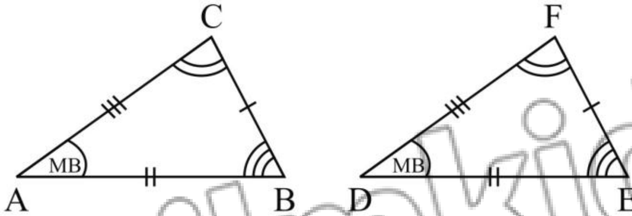


$\Delta ABC \cong \Delta DEF$

UNIT 10
CONGRUENT TRIANGLES

1 – 1 Correspondence between Two Triangles (U.B)

$(1-1)$ Correspondences can be established between ΔABC and ΔDEF is explained below:

In $\Delta ABC \longleftrightarrow \Delta DEF$, it means

(i)	$\angle A \longleftrightarrow \angle D$	($\angle A$ corresponds to $\angle D$)
(ii)	$\angle B \longleftrightarrow \angle E$	($\angle B$ corresponds to $\angle E$)
(iii)	$\angle C \longleftrightarrow \angle F$	($\angle C$ corresponds to $\angle F$)
(iv)	$\overline{AB} \longleftrightarrow \overline{DE}$	($\overline{AB}$ corresponds to $\overline{DE}$)
(v)	$\overline{BC} \longleftrightarrow \overline{EF}$	($\overline{BC}$ corresponds to $\overline{EF}$)
(vi)	$\overline{CA} \longleftrightarrow \overline{FD}$	($\overline{CA}$ corresponds to $\overline{FD}$)

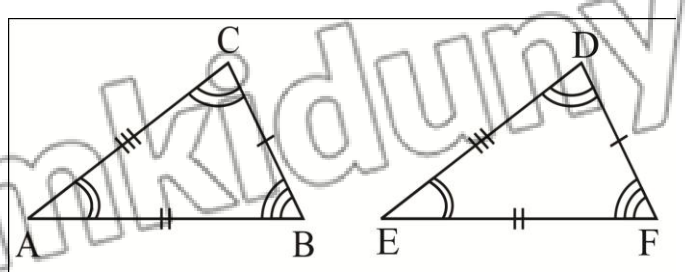
Congruency of Triangles (U.B)

Two triangles are said to be congruent written symbolically as $\cong$ if there exists a correspondence between them such that all the corresponding sides and angles are congruent.

For example:

(GRW 2013, FSD 2015, SWL 2017, RWP 2016, BWP 2016, MTN 2014, 16, D.G.K 2013)

Consider:

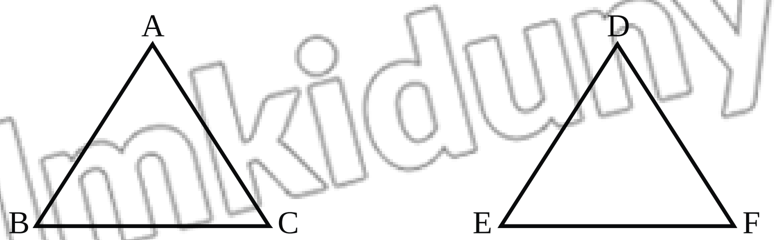


$\Delta ABC \cong \Delta DEF$

In $\Delta ABC \longleftrightarrow \Delta DEF$

If $\begin{cases} \overline{AB} \cong \overline{DE} \\ \overline{BC} \cong \overline{EF} \\ \overline{AC} \cong \overline{DF} \end{cases}$ and $\begin{cases} \angle A \cong \angle D \\ \angle B \cong \angle E \\ \angle C \cong \angle F \end{cases}$

Then, $\Delta ABC \cong \Delta DEF$



Note **(U.B)**

(i) These triangles are congruent w.r.t the above mentioned choice of the $(1-1)$ correspondence

(ii) $\Delta ABC \cong \Delta ABC$

(iii) $\Delta ABC \cong \Delta DEF \Leftrightarrow \Delta DEF \cong \Delta ABC$

(iv) If $\Delta ABC \cong \Delta DEF$ and $\Delta ABC \cong \Delta PQR$ then $\Delta DEF \cong \Delta PQR$

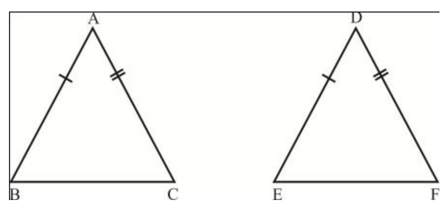
S.A.S Postulate

(LHR 2016, GRW 2017, FSD 2017, MTN 2017, RWP 2016) **(U.B)**

In any correspondence of two triangles, if two sides and their included angle of one triangle are congruent to the corresponding two sides and their included angle of the other then the triangles are congruent.

In $\Delta ABC \longleftrightarrow \Delta DEF$, shown in the following figure,

If $\begin{cases} \overline{AB} \cong \overline{DE} \\ \angle A \cong \angle D \\ \overline{AC} \cong \overline{DF} \end{cases}$

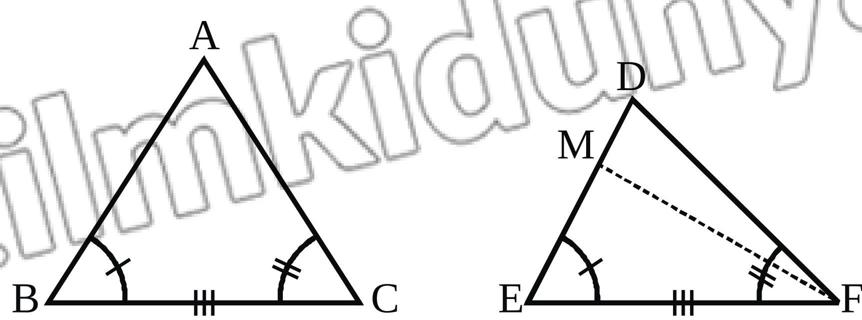


Then, $\Delta ABC \cong \Delta DEF$

Theorem 10.1.1

(A.B)

In any correspondence of two triangles, if one side and any two angles of one triangle are congruent to the corresponding side and angles of the other then the triangles are congruent.(A.S.A $\cong$ A.S.A.)



Given

In $\Delta ABC \leftrightarrow \Delta DEF$

$\angle B \cong \angle E$, $\overline{BC} \cong \overline{EF}$, $\angle C \cong \angle F$

To prove

$\Delta ABC \cong \Delta DEF$

Construction

Suppose $\overline{AB} \ncong \overline{DE}$. Take a point M on $\overline{DE}$ such that $\overline{AB} \cong \overline{ME}$. Join M to F

Proof

Statements	Reasons
In $\Delta ABC \leftrightarrow \Delta MEF$	
$\overline{AB} \cong \overline{ME}$ ____ (i)	Construction
$\overline{BC} \cong \overline{EF}$ ____ (ii)	Given
$\angle B \cong \angle E$ ____ (iii)	Given
$\therefore$ $\Delta ABC \cong \Delta MEF$	S.A.S postulate
So, $\angle C \cong \angle MFE$	(Corresponding angles of congruent triangles)
But $\angle C \cong \angle DFE$	Given
$\therefore$ $\angle DFE \cong \angle MFE$	Both congruent to $\angle C$
This is possible only if D and M are the same points and $\overline{ME} \cong \overline{DE}$	
So $\overline{AB} \cong \overline{DE}$ ____ (iv)	$\overline{AB} \cong \overline{ME}$ (construction) and $\overline{ME} \cong \overline{DE}$ (proved)
Thus from (ii), (iii) and (iv), we have $\Delta ABC \cong \Delta DEF$	S.A.S postulates

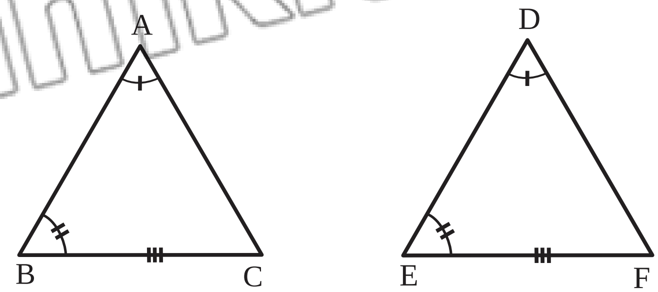
Corollary:

In any correspondence of two triangles, if one side and any two angles of one triangle are congruent to the corresponding side and angles of other, then the triangles are congruent. (S.A.A $\cong$ S.A.A.) **(K.B + U.B + A.P)**

Given:

In $\Delta ABC \leftrightarrow \Delta DEF$

$\overline{BC} \cong \overline{EF}$, $\angle A \cong \angle D$, $\angle B \cong \angle E$



To prove

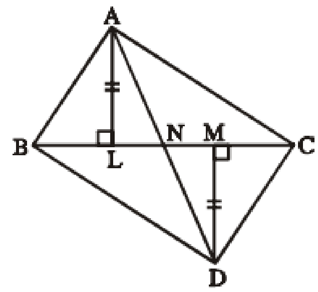
$\Delta ABC \cong \Delta DEF$

Proof

Statements	Reasons
In $\Delta ABC \leftrightarrow \Delta DEF$	
$\angle B \cong \angle E$	Given
$\overline{BC} \cong \overline{EF}$	Given
$\angle C \cong \angle F$	$\angle A \cong \angle D$, $\angle B \cong \angle E$, (Given)
$\therefore \Delta ABC \cong \Delta DEF$	A.S.A. ≅ A.S.A.

Example

If ΔABC and ΔDCB are on the opposite sides of common base $\overline{BC}$ such that $\overline{AL} \perp \overline{BC}$, $\overline{DM} \perp \overline{BC}$ and $\overline{AL} \cong \overline{DM}$, then $\overline{BC}$ bisects $\overline{AD}$.



Given

ΔABC and ΔDCB are on the opposite sides of $\overline{BC}$ such that $\overline{AL} \perp \overline{BC}$, $\overline{DM} \perp \overline{BC}$, $\overline{AL} \cong \overline{DM}$, and $\overline{AD}$ is cut by $\overline{BC}$ at N.

To prove

$\overline{AN} \cong \overline{DN}$

Proof

Statements	Reasons
In $\Delta ALN \leftrightarrow \Delta DMN$	
$\overline{AL} \cong \overline{DM}$	Given
$\angle ALN \cong \angle DMN$	Each angle is right angle
$\angle ANL \cong \angle DNM$	Vertical angels
$\therefore \Delta ALN \cong \Delta DMN$	S.A.A ≅ S.A.A
$\overline{AN} \cong \overline{DN}$	Corresponding sides of ≅ Δs.

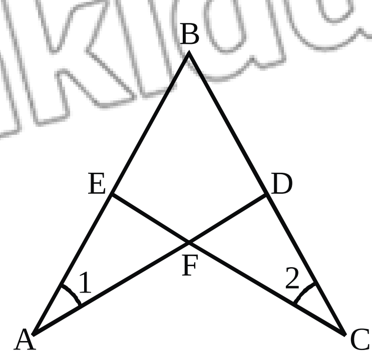
Exercise 10.1

Q.1 In the given figure **(K.B)**

$\angle 1 \cong \angle 2$ and $\overline{AB} \cong \overline{CB}$

Prove that

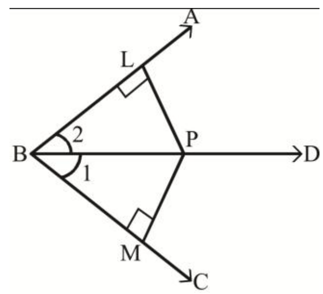
$\Delta ABD \cong \Delta CBE$



Proof

Statements	Reasons
In ΔABD ↔ ΔCBE	
$\overline{AB} \cong \overline{CB}$	Given
∠BAD ≅ ∠BCE	Given ∠1 ≅ ∠2
∠ABD ≅ ∠CBE	Common
ΔABD ≅ ΔCBE	S.A.A ≅ S.A.A

Q.2 From a point on the bisector of an angle, perpendiculars are drawn to the arms of the angle. Prove that these perpendiculars are equal in measure. (K.B)



Given

$\overline{BD}$ is bisector of ∠ABC. P is point on $\overline{BD}$ and $\overline{PL}$ and $\overline{PM}$ are perpendicular to $\overline{AB}$ and $\overline{CB}$ respectively .

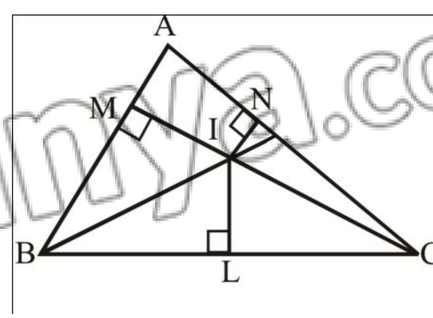
To prove

$\overline{PL} \cong \overline{PM}$

Proof

Statements	Reasons
In ΔBLP ↔ ΔBMP	
$\overline{BP} \cong \overline{BP}$	Common
∠BLP ≅ ∠BMP	Each is right angle (given)
∠LBP ≅ ∠MBP	Given $\overline{BD}$ is bisector of angle B
∴ ΔBLP ≅ ΔBMP	S.A.A ≅ S.A.A
So $\overline{PL} \cong \overline{PM}$	Corresponding sides of congruent triangles

Q.3 In a triangle ABC, the bisectors of ∠B and ∠C meet in a point I. Prove that I is equidistant from the three sides of ΔABC. (K.B)



Given

In ΔABC, the bisectors of ∠B and ∠C meet at I and $\overline{IL}$, $\overline{IM}$, and $\overline{IN}$ are perpendiculars to the three sides of ΔABC.

To prove

$\overline{IL} \cong \overline{IM} \cong \overline{IN}$

Proof

Statements	Reasons
In ΔILB ↔ ΔIMB	
$\overline{BI} \cong \overline{BI}$	Common
∠IBL ≅ ∠IBM	Given BI is bisector of ∠B
∠ILB ≅ ∠IMB	Given each angle is rights angles
ΔILB ≅ ΔIMB	S.A.A ≅ S.A.A

∴ $\overline{IL} \cong \overline{IM}$ ____ (i)	Corresponding sides of ≅ Δ's
Similarly	
ΔILC ≅ ΔINC	
So $\overline{IL} \cong \overline{IN}$ ____ (ii)	Corresponding sides of ≅ Δs
from (i) and (ii)	
$\overline{IL} \cong \overline{IM} \cong \overline{IN}$	
∴ I is equidistant from the three sides of ΔABC.	

Theorem 10.1.2 **(A.B)**

If two angles of a triangle are congruent then the sides opposite to them are also congruent.

Given

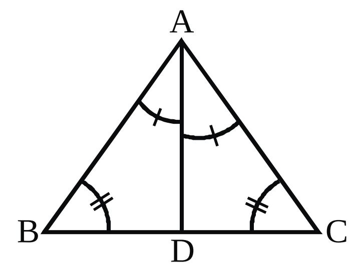
In ΔABC, ∠B ≅ ∠C

To prove

$\overline{AB} \cong \overline{AC}$

Construction

Draw the bisector of ∠A, meeting $\overline{BC}$ at point D



Proof

Statements	Reasons
In ΔABD ↔ ΔACD	
$\overline{AD} \cong \overline{AD}$	Common
∠B ≅ ∠C	Given
∠BAD ≅ ∠CAD	Construction
ΔABD ≅ ΔACD	S.A.A ≅ S.A.A
Hence $\overline{AB} \cong \overline{AC}$	(Corresponding sides of congruent triangles)

Example # 1 **(A.B)**

If one angle of a right triangle is of 30°, the hypotenuse is twice as long as the side opposite to the angle.

Given

In ΔABC, m∠B=90° and $m\angle C = 30°$

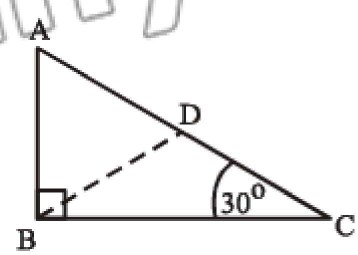
To prove

$m\overline{AC} = 2m\overline{AB}$

Construction

At B, construct ∠CBD of 30°.

Let $\overline{BD}$ cut $\overline{AC}$ at the point D.



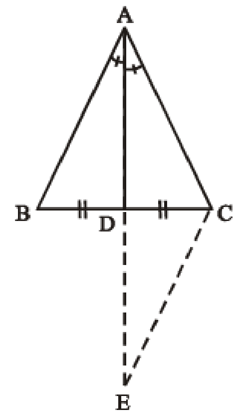
Proof

Statements	Reasons
In ΔABD, m∠A=60°	m∠ABC=90°, $m\angle C = 30°$
$m\angle ABD = m\angle ABC - m\angle CBD = 60°$	m∠ABC = 90°, m∠CBD = 30°
∴ m∠ADB = 60°	Sum of measures of ∠s of a Δ is 180°

$\therefore \Delta ABD$ is equilateral	Each of its angles is equal to 60°
$\therefore \overline{AB} \cong \overline{BD} \cong \overline{AD}$	Sides of equilateral Δ
In $\Delta BCD, \overline{BD} \cong \overline{CD}$	$\angle C = \angle CBD$ (each of 30°)
Thus $m\overline{AC} = m\overline{AD} + m\overline{CD}$ $= m\overline{AB} + m\overline{AB}$ $= 2(m\overline{AB})$	$\overline{AD} \cong \overline{AB}$ and $\overline{CD} \cong \overline{BD} \cong \overline{AB}$

Example # 2 **(A.B)**

If the bisectors of an angle of a triangle bisect the side opposite to it, the triangle is isosceles.



Given

In ΔABC, $\overline{AD}$ bisects $\angle A$ and $\overline{BD} \cong \overline{CD}$

To prove

$\overline{AB} \cong \overline{AC}$

Construction

Produce $\overline{AD}$ to E, and take $\overline{ED} \cong \overline{AD}$

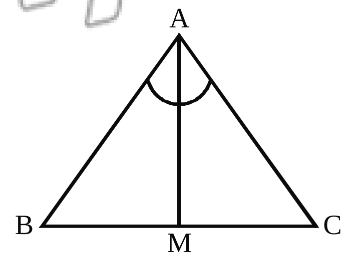
Joint C to E

Proof

Statements	Reasons
In $\Delta ADB \leftrightarrow \Delta EDC$	
$\overline{AD} \cong \overline{ED}$	Construction
$\angle ADB \cong \angle EDC$	Vertical angles
$\overline{BD} \cong \overline{CD}$	Given
$\therefore \Delta ADB \cong \Delta EDC$	S.A.S. Postulate
$\therefore \overline{AB} \cong \overline{EC}$ …(i)	Corresponding sides of congruent Δs
and $\angle BAD \cong \angle E$	Corresponding angles of congruent Δs
But $\angle BAD \cong \angle CAD$	Given
$\therefore \angle E \cong \angle CAD$	Each $\cong \angle BAD$
In $\Delta ACE, \overline{AC} \cong \overline{EC}$ …(ii)	$\angle E \cong \angle CAD$ (proved)
Hence $\overline{AB} \cong \overline{AC}$	From (i) and (ii)

Exercise 10.2

Q.1 Prove that any two medians of an equilateral triangle are equal in measure. (K.B)



Given

In ΔABC, $\overline{AB} \cong \overline{AC}$ and M is midpoint of $\overline{BC}$

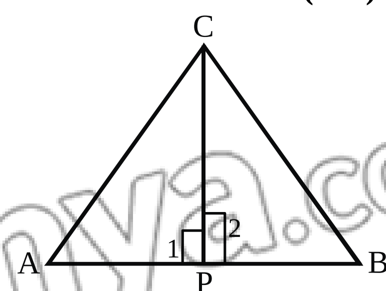
To prove

$\overline{AM}$ bisects $\angle A$ and $\overline{AM}$ is perpendicular to $\overline{BC}$

Proof

Statements	Reasons
In $\Delta ABM \leftrightarrow \Delta ACM$	
$\overline{AB} \cong \overline{AC}$	Given
$\overline{BM} \cong \overline{CM}$	Given M is midpoint of $\overline{BC}$
$\overline{AM} \cong \overline{AM}$	Common
$\therefore \Delta ABM \cong \Delta ACM$	S.S.S ≅ S.S.S
So $\angle BAM \cong \angle CAM$	Corresponding angles of congruent triangles
$m\angle AMB + m\angle AMC = 180^o$	
$\therefore$ $m\angle AMB = m\angle AMC = 90^o$	
i.e $\overline{AM}$ is perpendicular to $\overline{BC}$	

Q.2 Prove that a point which is equidistant from the end points of a line segment, is on the right bisector of line segment (U.B)



Given

$\overline{AB}$ is line segment. The point C is such that $\overline{CA} \cong \overline{CB}$

To prove

Point C lies on the right bisector of $\overline{AB}$

Construction

(i) Take P as midpoint of $\overline{AB}$ i.e. $\overline{AP} \cong \overline{BP}$

(ii) Join point C to A, P, B

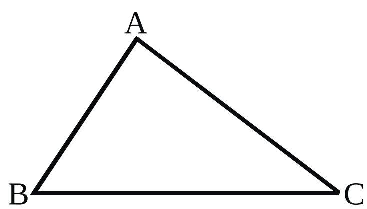
Proof:

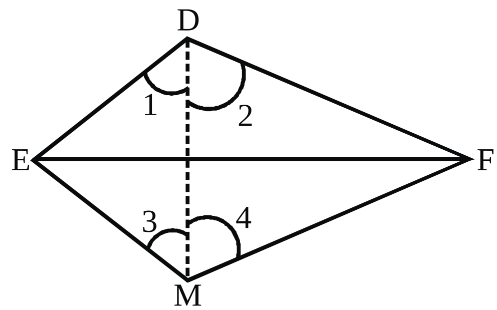
Statements	Reasons
In ΔABC	
$\overline{CA} \cong \overline{CB}$	Given
$\angle A \cong \angle B$	Corresponding angles of congruent triangles
In Δ CBP $\leftrightarrow \Delta CAP$	
$\overline{CB} \cong \overline{CA}$	
$\Delta CAP \cong \Delta CBP$	S.A.S ≅ S.A.S

$\therefore \angle 1 \cong \angle 2$	
$m\angle 1 + m\angle 2 = 180^o$	Adjacent supplementary angles
Thus $m\angle 1 = m\angle 2 = 90^o$	
Hence $\overline{CP}$ is right bisector of $\overline{AB}$ and point C lies on $\overline{CB}$	

Theorem 10.1.3 **(A.B + U.B)**

In a correspondence of two triangles if three sides of one triangle are congruent to the corresponding three sides of the other, then the two triangles are congruent. (S.S.S ≅ S.S.S)





Given:

In $\Delta ABC \leftrightarrow \Delta DEF$

$\overline{AB} \cong \overline{DE}, \overline{BC} \cong \overline{EF}$ and $\overline{CA} \cong \overline{FD}$

To prove

$\Delta ABC \cong \Delta DEF$

Construction:

Suppose that in ΔDEF the side $\overline{EF}$ is not smaller than any of the remaining two sides. On $\overline{EF}$ construct a ΔMEF in which, $\angle FEM \cong \angle B$ and $\overline{ME} \cong \overline{AB}$. Join D and M, as shown in the above figures we label some of the angles as 1, 2, 3, and 4

Proof:

Statements	Reasons
In $\Delta ABC \leftrightarrow \Delta MEF$	
$\overline{BC} \cong \overline{EF}$	Given
$\angle B \cong \angle FEM$	Construction
$\overline{AB} \cong \overline{ME}$	Construction
$\therefore \Delta ABC \cong \Delta MEF$	S.A.S Postulate
and $\overline{CA} \cong \overline{FM}$ ____ (i)	(Corresponding sides of congruent triangles)
also $\overline{CA} \cong \overline{FD}$ ___ (ii)	Given
$\therefore \overline{FM} \cong \overline{FD}$	{ From (i) and (ii) }

In ΔFDM $\angle 2 \cong \angle 4$ _____(iii) Similarly $\angle 1 \cong \angle 3$ ______(iv) $\therefore$ $m\angle 2 + m\angle 1 = m\angle 4 + m\angle 3$ $\therefore$ $m\angle EDF = m\angle EMF$	$\overline{FM} \cong \overline{FD}$ (proved) { from (iii) and iv }
Now in $\Delta DEF \leftrightarrow \Delta MEF$	
$\overline{FD} \cong \overline{FM}$	Proved
and $m\angle EDF \cong \angle EMF$	Proved
$\overline{DE} \cong \overline{ME}$	Each one $\cong \overline{AB}$
$\therefore$ $\Delta DEF \cong \Delta MEF$	S.A.S postulate
also $\Delta ABC \cong \Delta MEF$	Proved
Hence $\Delta ABC \cong \Delta DEF$	Each $\Delta \cong \Delta MEF$ (proved)

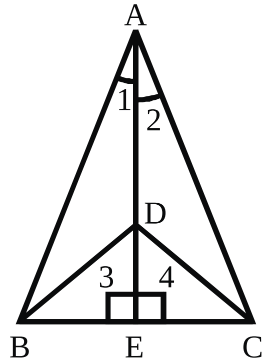
Corollary **(U.B)**

If two isosceles triangles are formed on the same side of their common base, the line through their vertices would be the right bisector of their common base.

Given

ΔABC and ΔDBC are formed on the same side of $\overline{BC}$ such that

$\overline{AB} \cong \overline{AC}, \overline{DB} \cong \overline{DC}, \overline{AD}$ meets $\overline{BC}$ at E.



To prove

$\overline{BE} \cong \overline{CE}. \overline{AE} \perp \overline{BC}$

Proof

Statements	Reasons
In $\Delta ADB \leftrightarrow \Delta ADC$	
$\overline{AB} \cong \overline{AC}$	Given
$\overline{DB} \cong \overline{DC}$	Given
$\overline{AD} \cong \overline{AD}$	Common
$\therefore \Delta ADB \cong \Delta ADC$	S.S.S $\cong$ S.S.S
$\therefore \angle 1 \cong \angle 2$	Corresponding angles of $\cong \Delta s$
In $\Delta ABE \leftrightarrow \Delta ACE$	
$\overline{AB} \cong \overline{AC}$	Given
$\angle 1 \cong \angle 2$	Proved
$\overline{AE} \cong \overline{AE}$	Common
$\Delta ABE \cong \Delta ACE$	S.A.S postulate
$\therefore \overline{BE} \cong \overline{CE}$	Corresponding sides of $\cong \Delta s$

$\angle 3 \cong \angle 4 \rightarrow (i)$	Corresponding angles of $\cong \Delta s$
$m\angle 3 + m\angle 4 = 180° \rightarrow (ii)$	Supplementary angles postulate
$m\angle 3 = m\angle 4 = 90° \rightarrow (iii)$	From (*i*) and (*ii*)
Hence $\overline{AE} \perp \overline{BC}$	From (*iii*)

Corollary:

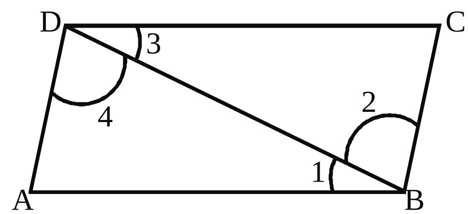
An equilateral triangle is an equiangular triangle.

Exercise 10.3

Q.1 In the figure, $\overline{AB} \cong \overline{DC}, \overline{AD} \cong \overline{BC}$ prove that $\angle A \cong \angle C$, $\angle ABC \cong \angle ADC$ (A.B + U.S)

Given

In the figure $\overline{AB} \cong \overline{DC}, \overline{AD} \cong \overline{BC}$



To prove

$\angle A \cong \angle C$

$\angle ABC \cong \angle ADC$

Proof

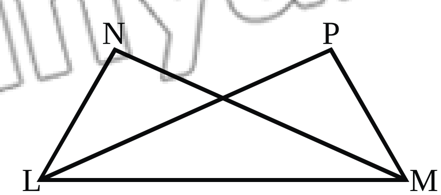
Statements	Reasons
In $\Delta ABD \leftrightarrow \Delta CDB$	
$\overline{AB} \cong \overline{DC}$	Given
$\overline{AD} \cong \overline{BC}$	Given
$\overline{BD} \cong \overline{BD}$	Common
$\Delta ABD \cong \Delta CDB$	S.S.S $\cong$ S.S.S
$\therefore$ Hence $\angle A \cong \angle C$	Corresponding angles of congruent triangles
$\angle 1 \cong \angle 3 \rightarrow (i)$	Corresponding angles of congruent triangles
$\angle 2 \cong \angle 4 \rightarrow (ii)$	Corresponding angles of congruent triangles
$m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	Adding (i) and (ii)
or $m\angle ABC = m\angle ADC$	Addition of angles postulate
$\angle ABC \cong \angle ADC$	

Q.2 In the figure $\overline{LN} \cong \overline{MP}$, $\overline{MN} \cong \overline{LP}$ prove that $\angle N \cong \angle P$, $\angle NML \cong \angle PLM$ (A.B + U.S)

Given

In the figure

$\overline{LN} \cong \overline{MP}$ and $\overline{LP} \cong \overline{MN}$



To prove

$\angle N \cong \angle P$ and $\angle NML \cong \angle PLM$

Proof

Statements	Reasons
$\Delta LMN \leftrightarrow \Delta MLP$	
$\overline{LN} \cong \overline{MP}$	Given
$\overline{LP} \cong \overline{MN}$	Given
$\overline{LM} \cong \overline{ML}$	Common

$\Delta LMN \cong \Delta MLP$	S.S.S ≅ S.S.S
$\angle N \cong \angle P$	Corresponding angles of congruent triangles
$\angle NML \cong \angle PLM$	Corresponding angles of congruent triangles

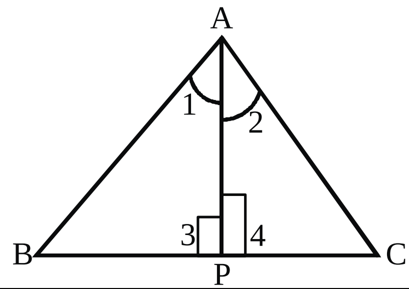
Q.3 Prove that median bisecting the base of an isosceles triangle bisects the vertex angle and it is perpendicular to the base **(A.B + U.S)**

Given

ΔABC

(i) $\overline{AB} \cong \overline{AC}$

(ii) Point P is mid point of $\overline{BC}$ *i.e.* $\overline{BP} \cong \overline{CP}$ P is joined to A, i.e. $\overline{AP}$ is median



To prove

$\angle 1 \cong \angle 2$

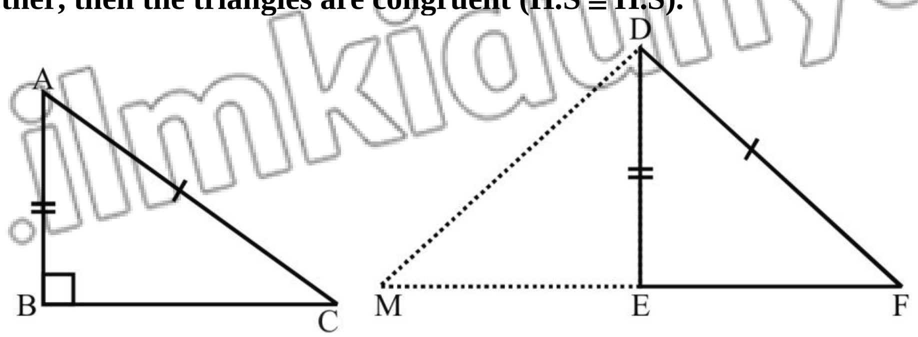
$\overline{AP} \perp \overline{BC}$

Proof

Statements	Reasons
$\Delta ABP \leftrightarrow \Delta ACP$	
$\overline{AB} \cong \overline{AC}$	Given
$\overline{BP} \cong \overline{CP}$	Given
$\overline{AP} \cong \overline{AP}$	Common
$\Delta ABP \cong \Delta ACP$	S.S.S ≅ S.S.S
$\angle 1 \cong \angle 2$	Corresponding angles of congruent triangles
$\angle 3 \cong \angle 4$ --------- (i)	Corresponding angles of congruent triangles
$m\angle 3 + m\angle 4 = 180^o$ ------- (ii)	Adjacent supplementary angles
Thus $m\angle 3 = m\angle 4 = 90^o$	From equation (i) and (ii)
Q $\overline{AP} \perp \overline{BC}$	

Theorem 10.1.4 **(A.B)**

If in the correspondence of the two right angled triangles, the hypotenuse and one side of one triangle are congruent to the hypotenuse and the corresponding side of the other, then the triangles are congruent (H.S ≅ H.S).



Given

$\Delta ABC \leftrightarrow \Delta DEF$

$\angle B \cong \angle E$ (right angles)

$\overline{CA} \cong \overline{FD}, \overline{AB} \cong \overline{DE}$

To Prove

$\Delta ABC \cong \Delta DEF$

Construction

Produce $\overline{FE}$ to a point M such that $\overline{EM} \cong \overline{BC}$ and join the points D and M.

Proof

Statements	Reasons
$m\angle DEF + \angle DEM = 180^\circ$ ------ (i)	Supplementary angles
Now $m\angle DEF = 90^\circ$ -------- (ii)	Given
$\therefore m\angle DEM = 90^\circ$	{ from (i) and (ii) }
In $\Delta ABC \leftrightarrow \Delta DEM$	
$\overline{BC} \cong \overline{EM}$	Construction
$\angle ABC \cong \angle DEM$	(Each angle equal to 90°)
$\overline{AB} \cong \overline{DE}$	Given
$\Delta ABC \cong \Delta DEM$	S.A.S postulate
and $\angle C = \angle M$	Corresponding angles of congruent triangles
$\overline{CA} \cong \overline{MD}$	Corresponding sides of congruent triangles
But $\overline{CA} \cong \overline{FD}$	Given
$\overline{MD} \cong \overline{FD}$	Each is congruent to $\overline{CA}$
In DMF	
$\angle F \cong \angle M$	$\overline{MD} \cong \overline{FD}$ (proved)
But $\angle C \cong \angle M$	(Proved)
$\angle C \cong \angle F$	Each is congruent to $\angle M$
	Given
$\angle ABC \cong \angle DEF$	Given
$\overline{AB} \cong \overline{DE}$	(Proved)
Hence $\Delta ABC \cong \Delta DEF$	(S.A.A $\cong$ S.A.A)

Example **(A.B + U.B)**

If perpendiculars from two vertices of a triangle to the opposite sides are congruent, then the triangle is isosceles.

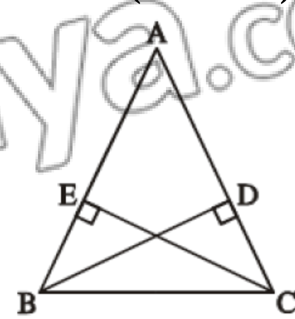
Given

In $\Delta ABC, \overline{BD} \perp \overline{AC}, \overline{CE} \perp \overline{AB}$

Such that $\overline{BD} \cong \overline{CE}$

To prove

$\overline{AB} \cong \overline{AC}$



Proof

Statements	Reasons
In $\Delta BCD \leftrightarrow \Delta CBE$	
$\angle BDC \cong \angle BEC$	$\overline{BD} \perp \overline{AC}, \overline{CE} \perp \overline{AB}$ (given) $\Rightarrow$ each angle $= 90^\circ$
$\overline{BC} \cong \overline{BC}$	Common hypotenuse
$\overline{BD} \cong \overline{CE}$	Given

$\Delta BCD \cong \Delta CBE$ $\angle BCD \cong \angle CBE$ Thus $\angle BCA \cong \angle CBA$ Hence $\overline{AB} \cong \overline{AC}$	H.S ≅ H.S Corresponding angles of congruent Δ s In ΔABC, $\angle BCA \cong \angle CBA$

Exercise 10.4

Q.1 In ΔPAB of figure $\overline{PQ} \perp \overline{AB}$ and $\overline{PA} \cong \overline{PB}$ prove that $\overline{AQ} \cong \overline{BQ}$ and $\angle APQ \cong \angle BPQ$

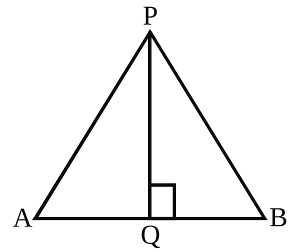
Given: **(A.B + U.B)**

In ΔPAB

$\overline{PQ} \perp \overline{AB}$ and $\overline{PA} \cong \overline{PB}$

To prove

$\overline{AQ} \cong \overline{BQ}$ and $\angle APQ \cong \angle BPQ$



Proof

Statements	Reasons
In ΔAPQ ↔ ΔBPQ	
$\overline{PA} \cong \overline{PB}$	Given
$\angle AQP \cong \angle BQP$	Given $\overline{PQ} \perp \overline{AB}$
$\overline{PQ} \cong \overline{PQ}$	Common
∴ ΔAPQ ≅ ΔBPQ	H.S ≅ H.S
So $\overline{AQ} \cong \overline{BQ}$	Corresponding sides of congruent triangles
and $\angle APQ \cong \angle BPQ$	Corresponding angles of congruent triangles

Q.2 In the figure $m\angle C = m\angle D = 90^o$ and $\overline{BC} \cong \overline{AD}$ prove that $\overline{AC} \cong \overline{BD}$ and $\angle BAC \cong \angle ABD$

Given **(A.B + U.B)**

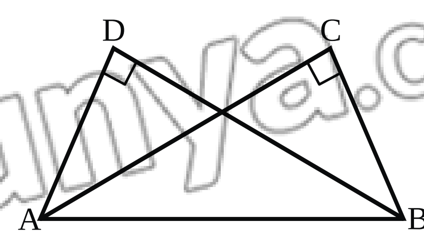
In the figure given $m\angle C = m\angle D = 90^o$

$\overline{BC} \cong \overline{AD}$

To Prove

$\overline{AC} \cong \overline{BD}$

$\angle BAC \cong \angle ABD$



Proof

Statements	Reasons
In ΔABD ↔ ΔBAC	
$\overline{AD} \cong \overline{BC}$	Given
$\angle D \cong \angle C$	Each is equal to 90^o
$\overline{AB} \cong \overline{BA}$	Common
Thus ΔABD ≅ ΔBAC	H-S ≅ H-S

$\therefore\ \overline{AC} \cong \overline{BD}$	Corresponding sides of congruent triangles
$\therefore\ \angle BAC \cong \angle ABD$	Corresponding angles of congruent triangles

Q.3 In the figure, $m\angle B = m\angle D = 90^o$ and $\overline{AD} \cong \overline{BC}$ prove that ABCD is a rectangle

(A.B + U.B)

Given

In the figure

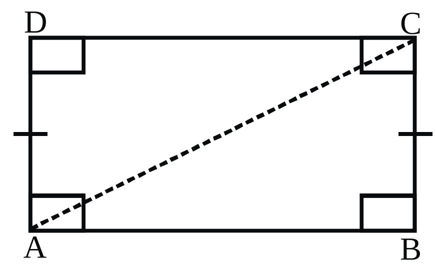
$m\angle B = m\angle D = 90^o$ and $\overline{AD} \cong \overline{BC}$

To prove

ABCD is a rectangle

Construction

Join *A* to *C*



Proof

Statements	Reasons
In $\Delta ABC \leftrightarrow \Delta CDA$	
$\angle B \cong \angle D$	Given each angle = 90^o
$\overline{AC} \cong \overline{CA}$	Common
$\overline{BC} \cong \overline{DA}$	Given
$\therefore \Delta ABC \cong \Delta CDA$	H-S ≅ H-S
$\overline{AB} \cong \overline{CD}$	Corresponding sides of congruent triangles
and $\angle ACB \cong \angle CAD$	Corresponding angles of congruent triangles
Hence ABCD is a rectangle	

Review Exercise 10

Q.1 Which of the following statements are true and which are false? (K.B + U.B)

(i) A ray has two end points. (False)

(ii) In a triangle there can be only one right angle. (True)

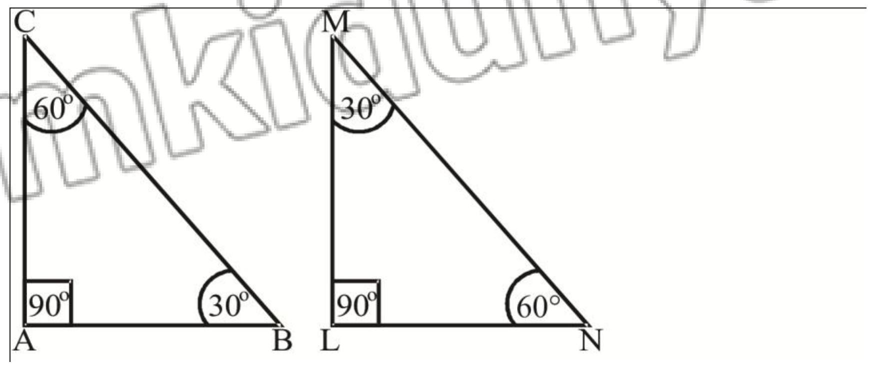
(iii) Three points are said to be collinear if they lie on same line. (True)

(iv) Two parallel lines intersect at a point. (False)

(v) Two lines can intersect only at one point. (True)

(vi) A triangle of congruent sides has non-congruent angles. (False)

Q.2 If $\Delta ABC \cong \Delta LMN$, then **(GRW 2013)** **(A.B + U.B)**

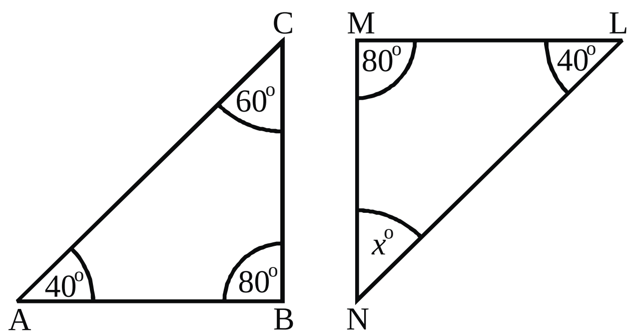


(i) $m\angle M \cong \underline{m\angle B} = 30^o$

(ii) $m\angle N \cong \underline{m\angle C} = 60^o$

(iii) $m\angle A \cong \underline{m\angle L} = 90^o$

Q.3 If $\Delta ABC \cong \Delta\angle LMN$ then find the value of *x*. **(A.B + K.B)**



$m\angle N = m\angle C = 60^o$

$m\angle N = x = 60^o$

Sum of three angles in a triangle is 180^o

So $x + 80^o + 40^o = 180^o$

$x + 120^o = 180^o$

$x = 180^o - 120^o$

$x = 60^o$

Q.4 Find the value of unknowns for the given congruent triangles. **(A.B + K.B)**

It is an isosceles triangle

$m\overline{AB} = m\overline{AC}$

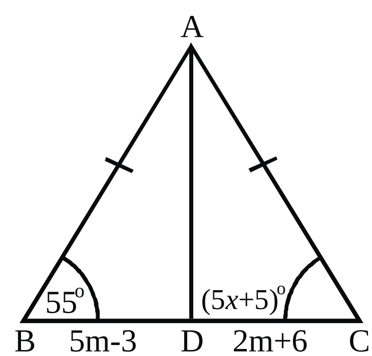
and $m\angle B = m\angle C$

when we draw a perpendicular from point A to BC, it bisects

So $m\overline{BD} \cong m\overline{DC}$

$5m - 3 = 2m + 6$

$5m - 2m = 6+3$



$$3m = 9$$
$$m = \frac{9}{3}$$
$$m = 3$$

Opposite angles are congruent

$\therefore$ $m\angle B = m\angle C$

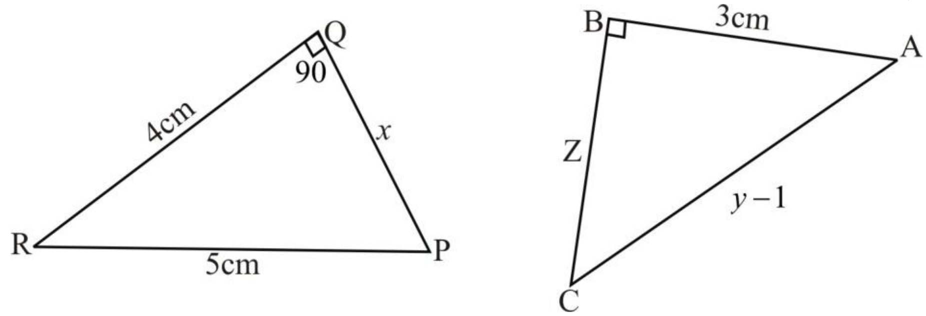
$55 = 5x + 5$

$55 - 5 = 5x$

$\frac{50}{5} = x$

$x = 10$

Q.5 If ΔPQR = ΔABC, then find the unknowns (A.B + K.B)



By definition of congruent triangles

$\overline{RP} = \overline{AC}$ and $\overline{AB} = \overline{QP}$ and $\overline{BC} = \overline{QR}$

$5 = y - 1$ $\quad 3cm = x$ $\quad z = 4cm$

$5 + 1 = y$ $\quad x = 3cm$

$y = 6cm$