

taking log

$$\log x=\log\frac{8.67\times3.94}{1.78}$$

$$=\log 8.67+\log 3.94-\log 1.78$$

$=0.9380+0.5955-0.2504$

$\log x = 1.2831$

taking antilog

$\text{Antilog}(\log x) = \text{Antilog } 1.2831$

$x = 19.19$ characteristic is 1.

Solution:-4(iii)

$$x = \frac{25.36 \times 3.4569}{9.87 \times 8.93}$$

$$\text{Let } \log x = \log \frac{25.36 \times 3.4569}{9.87 \times 8.93}$$

$\log x = \log 25.36 + \log 3.4569 - \log 9.87 - \log 8.9$

$=1.4041+0.5386-0.9943-0.9509$

$=1.9427-1.9452$

$\log x = -.0025$

taking antilog

$\text{Antilog}(\log x) = \text{Antilog } (-.0025)$

$=\text{Antilog } (-1+1-.0025)$

$=\text{Antilog}(\bar{1}.9975)$

$=$ ***.9942*** *Ans.*

5- *Prove that*

(i) $$\log\left(\frac{a^2}{bc}\right) + \log\left(\frac{b^2}{ca}\right) + \log\left(\frac{c^2}{ab}\right) = 0$$

(ii) $3\log 2 + 2\log 3 + \log 5 = \log 360$

(iii) $5\log 2 - \log 9 = \log 27$

(iv) $\log\left(\frac{75}{16}\right) + \log\left(\frac{32}{243}\right) - 2\log\left(\frac{5}{9}\right) = \log 2$

(v) $2\log\left(\frac{11}{3}\right) + \log\left(\frac{130}{77}\right) - \log\left(\frac{55}{91}\right) = \log 2$

Proof:

5(i) $log\left(\frac{a^2}{bc}\right) + log\left(\frac{b^2}{ca}\right) + log\left(\frac{c^2}{ab}\right)$

$= log\left(\frac{a^2}{bc} \times \frac{b^2}{ca} \times \frac{c^2}{ab}\right)$

$= log\ 1 = 0 = R.H.S$

Proof:

5(ii) $3log2 + 2log3 + log5 = 360$

$L.H.S$

$= 3log2 + 2log3 + log5$

$= log\ 2^3 + log\ 3^2 + log\ 5$

$= log\ 2^3 \times 3^2 \times 5 = log\ 8 \times 9 \times 5$

$= log\ 360 = R.H.S$

Proof:

5(iii) $5log3 - log9 = 27$

$5\log 3 - \log 9$

$= \log 3^5 - \log 9$

$= \log \frac{3^5}{9} = \log \frac{3^5}{3^2} = \log 3^{5-2}$

$= \log 3^3 = \log 27 = R.H.S$

Proof:

5(iv) $\log \frac{75}{16} + \log \frac{32}{243} - 2\log \frac{5}{9} = \log 2$

L.H.S

$= \log \frac{75}{16} + \log \frac{32}{243} - 2\log \frac{5}{9}$

$= \log \frac{75}{16} + \log \frac{32}{243} - \log \left(\frac{5}{9}\right)^2$

$= \log \frac{75}{16} \times \frac{32}{243} \times \frac{1}{\left(\frac{5}{9}\right)^2}$

$= \log \frac{75}{16} \times \frac{32}{243} \times \frac{9^2}{5^2}$

$= \log \frac{\cancel{75}^{3}}{\cancel{16}_{1}} \times \frac{\cancel{32}^{2}}{\cancel{243}_{\cancel{3}}} \times \frac{\cancel{81}^{1}}{\cancel{25}}$

$= \log 2 = R.H.S$

Proof:

5(v) $2\log\left(\frac{11}{13}\right)+\log\frac{130}{77}-\log\frac{55}{91}=\log 2$

L.H.S

$$2\log\left(\frac{11}{13}\right)+\log\frac{130}{77}-\log\frac{55}{91}$$

$$=\log\left(\frac{11}{13}\right)^2+\log\frac{130}{77}-\log\frac{55}{91}$$

$$=\log\left(\frac{11}{13}\right)^2\times\frac{130}{77}\times\frac{55}{91}$$

$$=\log\left(\frac{11}{13}\right)^2\times\frac{130}{77}\times\frac{1}{\frac{55}{91}}$$

$$=\log\frac{11\times 11}{13\times 13}\times\frac{130^{10}}{77_7}\times\frac{91^{13}}{55_5}$$

$\log 2$ = R.H.S.

6- **Show that:** $3\log 4+2\log 5-\frac{1}{3}\log 64-\frac{1}{2}\log 16=2$

$$3\log 4+2\log 5-\frac{1}{3}\log 64-\frac{1}{2}\log 16=2$$

L.H.S

$$=3\log 4+2\log 5-\frac{1}{3}\log 64-\frac{1}{2}\log 16=2$$

$$= \log 4^3 + \log 5^2 - \frac{1}{3}\log 4^3 - \frac{1}{2}\log 4^2$$

$$= \log 4^3 + \log 5^2 - \log(4^3)^{\frac{1}{3}} - \log(4^2)^{\frac{1}{2}}$$

$$= \log 4^3 + \log 5^2 - \log 4 - \log 4$$

$$= \log 4^3 \times 5^2 - 2\log 4$$

$$= \log 4^3 \times 5^2 - \log 4^2 = \log \frac{4^3 \times 5^2}{4^2}$$

$$= \log 4 \times 25 = \log 100 = \log 10^2$$

$$= = 2\log 10 = 2 = R.H.S$$

7- Show that: log(1 x 2 x 3)= log 1 + log 2 + log 3

$$\left.\begin{aligned} x &= \log_{10}{}^{1} \\ y &= \log_{10}{}^{2} \\ z &= \log_{10}{}^{3} \end{aligned}\right\} A$$

Therefore $10^x = 1$ *(i)*

$10^y = 2$ *(ii)*

$10^z = 3$ *(iii)*

$$10^x \times 10^y \times 10^z = 1 \times 2 \times 3$$

$$10^{x+y+z} = 1 \times 2 \times 3$$

$$1 \times 2 \times 3 = 10^{x+y+z}$$

$\log_{10}(1 \times 2 \times 3) = x+y+z$ *logarithmic form*

$$= log1 + log2 + log3 \qquad From \quad (A)$$

8- *Using logarithmic table evaluate the following:*

(i) $69.13 \times 0.34 \times 0.014$ *(ii)* $\dfrac{8.67 \times 3.94}{1.78}$

(iii) $\dfrac{4}{3} \times 3.142 \times (1.5)^3$ *(iv)* $\dfrac{(25.36)^2 \times (0.4569)}{847.5}$

(v) $\dfrac{0.9876 \times (16.42)^2}{(4.567)^{\frac{1}{3}}}$ *(vi)* $\sqrt{\dfrac{3\sqrt{0.0125} \times \sqrt{31.15}}{0.00081}}$

(vii) $\dfrac{(6.45)^3 \times (0.00034)^{\frac{1}{3}} \times (981.9)}{(9.37)^2 \times (8.93)^{\frac{1}{4}} \times (0.0617)}$

(viii) $\dfrac{(0.0437)^{\frac{2}{3}} \times (1.407)^2}{(0.0015)^{\frac{1}{3}} \times (1.235)^{\frac{1}{7}}}$

Solution:-8(i)

$$x = 69.13 \times 0.34 \times 0.014$$

Taking log

$$\log x = \log(69.13 \times 0.34 \times 0.014)$$

$$= \log 69.13 + \log 0.34 + \log 0.014$$

$$= 1.8397 + (\bar{1}.5315) + (\bar{2}.1461)$$

$$= 1.8397 + (-.4685) + (-1.8539)$$

$$= 1.8397 - .4685 - 1.8539$$

$$\log x = -0.4827$$

$$= +1 - 1 - 0.4827$$

$$\log x = \bar{1}.5173$$

Taking anti-log

$$Antilog(\log x) = Antilog\ \bar{1}.5173$$

$$x = .3291\ Ans.$$

Solution:- 8(ii)

Let $$x = \frac{8.67 \times 3.94}{1.78}$$

$$\log x = \log 8.67 + \log 3.94 - \log 1.78$$ Taking log

$$= 0.9380 + 0.5955 - 0.2504$$

$$\log x = 1.2831$$

$$Antilog(\log x) = Antilog\ 1.2831$$ *Taking anti-log*

$$= 19.19\ Ans.$$

Solution:-8(iii)

Let $$x = \frac{4}{3} \times 3.142 \times (1.5)^3$$

$$\log x = \log \frac{4}{3} \times 3.142 \times (1.5)^3$$ Taking log

$$= \log 4 - \log 3 + \log 3.142 + 3\log 1.5$$

$$= 0.6021 - 0.4771 + 0.4972 + 3(0.1761)$$

$$= 0.6021 - 0.4771 + 0.4972 + 0.5283$$

$$\log x = 1.1505$$

Taking anti-log

$$Antilog(\log x) = Antilog\ 1.1505$$

$$x = 14.15\ Ans.$$

Solution:-8(iv)

$$Let \quad x = \frac{(25.36)^2 \times (0.4569)}{847.5}$$

$$\log x = \log \frac{(25.36)^2 \times (0.4569)}{847.5} \text{ taking log}$$

$$= \log(25.36)^2 + \log(0.4569) - \log 847.5$$

$$= 2\log 25.36 + \log 0.4569 - \log 847.5$$

$$= 2(1.4041) + (\bar{1}.6599) - (2.928)$$

$$= 2.8082 - 1 + .6599 - 2.928$$

$$= -0.4599$$

$$= 1 - 1 - 0.4599$$

$$\log x = \bar{1}.5401$$

taking anti-log

$$Antilog(\log x) = Antilog\ \bar{1}.5401$$

$$x = .3468$$

Solution:-8(v)

$$\textbf{Let} \quad x = \frac{0.9876 \times (16.42)^2}{(4.567)^{\frac{1}{3}}}$$

Taking log

$$\log x = \log \frac{0.9876 \times (16.42)^2}{(4.567)^{\frac{1}{3}}}$$

$$= \log 0.9876 + \log(16.42)^2 - \log(4.567)^{\frac{1}{3}}$$

$$= \log 0.9876 + 2\log 16.42 - \frac{1}{3}\log 4.567$$

$$= \bar{1}.9946 + 2(1.2153) - \frac{1}{3}(0.6596)$$

$$= \bar{1}.9946 + 2.4306 - .2199$$

$$= -1 + .9946 + 2.4306 - .2199$$

$$\log x = 2.2053$$

Antilog(logx) = Antilog 2.2053 Taking anti-log

$$x = 160.4 \text{ Ans.}$$

Solution:- 8(vi)

Let $$x = \sqrt{\frac{\sqrt[3]{0.0125 \times \sqrt{31.15}}}{0.00081}}$$

$$= \left(\frac{3 \times \sqrt{0.0125} \times \sqrt{31.15}}{0.00081} \right)^{\frac{1}{2}}$$

$$= \left(\frac{3 \times (0.0125)^{\frac{1}{2}} \times (31.15)^{\frac{-1}{2}}}{0.00081} \right)^{\frac{1}{2}}$$

$$= \frac{3^{\frac{1}{2}} \times (0.0125)^{\frac{1}{2} \times \frac{1}{2}} \times (31.15)^{\frac{1}{2} \times \frac{1}{2}}}{(0.00081)^{\frac{1}{2}}}$$

$$= \frac{3^{\frac{1}{2}} \times (0.0125)^{\frac{1}{4}} \times (31.15)^{\frac{1}{4}}}{(0.00081)^{\frac{1}{2}}}$$

taking log

$$\log x = \log \frac{3^{\frac{1}{2}} \times (0.0125)^{\frac{1}{4}} \times (31.15)^{\frac{1}{4}}}{(0.00081)^{\frac{1}{2}}}$$

$$= \frac{1}{2}\log 3 + \log(.0125)^{\frac{1}{4}} + \log(31.15)^{\frac{1}{4}} - \log(.00081)^{\frac{1}{2}}$$

$$\frac{1}{2}\log 3 + \frac{1}{4}\log .0125 + \frac{1}{4}\log 31.15 - \frac{1}{2}\log(.00081)$$

$$\frac{0.4771}{2} \times \frac{1}{4}(\bar{2}.0969) + \frac{1}{4}(1.4935) - \frac{1}{2}(\bar{4}.9085)$$

$$\frac{0.4771}{2} + \frac{1}{4}(-2 + .0969) + .3734 - \frac{1}{2}(-4 + .9085)$$

$$\log x = 0.2386 + \frac{1}{4}(-1.9031)$$

$$+.3734 - \frac{1}{2}(-3.0915)$$

$$=0.2386-.4758+.3734+1.5458$$

$$\log x = 1.6820$$

taking anti-log

$$Antilog(\log x) = Antilog\ 1.6820$$

$$x = 48.08\ Ans.$$

Solution:-8(vii)

$$\textbf{Let} \qquad x = \frac{(6.45)^3 \times (.00034)^{\frac{1}{3}} \times (981.9)}{(9.37)^2 \times (8.93)^{\frac{1}{4}} \times (.0617)}$$

taking log

$$\log x = \log(6.45)^3 + \log(.00034)^{\frac{1}{3}}$$
$$+\log 981.9 - \log(9.37)^2 -$$
$$\log(8.93)^{\frac{1}{4}} - \log(.0617)$$
$$= 3\log 6.45 + \frac{1}{3}\log(.00034)$$
$$+\log(981.9) - 2\log(9.37)$$
$$-\frac{1}{4}\log 8.93 - \log .0617$$

$$= 3(0.8096) + \frac{1}{3}(\bar{4}.5315) + 2.9921 - 2(0.9717) - \frac{1}{4}(0.9509) - (\bar{2}.7903)$$

$$= 3(0.8096) + \frac{1}{3}(-3.4685) + 2.9921 - 2(0.9717) - \frac{1}{4}(0.9509) - (-1.2097)$$

$$= \begin{matrix} 2.4288 - 1.1562 + 2.9921 \\ -1.9434 - .2377 + 1.2097 \end{matrix}$$

$$\log x = 3.2933$$

taking antilog

$$Antilog(\log x) = Antilog 3.2933$$

$$= 19.64$$

Solution:- 8(viii)

Lets

$$x = \frac{(0.0437)^{\frac{2}{3}} \times (1.407)^2}{(0.0015)^{\frac{1}{3}} \times (1.235)^{\frac{1}{7}}}$$

$$\log x = \log(.0437)^{\frac{2}{3}} + \log(1.407)^2 - \log(.0015)^{\frac{1}{3}} - \log(1.235)^{\frac{1}{7}}$$

$$= \frac{2}{3}\log(.0437) + 2\log(1.407) - \frac{1}{3}\log(.0015) - \frac{1}{7}\log(1.235)$$

$$= \frac{2}{3}(\bar{2}.6405) + 2(0.1483) - \frac{1}{3}(\bar{3}.3761) - \frac{1}{7}(0.0917)$$

$$= \frac{2}{3}(-1.3595) + 2(0.1483) - \frac{1}{3}(-2.8239) - \frac{1}{7}(0.0917)$$

$$= \frac{-2.7190}{3} + .2966 + 0.9413 - 0.0131$$

$$= -.9063 + .2966 + 0.9413 - 0.0131$$

$$\log x = 0.3185$$

Taking antilog

$$\text{Antilog}(\log x) = \text{Antilog}\ (0.3185)$$

$$= 2.082 \text{ Ans.}$$

9- *If* $v = \sqrt{\frac{gl}{2\pi}}$ *find v. when*

$$l = 150, g = 32.16, \pi = 3.142$$

Solution:-

$$v = \sqrt{\frac{gl}{2\pi}}$$

$g = 32.16$ and $l = 150\ \pi = 3.142$

$$v = \sqrt{\frac{(32.16)(150)}{2(3.142)}}$$

taking log

$$\log v = \log \sqrt{\frac{(32.16)(150)}{2(3.142)}}$$

$$= \log \left(\frac{32.16 \times 150}{2 \times (3.142)} \right)^{\frac{1}{2}}$$

$$= \frac{1}{2} \log \frac{32.16 \times 150}{2 \times (3.142}$$

$$= \frac{1}{2} [\log 32.16 + \log 150 - \log 2 - \log 3.142]$$

$$= \frac{1}{2} [1.5073 + 2.1761 - 3010 - 0.4972]$$

$$= \frac{1}{2} [2.8851]$$

$$\log v = 1.4426$$

taking antilog

$$Antilog(\log v) = Antilog\ 1.4426$$

10- **If** $H = \frac{I^2 Rt}{4.2}$ **find H. When** $I = 1.3, R = 6.7$ **and** $t = 25$

Solution:-

$$H = \frac{I^2 Rt}{4.2}$$

Putting values $I = 1.3 \quad R = 6.7 \quad t = 25$

$$H = \frac{(1.3)^2(6.7)(25)}{4.2}$$

$$\log H = \log \frac{(1.3)^2(6.7)(25)}{4.2}$$

$$= \log \frac{(1.3)^2(6.7)(25)}{4.2}$$

$$= \log(1.3)^2 + \log 6.7 + \log 25 - \log 4.2$$

$$= 2\log 1.3 + \log 6.7 + \log 25 - \log 4.2$$

$$= 2(0.1139) + 0.8261 + 1.3979 - 0.6232$$

$$= .2278 + 0.8261 + 1.3979 - 0.6232$$

$$\log H = 1.8286$$

taking antilog

$$\text{Antilog}(\log H) = \text{Antilog } 1.8286$$

$$H = 67.39 \text{ Ans.}$$

11- **Find h, if** $h = \frac{v}{\pi(R^2 - r^2)}$, **when**

$v = 1190, R = 83.6, r = 62.4$ and $\pi = 3.14$

Solution:-

$$h = \frac{v}{\pi(R+r)(R-r)}$$

Putting values r, R, π, v

$$h = \frac{1190}{3.14(83.6+62.4)(83.6-62.4)}$$

$$h = \frac{1190}{3.14(146)(21.2)}$$

taking log

$$\log h = \log \frac{1190}{3.14 \times 146 \times 21.2}$$

$$\log 1190 - \log 3.14 - \log 146$$

$$= -\log 21.2$$

$$= 3.0755 - 0.4969 - 2.1644 - 1.3263$$

$$= -0.9121$$

$$= \bar{1} - 1 - 0.9121$$

$$\log h = 1.0879$$

taking antilog

$$\text{Antilog } h = \text{Antilog } \bar{1}.0879$$

$$h = .1224 \text{ Ans.}$$

Exercise 7.1

1- Write the first three terms of the following:

(i) $a_n = n + 3$ (ii) $a_n = (-1)^n n^3$ (iii) $a_n = 3n + 5$

(iv) $a_n = \frac{n+1}{2n+5}$ (v) $a_n = \frac{1}{(2n-1)^2}$ (vi) $a_n = n + 3 = 2$

(vii) $a_n = \frac{1}{3^n}$ (viii) $a_n = 3n - 5$ (ix) $a_n = (n+1)a_{n-1}, a_1 = 1$

Solution:(i)

$a_n = n + 3$

Putting values $n = 1, 2, 3$

$a_1 = 1 + 3 = 4$

$a_2 = 2 + 3 = 5$

$a_3 = 3 + 3 = 6$

First three terms 4,5,6

Solution:(ii)

$a_n = (-1)^n n^3$

Putting values $n = 1, 2, 3$

$a_1 = (-1)^1(1)^3$

$= (-1)(1) = -1$

$a_2 = (-1)^2(2)^3$

$= (+1)(8) = 8$

$= (+1)(8) = 8$

$a_3 = (-1)^3(3)^3$

$= (-1)(27) = -27$

First three terms $-1, 8-27$

Solution:(iii)

Putting $n = 1, 2, 3$

$$a_1 = 3(1) + 5$$
$$= 3 + 5 = 8$$
$$a_2 = 3(2) + 5$$
$$= 6 + 5 = 11$$
$$a_3 = 3(3) + 5$$
$$= 9 + 5 = 14$$

First three terms are *8, 11, 14.*

Solution:(iv)

$$a_n = \frac{n+1}{2n+5}$$

Putting $n = 1, 2, 3$

$$a_1 = \frac{1+1}{2(1)+5} = \frac{2}{2+5} = \frac{2}{7}$$

$$a_2 = \frac{2+1}{2(2)+5} = \frac{3}{4+5} = \frac{3}{9}$$

$$a_3 = \frac{3+1}{2(3)+5} = \frac{4}{6+5} = \frac{4}{11}$$

First three terms are $\frac{2}{7}, \frac{3}{9}, \frac{4}{11}$

Solution:(v)

$$a_n = \frac{1}{(2n-1)^2}$$

Putting $n = 1, 2, 3$

$$a_1 = \frac{1}{\{2(1)-1\}^2} = \frac{1}{(2-1)^2} = \frac{1}{(1)^2} = 1 \quad (i)$$

$$a_2 = \frac{1}{\{2(2)-1\}^2} = \frac{1}{(4-1)^2} = \frac{1}{(3)^2} = \frac{1}{9} \quad (i$$

$$a_3 = \frac{1}{\{2(3)-1\}^2} = \frac{1}{(6-1)^2} = \frac{1}{(5)^2} = \frac{1}{25} \quad (ii$$

First three terms are $1, \frac{1}{9}, \frac{1}{25}$

Solution:(vi)

$$a_n = n + 3$$

Putting $n = 1, 2, 3$

$$a_1 = 1 + 3 = 4$$
$$a_2 = 2 + 3 = 5$$
$$a_3 = 3 + 3 = 6$$

First three terms are 4, 5, 6

Solution:(vii)

$$a_n = \frac{1}{3^n}$$

Putting $n = 1, 2, 3$

$$a_1 = \frac{1}{3^1} = \frac{1}{3}$$

$$a_2 = \frac{1}{3^2} = \frac{1}{9}$$

$$a_3 = \frac{1}{3^3} = \frac{1}{27}$$

First three terms are $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}$

Solution:(viii)

$$a_n = 3n - 5$$

Putting $n = 1, 2, 3$

$$a_1 = 3(1) - 5 = 3 - 5 = -2$$

$a_2 = 3(2) - 5 = 6 - 5 = 1$

$a_3 = 3(3) - 5 = 9 - 5 = 4$

First three terms are $-2, 1, 4$

Solution:(ix)

$a_n = (n + 1)$

Putting $n = 2, 3$

$a_1 = 1$

$a_2 = 2 + 1 = 3$

$a_3 = 3 + 1 = 4$

First three terms are *1, 3, 4*

2- ***Find the terms indicated in the following sequences.***

(i) $2,6,11,17,\ldots a_8$ *(ii)* $1,3,12,60,\ldots a_7$ *(iii)* $1,\frac{1}{3},\frac{1}{9},\frac{1}{27},\ldots a_6$

(iv) $-1,1,3,5,\ldots,a_9$ *(v)* $\frac{1}{3},\frac{2}{5},\ldots,a_5$ *(vi)* $1,-3,5,-7,\ldots,a_9$

2(i) 2, 6, 11, 17, ... a_8

Solution:

$a_1 = 2$

$a_2 = 6$

$a_3 = 11$

$a_4 = 17$

$a_2 - a_1 = 6 - 2 = 4$

$a_3 - a_2 = 11 - 6 = 5$

$a_4 - a_3 = 17 - 11 = 6$

$a_5 - a_4 = 24 - 17 = 7$

$a_6 - a_5 = 32 - 24 = 8$

$a_7 - a_6 = 41 - 32 = 9$

$a_8 - a_7 = 51 - 41 = 10$

$$a_8 = 51$$

2(ii) **1, 3, 12, 60, ... a_7**

Solution:

$$a_1 = 1$$
$$a_2 = 3 \quad = 1 \times 3$$
$$a_3 = 12 \quad = 3 \times 4$$
$$a_4 = 60 \quad = 12 \times 5$$
$$a_5 = 360 \quad = 60 \times 6$$
$$a_6 = 2520 \quad = 360 \times 7$$
$$a_7 = \quad = 2520 \times 8 = 20160$$

2(iii) $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots a_6$

Solution:-

$$a_1 = 1$$
$$a_2 = \frac{1}{3} \quad = 1 \times \frac{1}{3}$$
$$a_3 = \frac{1}{9} \quad = \frac{1}{3} \times \frac{1}{3}$$
$$a_4 = \frac{1}{27} \quad = \frac{1}{9} \times \frac{1}{3}$$
$$a_5 = \frac{1}{81} \quad = \frac{1}{27} \times \frac{1}{3}$$
$$a_6 = \frac{1}{243} \quad = \frac{1}{81} \times \frac{1}{3} = \frac{1}{243}$$

2(iv) **–1, 1, 3, 5, ... a_9**

Solution:

$a_1 = -1$

$a_2 = 1$

$a_3 = 3$

$a_4 = 5$

$a_2 - a_1 = 1 - (-1)$

$= 1 + 1 = 2$

$a_3 - a_2 = 3 - 1 = 2$

$a_4 - a_3 = 5 - 3 = 2$

$a_5 = 5 + 2 = 7$

$a_6 = 7 + 2 = 9$

$a_7 = 9 + 2 = 11$

$a_8 = 11 + 2 = 13$

$a_9 = 13 + 2 = 15$

2(v) $\frac{1}{3}, \frac{2}{5}, \ldots, a_5$

Solution:

$\frac{1}{3}, \frac{2}{5}, \ldots, a_5$

$a_1 = \frac{1}{3}$

$a_2 = \frac{2}{5}$

$a_3 = \frac{3}{7}$

$a_4 = \frac{4}{9}$

$a_5 = \frac{5}{11}$

2(vi) ***1, -3, 5, -7, ..., a_9***

Solution:

$a_1 = 1$

$a_2 = -3$

$a_3 = 5$

$a_4 = -7$

$a_5 = 9$

$a_6 = -11$

$a_7 = 13$

$a_8 = -15$

$a_9 = 17$

3- Find the next four terms of the following sequences.

(i) 12,16,21,27,... (ii) 1,3,7,15,31,... (iii) -1,2,12,40,...
(iv) 9,11,14,17,19,22... (v) 4,8,12,16,... (vi) -2,0,2,4,6,8,10,...

3(i) 12, 16, 21, 27,...

Solution:

$16 - 12 = 4$
$21 - 16 = 5$
$27 - 21 = 6$
$34 - 27 = 7$
$42 - 34 = 8$
$51 - 42 = 9$
$61 - 51 = 10$

$27 + 7 = 34$
$34 + 8 = 42$
$42 + 9 = 51$
$51 + 10 = 61$
34,42,51,61

3(ii) 1,3,7,15,31,...

Solution:

$3 - 1 = 2$
$7 - 3 = 4$
$15 - 7 = 8$
$31 - 15 = 16$
$63 - 31 = 32$
$127 - 63 = 64$

$31 + 32 = 63$
$63 + 64 = 127$
$127 + 128 = 255$

$225 - 127 = 128$

$511 - 255 = 256$ $\qquad$ $255 + 256 = 511$

$1,(1+2),(3+2^2)+(7+2^3)+(15+2^4)$
$+(31+2^5)+(63+2^6)+(127+2^7)....$

3(iii) -2,2,12,40,.....

Solution:

$2-(-2)=2+2=4=3^1+1$
$12-2=10=3^2+1$
$40-12=28=3^3+1$

$112-40=82=3^4+1$
$356-112=244=3^5+1$
$1086-356=730=3^6+1$
$3274-1086=2188=3^7+1$

$112, 356, 1086, 3274$

3(iv) 9, $\underline{9}$ $\underline{11}$ $\overset{3}{\underline{14}}$ $\overset{3}{\underline{17}}$ $\underline{19}$ $\overset{3}{\underline{22}}$ $\overset{3}{\underline{25}}$ $\underline{27}$ $\overset{3}{\underline{30}}$ $\overset{3}{\underline{33}}$

3(v) 4, 8, 12, 16,..... $\qquad$ 2 $\qquad$ 2

Solution:

$8-4=4$
$12-8=4$
$16-12=4$
$20-16=4$
$24-20=4$
$28-24=4$
$32-28=4$

3(vi) -2, 0, 2, 4, 6, 8, 10

Solution:

$0-(-2)=0+2=2$
$2-0=2$
$4-2=2$
$6-4=2$
$8-6=2$
$10-8=2$
$12-10=2$
$14-12=2$
$16-14=2$
$18-16=2$

-2, 0, 2, 4, 6, 8, 10, ...

Exercise 7.2

1- ***Find the speçified term of the following A.P.***

(i) 3, 7, 11, ...61st term. *(ii)* $-4, -7, -10, \ldots a_{19}$

(iii) 6, 4, 2, ...45th term. *(iv)* $9, 14, 19, \ldots a_{14}$

(v) $11, 6, 1, \ldots a_{18}$

Solution:-

61^{st} 3, 7, 11,.....

Here $a_1 = 3$

$d = 7 - 3 = 4$

$a_{61} = ?$

$n = 61$

We know that $a_n = a+(n-1)d$

Putting values a, d and n in the formula

$$a_{61} = 3+(61-1)(4)$$
$$= 3 + 60 \times 4$$
$$= 3+240$$
$$= 243$$

1(ii) -4, -7, -10, ... a_{19}

Solution:

Here $a = a_1 = -4$

$d = (-7) - (-4)$

$= -7 + 4 = -3$

$a_{19} = ?$

$n = 19$

We know that $a_n = a+(n-1)d$

Putting values of a, d and n in the formula

$$a_{19} = -4+(19-1)(-3)$$
$$= -4+18(-3)$$
$$= -4 - 54$$
$$= -58$$

$a_{19} = -58$

Solution:

Here $a = a_1 = 4$

$d = 4 - 6 = -2$

$a_{45} = ?$

$n = 45$

We know that $a_n = a+(n-1)d$

Putting values of a, d and n in the formula

$$a_{45} = 6+(45-1)(-2)$$
$$= 6+(44)(-2)$$
$$= 6-88$$
$$= -82$$

Hence 45th term is -82

1(iv) *9, 14, 19, ...a_{14}*

Solution:

Here $a = a_1 = 9$

$d = 14 - 9 = 5$

$a_{14} = ?$

$n = 14$

We know that $a_n = a+(n-1)d$

Putting values of a, d and n in the formula

$$a_{14} = 9+(14-1)(5)$$
$$= 9+(13)(5)$$
$$= 9 + 65$$
$$= 74$$

1(v) *11, 6, 1, ...a_{18}*

Solution:

$a = a_1 = 11$

$d = 6 - 11 = -5$

$a_{18} = ?$

$n = 18$

We know that $a_n = a+(n-1)d$

Putting values in a, d and n in the formula

$$a_{18} = 11+(18-1)(-5)$$
$$= 11+(17)(-5)$$

$= 11 - 85$

$= -74$

2- ***Find the missing element using the formula of A.P***

$a_n = a + (n - 1)\, d$

(i) $a = 2,\ a_n = 402,\ n = 26$

(ii) $a_n = 81,\ d = -3,\ n = 18$

(iii) $a = 5,\ a_n = 61,\ n = 15$

(iv) $a = 16,\ a_n = 0,\ d = -\frac{1}{4}$

(v) $a = 10,\ a_n = 400,\ d = 5$

(vi) $a_n = 261,\ d = 4,\ n = 18$

Solution: (i) $a_n = a + (n - 1)\, d$

$a = 2$

$a_n = 402$

$n = 26$

$d = ?$

Here we find *d?*

$a_n = a + (n - 1)\, d$

Putting values of $a,\ a_n,\ n$

$402 = 2 + (26 - 1)\, d$

$402 = 2 + 25\, d$

$2 + 25\, d = 402$ *OR*

$25\, d = 402 - 2$

$25\, d = 400$

Hence $d = \frac{400}{25} = 16$

Solution: (ii) $a_n = a + (n - 1)\, d$

$a_n = 81$

$d = -3$

$n = 18$

Here *a* is unknown

$a_n = a + (n - 1)\, d$

Putting values of $a,\ a_n,\ n$

$$81 = a + (18 - 1)(-3)$$
$$81 = a + (17)(-3)$$
$$81 = a - 51$$
$$a - 51 = 81 \quad OR$$

Therefore $a = 81 + 51$
$$= 132$$

Solution: (iii) $a_n = a + (n - 1)\,d$ *formula*

$a_n = 5$ *and*

$a_n = 61$

$n = 15$

Here d is unknown

$$a_n = a + (n - 1)\,d$$

Putting values of a, a_n, and n.

$$61 = 5 + (15 - 1)\,d$$
$$61 = 5 + 14\,d$$
$$5 + 14\,d = 61 \quad OR$$
$$14\,d = 61 - 5$$
$$14\,d = 56 \quad Therefore$$
$$d = \frac{56}{14}$$
$$d = 4$$

Solution: (iv) $a_n = a + (n - 1)\,d$ *formula*

$a = 16$ *and*

$a_n = 0$

$$d = -\frac{1}{4}$$

Here n is unknown.

$$a_n = a + (n - 1)\,d$$

Putting values of a, a_n and d.

$$0 = 16 + (n - 1)\left(-\frac{1}{4}\right)$$

Multiplying by 4.

$$0 = 64 + (n - 1)(-1)$$

$$0 = 64 - n + 1$$
$$n = 64 + 1 \qquad OR$$
$$n = 65 \qquad OR$$

Solution: ***(v)*** $a_n = a + (n - 1)\,d$ formula

$$a = 10 \qquad and$$
$$a_n = 400$$
$$d = 5$$

Here n is unknown

$$a_n = a + (n - 1)\,d$$

Putting values of a, an and d.

$$400 = 10 + (n - 1)(5)$$
$$400 - 10 = 5n - 5$$
$$5n - 5 = 390 \qquad OR$$
$$5_n = 390 + 5$$
$$5_n = 395$$
$$n = \frac{395}{5}$$
$$= 79$$

Solution: ***(vi)*** $a_n = a + (n - 1)\,d$ fromula

$$a_n = 261 \qquad and$$
$$d = 4$$
$$n = 18$$

Here a is unknown

$$a_n = a + (n - 1)\,d$$

Putting values of a_n, d and n.

$$261 = a + (18 - 1)(4)$$
$$= a + 17 \times 4$$
$$261 = a + 68$$
$$261 - 68 = a$$
$$193 = a \qquad OR$$
$$a = 193$$

3- ***Find the 15^{th} terms of an A.P where the 3^{rd} term is 8 and the common difference is $\frac{1}{3}$.***

Solution:

$$a_3 = 8$$

$$d = \frac{1}{3}$$

$$a_{15} = ?$$

First we find a

$$a_n = a + (n-1)\,d$$

$$a_3 = a + (3-1)\,d$$

$$8 = a + (2)\left(\frac{1}{3}\right)$$

$$8 = a + \left(\frac{2}{3}\right)$$

$$a = 8 - \frac{2}{3} \quad OR$$

$$a = 8 - \frac{2}{3}$$

$$a = \frac{24-2}{3} = \frac{22}{3} \quad (A)$$

$a_n = a + (n-1)\,d$ formula

$$a_{15} = \frac{22}{3} + (15-1)\left(\frac{1}{3}\right) \quad \text{from A}$$

$$= \frac{22}{3} + (14)\frac{1}{3}$$

$$= \frac{22}{3} + \frac{14}{3}$$

$$= \frac{22+14}{3}$$

$$= \frac{36}{3} = 12$$

4- *Which term of an A.P 6, 2, −2..... is− 146?*

Solution:

Here term ..... 6, 2, -2, ...

$a = 6$

$d = 2 - 6 = -4$

$an = -146$

$n = ?$

$a_n = a + (n-1)\,d$ *formula*

Putting values a, a_n and d in formula.

$-146 = 6 + (n-1)(-4)$ *formula*

$-146 = 6 - 4n + 4$

$4n = 6 + 4 + 146$ *OR*

$4n = 156$

$$n = \frac{156}{4}$$

$n = 39$

5- *Which term of an A.P 5,2, –1 .... is − 134?*

Solution:

$a = 5$

$d = 2 - 5 = -3$

$n = -118$

$n = ?$

$a_n = a + (n-1)\,d$ formula

Putting values of a, d and n.

$-118 = 5 + (n-1)(-3)$

$-118 = 5 + 3n + 3$

$3n = 5 + 3 + 118$ Or

$3n = 126$

$$n = \frac{126}{3}$$

$n = 42$

6- ***How many terms are there in an A.P, in which $a_1 = a = 11$, $a_n = 68$, $d = 3$.***

Solution:- ***Here term.....***

$d = 3$

$a_n = 68$

$a = 11$

$n = ?$

$a_n = a + (n-1)\,d$ *formula*

Putting values of a, a_n and d in the formula.

$68 = 11 + (n-1)(3)$

$68 = 11 + 3n - 3$

$68 = 8 + 3n$

$8 + 3n = 68$

$3n = 68 - 8$ Or

$3n = 60$

$$n = \frac{60}{3}$$

$n = 20$

7- ***Find the 11^{th} term of a_n A.P $2 - x$, $3 - 2x$, $4 - 3x$.....***

Solution:- ***In term of a_n A.P $2 - x$, $3 - 2x$, $4 - 3x$.....***

$a = 2 - x$

$d = (3 - 2x) - (2 - x)$

$d = 3 - 2x - 2 + x = 1 - x$

$a_{11} = ?$

$\therefore n = 11$

$a_n = a + (n-1)\,d$ ***formula***

$a_{11} = (2 - x) + (11 - 1)(1 - x)$

$$= 2 - x + 10(1 - x)$$
$$= 2 - x + 10 - 10x$$

Hence $a_{11} = 12 - 11x$

8- Find the n^{th} term of an A.P, where $a_{n-5} = 3n + 9$

$a_n = ?$

Solution:-

$a_{n-5} = 3n + 9$ We put $n + 5$ in place of n.

$a_{n+5-5} = 3(n + 5) + 9$

$a_n = 3n + 15 + 9$

$a_n = 3n + 24$

9- Find the n^{th} term of an A.P: $\left(\frac{3}{4}\right)^2, \left(\frac{3}{7}\right)^2, \left(\frac{3}{10}\right)^2 \ldots\ldots$

Solution:

$a = 4$

$d = 7 - 4 = 3$

$a_n = ?$

$a_n = a + (n - 1)\,d$ ***formula***

$= 4 + (n - 1)(3)$

$= a + 3n - 3$

$= 4 + 3n - 3$

$= 3n + 1$

The nth term of the sequence $\left(\frac{3}{3n+1}\right)^2$

10- If the n^{th} term of an A.P is $3n - 5$. Find the A.P

Solution:-

$a_n = 3n - 5$

Putting $n = 1, 2, 3, 4, \ldots.$

$a_1 = 3(1) - 5 = 3 - 5 = -2$

$a_2 = 3(2) - 5 = 6 - 5 = 1$

$a_3 = 3(3) - 5 = 9 - 5 = 4$

$a_4 = 3(4) - 5 = 12 - 5 = 7$

$-2, 1, 4, 7\ldots\ldots$ *is required*

Exercise 7.3

1- Find A.M between:

(i) $-3, 7$ (ii) $x-1, x+7$

(iii) $\sqrt{7}, 3\sqrt{7}$ (iv) x^2+x+1, x^2-x+1

Solution:-(i) $-3, 7$

Here $a = -3$

$b = 7$

(A. M) $A = \dfrac{-3+7}{2}$

Here $A = \dfrac{4}{2} = 2$

Solution:-(ii) $x-1, x+7$

Here $a = x-1$

$b = x+7$

(A. M) $A = \dfrac{a+b}{2}$

$= \dfrac{(x-1)+(x+7)}{2}$

$= \dfrac{x-1+x+7}{2}$

$= \dfrac{2x+6}{2}$

$= \dfrac{2(x+3)}{2}$

$A = x+3$

Solution:-(iii) $\sqrt{7}, 3\sqrt{7}$

Here $a = \sqrt{7}$

$b = 3\sqrt{7}$

(A.M) $A = \dfrac{a+b}{2}$

$= \dfrac{\sqrt{7}+3\sqrt{7}}{2}$

$= \dfrac{\sqrt{7}(1+3)}{2}$

$= \dfrac{\sqrt{7}\ \ 4}{2}$

Hence $A = 2\sqrt{7}$

Solution:-(iv) $x^2 + x + 1, x^2 - x + 1$

Here $a = x^2 + x + 1$

$b = x^2 - x + 1$

(A.M) $A = \dfrac{a+b}{2}$

$= \dfrac{(x^2+x+1)+(x^2-x+1)}{2}$

$= \dfrac{x^2+x+1+x^2-x+1}{2}$

$= \dfrac{2x^2+2}{2}$

$= \dfrac{2(x^2+1)}{2}$

Hence $A = x^2 + 1$

2- If 3 and 6 are two A.Ms between a and b, find a and b.

Solution:-

Here A. Ms between $a, 3, 6, b$

$$d = 6 - 3 = 3$$

$$b = a_4 = a_3 + d$$

$$= 6 + 3$$

$$b = 9$$

and

$$a = a_2 - d$$

$$= 3 - 3$$

$$a = 0$$

$$a = 0,\ b = 9$$

We can also find a and b using this method.

$$a + d = 3$$

$$a + 3 = 3$$

$$a = 3 - 3$$

$$a = 0$$

$$6 + d = b$$

and

$$6 + 3 = b$$

$$9 = b$$

$$b = 9$$

3- ***Find three A.Ms between 11 and 19.***

Solution:-

Here

$$a_1 = 11$$

$$a_5 = 19$$

$$n = 5$$

Now

$$a_n = a + (n - 1)d$$

$$a_5 = a + (5 - 1)d$$

Now

$$a_n = 11 + 4d$$

$$19 = 11 + 4d$$

$$19 - 11 = 4d$$

$$8 = 4d$$

$$d = 2$$

$$A_1 = a + d$$

$$= 11 + 2 = 13$$

$$A_2 = A_1 + d$$

$= 13 + 2 = 15$

$A_3 = A_2 + d$

$= 15 + 2 = 17$

required three A. Ms are 13, 15, 17.

4- *Find three A.Ms between $2\sqrt{3}$ and $6\sqrt{3}$*

Solution:-

Let A_1, A_2, A_3 be three A.Ms between $2\sqrt{3}$ and $6\sqrt{3}$. Such that $2\sqrt{3}, A_1, A_2, A_3, 6\sqrt{3}$ is an A.P.

$a_1 = 2\sqrt{3}$

$a_5 = 6\sqrt{3}$

$n = 5$

Now $a_n = a + (n - 1)d$

$6\sqrt{3} - 2\sqrt{3} = 4d$

Or $4d = 6\sqrt{3} - 2\sqrt{3}$

$= \sqrt{3}(6 - 2)$

$4d = \sqrt{3}(4)$

$d = \sqrt{3}$

$A_1 = a + d$

Now $= 2\sqrt{3} + \sqrt{3}$

$\sqrt{3} + (2 + 1)$

$A_1 = 3\sqrt{3}$

and $A_2 = a + 2d$

$= 2\sqrt{3} + 2\sqrt{3}$

$= \sqrt{3}(2 + 2) = 4\sqrt{3}\ A_2$

$A_3 = a + 3d$

$$= 2\sqrt{3} + 3\sqrt{3}$$

$$= \sqrt{3}(2+3)$$

$$A_3 = 5\sqrt{3}$$

$$3\sqrt{3}, 4\sqrt{3}, 5\sqrt{3}$$

Thus $3\sqrt{3}, 4\sqrt{3}, 5\sqrt{3}$ are the required three A.Ms between $6\sqrt{3}$ and $2\sqrt{3}$.

5- Find six A.Ms between 5 and 8.

Solution:-

Let $A_1, A_2, A_3, A_4, A_5, A_6$ be six A.Ms between 5 and 8. Such that $5, A_1, A_2, A_3, A_4, A_5, A_6, 8$ *is an A.P.*

Here $a_1 = 5$

$a_8 = 8$

$n = 8$

$a_n = a + (n-1)d$

$8 = 5 + (8-1)d$

$8 - 5 = 7d$

$3 = 7d$

$7d = 3$ Or

$d = \frac{3}{7}$ Or

Now $A_1 = a + d$

$$= 5 + \frac{3}{7} = \frac{38}{7}$$

$A_2 = a + 2d$

$$= 5 + 2 \times \frac{3}{7}$$

$$= 5 + \frac{6}{7} = \frac{41}{7}$$

$A_3 = a + 3d$

$$= 5 + 3 \times \frac{3}{7}$$

$$= 5 + \frac{9}{7} = \frac{44}{7}$$

$A_4 = a + 4d$

$$= 5 + 4 \times \frac{3}{7}$$

$$= 5 + \frac{12}{7} = \frac{47}{7}$$

$A_5 = a + 5d$

$$= 5 + 5\left(\frac{3}{7}\right)$$

$$= 5 + \frac{15}{7} = \frac{50}{7}$$

$A_6 = a + 6d$

$$= 5 + 6\left(\frac{3}{7}\right)$$

$$= 5 + \frac{18}{7}$$

$$= \frac{35 + 18}{7} = \frac{53}{7}$$

Thus $\frac{38}{7}, \frac{41}{7}, \frac{44}{7}, \frac{47}{7}, \frac{50}{7}, \frac{53}{7}$ are six required A.Ms, between 5 and 8.

6. *Find seven A.Ms between 8 and 12.*

Solution:-

Let $A_1, A_2, A_3, A_4, A_5, A_6, A_7$ be seven required A.Ms, between 8 and 12.

Such that $8\ A_1, A_2, A_3, A_4, A_5, A_6, A_7, 12$ is an A. P.

$$a_1 = 8$$
$$a_9 = 12$$
$$n = 9$$
$$a_n = a + (n-1)d$$
$$12 = 8 + (a-1)d$$
$$12 - 8 = 8d$$
$$4 = 8d$$
$$8d = 4 \qquad \text{Or}$$
$$d = \frac{4}{8}$$
$$d = \frac{1}{2} \qquad \text{Or}$$
$$A_1 = a + d$$
$$= 8 + \frac{1}{2}$$
$$= \frac{16+1}{2} = \frac{17}{2}$$

Now

$$A_2 = 8 + 2d$$
$$= 8 + 2(\frac{1}{2})$$
$$= 8 + \frac{2}{2}$$
$$= \frac{16+2}{2} = \frac{18}{2}$$
$$A_3 = a + 3d$$
$$= 8 + 3(\frac{1}{2})$$
$$= 8 + \frac{3}{2}$$

$$= \frac{16+3}{2} = \frac{19}{2}$$

$A_4 = a + 4d$

$$= 8 + 4(\frac{1}{2}) \quad = \frac{8}{1} + \frac{4}{2}$$

$$= \frac{16+4}{2} = \frac{20}{2}$$

$A_5 = a + 5d$

$$= 8 + 5(\frac{1}{2})$$

$$= \frac{16+5}{2} = \frac{21}{2}$$

$A_6 = 8 + 6d$

$$= 8 + 6(\frac{1}{2})$$

$$= \frac{16+6}{2} = \frac{22}{2}$$

$A_7 = a + 7d$

$$= 8 + 7(\frac{1}{2})$$

$$= \frac{16+7}{2} = \frac{23}{2}$$

Thus $\frac{17}{2}, \frac{18}{2}, \frac{19}{2}, \frac{20}{2}, \frac{21}{2}, \frac{22}{2}, \frac{23}{2}$, are the required seven A.Ms between 8 and 12.

7- ***If the A.M between 5 and b is 10, then find the value of b.***

Solution:-

a, A, b is an A. P.

$$A = \frac{a+b}{2}$$

Here $a = 5$

$b = b$

$A = 10$

$$A = \frac{a+b}{2}$$

$$10 = \frac{5+b}{2}$$

Or $20 = 5 + b$

Or $5 + b = 20$

$b = 20 - 5$

$b = 15$

8- ***If the A.M between a and 10 is 40, then find the value of "a".***

Solution:-

a, A, b is an A. P.

$$A = \frac{a+b}{2}$$

Here $a = a$

$b = 10$

$A = 40$

$$A = \frac{a+b}{2}$$ and

$$40 = \frac{a+10}{2}$$

$$40 \times 2 = a + 10$$

$$80 = a + 10$$

$$a + 10 = 80$$

$$a = 80 - 10 \qquad \text{Or}$$

$$a = 70$$

9. ***If the three A.Ms between a and b are 5,9 and 13, find a and b.***

Solution:-

5, 9 and 13 is an A.P between a and b.

$a, 5, 9, 13, b$ is A. Ms

$$d = 9 - 5 = 4$$

$$= 13 - 9 = 4$$

$$a + d = 5$$

$$a + 4 = 5$$

$$a = 5 - 4$$

$$a = 1 \qquad (i)$$

$$13 + d = b \qquad \text{and}$$

$$13 + 4 = b$$

$$17 = b$$

$$b = 17 \qquad (ii)$$

$$a = 1, b = 17$$

Exerise 7.4

1- Find the 7^{th} term of a G.P 2,8,32.....

Solution: G. P sequence is 2, 8, 32, ...

Given $a = 2$

$$r = \frac{8}{2} = 4$$

$n = 7$

$a_n = ar^{n-1}$ **formula**

$a_7 = ?$

$a_7 = 2(4)^{7-1}$

$= 2(4)^6$

$= 2 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4$

$= 2 \times 4096$

$a_7 = 8192$

2- Find the 11^{th} term of a G.P 2,6,18 ....

Solution:- G. P sequence is 2, 6, 18,.....

Given $a = 2$

$$r = \frac{6}{2} = 3$$

$n = 11$

$a_{11} = ?$

$a_n = ar^{n-1}$ **formula**

$a_{11} = 2 \times 3^{11-1}$

$= 2 \times 3^{10}$

$= 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

$= 2 \times 9 \times 9 \times 9 \times 9$

$= 2 \times 81 \times 81 \times 9$

$a_{11} = 118098$

3- ***Find the 6^{th} term of a G.P $-\frac{3}{2}, 3, -6.....$***

Solution:-

In term of a G. P $-\frac{3}{2}, 3, -6.....$

$$a = -\frac{3}{2}$$

$$r = -\frac{6}{3} = -2$$

$$n = 6$$

$$a_6 = ?$$

$a_n = ar^{n-1}$ ***formula***

$$a_6 = \left(-\frac{3}{2}\right)(-2)^{6-1} \quad \text{Now}$$

$$= \left(-\frac{3}{2}\right)(-2)(-2)(-2)(-2)(-2)$$

$$= \left[\left(-\frac{3}{2}\right)(-2)\right]\left[(-2)(-2)\right]\left[(-2)(-2)\right]$$

$$= 3 \times 4 \times 4$$

$$= 48$$

4- ***Find the 5^{th} term of a G.P 4, –12, 36....***

Solution:-

In term of a G. P 4, -12, 36,....

$$a = 4$$

$$d = \frac{-12}{4} = -3$$

$$n = 5$$

$a_5 = ?$

$a_n = ar^{n-1}$ ***formula***

$a_5 = 4(-3)^{5-1}$

$= 4(-3)^4$

$= 4(-3)(-3)(-3)(-3)$

$= 4 \times 9 \times 9$

$a_5 = 324$

5- *Find the missing elements of the G.P*

(i) $r = 10,\ a_n = 100,\ a = 1$

(ii) $a_n = 400,\ r = 2,\ a = 25$

(iii) $a = 128,\ r = \frac{1}{2},\ a_n = \frac{1}{4}$

Solution:5(i)

$r = 10$

$a_n = 100$

$a = 1$

$n = ?$

$a_n = ar^{n-1}$ ***formula***

Putting values of a ' a_n ' r

$100 = (1)(10)^{n-1}$

$(10)^2 = (10)^{n-1}$

$(10)^2 = (10)^{n-1}$

Therefore $n-1 = 2$

$n = 2 + 1 = 3$

Solution:5(ii)

$a_n = 400$

$r = 2$

$a = 25$

$n = ?$

$a^n = ar^{n-1}$ *formula*

Putting values a ' r ' a_n

$400 = 25(2)^{n-1}$

$$\frac{400}{25} = 2^{n-1}$$

$$16 = 2^{n-1}$$

$$2^4 = 2^{n-1}$$

Therefore $n-1 = 4$

$$n = 4 + 1 = 5$$

Solution:5(iii)

$$a = 128$$

$$r = \frac{1}{2}$$

$$a_n = \frac{1}{4}$$

$$n = ?$$

$a^n = ar^{n-1}$ *Formula*

Putting values a, r, a_n

$$\frac{1}{4} = 128\left(\frac{1}{2}\right)^{n-1}$$

$$\frac{1}{4\times 128} = \left(\frac{1}{2}\right)^{n-1}$$

$$\frac{1}{32\times 4\times 4} = \left(\frac{1}{2}\right)^{n-1}$$

$$\frac{1}{2^2\times 2^2\times 2^2\times 2^3} = \left(\frac{1}{2}\right)^{n-1}$$

$$\frac{1}{2^9} = \left(\frac{1}{2}\right)^{n-1}$$

$$\left(\frac{1}{2}\right)^9 = \left(\frac{1}{2}\right)^{n-1}$$

$$n-1=9$$
$$n=9+1$$

Hence $n=10$

6- *Find the 11^{th} term of a G.P whose 5^{th} term is 9 and common ratio is 2.*

Solution:

$r=2$

$a_5=9$

$a_{11}=?$

$a_n=ar^{n-1}$ *formula*

$a_5=ar^{5-1}=ar^4$ *Putting values of a_5 and r.*

a_5r

$9=a(2)^4$

$9=a\times 16$

$$\frac{9}{16}=a$$

$$a=\frac{9}{16}$$

Now we want to find $a_{11}=ar^{11-1}$

putting values of a and r.

$$a_{11}=\frac{9}{16}\times 2^{10}$$

$$a_{11}=\frac{9}{16}\times 1024$$

$$a_{11}=9\times 64$$

$$=576$$

7- *Find the 13^{th} term of a G.P whose 7^{th} term is 25 and common ratio is 3.*

Solution:

$a_7=25$

$r=3$

$a_{13} = ?$

$a_n = ar^{n-1}$ ***formula***

$a_7 = ar^{7-1} = ar^6$ putting values of a_7 and r.

$25 = a(3)^6$

$25 = a \times 729$

$$\frac{25}{729} = a$$

$$a = \frac{25}{729}$$

$a_{13} = ar^{13-1}$ ***Now***

Putting values of a, a_{13}

$$= \frac{25}{729}(3)^{13-1}$$

$$= \frac{25}{729} \times 3^{12}$$

$$= \frac{25}{3^6} \times 3^6 \times 3^6$$

$$= 25 \times 3^6$$

$$= 25 \times 729 = 18225$$

8- ***If a, b, c, d are in G.P, show that, $a-b, b-c, c-d$, are in G.P***

Solution:-

a, b, c, d is G.P *therefore*

$$(A)\quad \frac{b}{a}=\frac{c}{b}=\frac{d}{c}$$

$$\frac{c}{b}=\frac{d}{c} \qquad \text{and}$$

$$\frac{b}{a}=\frac{d}{c}$$

If $a-b, b-c, c-d$ is G.P.

Then $$\frac{b-c}{a-b}=\frac{c-d}{b-c}$$

$$(b-c)(b-c)=(a-b)(c-d)$$

$$b^2-bc-bc+c^2=ac-ad-bc+bd$$

$$(i)\quad b^2=ca \qquad \left(\frac{b}{a}=\frac{c}{b}\right)$$

$$\left(\frac{c}{b}=\frac{d}{c}\right) \quad c^2=bd \quad (ii)$$

$$\left(\frac{b}{a}=\frac{d}{c}\right) \quad bc=ad \quad (iii)$$

$$b^2-bc-bc+c^2$$

$$=ca-bc-ad+bd \quad \text{From } (i),(ii),(iii)$$

$a-b,\ b-c,\ c-d$ is G.P

Second Law:

a, b, c, d is G.P

$$\frac{b}{a}=\frac{c}{b}=\frac{d}{c}$$ Therefore

$$\frac{a}{b}=\frac{b}{c}=\frac{c}{d} \quad (A) \quad \text{Or}$$

Now if $a-b,\ b-c,\ c-d$ *is G.P*

Then $\dfrac{b-c}{a-b}=\dfrac{c-d}{b-c}$ prove

Now $\dfrac{a}{b}=\dfrac{b}{c}$ from A

$$\frac{a}{b}-1=\frac{b}{c}-1$$

$$\frac{a-b}{b}=\frac{b-c}{c}$$

$$\frac{a-b}{b-c}=\frac{b}{c}$$

$$\frac{b-c}{a-b}=\frac{c}{b} \qquad (i)$$

$$\frac{b}{c}=\frac{c}{d}$$

$$\frac{b}{c}-1=\frac{c}{d}-1$$

$$\frac{b-c}{c}=\frac{c-d}{d}$$

$$\frac{b-c}{c-d}=\frac{c}{d}$$

$$\frac{c-d}{b-c}=\frac{d}{c}$$

$$\frac{c-d}{b-c}=\frac{c}{b}\ (ii)\left(\frac{c}{b}=\frac{d}{c}\right) \quad (A)$$

$$\frac{b-c}{a-b}=\frac{c-d}{b-c} \qquad (i),(ii)$$

9- ***Find the n^{th} term of a G.P, if $\frac{a_5}{a_3}=\frac{4}{9}$ and a***

Solution:

$$\frac{a_5}{a_3}=\frac{4}{9}$$

$$\frac{ar^{5-1}}{ar^{3-1}}=\frac{4}{9}$$

$$\frac{r^4}{r^2}=\frac{4}{9}$$

$$r^2=\left(\frac{2}{3}\right)^2$$

$$r=\frac{2}{3} \qquad (A)$$

$$a_2=ar^{2-1} \qquad (i)$$

$$a_2=\frac{4}{9} \qquad (ii)$$

$$ar=\frac{4}{9} \qquad \text{from } (i) \text{ and } (ii)$$

$$a\times\frac{2}{3}=\frac{4}{9} \qquad \text{from } (A)$$

$$a=\frac{4}{9}\times\frac{3}{2}$$

$$a=\frac{2}{3}$$

Now $a_n = ar^{n-1}$

$$= \frac{2}{3}\left(\frac{2}{3}\right)^{n-1}$$

$$a_n = \left(\frac{2}{3}\right)^{n-1+1} = \left(\frac{2}{3}\right)^n$$

10- ***Find three consecutive numbers in G.P, whose sum is 26 and their product is 216.***

Solution:-

Three consecutive numbers according to the given condition a, ar, ar^2.

$$a + ar + ar^2 = 26$$

$$(a)(ar)(ar^2) = 216 \quad \text{and}$$

$$a^3r^3 = 216$$

$$(ar)^3 = (6)^3$$

$$ar = 6 \qquad (A)$$

$$a = \frac{6}{r} \qquad \text{Putting (i)}$$

$$a + ar + ar^2 = 26$$

$$a(1 + r + r^2) = 26$$

$$a = \frac{6}{r} \qquad (i)$$

$$(1 + r + r^2) = 26\frac{6}{r}$$

$$6(1 + r + r^2) = 26r$$

$$6 + 6r + 6r^2 - 26r = 0$$

$$6r^2 - 20r + 6 = 0 \quad \text{Dividing by 2}$$

$$3r^2 - 10r + 3 = 0$$

$$3r^2 - r - 9r + 3 = 0$$

$$r(3r - 1) - 3(3r - 1) = 0$$

$$(3r - 1)(r - 1) = 0$$

$$3r-1=0$$
$$3r=1$$
$$r=\frac{1}{3} \qquad (ii)$$

and $r-3=0$

$r=3$ then (B)

$ar=6$ Now (A)

$a(3)=6$ Now (B)

$$a=\frac{6}{3}$$
$$a=2 \qquad (C)$$

Take 3 or $a=2, r=\frac{1}{3}$

$$a=2$$
$$ar=2\times\frac{1}{3}=\frac{2}{3}$$
$$ar^2=2\times\left(\frac{1}{3}\right)^2=2\times\frac{1}{9}$$
$$=\frac{2}{9}$$

Three consecutive numbers are $2, \frac{2}{3}, \frac{2}{9}$

If $r=3$

Then $a=2$

$ar=2\times3=6$

$ar^2=2\times3^2=2\times9=18$

Consecutive number 2, 6, 18

11- ***Find the 30th term of a G.P*** $x, 1, \frac{1}{x}....$

Solution:

G. P is $x, 1, \frac{1}{x} \ldots.$

$$a = x \qquad (i)$$

$$r = \frac{1}{x} \qquad (ii)$$

$$a_n = ar^{n-1}$$

$$a_{30} = ar^{30-1}$$

from (i) and (ii)

$$= x \times \left(\frac{1}{x}\right)^{29}$$

$$= x \times x^{-29}$$

$$= x^{1-29}$$

$$= x^{-28}$$

$$= x^{\frac{1}{28}}$$

12- Find the p^{th} term of a G.P $x, x^3, x^5, \ldots\ldots$

Solution:-

G.P is $x, x^3, x^5, \ldots\ldots$

Therefore $a = x$

$$r = \frac{x^3}{x}$$

$$x^2 =$$

$$a_n = ar^{n-1} \qquad \text{formula}$$

$$a_p = (x)(x^2)^{p-1}$$

$$= (x)x^{2(p-1)}$$

$$= x\, x^{2p-2}$$

$$= x^{2p-2+1}$$

$$a_p = x^{2p-1}$$

Exercise 7.5

G.M between a, b is G. $G = \pm\sqrt{ab}$

1- ***Find G.M between: (i) 9 and 5 (ii) 4 and 9 (iii) –2 and –8.***

(i) 9 and 5 (ii) 4 and 9

(iii) -2 and -8

Solution:(i)

Given 9 and 5

$$G = \pm\sqrt{ab}$$
$$= \pm\sqrt{9\times 5}$$
$$= \pm 3\sqrt{5}$$

Solution:- (ii)

4 and 9

$$G = \pm\sqrt{ab}$$
$$= \pm\sqrt{(4)(9)}$$
$$= \pm\sqrt{36}$$
$$= \pm 6$$

Solution:- (iii)

-2 and -8

$$G = \pm\sqrt{ab}$$
$$= \pm\sqrt{(-2)(-8)}$$
$$= \pm\sqrt{16}$$
$$= \pm 4$$

2- ***Insert two G.Ms between:(i) 1 and 8 (ii) 3 and 81***

(i) 1 and 8 (ii) 3 and 81

Solution:-

Let G_1 and G_2 be the two G.Ms between *1* and *8* such that *1*, G_1, G_2 *8* is a *G.P.*

Here $a = 1$

$n = 4$

$a_4 = 8$

Since $a_n = ar^{n-1}$

$a_4 = ar^{4-1}$

$8 = ar^3$

$8 = 1 \times r^3$

$2^3 = r^3$

$r = 2$

Thus $G_1 = ar$

$= 1 \times 2 = 2$

$G_2 = ar^2$

$= 1 \times 2^2$

$= 1 \times 4 = 4$

Solution:- (ii) 3 and 81

Let G_1 and G_2 be the two GMs between 3 and 81, such that 3, G_1, G_2, 81 is a G.P.

$n = 4$

Here $a = 3$

$n = 4$

$a_4 = 81$

Since $a_n = ar^{n-1}$

$a_4 = ar^{4-1}$

$81 = 3r^3$

$r^3 = \frac{81}{3} = 27$

$r^3 = 3^3$

$r = 3$

Thus $G_1 = ar$

$= 3 \times 3$

$= 9$

$G_2 = ar^2$

$= 3(3)^2$

$= 3 \times 9 = 27$

3- ***Insert three G.Ms between:(i) 1 and 16 (ii) 2 and 32***

(i) 1 and 16

(ii) 2 and 32

Solution:-

Let G_1, G_2 and G_3 be the three G.Ms between 1 and 16, that are required. G.Ms 1, G_1, G_2, G_3 16 is a G. P.

Here $a = 1$

$n = 5$

$a_5 = 16$

Since $a_n = ar^{n-1}$

$a_5 = (1)r^{5-1}$

$16 = r^4$

$r^3 = r^4$

Thus $2^4 = r^4$

$r = 2$

$G_1 = ar$

$= 1 \times 2 = 2$

$G_2 = ar^2 = 1 \times 2^2 = 4$

$G_3 = ar^3 = 1 \times 2^3 = 8$

2, 4, 8 are required three G. Ms

Solution:- 2 and 32

Let G_1, G_2, G_3 be the three G.Ms between 2 and 32 that are required. G.Ms 2, G_1, G_2 and G_3 32 is a G. P.

Here $a = 2$

$n = 5$

$a_5 = 32$

Since $a_n = ar^{n-1}$

$a_5 = ar^{5-1}$

$32 = 2r^4$

$r^4 = 16$

Thus $r^4 = 2^4$

$r = 2$

Now $G_1 = ar$

$= 2 \times 2 = 4$

$G_2 = ar^2$

$= 2 \times 2^2 = 8$

$G_3 = ar^3$

$= 2 \times 2^3 = 16$

4, 8 and 16 are required three G.Ms.

4- *Insert four real geometric means between 3 and 96.*

Solution:- ***3 and 96***

Let G_1, G_2, G_3 and G_4 be the four G.Ms between 3 and 96 that are required. G. Ms 3, G_1, G_2, G_3 and G_4, 96 is a G. P.

Here $a = 3$

$n = 6$

$a_6 = 96$

Since $a_n = ar^{n-1}$

$a_6 = ar^{6-1}$

$96 = 3r^5$

$$\frac{96}{3} = r^5$$

$$r^5 = \frac{96}{3} = 32$$

$r^5 = 2^5$

$r = 2$

$G_1 = ar$

$= 3 \times 2 = 6$

$G_2 = 3r^2$

$= 3(2)^2 = 12$

$G_3 = 3r^3$

$= 3(2)^3 = 24$

$G_4 = 3(r)^4$

$= 3(2)^4 = 48$

6,12,24,48 are four required G.Ms.

5- ***The A.M between two numbers is 5 and their positive G.M is 4. Find the numbers.***

Solution:-

Let numbers are a and b.

(i) A is A.M $A = \frac{a+b}{2}$

(ii) G is G.M $G = \sqrt{ab}$

According to condition A = 5, G = 4

$$5 = \frac{a+b}{2}$$ from (i) and (ii)

$$4 = \sqrt{ab}$$

$$16 = ab$$ *from (iii)*

$$10 = a + b$$ *from (iv)*

$$10 - b = a$$ Or

$$a = 10 - b$$ Or

Putting values of a in *(iii)*

$$16 = (10 - b)(b)$$

$$16 = 10b - b^2$$

$$b^2 - 10b + 16 = 0$$ Or

$$b^2 - 2b - 8b + 16 = 0$$

$$b(b-2) - 8(b-2) = 0$$

$$(b-2)(b-8) = 0$$

$$b - 2 = 0$$

$$b = 2$$

$$b - 8 = 0$$

$$b = 8$$

$$a + b = 10$$ Now from *(iv)*

$$a + 2 = 10$$

$$a = 10 - 2 = 8$$

$$a + 8 = 10$$ b = 8

$$a = 10 - 8 = 2$$

Hence Numbers are 2,8 or 8,2

6- ***The positive G.M between two numbers is 6 and the A.M between them is 10. Find the numbers.***

Solution:-

$$\text{A is A.M} = \frac{a+b}{2} \qquad \text{(i)}$$

$$\text{G is G.M} = \sqrt{ab} \qquad \text{(ii)}$$

From (i) and (ii)

$$10 = \frac{a+b}{2} \quad \text{Or}$$

$$20 = a + b \qquad \text{(iii)}$$

$$6 = \sqrt{ab} \quad \text{Or}$$

$$36 = ab \qquad \text{(iv)}$$

$$a = 20 - b \qquad \text{from (iii)}$$

Putting values of a in (iv)

$$36 = (20 - b)(b)$$

$$36 = 20b - b^2$$

$$b^2 - 20b + 36 = 0 \quad \text{Or}$$

$$b^2 - 2b\ b - 18b + 36 = 0$$

$$b(b-2) - 18(b-2) = 0$$

$$(b-2)(b-18) = 0$$

If $b - 2 = 0$

$$b - 18 = 0$$

$$b - 2 = 0$$

$$b = 2$$

Putting b = 2 in (iv)

$$[redacted] = 2a$$

$$a = 18$$

$b - 18 = 0$ *and*

$$b = 18$$

Putting b = 18 in (iv)

$$36 = 18a$$

$$a = 2$$

Thus numbers are 2, 18 or 18, 2

7- ***Show that the A.M between the two numbers 4 and 8 is greater than their geometric mean.***

Solution:-

Here $a = 4$

$b = 8$

$$(A.M) = \frac{4+8}{2} = 6$$

$$(G.M) = \sqrt{ab}$$

$$G = \sqrt{4 \times 8}$$

$$= \sqrt{32}$$

Now $\sqrt{32}$ *is less than 6.*

Proved $6 > \sqrt{32}$

Therefore $A > G$

8- ***Insert four geometric means between 160 and 5.***

Solution:-

numbers are 5 and 160.

Let G_1, G_2, G_3 and G_4 the four G.Ms between 160 and 5.

Here $n = 6$

$a = 5$

$a_6 = 160$

Since $a_n = ar^{n-1}$

$a_6 = 5r^{6-1}$

$160 = 5r^5$

$$r^5 = \frac{160}{5}$$

$r^5 = 32 = 2^5$

$r = 2$

$G_1 = ar$

$$= 5 \times 2 = 10$$

$$G_2 = ar^2$$

$$= 5 \times 2^2 = 20$$

$$G_3 = ar^3$$

$$= 5 \times 2^3 = 40$$

$$G_4 = 5 \times 2^4 = 80$$

Hence required G.Ms are 10, 20, 40, 80.

9. ***Insert three geometric means between 486 and 6.***

Solution:- ***Let G_1, G_2 and G_3 the three G. Ms between 186 and 6.***

Here $n = 5$

$a = 486$

$a_5 = 6$

Since $a_n = ar^{n-1}$

$a_5 = ar^{5-1}$

$6 = 486r^4$

$$\frac{6}{486} = r^4$$

$$\frac{1}{81} = r^4$$

$$r^4 = \frac{1}{81} = \left(\frac{1}{3}\right)^4$$

$$r = \frac{1}{3}$$

$$G_1 = ar$$

$$= 486 \times \frac{1}{3} = 162$$

$$G_2 = 486 \times \left(\frac{1}{3}\right)^2$$

$$= 486 \times \frac{1}{9} = 54$$

$$G_3 = 486 \times \left(\frac{1}{3}\right)^3$$

$$= 486 \times \frac{1}{27} = 18$$

Hence required G.Ms are 162, 54, 18.

10- *Insert four geometric means between $\frac{1}{8}$ and 128.*

Solution:- $\frac{1}{8}$, 128

Let G_1, G_2, G_3, G_4 be the four G.Ms between $\frac{1}{8}$ and 128.

Here $n = 6$

$$a = \frac{1}{8}$$

$$a_6 = 128$$

$$a_n = ar^{n-1}$$

$$a_6 = ar^{6-1}$$

$$128 = \frac{1}{8} r^5$$

$$128 \times 8 = r^5$$

$$1024 = r^5$$

$$4^5 = r^5$$

$$r = 4$$

$$G_1 = ar$$

$$= \frac{1}{8} \times 4 = \frac{1}{2}$$

$$G_2 = ar^2$$

$$= \frac{1}{8} \times 4^2$$

$$= \frac{1}{8} \times 16 = 2$$

$$G_3 = ar^3$$

$$= \frac{1}{8} \times 4^3$$

$$= \frac{1}{8} \times 64 = 8$$

$$G_4 = \frac{1}{8} r^4$$

$$= \frac{1}{8} \times 4^4 = 32$$

$\frac{1}{2}$, 2, 8, 32 are required G.Ms.

11- Insert six geometric means between 56and $-\frac{7}{16}$

Solution:- 56, $-\frac{7}{16}$

Let $G_1, G_2, G_3, G_4, G_5, G_6$ be the six G.Ms between 56 and $-\frac{7}{16}$

Here $n = 8$

$a = 56$

$a_6 = -\frac{7}{16}$

Since $a_n = ar^{n-1}$

$a_6 = ar^{a-1}$

$$-\frac{7}{16} = 56r^7$$

$$r^7 = -\frac{7}{16 \times 56}$$

$$= -\frac{1}{16 \times 8}$$

$$= -\frac{1}{128}$$

$$r^7 = \left(-\frac{1}{2}\right)^7$$

$$r = -\frac{1}{2}$$

$$G_1 = ar$$

$$= 56\left(-\frac{1}{2}\right) = -28$$

$$G_2 = ar^2$$

$$= 56\left(-\frac{1}{2}\right)^2$$

$$= 56 \times \frac{1}{4} = 14$$

$$G_3 = ar^3$$

$$= 56 \times \left(-\frac{1}{2}\right)^3$$

$$= 56 \times \left(-\frac{1}{8}\right) = -7$$

$$G_4 = ar^4$$

$$= 56\left(-\frac{1}{2}\right)^4$$

$$= 56 \times \frac{1}{16} = \frac{7}{2}$$

$$G_5 = ar^5$$

$$= a\left(-\frac{1}{2}\right)^5$$

$$= 56 \times \left(-\frac{1}{32}\right) = -\frac{7}{4}$$

$$G_6 = ar^6$$

$$= 56 \times \left(-\frac{1}{2}\right)^6$$

$$= 56 \times \frac{1}{64} = \frac{7}{8}$$

Six required G. Ms are $-28, 14, -7, \frac{7}{2}, -\frac{7}{4}, \frac{7}{8}$.

12- Insert five geometric means between $\frac{32}{81}$ and $\frac{9}{2}$

Solution:-

Numbers are $\frac{32}{81}, \frac{9}{2}$

Let $G_1 G_2 G_3 G_4 G_5$ *be the five G. Ms between* $\frac{32}{81}$

and $\frac{9}{2}$.

Here $n = 7$

$$a = \frac{32}{81}$$

$$a_6 = +\frac{9}{2}$$

Since $a_n = ar^{n-1}$

$$a_n = ar^{6-1}$$

$$\frac{9}{2} = \frac{32}{81}r^5$$

$$r^5 = \frac{9}{2} \times \frac{32}{81}$$

$$r^5 = \frac{729}{64} = \left(\frac{3}{2}\right)^5$$

Therefore $r = \frac{3}{2}$

$$G_1 = ar$$

$$= \frac{32}{81} \times \frac{3}{2} = \frac{16}{27}$$

$$G_2 = ar^2$$

$$= \frac{32}{81} \times \left(\frac{3}{2}\right)^2$$

$$= \frac{32}{81} \times \frac{9}{4} = \frac{8}{9}$$

$$G_3 = ar^3$$

$$= \frac{32}{81} \times \left(\frac{3}{2}\right)^3$$

$$= \frac{32}{81} \times \frac{27}{8} = \frac{4}{3}$$

$$G_4 = ar^4$$

$$= \frac{32}{81} \times \left(\frac{3}{2}\right)^4$$

$$= \frac{32}{81} \times \frac{81}{16} = 2$$

$$G_5 = ar^5$$

$$= \frac{32}{81} \times \left(\frac{3}{2}\right)^5$$

$$= \frac{32}{81} \times \frac{243}{32} = 3$$

Five required G.Ms are $\frac{16}{27}, \frac{8}{9}, \frac{4}{3}, 2, 3$

Exercise 8.1

1- *If* $A=\{1,4,7,8\}, B=\{4,6,8,9\}$ *and* $C=\{3,4,5,7\}$

find:

(i) $A \cup B$ (ii) $B \cup C$

(iii) $A \cap C$ (iv) $A \cap (B \cap C)$

(v) $(A \cup B) \cup C$ (vi) $(A \cap B) \cap C$

Solution:1(i)

$A = \{1, 4, 7, 8\}$

$B = \{4, 6, 8, 9\}$

$A \cup B = ?$

$A \cup B = \{1, 4, 7, 8\} \cup \{4, 6, 8, 9\}$

$= \{1, 4, 6, 7, 8, 9\}$

Solution:1(ii)

Taking common elements once a time

$B = \{4, 6, 8, 9\}$

$C = \{3, 4, 5, 7\}$

$B \cup C = ?$

$B \cup C = \{4, 6, 8, 9\} \cup \{3, 4, 5, 7\}$

$B \cup C = \{3, 4, 5, 6, 7, 8, 9\}$

Solution:1(iii)

$A = \{1, 4, 7, 8\}$

$C = \{3, 4, 5, 7\}$

$A \cap C = ?$

$A \cap C = \{4, 7\}$

Solution:1(iv)

$A = \{1, 4, 7, 8\}$

$B = \{4, 6, 8, 9,\}$

$C = \{3, 4, 5, 7\}$

$A \cap (B \cap C) = ?$

$B = \{4, 6, 8, 9\}$

$C = \{3, 4, 5, 7\}$

$$B \cap C = \{4, 6, 8, 9\} \cap \{3, 4, 5, 7\}$$
$$= \{4\}$$
$$A \cap (B \cap C) = \{1, 4, 7, 8\} \cap \{4\}$$
$$= \{4\}$$

Solution:1(v)

$$(A \cup B) \cup C = ?$$
$$A = \{1, 4, 7, 8\}$$
$$B = \{4, 6, 8, 9\}$$
$$C = \{3, 4, 5, 7\}$$
$$A = \{1, 4, 7, 8\}$$
$$B = \{4, 6, 8, 9\}$$
$$A \cup B = \{1, 4, 7, 8\} \cup \{4, 6, 8, 9\}$$
$$A \cup B = \{1, 4, 7, 8, 9\}$$
$$(A \cup B) \cup C = \{1, 4, 6, 7, 8, 9\} \cup \{3, 4, 5, 7\}$$
$$(A \cup B) \cup C = \{1, 3, 4, 5, 6, 7, 8, 9\}$$

Solution:1(vi)

$$(A \cap B) \cap C = ?$$
$$A = \{1, 4, 7, 8\}$$
$$B = \{4, 6, 8, 9\}$$
$$C = \{3, 4, 5, 7\}$$
$$A = \{1, 4, 7, 8\}$$
$$B = \{4, 6, 8, 9\}$$
$$A \cap B = \{1, 4, 7, 8\} \cap \{4, 6, 8, 9\}$$
$$= \{4\}$$
$$(A \cap B) \cap C = \{4\} \cap \{3, 4, 5, 7\}$$
$$= \{4\}$$

2- *If* $A = \{1, 7, 11, 15, 17, 21\}, B = \{11, 17, 19, 23\}$ *and* $C = \{2, 3, 5\}$ *verify that:*

$$(A \cap B) \cap C = A \cap (B \cap C)$$

Solution: L.H.S

$(A \cap B) \cap C = ?$

$A \cap B = \{1,7,11,15,17,21\} \cap \{11,17,19,23\}$

$= \{11,17\}$ (iv)

$(A \cap B) \cap C = \{11,17\} \cap \{2,3,5\}$ (iv),(iii)

$= \{\cdot\}$

$= \phi$ empty set (No element is common)

R.H.S

$= A \cap (B \cap C) = ?$

$B \cap C = \{11,17,19,23\} \cap \{2,3,5\}$

$= \{\ \}$ (v) No element is common

$A \cap (B \cap C) = \{1,7,11,15,17,21\}$ $\{\cap\}$ (i)(v)

$= \{\ \}$

$= \phi$

L.H.S = R.H.S

$(A \cap B) \cap C = A \cap (B \cap C)$

3- If A = {2, 4, 6}, B = {3, 6, 9, 12} and C = {4, 6, 8, 10} verify that:

$$A \cup (B \cup C) = (A \cup B) \cup C$$

Solution:

$(A \cup B) \cup C = ?$

$A \cup B = \{2, 4, 6\} \cup \{3, 6, 9, 12\}$

$= \{2, 3, 4, 6, 9, 12\}$

Taking common elements once a time

$(A \cup B) \cup C = \{2, 3, 4, 6, 9, 12\} \cup \{4, 6, 8, 10\}$

$= \{2, 3, 4, 6, 8, 9, 10, 12\}$

$A \cup (B \cup C) = ?$

$(B \cup C) = \{2,4,6\} \cup \{3,4,6,8,9,10,12\}$

$= \{3,4,6,8,9,10,12\}$

$$A \cup (B \cup C) = \{2,4,6\} \cup \{3,4,6,8,9,10,12\}$$

$$= \{2,3,4,6,8,9,10,12\}$$

L.H.S = R.H.S

4- *If* $A = \{2,3,5,7,9\}$, $B = \{1,3,5,7\}$ *and* $C = \{2,3,4,5,6\}$ ***verify that:***

$$(A \cap B) \cap C = A \cap (B \cap C)$$

$$(A \cap B) \cap C = ?$$

$$(A \cap B) = \{2, 3, 5, 7, 9\} \cap \{1, 3, 5, 7\}$$

(Only common elements) $= \{3, 5, 7\}$

$$(A \cap B) \cap C = \{3, 5, 7\} \cap \{2, 3, 4, 5, 6\}$$

(Only common elements)

$$= \{3, 5\}$$

$$A \cap (B \cap C) = ?$$

$$A \cap B = \{1, 3, 5, 7\} \cap \{2, 3, 4, 5, 6\}$$

(Only common elements)

$$= \{3, 5\}$$

$$A \cap (B \cap C) = \{2,3,5,7,9\} \cap \{3, 5\}$$

$$A \cap (B \cap C) = \{3, 5\}$$

$$A \cap (B \cap C) = (A \cap B) \cap C$$

5- *If* $U = \{7,8,9,10,11,12,13,14\}$ $A = \{7,10,13,14\}$ ***and*** $B = \{7,8,11,12\}$ ***verify***

$$(A \cap B)' = A' \cup B'$$

Solution:-

$U = \{7, 8, 9, 10, 11, 12, 13\}$

$A = \{7,10,13,14\}$

$B = \{7,8,11,12\}$

(Only common elements)

$A \cap B = \{7, 10, 13, 14\} \cap \{7, 8, 11, 12\}$

$A \cap B \quad \{7\}$

$(A \cap B)' \equiv \cup - (A \cap B)$

$= \{7,8,9,10,11,12,13,14\} - \{7\}$

$= \{8, 9, 10, 11, 12, 13, 14\}$

$A' \cup B'$

$A = \{7,10,13,14\}$

First we find $A' = u - B$

$= \{7,8,9,10,11,12,13,14\} - \{7,10,13,14\}$

elements of A are left

$= \{8,9,11,12\}$

Now $B = \{7,8,11,12\}$

$B' = u - A$

$= \{7,8,9,10,11,12,13,14\} - \{7,8,11,12\}$

elements of B are left

$B' = \{9,10,13,14\}$

$A' \cup B'$

$= \{8,9,11,12\} \cup \{9,10,13,14\}$

$= \{8,9,10,11,12,13,14\}$ (M)

$(A \cap B)' = A' \cup B'$

6- $U = \{4,6,8,9,10\}$, $A = \{4,6\}$ **and** $B = \{6,8,9\}$ **then verify De Morgan's Law**

Solution:-

First we write De Morgan's Laws

$(i) \quad (A \cup B)' = A^c \cap B^c$

$(ii) \quad (A \cap B)' = A' \cup B'$

(i) $u = \{4,6,8,9,10\}$

(ii) $A=\{4,6\}$

(iii) $B=\{6,8,9\}$

1st Law

$$(A\cup B)^c = A^c \cup B^c$$

L.H.S $A\cup B = \{4,6\}\cup\{6,8,9\}$

$=\{4,6,8,9\}$ *(iv)*

Now $(A\cup B)^c = u-(A\cup B)$

$=\{4,6,8,9,10\}-\{4,6,8,9\}$

$=\{10\}$

$A^c = u-A$

From (i) and (ii) $=\{4,6,8,9,10\}-\{4,6\}$

$=\{8,9,10\}$

$B^c = u-B$

$=\{4,6,8,9,10\}-\{6,8,9\}$

$B^c = \{4,10\}$

$A^c \cap B^c = \{8,9,10\}\cap\{4,10\}$

$=\{10\}$

Result (v) and (viii)

$$(A\cup B)^c = A^c \cap B^c$$

$$(A\cap B)^c = A^c \cup B^c$$

2nd Law

L.H.S $A=\{4,6\}$

$B=\{6,8,9\}$

$A\cap B = \{4,6\}\cap\{6,8,9\}$

$=\{6\}$

Now $(A\cap B)^c = u-(A\cap B)$

$=\{4,6,8,9,10\}-\{6\}$

$= \{4,8,9,10\}$

$A^C = u - A$

$R.H.S = \{4,6,8,9,10\} - \{4,6\}$

$= \{8,9,10\}$

and $B^C = u - B$

$= \{4,6,8,9,10\} - \{6,8,9\}$

$= \{4,10\}$

$A^C \cup B^C = \{8,9,10\} \cup \{4,10\}$

$P = \{4,8,9,10\}$

Proved

$(A \cap B)^C = A^C \cup B^C$

7- If $U = \{1,2,3,4,5,6,7,8,9\}$, $A = \{2,3,6,9\}$ **and** $B = \{1,3,6,7,8\}$ **then verify** $(A \cup B)' = A' \cap B'$

L.H.S

$A \cup B = \{2,3,6,9\} \cup \{1,3,6,7,8\}$

$= \{1,2,3,6,7,8,9\}$

$(A \cup B)' = u - (A \cup B)$

$= \{1,2,3,4,5,6,7,8,9\} - \{1,2,3,6,7,8,9\}$iv,i

$= \{4,5\}$

R. H. S $A' = u - A$

$= \{1,2,3,4,5,6,7,8,9\} - \{2,3,6,9\}$

$= \{1,4,5,7,8\}$

$B' = u - B$

$= \{1,2,3,4,5,6,7,8,9\} - \{1,3,6,7,8\}$

$B' = \{2,4,5,9\}$

Now $A' \cap B' = \{1,4,5,7,8\} \cap \{2,4,5,9\}$

$= \{4,5\}$....

$(A \cup B)' = A' \cap B'$

$L.H.S = R.H.S$

8- *Fill in the blanks:*

(i) $A \cup A =$ _____ (ii) $A \cap A =$ _____

(iii) $A \cup \Phi =$ _____ (iv) $A \cap \Phi =$ _____

(v) $\Phi \cup \Phi =$ _____ (vi) $(A \cap B)' =$ _____

(vii) $(A \cup B)' =$ _____ (viii) $(A')' =$ _____

(ix) $\Phi \cap \Phi' =$ _____ (x) $A \cap A' =$ _____

Answers:

(i) A *(ii)* A *(iii)* A *(iv)* ϕ *(v)* ϕ

(vi) $A' \cup B'$ *(vii)* $A' \cap B'$ *(viii)* A *(ix)* ϕ *(x)* ϕ

Exercise 8.2

1- *If* $A=\{3,5,6\}$, $B=\{1,3\}$, *Find* $A\times B$ *and* $B\times A$ *and also the domains and range of the two binary relations established at our own for each case.*

Solution:-

$$A=\{3,5,6\}$$
$$B=\{1,3\}$$
$$A\times B=\{3,5,6\}\times\{1,3\}$$
$$=\{(3,1),(3,3),(5,1),(5,3),(6,1),(6,3)\}$$
$$R_1=\{(3,1),(5,3)\}$$
$$Dom\ R_1=\{3,5\}$$
$$Range\ R_1=\{1,3\}$$

Let $R_2=\{(3,1),(3,3),(5,3)\}$

$Dom\ R_2=\{3,5\}$

$Range\ R_2=\{1,3\}$

Now $A=\{3,5,6\}$

$B=\{1,3\}$

Then $B\times A=\{1,3\}\times\{3,5,6\}$

$=\{(1,3),(1,5),(1,6),(3,3),(3,5),(3,6)\}$

Let $R_3=\{(1,3),(3,3)\}$

$Dom\ R_3=\{1,3\}$

$Range\ R_3=\{3\}$

Let $R_4=\{(1,5),(1,6),(3,5),(3,6)\}$

$Dom\ R_4=\{1,3\}$

$Range\ R_4=\{5,6\}$

2- *If* $A=\{-2,1,4\}$, *then write two binary relations A also write their domains and range.*

Solution:-

$$A=\{-2,1,4\}$$
$$A\times A=\{-2,1,4\}\times\{-2,1,4\}$$

$= \{(-2,-2),(-2,1),(-2,4),(1,-2),(1,1),(1,4),$
$(4,-2),(4,1),(4,4)\}$

$R_1 = \{(-2,-2),(1,-2),(4,4)\}$

Now $Dom\ R_1 = \{-2,1,4\}$

$Range\ R_1 = \{-2,4\}$

Let $R_2 = \{(-2,1),(-2,4),(1,1),(4,1)\}$

Now $Dom\ R_2 = \{-2,1,4\}$

$Range\ R_2 = \{1,4\}$

3- *Write the number of binary relations possible in each of following cases.*

(i) In $C \times C$ when the number of elements in C is 3.

(ii) In $A \times B$ if the number of elements in set A is 3 and in set B is 4.

Solution:3(ii)

Number of elements C = 3

Number of Binary relation C × C = ?

Number of elements C × C = 3 × 3 = 9

Formula

If number of elements in characteristic in cartesian product is 'n' then number of binary relation will be 2^n

Number of Binary relation C × C = 2^9

Number of elements of A = 3

Number of elements of B = 4

Number of elements A × B = 3 × 4 = 12

Number of Binary relation A × B = 2^{12}

4- *If* $L = \{1,2,3\}$ ***and*** $M = \{2,3,4\}$ ***then, write a binary relation R such that*** $R = \{(x,y) \setminus x \in L, y \in M \wedge y \leq x\}$. ***Also write Dom (R) and Range (R).***

Solution:-

$L = \{1,2,3\}$

$M = \{2,3,4\}$

$$R = \{(x,y) / x \in L, y \in M \wedge y \leq x\}$$

Now $R = \{(x,y) / x \in L, y \in M \wedge y \leq x\}$

The explanation of this condition is that we have to write such kind of binary relation, whose first elements are the number of L. And in the 2nd condition is that, second elements should be member of M. According to the condition 1st elements are smaller or equal.

Now $L \times M = \{1,2,3\} \times \{2,3,4\}$

$$= \begin{Bmatrix} (1,2),(1,3),(1,4),(2,2),(2,3), \\ (2,4),(3,2),(3,3),(3,4) \end{Bmatrix}$$

$R = \{(2,2),(3,2),(3,3)\}$

Dom $R = \{2,3\}$

Range $R = \{2,3\}$

5- If $X = \{0,3,5\}$ and $Y = \{2,4,8\}$, then establish any four binary relations in $X \times Y$.

Solution:-

$X \times Y = \{0,3,5\} \times \{2,4,8\}$

$= \{(0,2),(0,4),(0,8),(3,2),(3,4),$

$(3,8),(5,2),(5,4),(5,8)\}$

Now $R_1 = \{(0,2)\}$

$R_2 = \{(0,8),(3,4)\}$

$R_3 = \{(3,8)\}$

$R_4 = \{(0,4),(3,2),(3,8),(5,2),(5,8)\}$

6- *If* $A = \{a,b,c\}$ ***and*** $B = \{2,4,6\}$ ***and*** $f = \{(a,4),(b,4),(c,4)\}$ ***is a binary relation***

from $A \times B$, then show that 'f' is A into B function.

Solution:- Proof

$A = \{a, b, c\}$

$B = \{2, 4, 6\}$

$f = \{(a, 4),(b, 4),(c, 4)\}$

$Dom\ f = \{a, b, c\} = A$

i) *The first condition of function is proved.*

ii) *There is no repetition in the first element of ordered pairs contained in "f". i.e. a, b, c is not repeated. The second condition of function is proved. The "f" is called a function from A to B and expressed as* $f: A \to B$.

$Range\ f = \{4\}$

$Range\ f = \{4\} \neq B$

$Range\ f = \{4\} \subset C$ *Hence 'f' A into B is function*

7- If $A = \{l, m, n\}$ and $B = \{1, 2, 3\}$ and $g = \{(l, 3),(m, 1),(n, 1)\}$ is a binary relation from $A \times B$, then show that 'g' is A into B function.

Solution:-

$A = \{l, m, n\}$

$B = \{1, 2, 3\}$

$g = \{(l, 3),(m, 1),(n, 1)\}$

$Dom\ g = \{l, m, n\} = A$

i) *The first condition of function is proved.*

ii) *There is no repetition in the first element of ordered pair contained in "g". i.e. l, m, n is not repeated.*

iii) *The second condition of "f" is proved.*

Then "g" is called function from A to B and expressed as $f: A \to B$.

$$\text{Range } g = \{3, 1\}$$

$$\text{Range } g = \{3,1\} \neq B$$

$$\text{Range } g = \{3,1\} \subset B$$

But Hence 'g' is A into B function.

8- ***If*** $A = \{1,3,5\}$ ***and*** $B = \{x, y, z\}$ ***and*** $g = \{(1,x),(3,y),(5,z)\}$ ***is a binary relation from*** $A \times B$, ***then show that 'g' is A onto B function.***

Solution:-

$$A = \{1,3,5\}$$

$$B = \{x,y,z\}$$

$$g = \{(1, x),(3, y),(5, z)\}$$

$$\text{Dom } g = \{1,3,5\} = A$$

i) *The first condition of function is proved.*

There is no repetition in the first element of ordered pair contained in 'g' i.e. 1, 2, 3 is not repeated.

Thus second condition of function is proved.

$$\text{Range } g = \{x,y,z\}$$

$$\text{Range } g = \{x,y,z\} = B$$

Thus "g" is A on to B function.

Hence "g" is called function from A to B and expressed as $f: A \to B$.

Exercise 9.1

1- ***Represent the points on the graph whose co-ordinates are given below.***

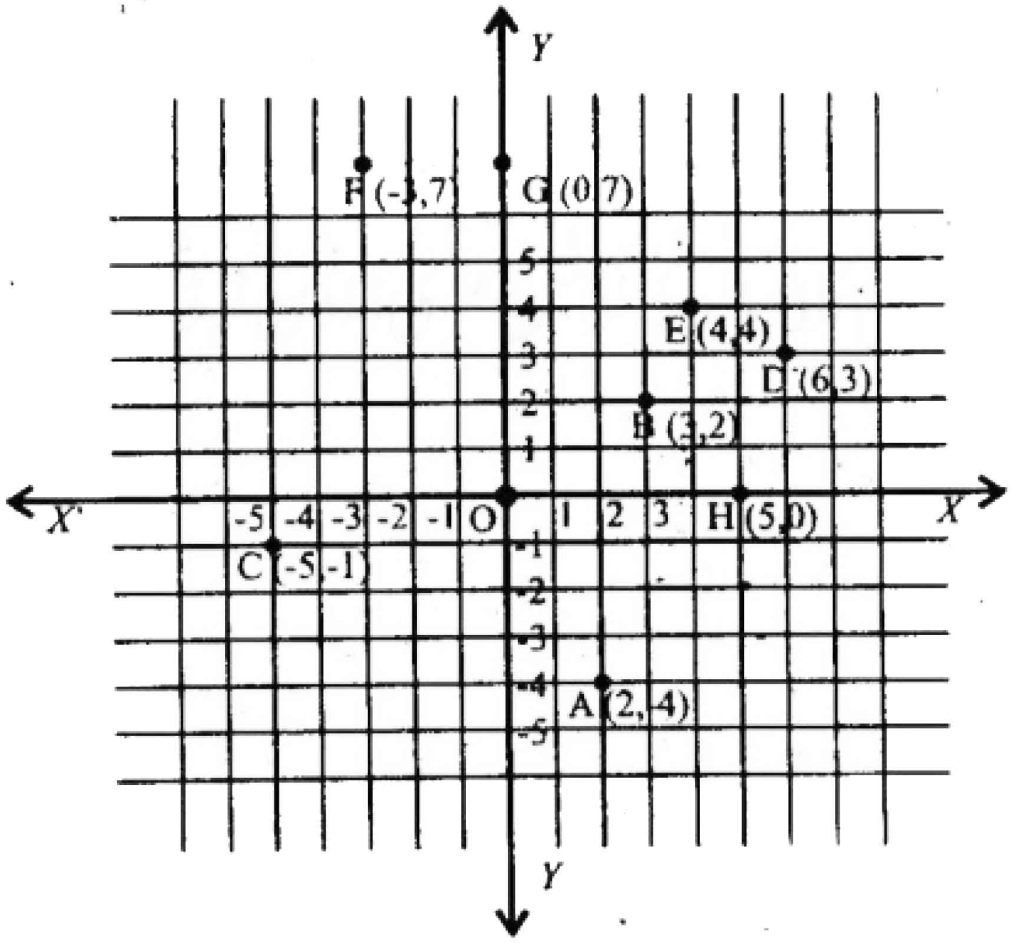
(i) $A(2,-4)$ *(ii)* $B(3,2)$

(iii) $C(-5,-1)$ *(iv)* $D(6,3)$

(v) $E(4,4)$ *(vi)* $F(-3,7)$

(vii) $G(0,7)$ *(viii)* $H(5,0)$

Solution:-



2- ***Write down the co-ordinates of:***

(i) Origin

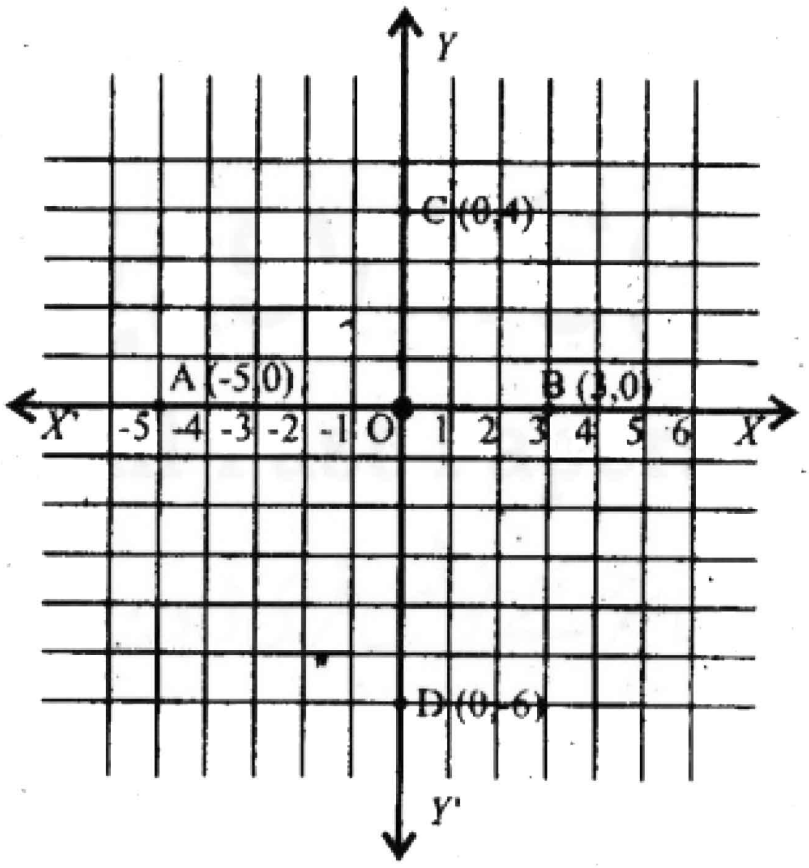
(ii) A point lying on the left hand side of x-axis and at a distance of 5 units from the origin.

(iii) A point lying to the right hand side of the origin on

x-axis at a distance of 3 units from the origin.

(iv) A point lying above x-axis and on y-axis at a distance of 4 units.

(v) A point lying below x-axis and on y-axis at a distance of 6 units.



(i) (0, 0) (ii) (-5, 0) (iii) (3, 0)

(iv) (0,4) (v) (0, -6)

3- ***Draw the figures with help of the following points on the graph paper.***

(i) *A(7,2), B(−6,−3), C(5,3)*

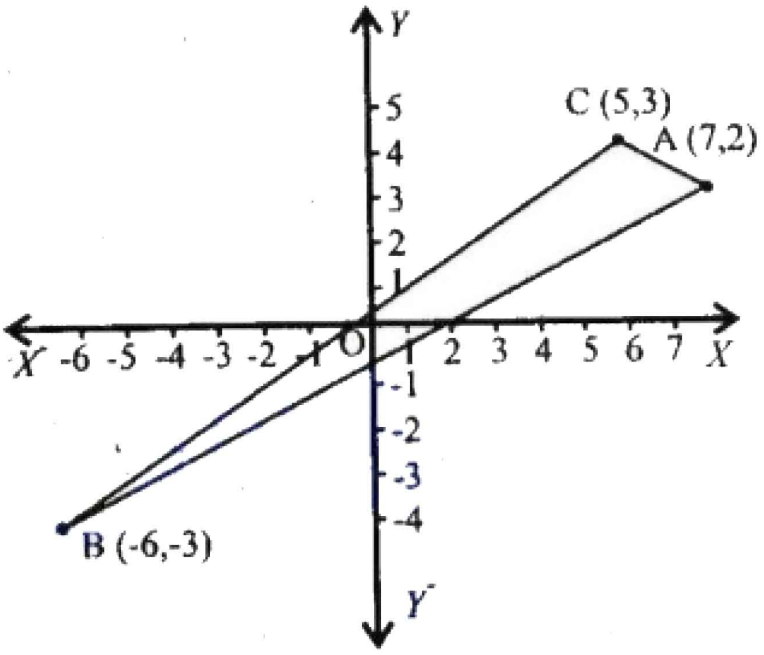
(ii) *A(0,−7), B(3,−2), C(4,0) D(5,6), E(7,8)*

(iii) *A(4,0), B(0,4), C(−4,0), D(0,−4)*

(iv) $A(10,6)$, $B(-10,6)$, $C(-10,-6)$, $D(10,-6)$

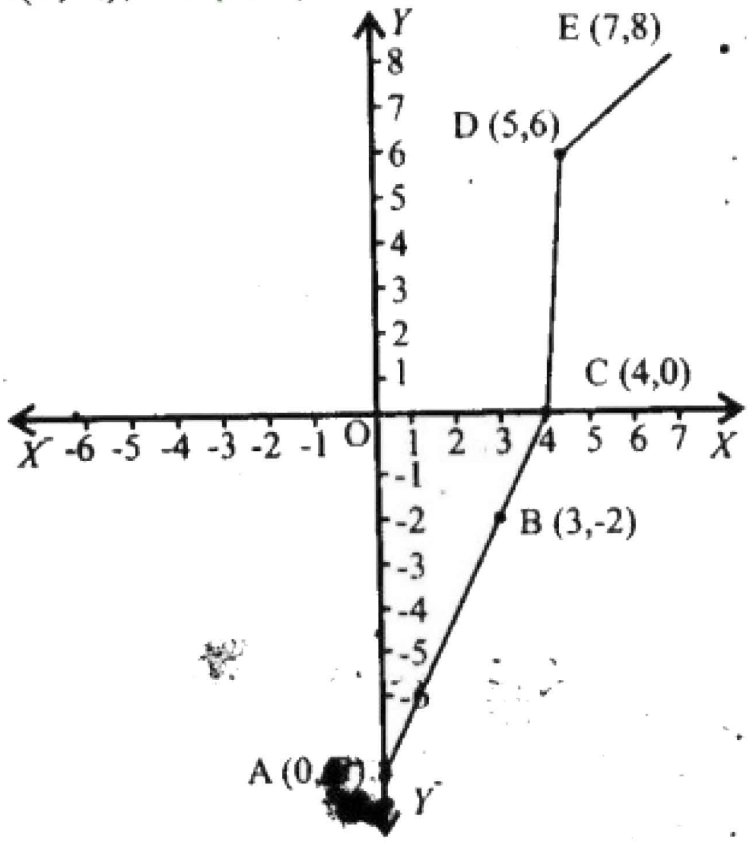
Solution:3(i)

A(7,2), B(-6,-3), C(5,3)



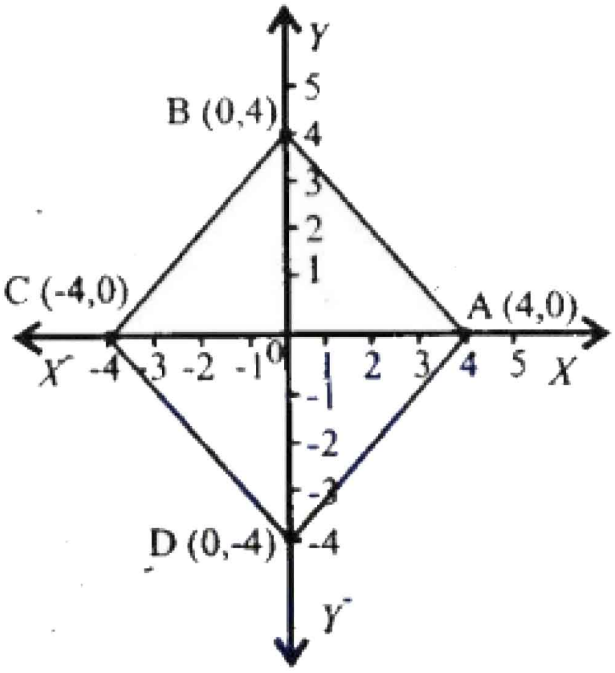
Solution:3(ii)

A(0,-7), B(3,-2), C(4,0), D(5,6), E(7,8) :



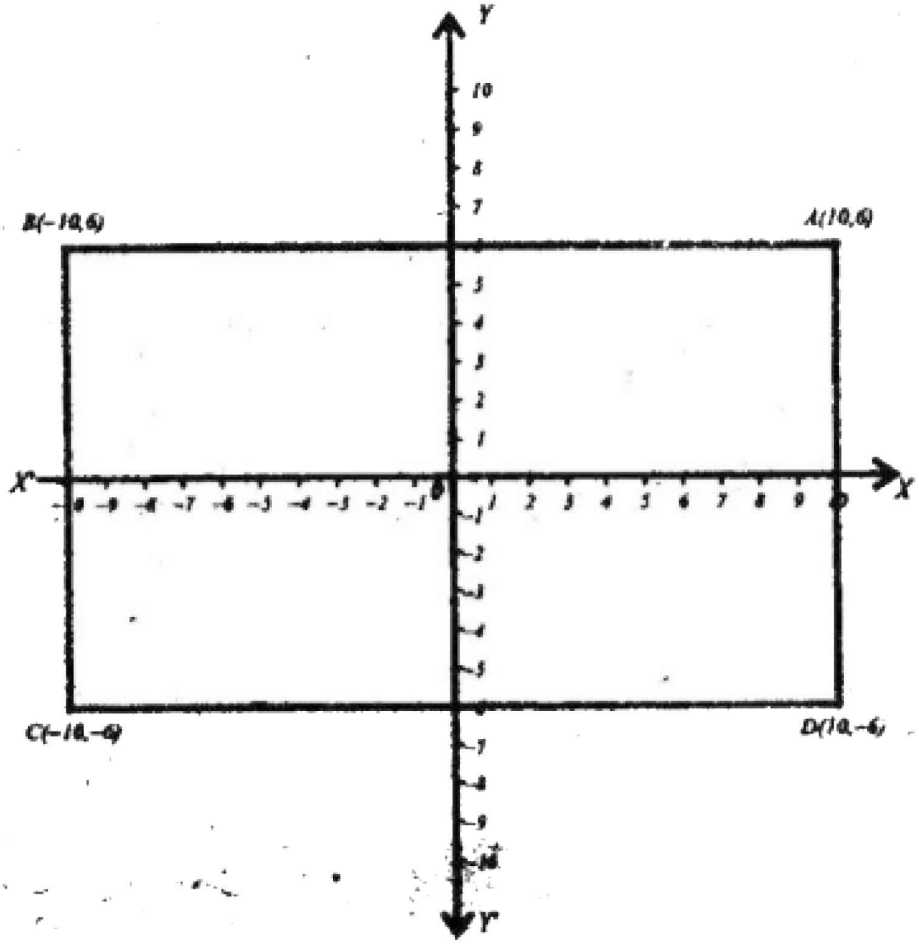
Solution:3(iii)

A(4,0), B(0,4), C(-4,0), D(0,-4) :



Solution:3(iv)

A(10,6), B(-10,6), C(-10,-6), D(10,-6) :



Exercise 9.2

Draw the graph of:

1. $y = 3x$ **2.** $y = x + 7$ **3.** $y = 2x - 3$

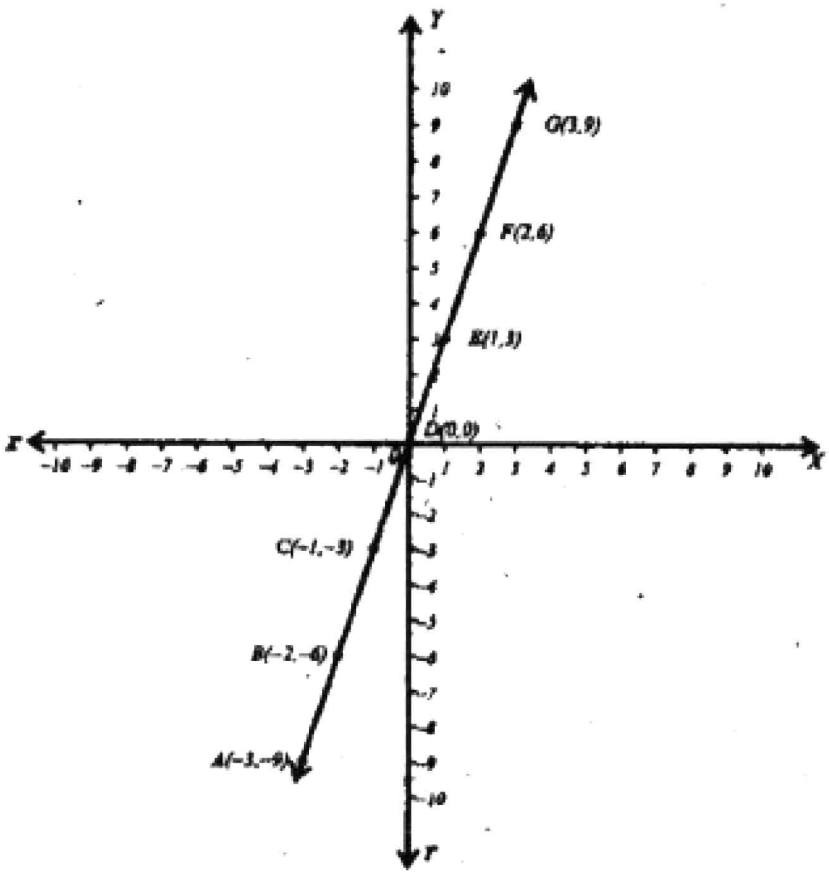
4. $y = 4x + 1$ **5.** $y = -\frac{x}{2} - \frac{3}{2}$ **6.** $y = x - 1$

7. $y = 2x - 3$ **8.** $y = 3x + 5$ **9.** $y = \frac{x}{2}$

Solution: (i) .y = 3x

Put x = 0 in equaion (i) then y = 0. The graph shows first point is (0, 0). Now we will find different values putting $x = \pm 1, \pm 2, \pm 3$ *from the table.*

x =	-3	-2	-1	0	1	2	3
y =	-9	-6	-3	0	3	6	9



Solution: $y = x + 7$

Put $x = 0$ in equation. We will find ordered pair (0, 7).

Now put $x = 1, 2, 3$ and we will find values of y.

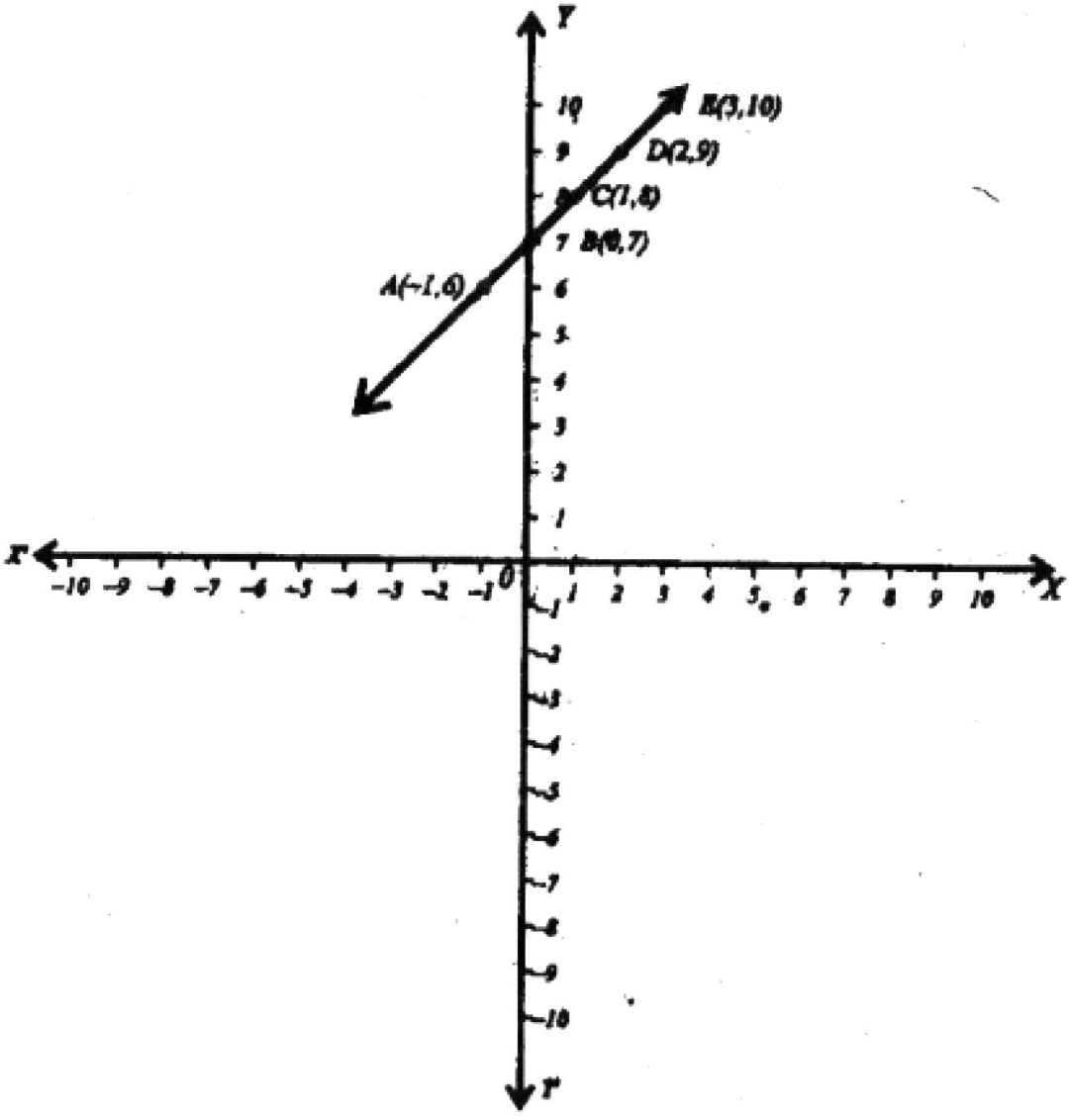
$y = x + 7$ $\quad$ $y = x + 7$ $\quad$ $y = x + 7$

$= 1 + 7$ $\quad$ $= 2 + 7$ $\quad$ $= 3 + 7$

$= 8$ $\quad$ $= 9$ $\quad$ $= 10$

Table of values of x and y is as under.

$x =$	0	1	2	3	-1
$y =$	7	8	9	10	6



Solution: $y = 2x - 3$

First we will put $x = 0$ in equation

$y = 2(0) - 3$

$y = (0, -3)$

Similarly we will get values of y putting values of x as

$x = -2, -1, 1, 2, 3$

$y = 2x - 3$
$= 2(-2) - 3$
$= -4 - 3$
$= -7$

$y = 2x - 3$
$= 2(-1) - 3$
$= -2 - 3$
$= -5$

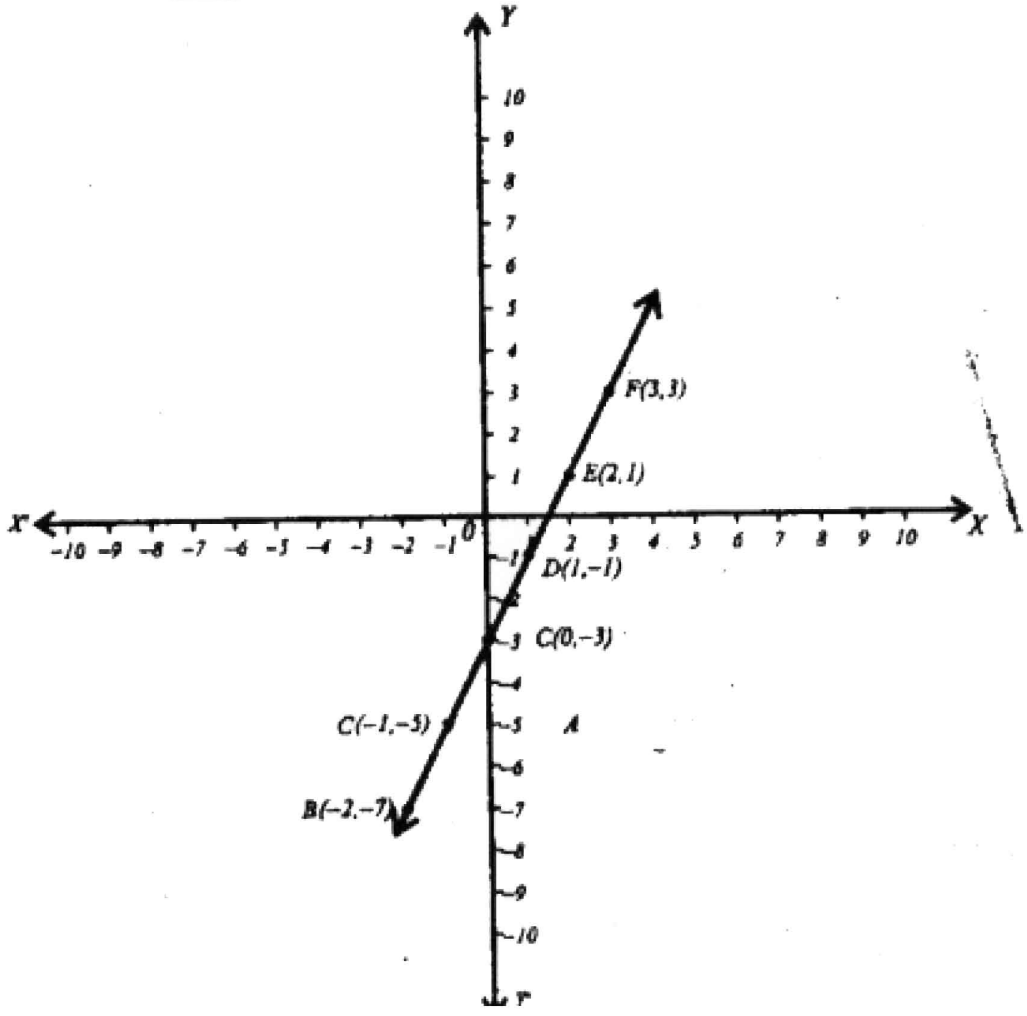
$y = 2x - 3$
$= 2(1) - 3$
$= 2 - 3$
$= -1$

$y = 2x - 3$
$= 2(2) - 3$
$= 4 - 3$
$= 1$

$y = 2x - 3$
$= 2(3) - 3$
$= 6 - 3$
$= 3$

Table of values of x, y is as under.

$x =$	0	-2	-1	1	2	3
$y =$	-3	-7	-5	-1	1	3



Solution: $y = 4x + 1$

Putting values of $x = 0$

$y = 4(0) + 1$

$y = 0 + 1$

We will get orderd pair (0, 1)

$y = 4x + 1$
$= 4(-2) + 1$
$= -8 + 1$
$= -7$

$y = 4x + 1$
$= 4(-1) + 1$
$= -4 + 1$
$= -3$

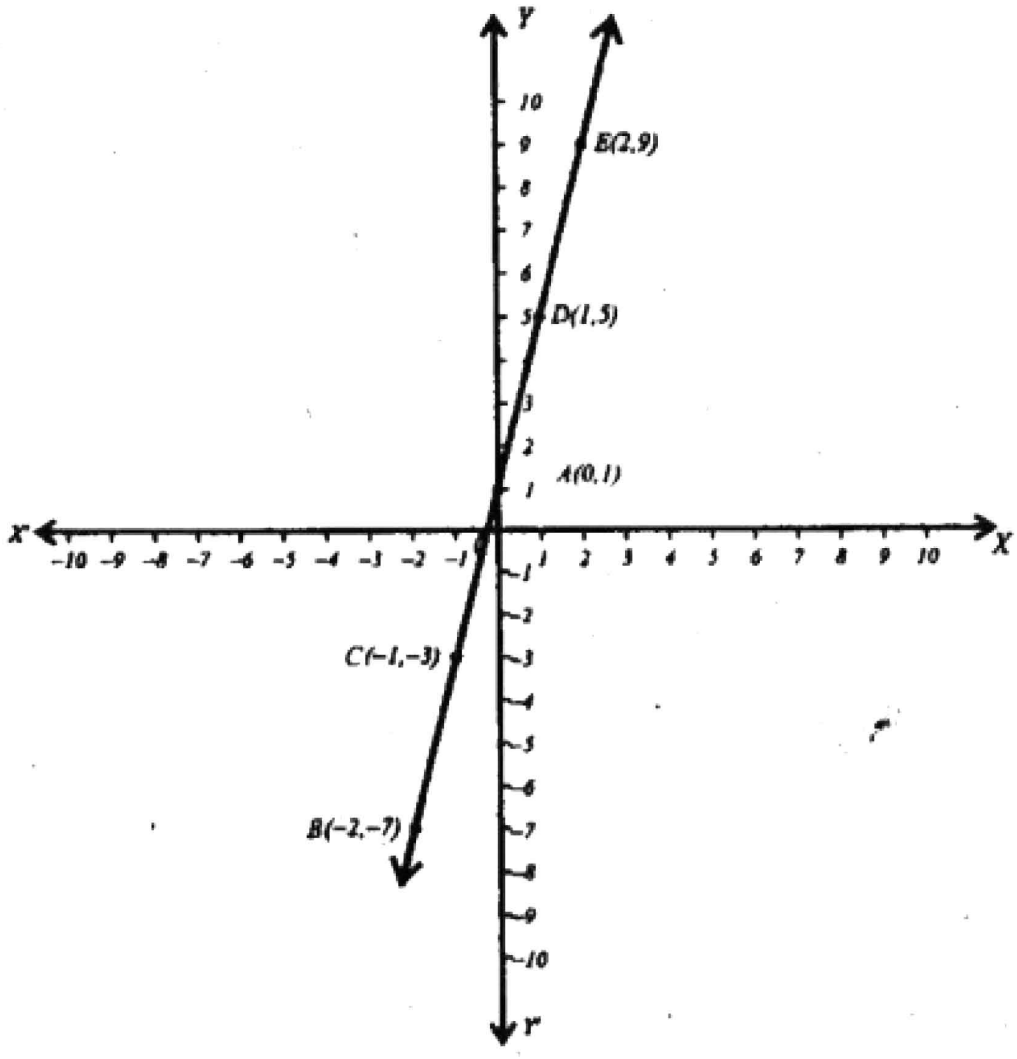
$y = 4x\ 1$
$= 4(1) + 1$
$= 4 + 1$
$= 5$

$y = 4x + 1$
$= 4\ (2) + 1$
$= 8 + 1$
$= 9$

$y = 4x + 1$
$= 4\ (3) + 1$
$= 12 + 1$
$= 13$

Now put values of x = -2, -1, 1, 2, 3 in equation. Table of values of x and y is as under.

$x =$	0	-2	-1	1	2	3
$y =$	1	-7	-3	5	9	13



Solution:

$$y = -\frac{x}{2} - \frac{3}{2}$$

$$= \frac{-x-3}{2}$$

Putting values of $x = -1, -3, 1, -13$ in equation.

$$y = \frac{-x-3}{2} \qquad y = \frac{-x-3}{2} \qquad y = \frac{-x-3}{2}$$

$$= \frac{-(-1)-3}{2} \qquad = \frac{-(-3)-3}{2} \qquad = \frac{-1-3}{2}$$

$$= \frac{1-3}{2}$$

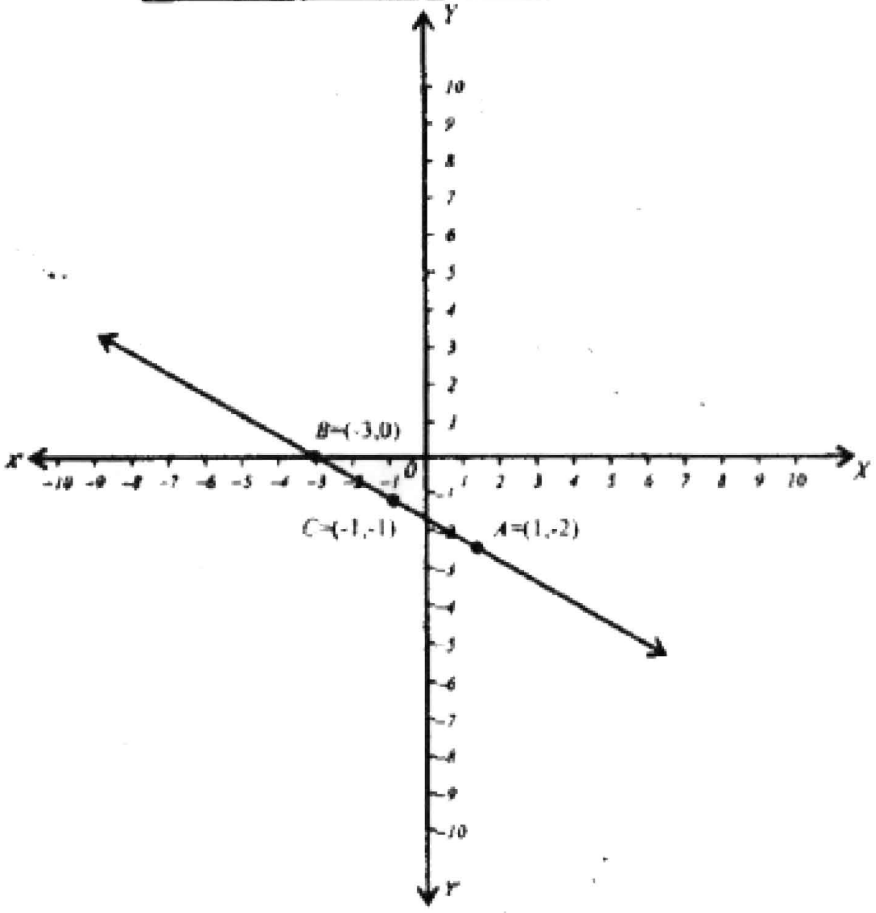
$$= \frac{-2}{2} = -1 \qquad = \frac{3-3}{2} = 0 \qquad = \frac{-4}{2} = -2$$

$$y = \frac{-x-3}{2}$$

$$= \frac{-(-13)-3}{2} = \frac{13-3}{2} = \frac{10}{2} = 5$$

Table of values of x and y is as under.

$x =$	-1	-3	1	-13
$y =$	-1	0	-2	5



Solution: $y = x - 1$

Putting values of x = 0, -1, 2, 8 in equation.

$y = x - 1$ $\quad$ $y = x - 1$ $\quad$ $y = x - 1$

$= 0 - 1$ $\quad$ $y = -1 - 1$ $\quad$ $= 1 - 1$

$= -1$ $\quad$ $= -2$ $\quad$ $= 0$

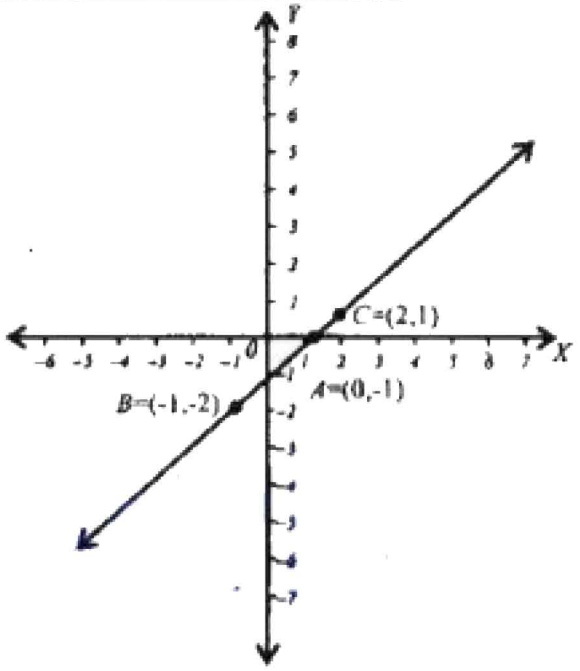
$y = x - 1$ $\quad$ $y = x - 1$

$= 2 - 1$ $\quad$ $= 8 - 1$

$= 1$ $\quad$ $= 7$

Table of values of x and y is as under.

$x =$	0	-1	1	2	8
$y =$	-1	-2	0	1	7



Solution: $y = 2x - 3$

Putting values of x = 0, -3, -2, -1, 2, 3 in equation.

$y = 2(0) - 3$

$= 0 - 3$

$= -3$

$y = 2(-3)-3$

$y = -6 -3$

$= -9$

$y = 2(-2)-3$

$= -4 -3$

$= -4 -3$

$y = 2(-1)-3$

$= -2 -3$

$= -5$

$y = 2(1)-3$

$= 2 -3$

$= -1$

$y = 2(2)-3$

$= -7$

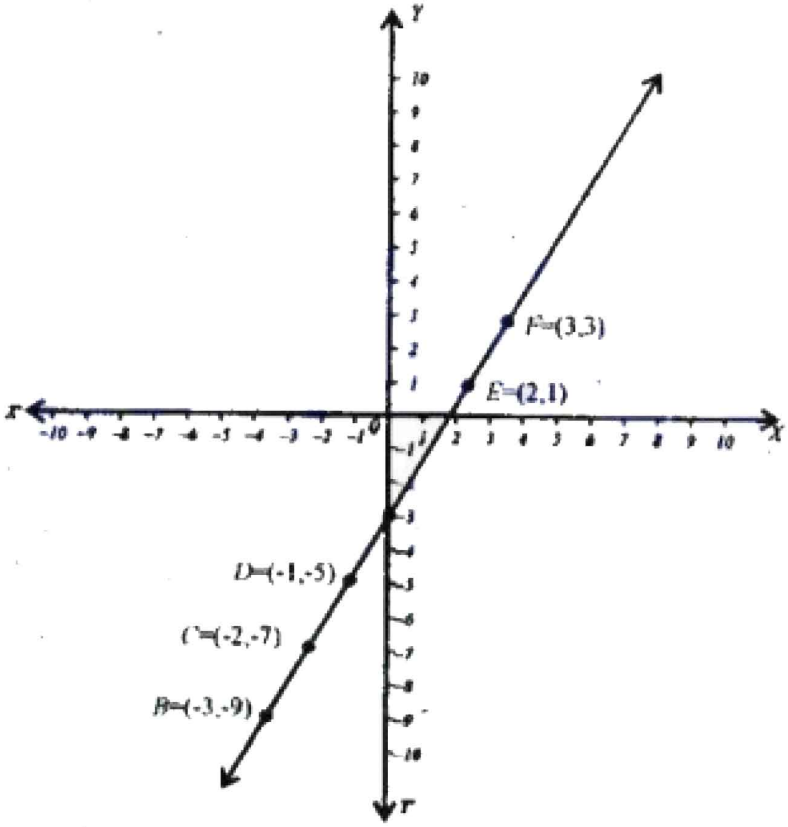
$= 1$

$y = 2(3)-3$

$= 6 - 3 = 3$

Table of values of x and y is as under.

$x =$	0	-3	-2	-1	1	2	3
$y =$	-3	-9	-7	-5	-1	1	3



Solution: $y = 3x + 5$

Putting values of x = 0, -1, -2, 1 in equation.

$y = 3x + 5$ $\quad$ $y = 3x + 5$ $\quad$ $y = 3x + 5$

$y = 3(0) + 5$ $\quad$ $y = 3(-1) + 5$ $\quad$ $y = 3(-2) + 5$

$= 0 + 5$ $\quad$ $= -3 + 5$ $\quad$ $= -6 + 5$

$= 5$ $\quad$ $= 2$ $\quad$ $= -1$

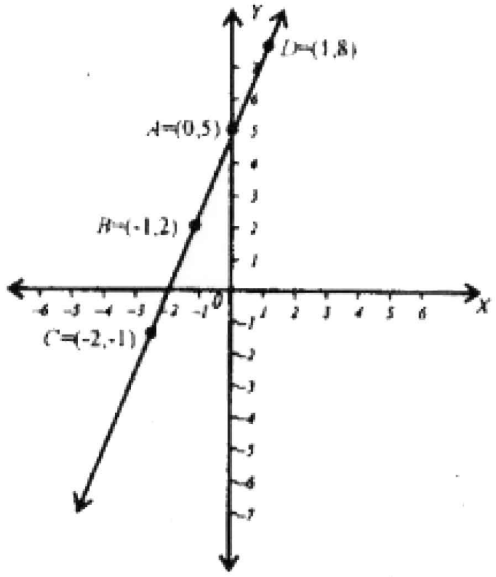
$y = 3x + 5$

$= 3(1) + 5$

$= 3 + 5 = 8$

Table of values of x and y is as under.

$x =$	0	-1	-2	1
$y =$	5	2	-1	8



Solution: $y = \frac{x}{2}$

Putting values of $x = 0, 4, 8, 8, 10, 12$ in equation.

$$y = \frac{x}{2} \qquad y = \frac{x}{2} \qquad y = \frac{x}{2} \qquad y = \frac{x}{2}$$

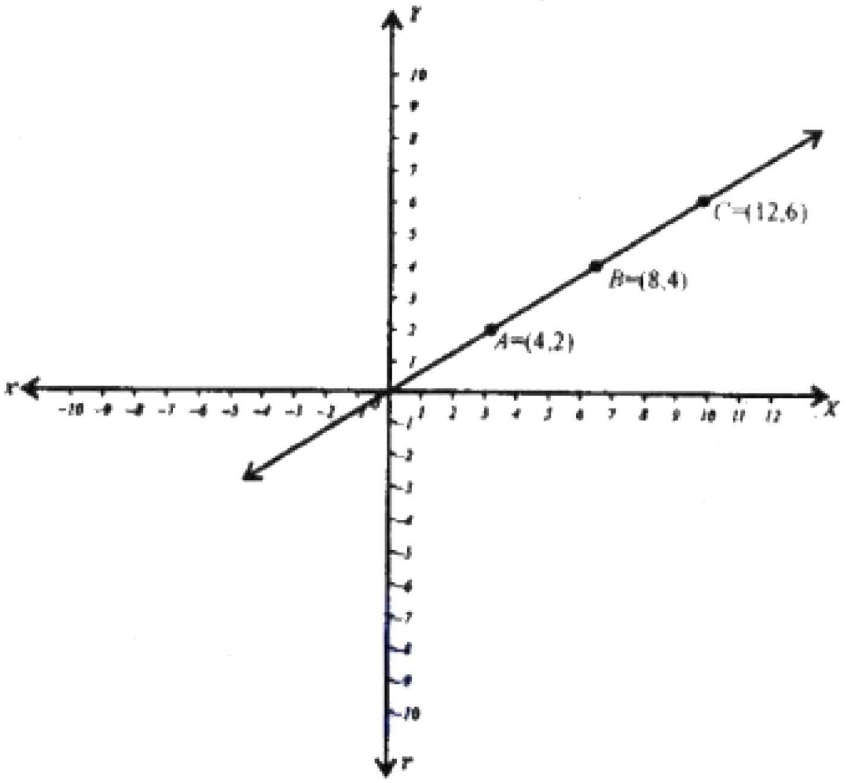
$$= \frac{0}{2} = 0 \qquad = \frac{4}{2} = 2 \qquad = \frac{8}{2} = 4 \qquad = \frac{10}{2} = 5$$

$$y = \frac{x}{2}$$

$$= \frac{12}{2} = 6$$

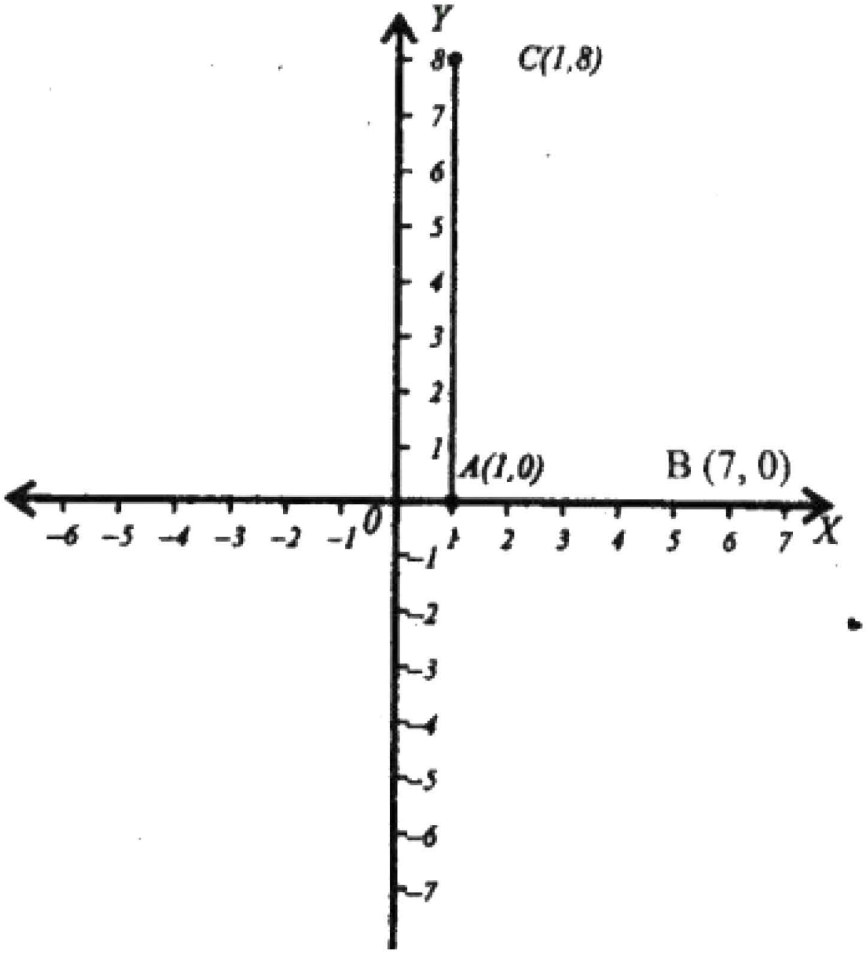
Table of values of x and y is as under.

$x =$	0	4	8	12	10	12



10- Draw the graph by plotting the points

A(1,0), B(7,0), C(1,8)



11- Draw the graph from the given tables.

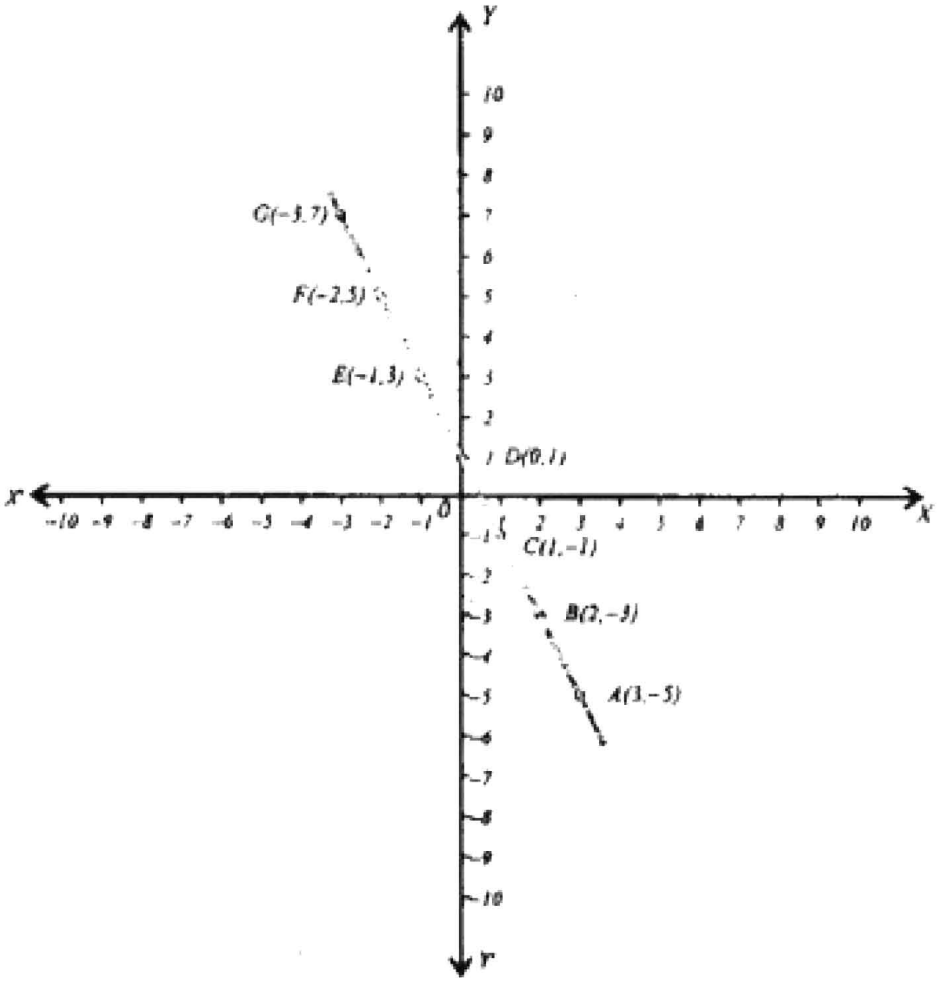
(i)

x	3	2	1	0	- 1	- 2	- 3
y	- 5	- 3	- 1	1	3	5	7

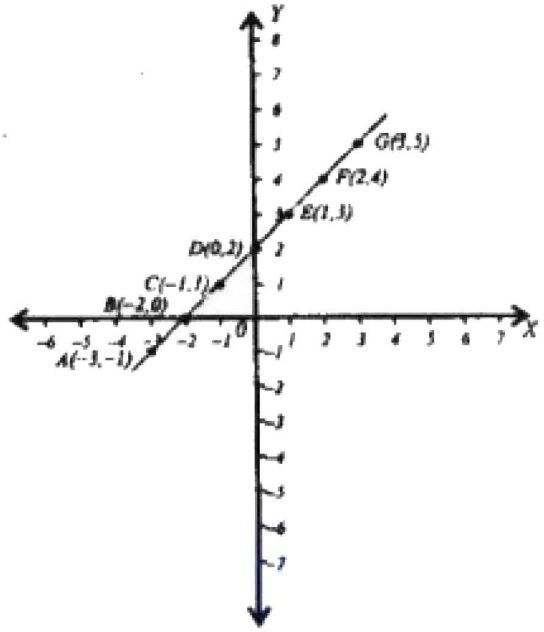
(ii)

x	- 3	- 2	- 1	0	1	2	3
y	- 1	0	1	2	3	4	5

Solution:

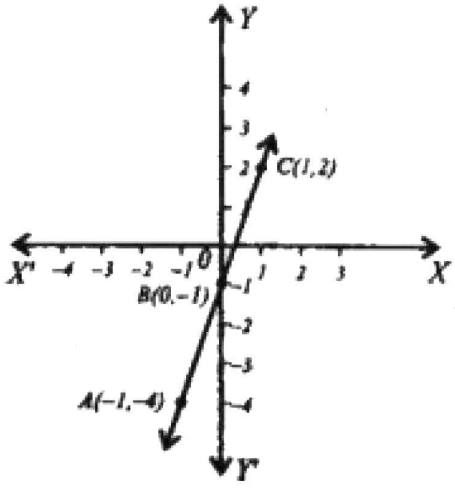


Solution:11(ii)



12- Identify thorugh the given graphs the domain and the range of a function.

Solution:



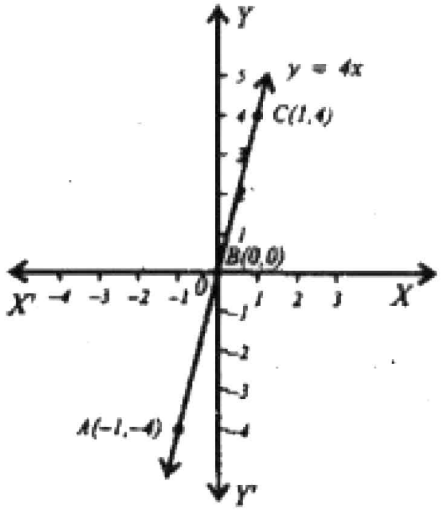
$$y = 3x - 1$$

$$f = \{(-1, -4), (0, -1), (1, 2)\}$$

$$\text{Dom } f = \{-1, 0, 1\}$$

$$\text{Range } f = \{-4, -1, 2\}$$

Solution:13



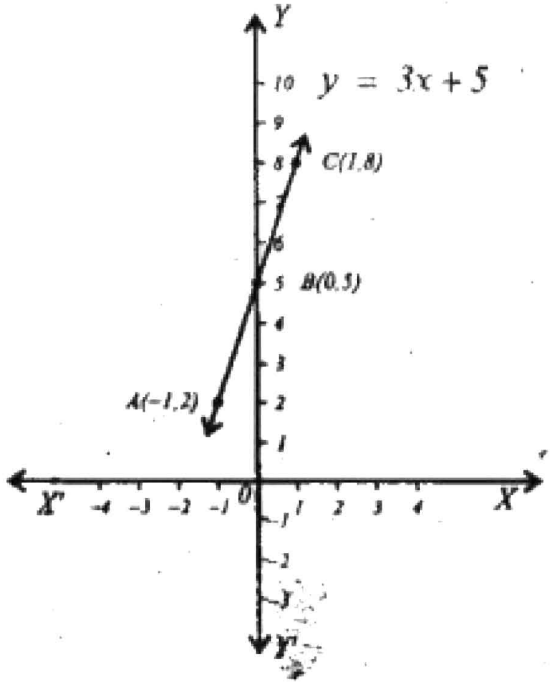
$$y = 4x$$

$$f = \{(-1, -4), (0, 0), (1, 4)\}$$

$$\text{Dom } f = \{-1, 0, 1\}$$

$$\text{Range } f = \{-4, 0, 4\}$$

Solution:14



$$f = \{(-1, 2), (0, 5), (1, 8)\}$$

$$\text{Dom } f = \{-1, 0, 1\}$$

$$\text{Range } f = \{2, 5, 8\}$$

Exercise 9.3

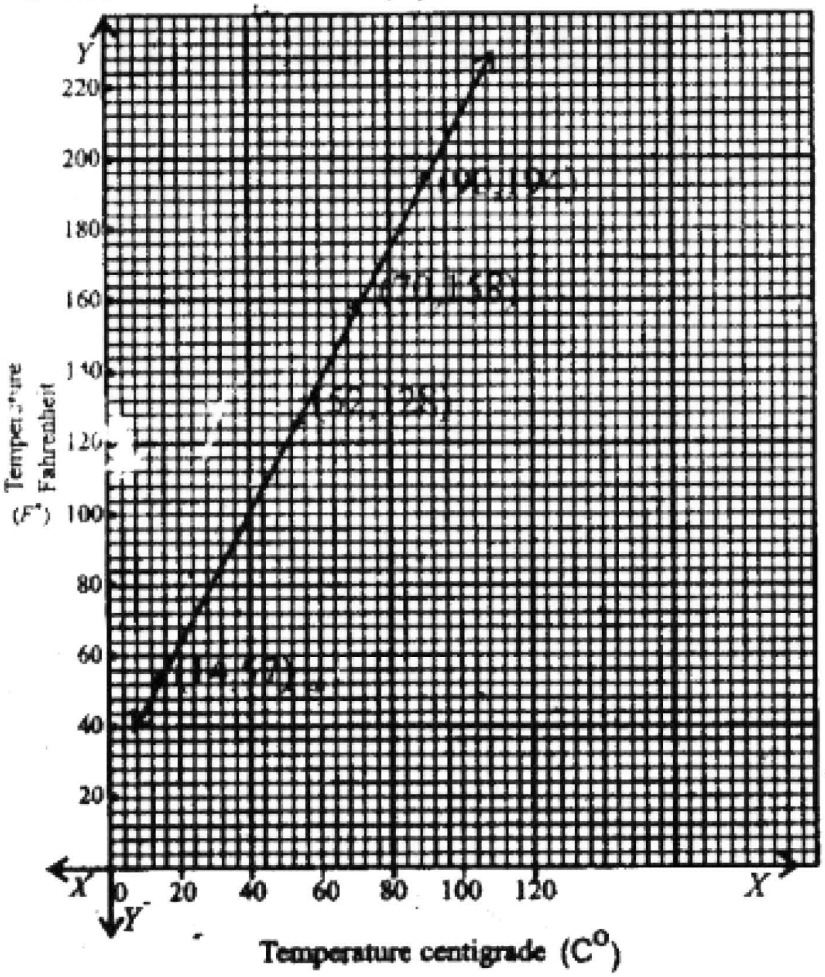
1- ***The table given temperatures in degrees Fahrenheit ^{o}F and the equivalent values in degrees Centigrade ^{o}C.***

Temperature in ^{o}F	57	126	158	194
Temperature in ^{o}C	14	52	70	90

Plot these points on a graph paper for centigrade values from 0 to 100 and Fahrenheit value from 0 to 220. Let 5 small sqaures represent 20 units on each axis. Use your graph to convert the following:

(a) $97^{o}F$ into ^{o}C (b) $127^{o}F$ into ^{o}C

(c) $25^{o}C$ into ^{o}F (d) $80^{o}C$ into ^{o}F



(a) $F^{o} = 97^{o}$, $C^{o} =$ (b) $F^{o} = 127^{o}$, $C = 36^{o}$

(c) $C = 25^{o}$, $F =$ (d) $C = 80^{o}$, $F =$

(a) $F = 97^{o}$ then $C = 36^{o}$

(b) $F = 127^{O}$ then $C = 53^{O}$

(c) $C = 25^{O}$ then $F = 77^{O}$

(d) $C = 80^{O}$ then $F = 176^{O}$

2- ***The table shows the conversion from US Dollar ($) to Pounds (£) for various amounts of money.***

$	50	100	200
£	35	70	140

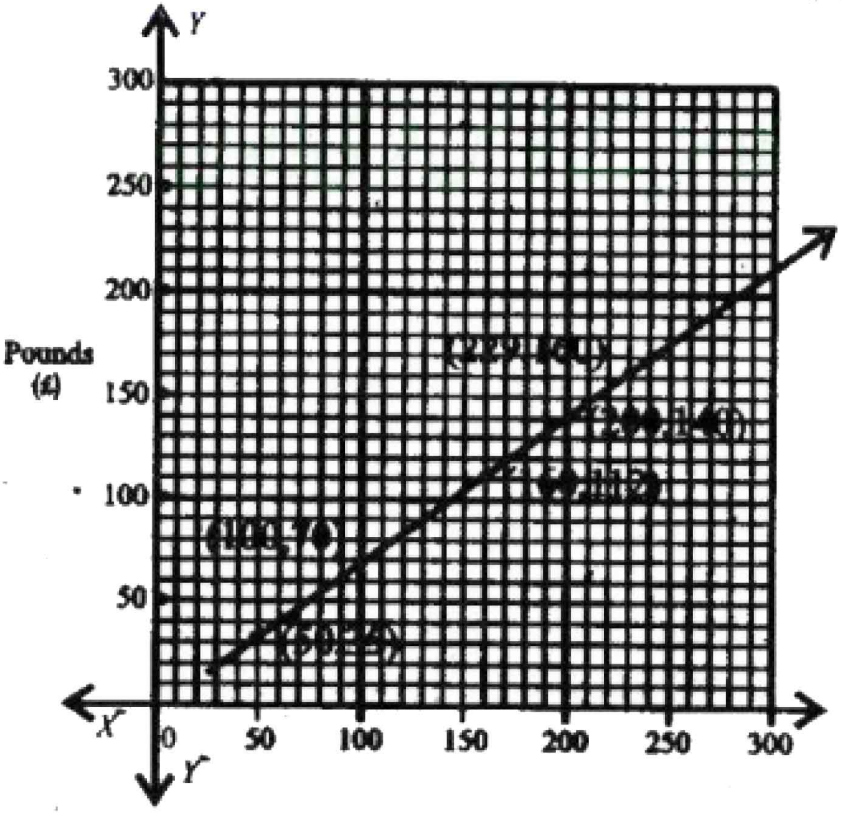
Plot these points on a graph paper and draw a straight line to pass through them. Let 5 small squares represent 50 units on each axis. Use your graph to convert the following:

(a) 160 dollars into £ **(b)** 96 dollars into £

(c) 120 £ into dollars **(d)** 76£ into dollars

$160 = £ ... (b) $160 = £ .... (a)

£160 = $ ... (d) £160 = $ ... (c)



From graph:

(a) $160 = £ 112

(b) $160 = £ 112

(c) £160 = $ 229

(d) £160 = $ 229

3- ***The table below gives various distances in kilometers with the equivalent values in miles.***

Kilometers	0	100	200	300
Miles	0	62.5	125	187.5

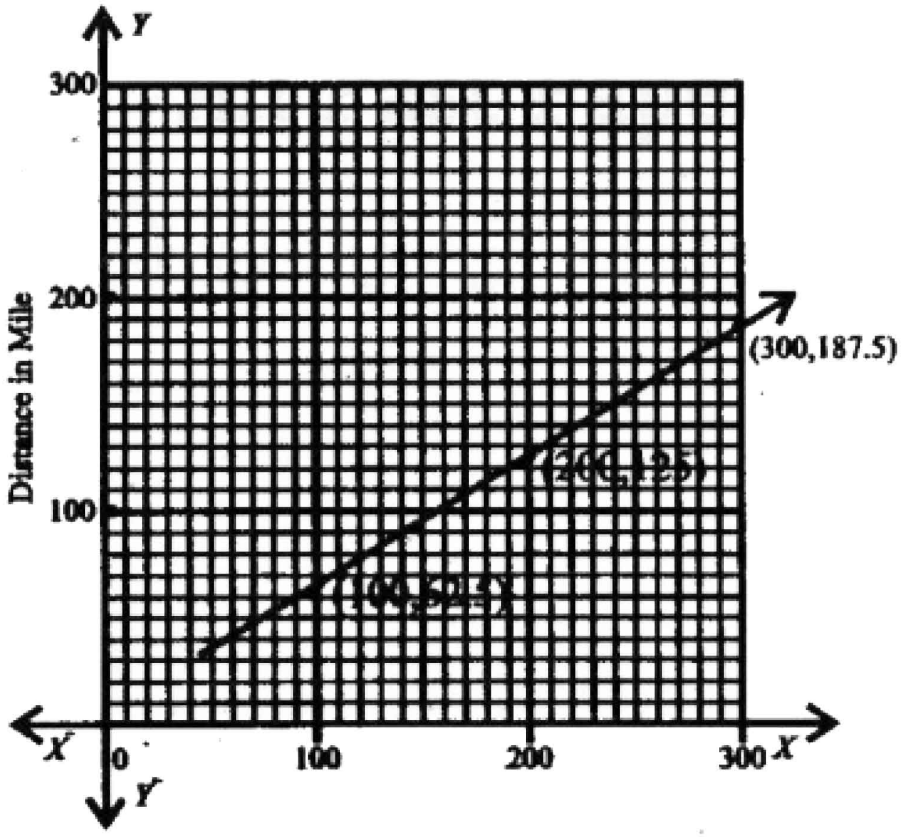
Plot these values on a graph paper taking 10 small squares equal to 100 kilometers on x-axis and 10 small squares equal to 100 miles on y-axis. Use your graph to convert the following:

(a) 140 kilometers into miles.

(b) 175 kilometers into miles.

(c) 50 miles into kilometers.

(d) 100 miles into kilometers.



Distance in Kilometer

From graph:

Answers:

(a) 140 km = 87.5 miles

(b) 175 km = 109.4 miles

(c) 50 miles = 80 km

(d) 100 miles = 160 km

4- Use the graph in article 9.2.3 to convert:

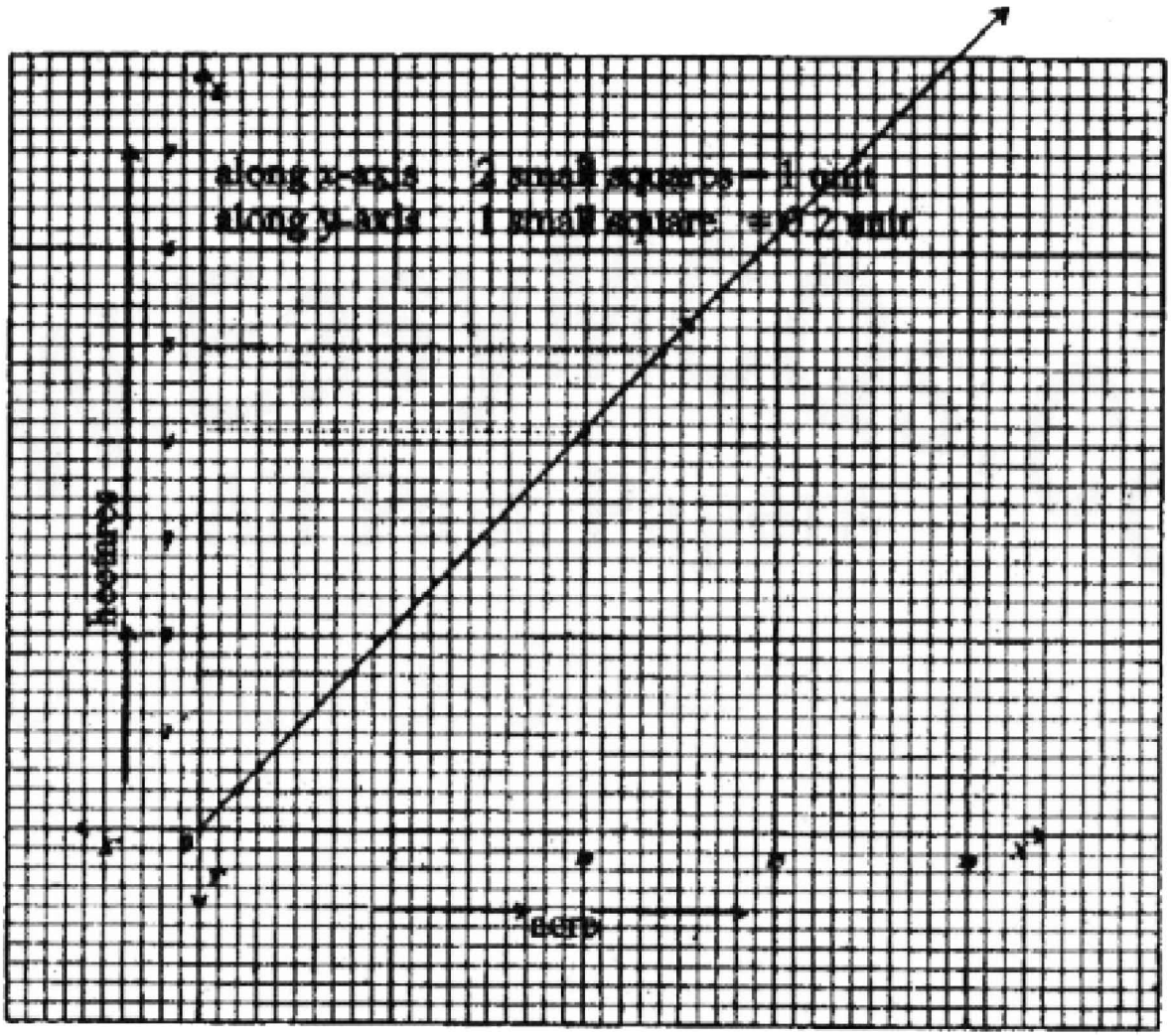
(a) 6 acres = ... Hectares

(b) 18 acres = ... Hectares

(c) 6.0702 hectare = ...acres.

(d) 24 acres = ... Hectares

(e) 11.3311 hectares = ... acres.



kilometers

From graph:

(a) 6 acres = 2.43 hectares

(b) 18 acres = 7.3 hectares

(c) 6.0702 hectares = 15 acres

(d) 24 acres = 9.7 hectares

(e) 11.3311 hecatres = 28.0 acres

Exercise 13.1

1- ***Fifty junior school children joined the school's computer club. Their ages were recorded.***

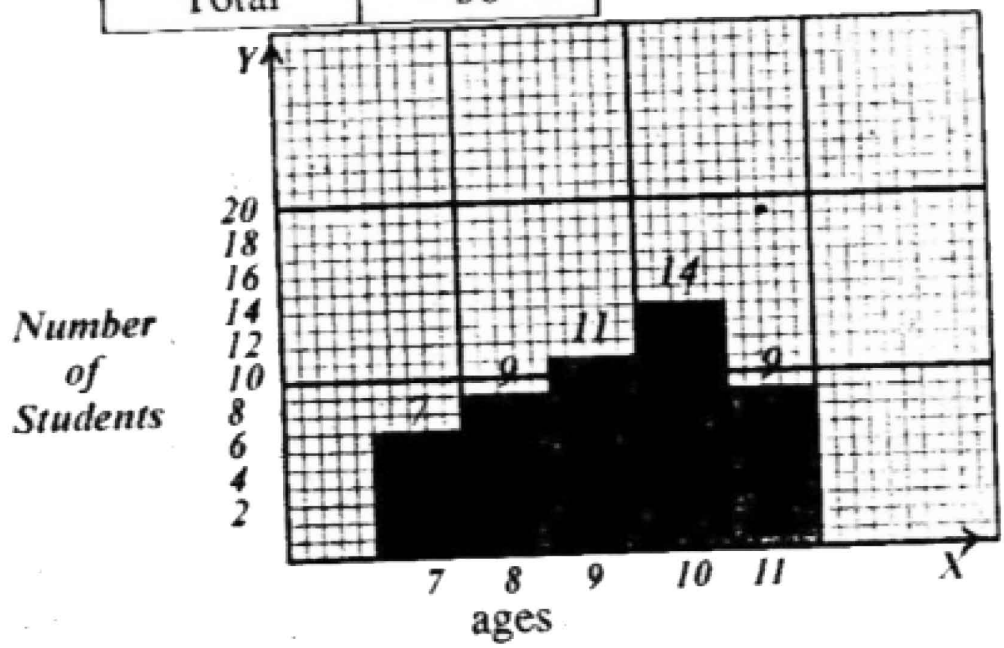
10	8	9	10	7	8	8	11	10	9
7	8	9	9	10	11	11	10	9	8
8	7	9	7	10	7	10	8	9	11
10	11	8	10	9	8	9	7	11	10
9	10	10	11	10	11	7	11	10	9

Make a frequency table showing the number of each age and illustrate this information with a bar chart.

Solution:-

Frequency table of 50 students ages.

Students ages	Numbers
7	7
8	9
9	11
10	14
11	9
Total	50



2- ***The local fish and chip shop had 56 customers on Saturday evening. They spent the following amount in rupess.***

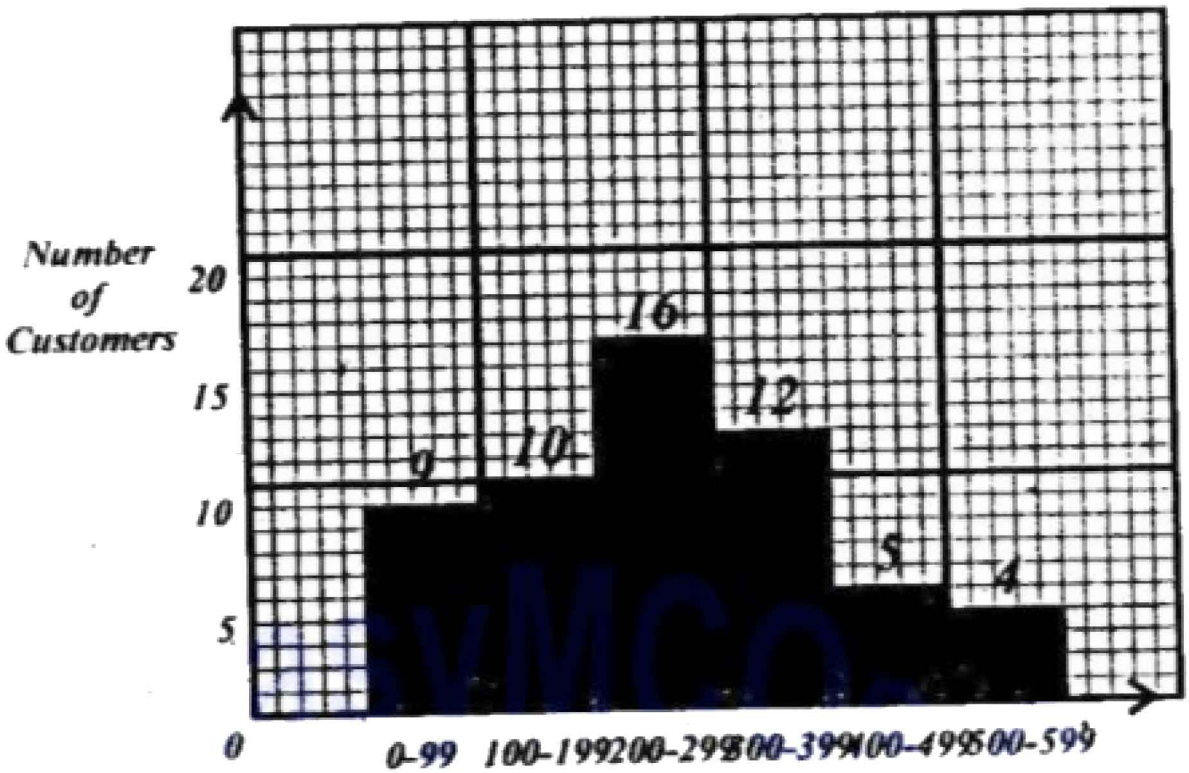
270	110	45	96	250	490	325	45
382	136	125	450	420	380	150	250
85	250	320	525	218	210	216	120
155	430	250	40	510	150	510	245
320	120	316	150	260	45	180	310
273	280	85	280	318	45	210	282
462	316	218	316	325	45	560	315

Use groups Rs. 0-99, Rs. 100-199, Rs. 200-299, Rs. 300-399, Rs. 400-499, Rs. 500-599 to make a frequency table illustrate the data with a bar chart.

Solution:-

Group frequency table

Income (Rs)	Tally	Frequency
0 - 99	𝍸 |	9
100 - 199	𝍸 𝍸	10
200 - 299	𝍸 𝍸 𝍸 |	16
300 - 399	𝍸 𝍸 ||	12
400 - 499	𝍸	5
500 - 599	||||	4
Total		56



Total amount spent (Rs)

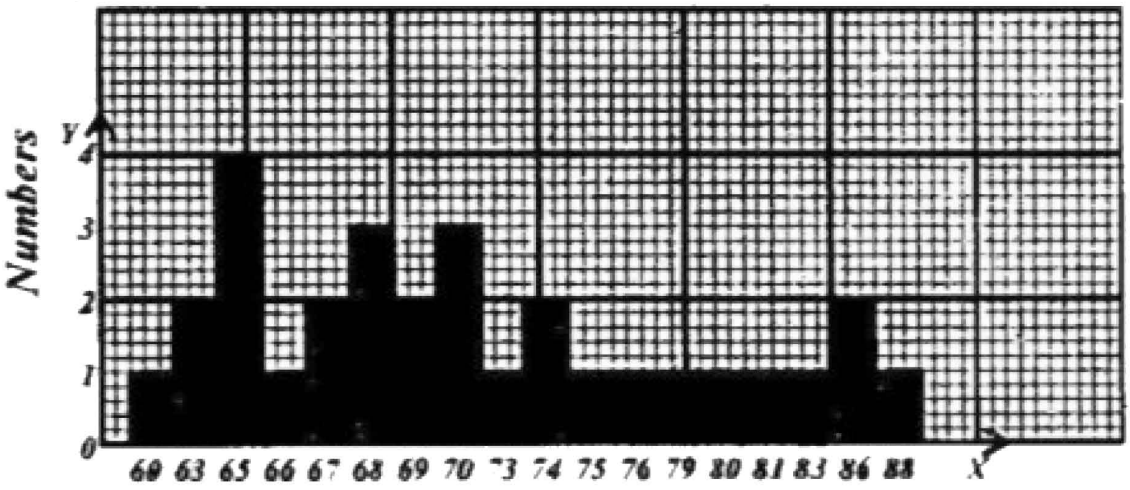
3- ***The weights to the nearest gram of 30 bags of popcorn sold at a festival are given as:***

69	83	75	65	68	68	73	70	80	79
70	76	63	86	69	65	66	74	86	68
70	60	67	74	65	65	67	88	81	63

Make a frequency table, illustrate the data with a bar chart.

Solution:-

Weights of Bags (gm)	Numbers
60	1
63	2
65	4
66	1
67	2
68	3
69	2
70	3
73	1
74	2
75	1
76	1
79	1
80	1
81	1
83	1
86	2
88	1
Total	30



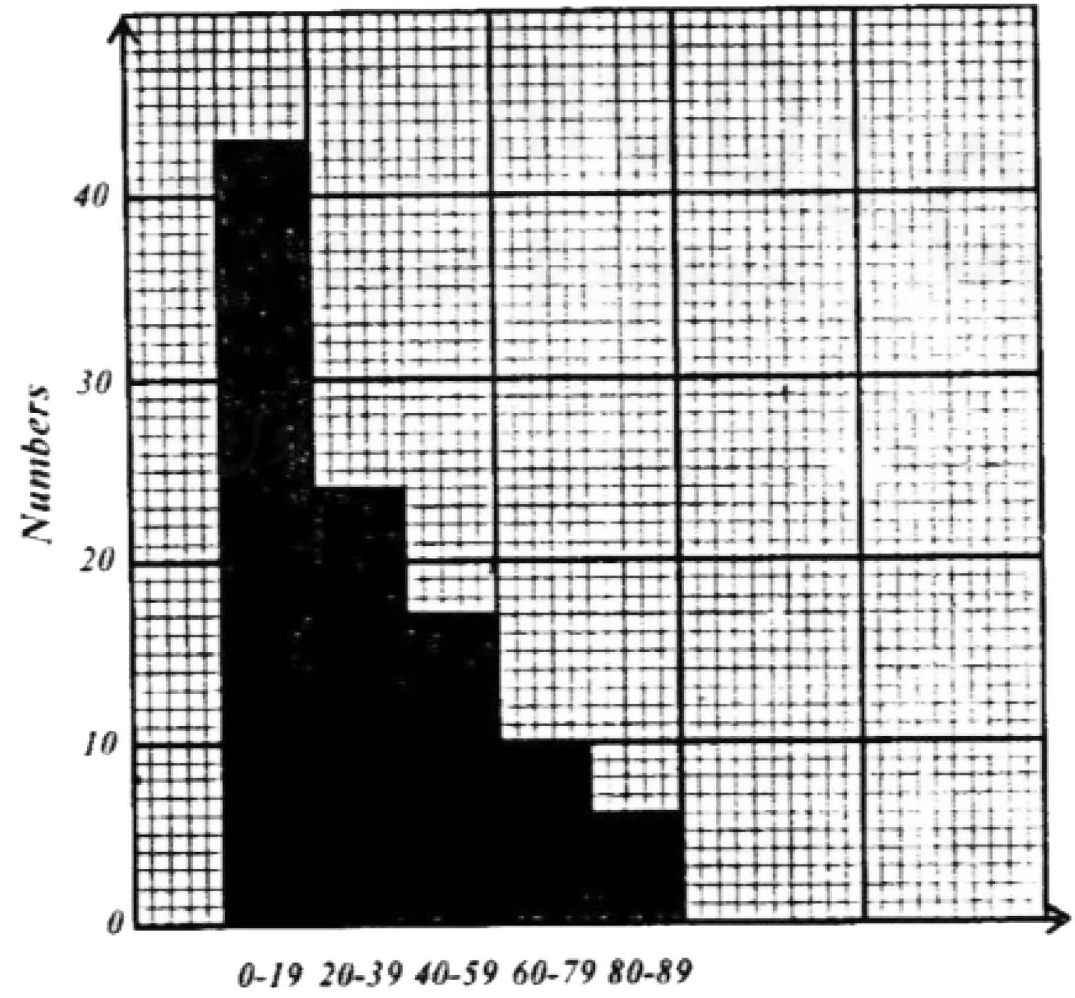
Weights (gm)

Exercise 10.2

1- ***Draw a histogram to represent the frequency table in each of the following tables.***

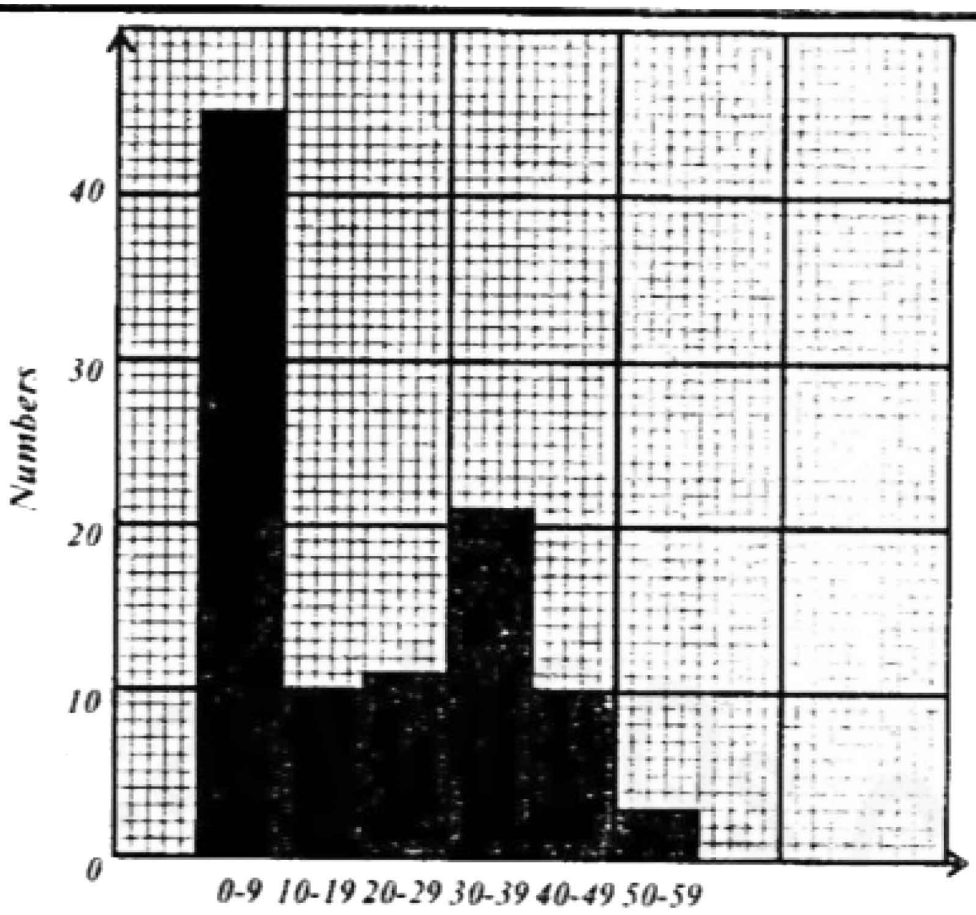
(i) The table shows the distribution of ages 100 people attending a school function.

Age(years)	*0-19*	*20-39*	*40-59*	*60-79*	*80-89*
Frequency	*43*	*24*	*17*	*10*	*6*



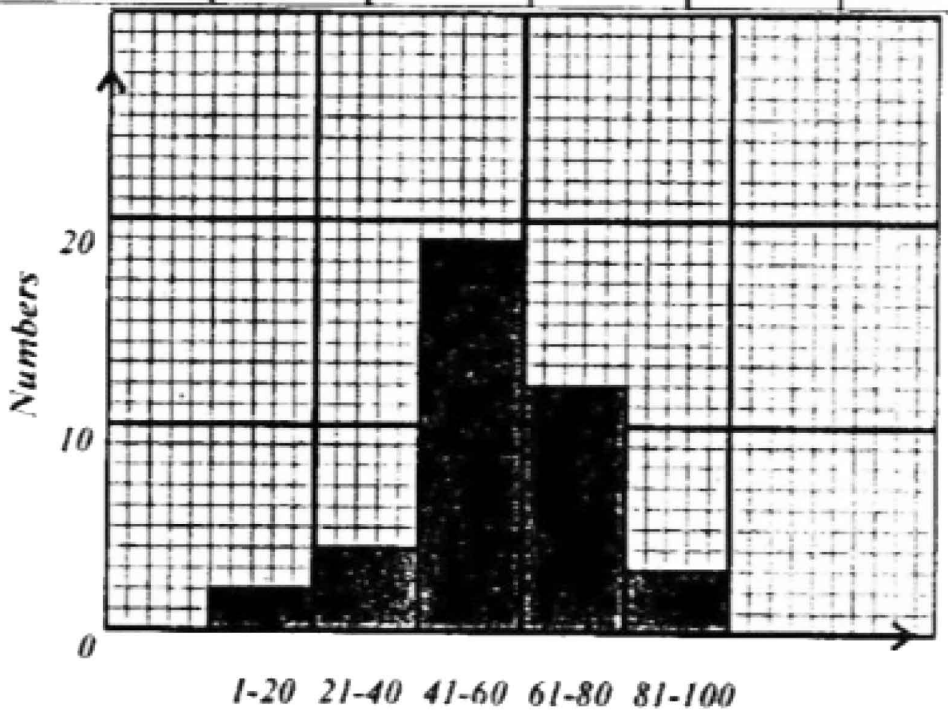
(ii) The table shows the results of a surves on the weekly earnings of 100 sixteen-years old boys.

Weekly income	*0-9*	*10-19*	*20-29*	*30-39*	*40-49*	*50-59*
Frequency	*45*	*10*	*11*	*21*	*10*	*3*



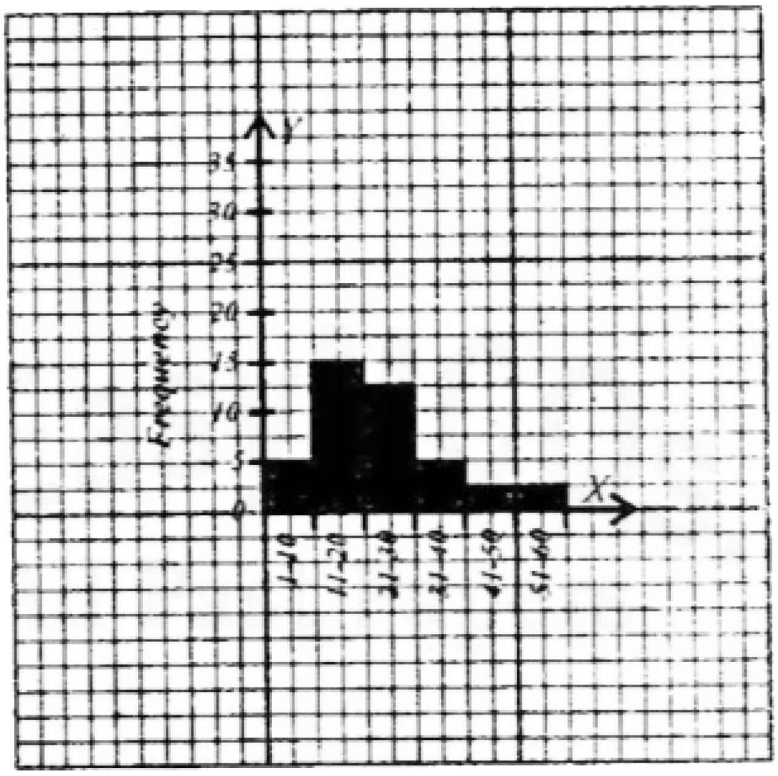
(iii) The table shows the distribution of the average marks of 40 children in the end-of-year examinations.

Average marks	1-20	21-40	41-60	61-80	81-100
Frequency	2	4	19	12	3



2- ***Following histogram shows the distribution of the times taken by 50 children to go to school. Construct a frequency table from the histogram.***

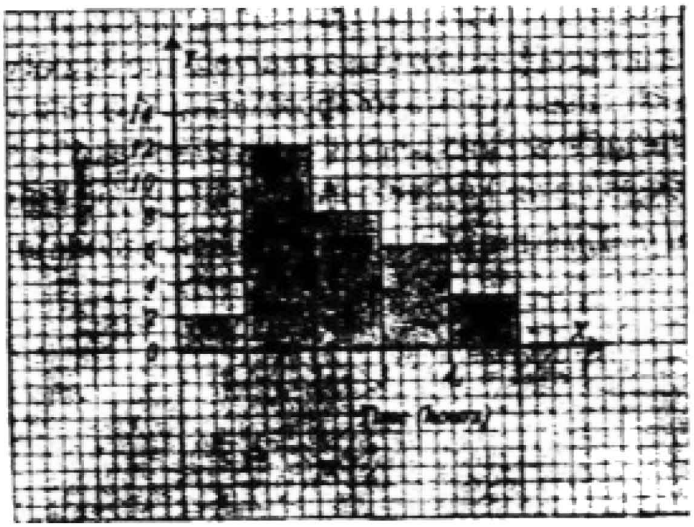
Time (min)	1-10	11-20	21-30	31-40	41-50	51-60
Frequency	5	15	13	5	3	3



Time (minutes)

3- ***Following histogram is based on the number of hours that 30 children spent watching television on a particular Saturday. Construct a frequency table from the histogram.***

Time (hours)	0 - 1	1 - 2	2 - 3	3 - 4	4 - 5
Frequency	2	12	8	6	4



Frequency Table

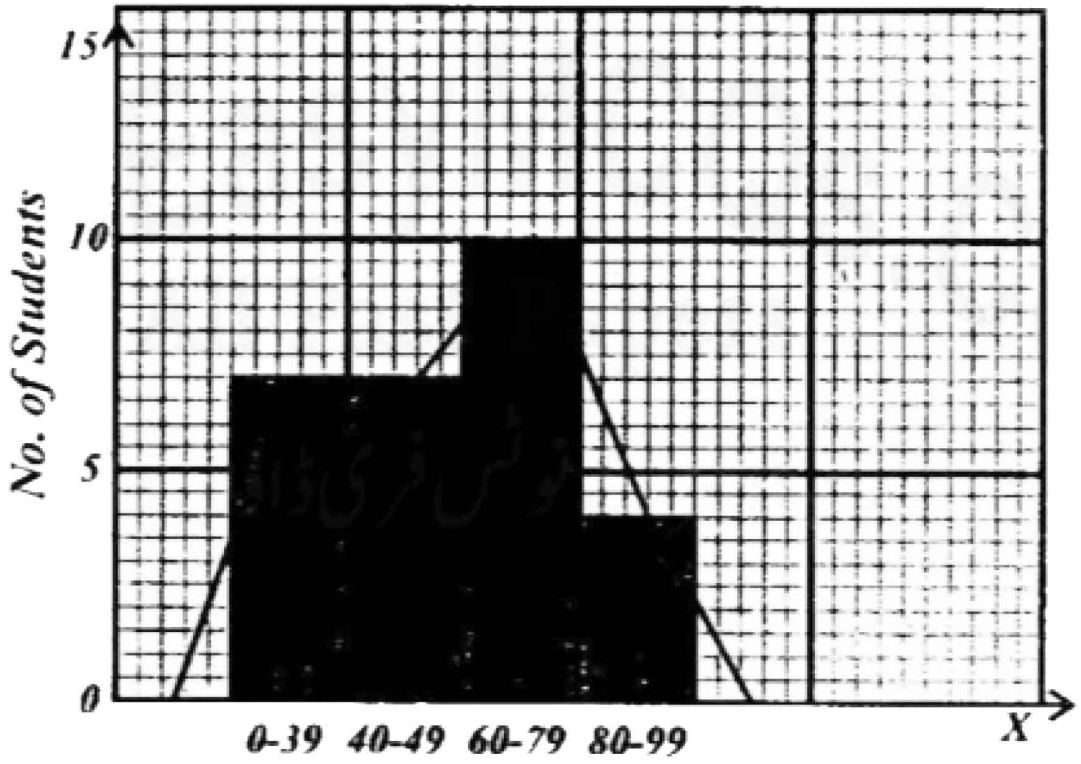
Exercise 10.3

1- ***Represent the given data using frequency polygon.***

(i) The table shows the distribution of marks of 30 children in a test.

Marks	0-39	40-49	60-79	80-99
Frequency	8	8	10	4

Solution:-



(i) obtained marks are takes along x-axis

(ii) 2 small squares represent 1 big square along y-axis.

(iii) Mid-point of every bar join with mid point of next bar.

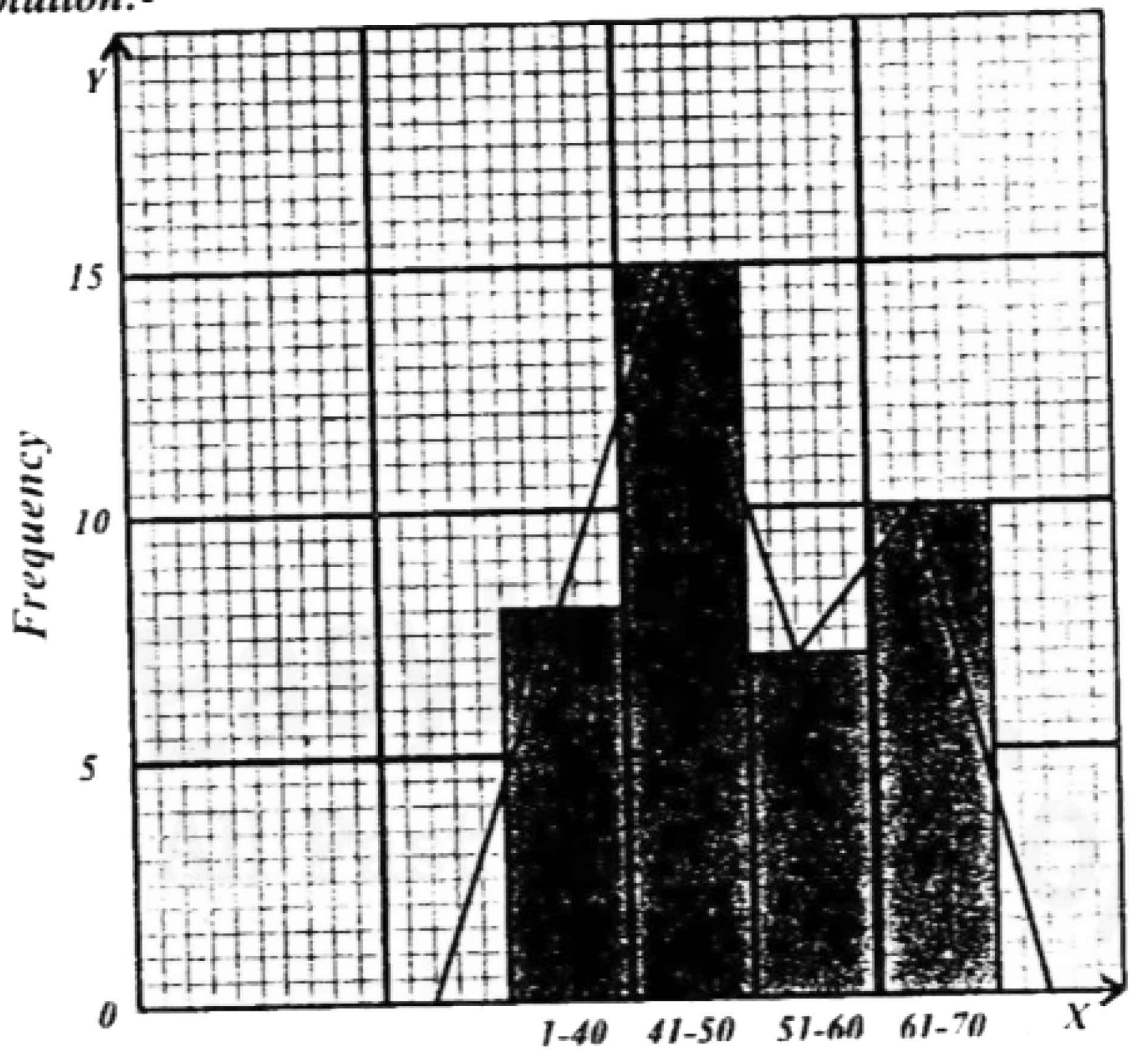
(iv) Extreme right mid point is joined 2.5 units on x-axis.

(v) Extreme left mid point is joined 2.5 units along x-axis.

(ii) ***The table shows the distribution of time (in seconds) taken for 40 children to complete the obstacle race.***

Time(sec)	*1-40*	*41-50*	*51-60*	*61-70*
Frequency	*8*	*15*	*7*	*10*

Solution:-



1- Construct a bar graph from given data while along y-axis. Represent two small squares equal one unit

2- Centre of every bar is joined by the centre of another bar.

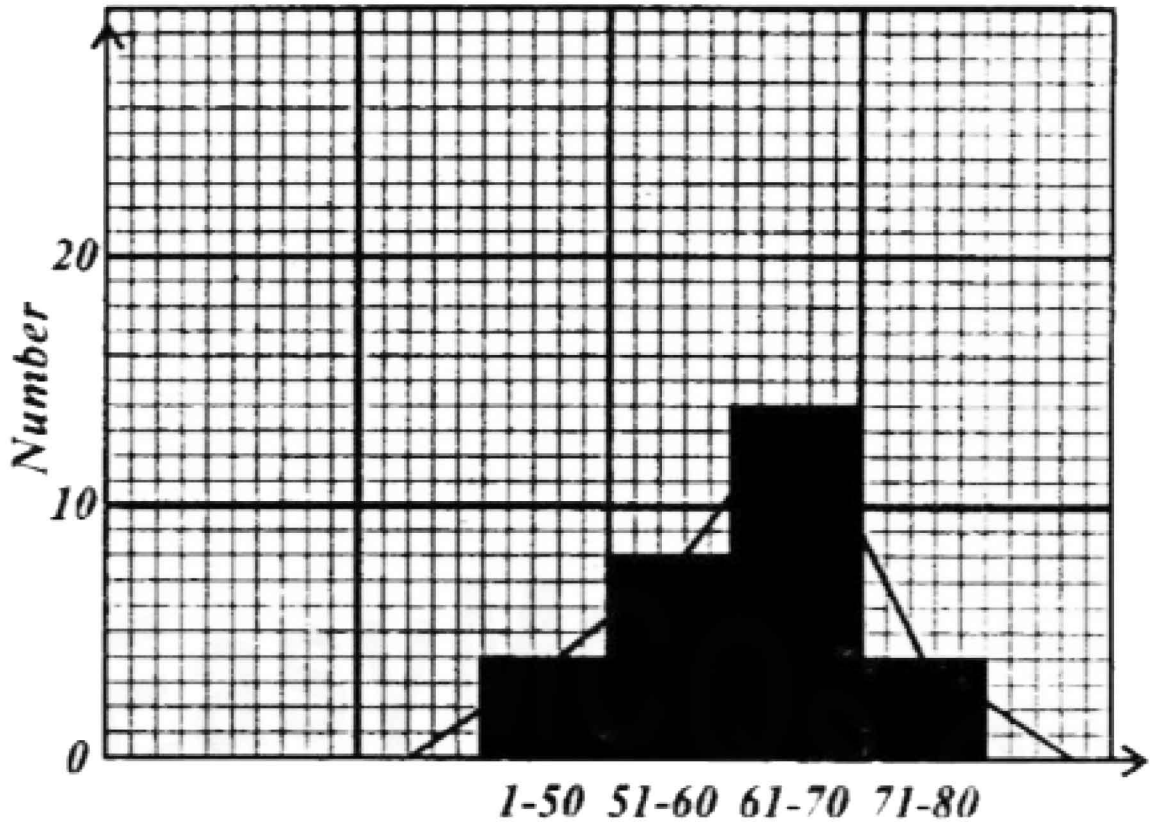
3- Centre point of left bar graph is joined by centre point of left bar graph along x-axis with the distance of 2.5

4- Centre point of right bar graph is joined by centre point of left bar graph along x-axis with the distance of 2.5.

(iii) The table shows the distribution of weights of 30 bags of chips from a fish and chip shop.

Weights(gm)	*1-50*	*51-60*	*61-70*	*71-80*
Frequency	*4*	*8*	*14*	*4*

Solution:-



1- y-axis represent small square graph along equal 1 unit.

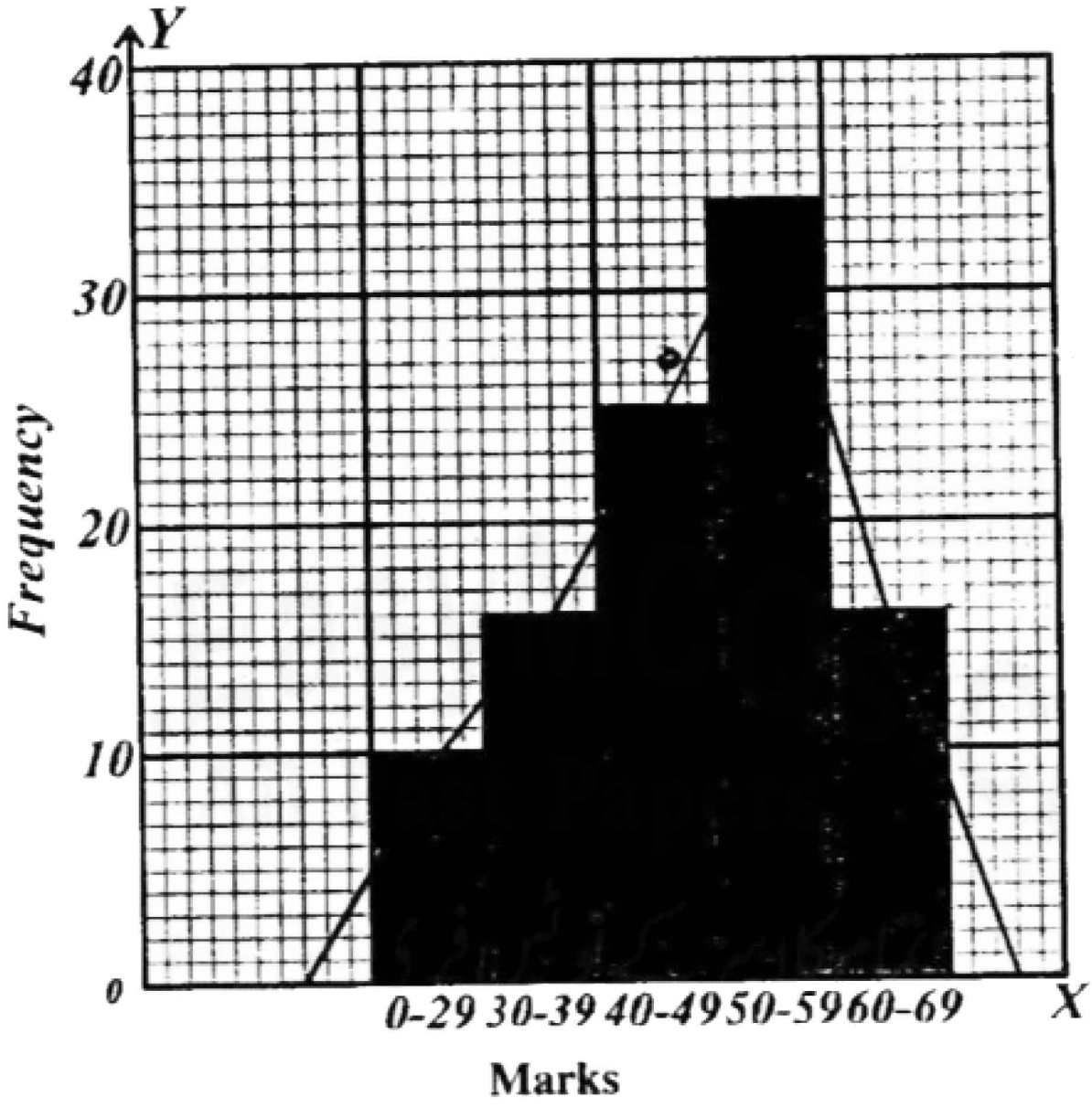
2- Bar graph is completed by given data.

3- Every centre point of bar graph is joined by another centre point of bar graph and completed frequency polygon.

(iv) ***The table shows the distribution of marks of 100 students in an end-of-terms mathematics examination.***

Marks	*0-29*	*30-39*	*40-49*	*50-59*	*60-99*
Frequency	*10*	*15*	*25*	*34*	*16*

Solution:-



(i) y-axis represent small square equal 1 unit.

(ii) Bar graph is completed by given data.

(iii) Every centre point of bar graph is joined by another centre point of bar graph and completed frequency polygon.

Exercise 10.4

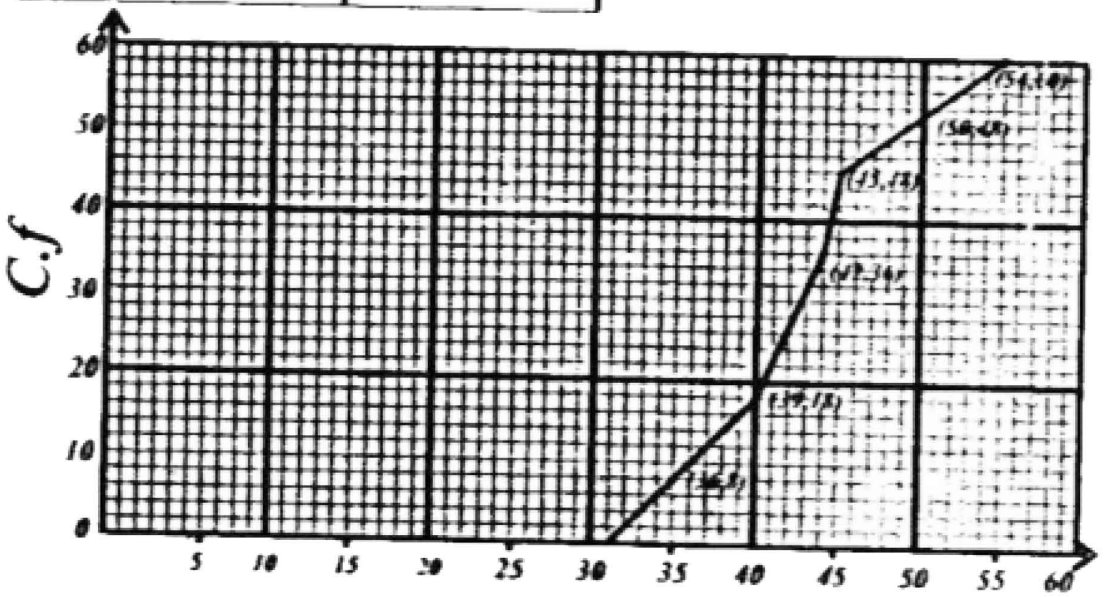
1- ***Construct a cumulative frequency polygon (that, is, an ogive) for the given data.***

(i) The table shows the distribution of weights (in kilograms) of 60 boys often years of age.

Weight (kg)	*31-36*	*37-39*	*40-42*	*43-45*	*46-54*
Frequency	*8*	*10*	*18*	*12*	*12*

Solution:-

Class intervals	Frequency	Cumulative Frequency
31 - 36	8	8
37 - 39	10	8 + 10 = 18
40 - 42	18	18 + 18 = 36
43 - 45	12	36 + 12 = 48
46 - 54	12	48 + 12 = 60
Total	60	



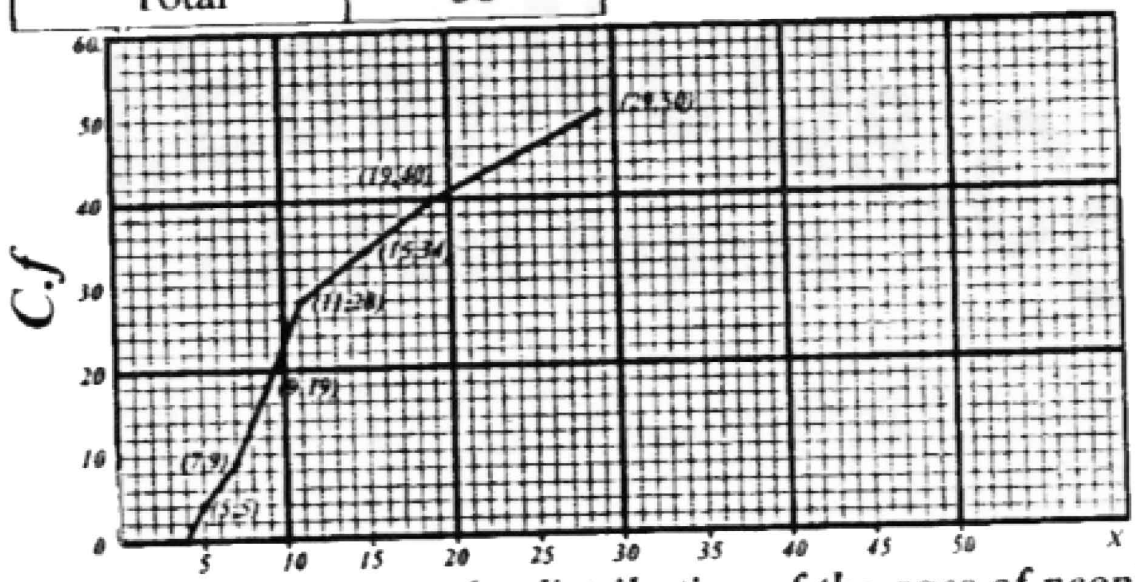
Class intervals (Ogive)

(ii) ***The table shows the distribution of times taken (in minutes) for 50 children of five years age to eat their school dinners.***

Time (min)	*4-5*	*6-7*	*8-9*	*10-11*	*12-15*	*16-19*	*20-29*
Frequency	5	4	10	0	6	6	19

Solution:-

Class intervals	Frequency	Cumulative Frequency
4 - 5	5	5
6 - 7	4	5 + 4 = 9
8 - 9	10	9 + 10 = 19
10 - 11	9	19 + 9 = 28
12 - 15	6	28 + 6 = 34
16 - 19	6	34 + 6 = 40
20 - 29	10	40 + 10 = 50
Total	50	

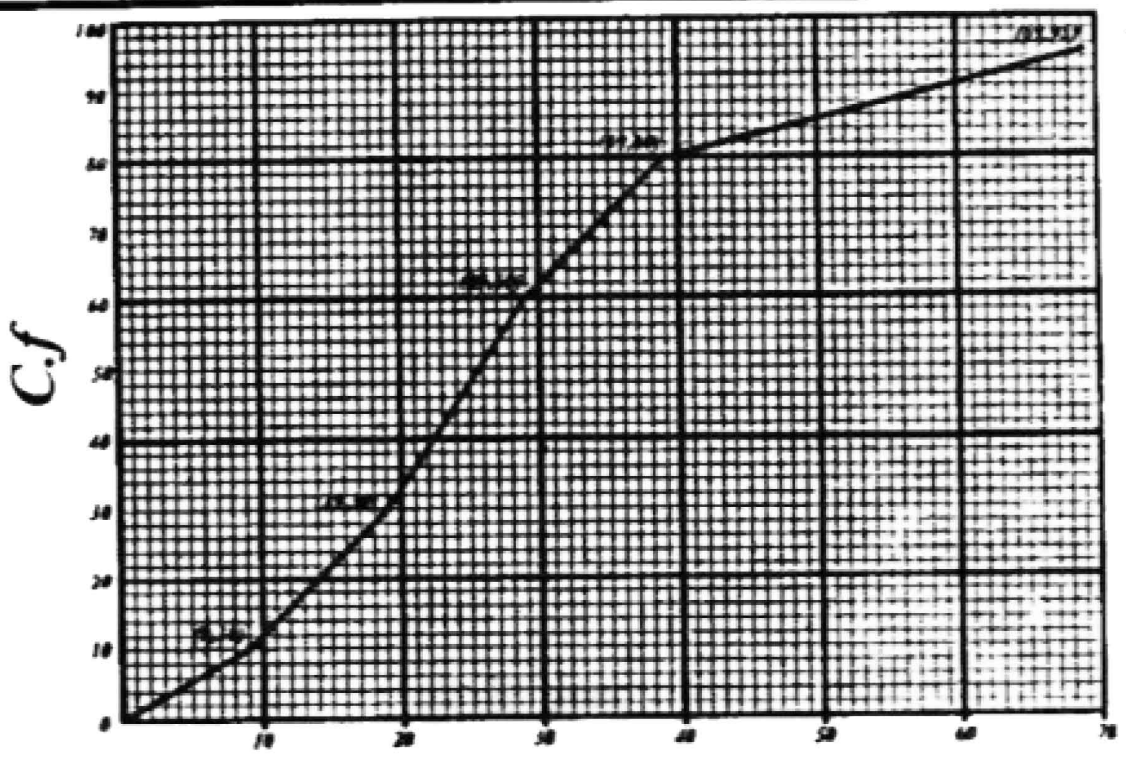


(iii) The table shows the distribution of the ages of people boarding buses at the bus station between 08.30 to 09.00 in the morning.

Age (years)	*0-9*	*10-19*	*20-29*	*30-39*	*40-69*
Frequency	*10*	*20*	*30*	*20*	*15*

Solution:-

Class intervals	Frequency	Cumulative frequency
0 - 9	10	10
10 - 19	20	10 + 20 = 30
20 - 29	30	30 + 30 = 60
30 - 39	20	60 + 20 = 80
40 - 69	15	80 + 15 = 95
Total	95	



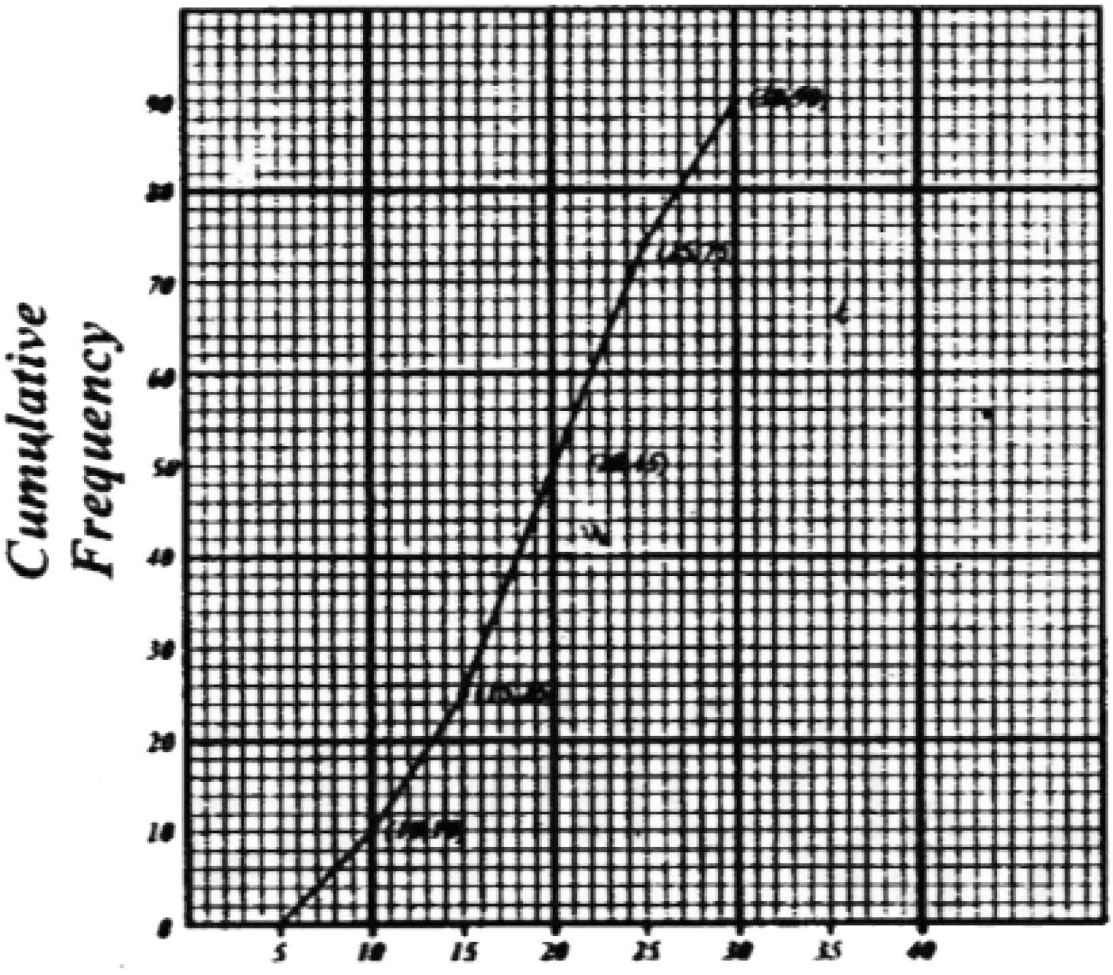
(Ogive)

(iv)

Classes	*5-10*	*10-15*	*15-20*	*20-25*	*25-30*
Frequency	*10*	*15*	*20*	*30*	*15*

Solution:-

Class intervals	frequency	Cumulative frequency
5 - 10	10	10
10 - 15	15	10 + 15 = 25
15 - 20	20	25 + 20 = 45
20 - 25	30	45 + 30 = 75
25 - 30	15	75 + 15 = 90
Total	90	



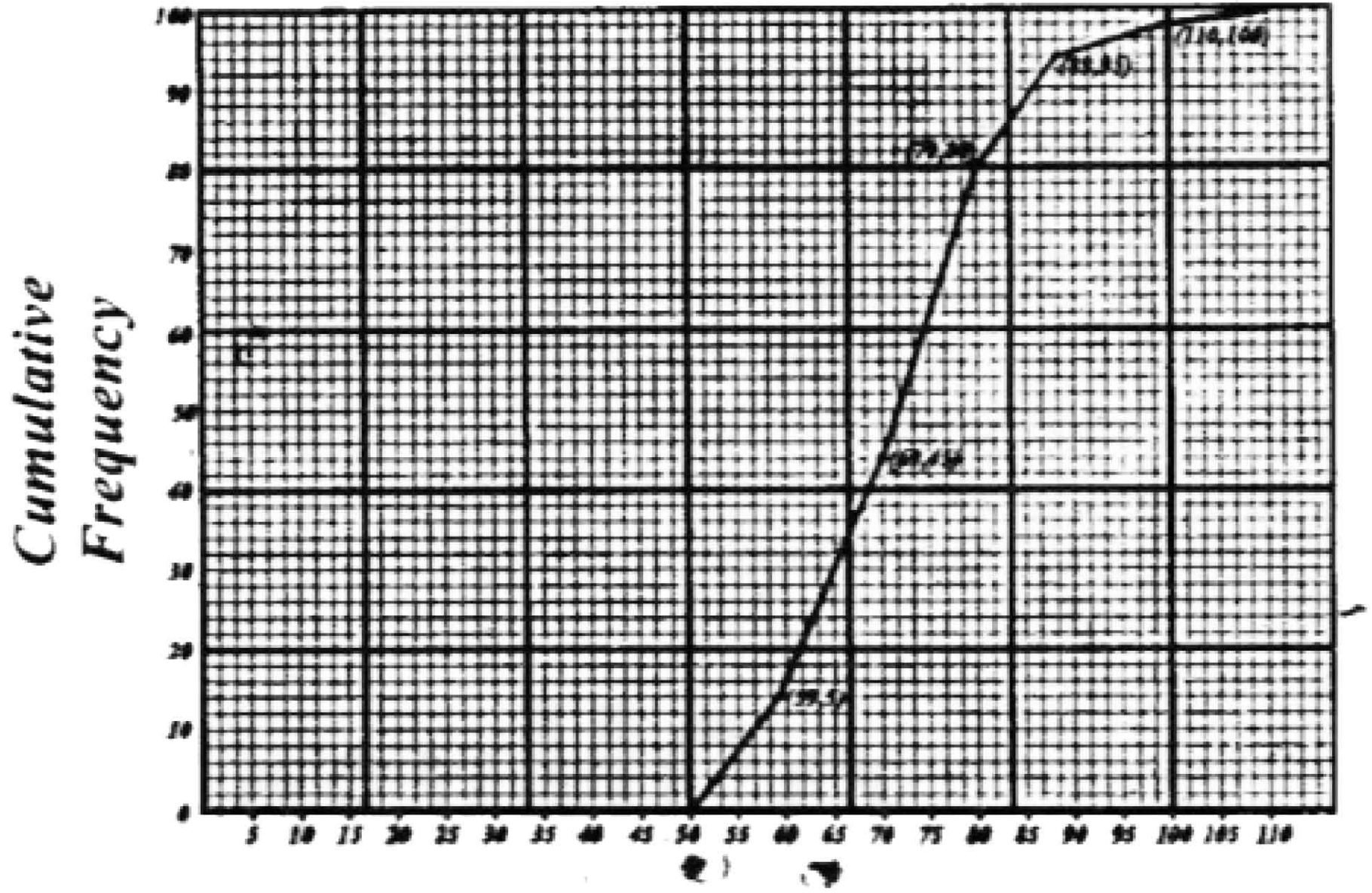
(Ogive)

(v) The table gives the distribution of weights (kilograms) of 100 people.

Weight (kilograms)	*50-59*	*60-69*	*70-79*	*80-89*	*90-99*	*100-109*
Frequency	*15*	*30*	*35*	*15*	*3*	*2*

Solution:-

Class intervals	Frequency	Cumulative frequency
50 - 59	15	15
60 - 69	30	15 + 30 = 45
70 - 79	35	45 + 35 = 80
80 - 89	15	80 + 15 = 95
90 - 99	3	95 + 3 = 98
100 - 109	2	98 + 2 = 100
Total	100	


Cumulative
Frequency
5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100 105 110

(Ogive)