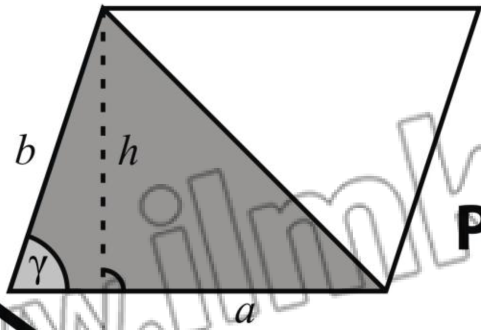


UNIT 11

PARALLELOGRAMS AND TRIANGLES

Parallelogram

(U.B)

A quadrilateral having opposite sides parallel is a parallelogram.

In a parallelograms

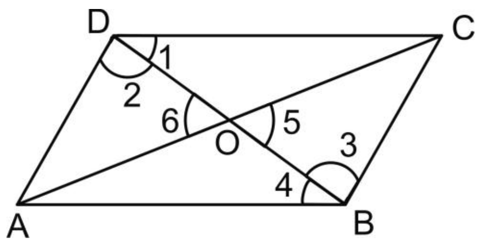
(i) Opposite sides are congruent

(ii) Opposite angles are congruent

(iii) The diagonals bisect each other

For example

In the given figure ABCD is a parallelogram.



Medians

A line segment joining a vertex of a triangle to the mid-point of the opposite side is called median of the triangle.

For example

In the given figure, AP is a median of triangle ABC.

Trisection

The process to divide a line segment into three equal parts is called trisection.

Theorem 11.1.1

(A.B)

In a parallelograms

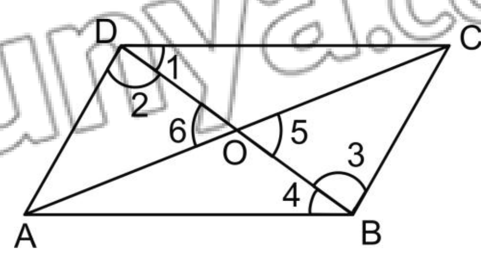
(iv) Opposite sides are congruent

(v) Opposite angles are congruent

(vi) The diagonals bisect each other

Given

In a quadrilateral $ABCD$, $\overline{AB} \parallel \overline{DC}, \overline{BC} \parallel \overline{AD}$ and the diagonals $\overline{AC}, \overline{BD}$ meet each other at point O.



To Prove

(i) $\overline{AB} \cong \overline{DC}, \overline{AD} \cong \overline{BC}$

(ii) $\angle ADC \cong \angle ABC, \ \angle BAD \cong \angle BCD$

(iii) $\overline{OA} \cong \overline{OC}, \ \overline{OB} \cong \overline{OD}$

Construction

In the figure as shown, we label the angles as $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$, $\angle 5$ and $\angle 6$.

Proof

Statements	Reasons
(i) In $\Delta ABD \leftrightarrow \Delta CDB$	
$\angle 4 \cong \angle 1$	Alternate angles
$\overline{BD} \cong \overline{BD}$	Common
$\angle 2 \cong \angle 3$	Alternate angles
$\therefore \Delta ABD \cong \Delta CDB$	$A.S.A \cong A.S.A$
$So, \overline{AB}, \overline{DC}, \overline{AD} \cong \overline{BC}$	(Corresponding sides of congruent triangles)
and $\angle A \cong \angle C$	(Corresponding angles of congruent triangles)
(ii) Since	
and $\angle 1 \cong \angle 4$................(a)	Proved
$\angle 2 \cong \angle 3$................(b)	Proved
$\therefore m\angle 1 + m\angle 2 = m\angle 4 + m\angle 3$	From (a) and (b)
or m$\angle$ADC = m$\angle$ABC	
or $\angle ADC \cong \angle ABC$	
and $\angle BAD \cong m\angle BCD$	Proved in (i)
(iii) In ΔBOC $\leftrightarrow \Delta$DOA	
$\overline{BC} \cong \overline{AD}$	Proved in (i)
$\angle 5 \cong \angle 6$	Vertical angles
$\angle 3 \cong \angle 2$	Proved
$\because \Delta BOC \cong \Delta DOA$	(A.A.S $\cong$ A.A.S)
Hence $\overline{OC} \cong \overline{OA}, \overline{OB} \cong \overline{OD}$	(Corresponding sides of congruent triangles)

Corollary **(U.B + A.B)**

Each diagonal of a parallelogram bisects it into two congruent triangles.

Example

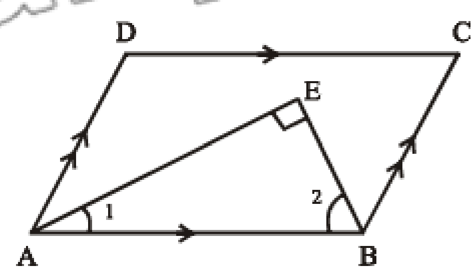
The bisectors of two angles on the same side of a parallelogram cut each other at right angles.

Given

A parallelogram ABCD, in which

$\overline{AB} \parallel \overline{DC}, \overline{AD} \parallel \overline{BC}$

The bisectors of $\angle A$ and $\angle B$ cut each other at E.



To Prove

m$\angle$E=90°

Construction:

Name the angles $\angle 1$ and $\angle 2$ as shown in the figure.

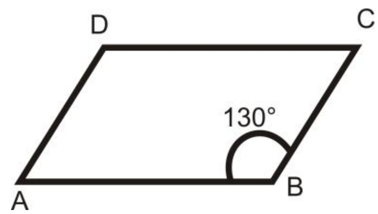
Proof

Statements	Reasons
$m\angle 1 + m\angle 2$ $= \frac{1}{2}(m\angle BAD + m\angle ABC)$ $= \frac{1}{2}(180°) = 90°$ Hence in $\Delta ABE, m\angle E = 90°$	$\begin{cases} m\angle 1 = \frac{1}{2} m\angle BAD \\ m\angle 2 = \frac{1}{2} m\angle ABC \end{cases}$ $m\angle 1 + m\angle 2 = 90°$ (proved)

Exercise 11.1

Q.1 One angle of a parallelogram in 130°. Find the measures of its remaining angles.
(GRW 2017, SWL 2015, 17, MTN 2014, BWP 2017, SGD 2016, D.G.K 2016) (K.B + U.B)

In parallelogram



m∠B = 130°

∠D = ∠B ∵ (Opposite angles of a parallelogram)

So m∠D = m∠B = 130°

We know that

∠A+∠B =180 ∵ (sum of int. ∠s on same side of a parallelogram is 180°)

∠A+130 =180

∠A = 180–130

∠A = 50°

If ∠D = ∠B ∵ opposite angles are equal.

Then

∠C = ∠A

∠C = 50°

Q.2 One exterior angle formed on producing one side of a parallelogram is 40°. Find the measures of its interior angles. (K.B + U.B)

Solution:

ABCD is a parallelogram. $\overline{BA}$ is produced towards *A*.

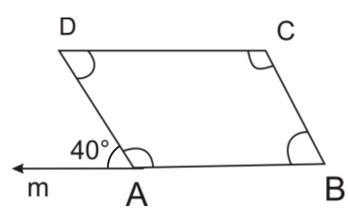
m∠DAM = 40°

m∠DAB = ?

m∠D = ?

m∠B = ?

m∠C = ?

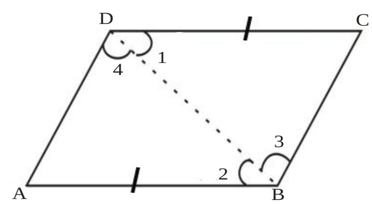


$m\angle DAM + m\angle DAB = 180°$ $\because$ supplementary angles
$40° + m\angle DAB = 180°$
$m\angle DAB = 180° - 40°$
$m\angle DAB = 140°$
$m\angle DAB + m\angle B = 180°$ $\because$ angles on same sides of a parallelogram
$140° + m\angle B = 180°$
$m\angle B = 180° - 140°$
$m\angle B = 40°$
$m\angle D = m\angle B = 40°$ $\because$ opposite angles are equal
$m\angle D = 40°$
$m\angle C = m\angle DAB$ $\because$ opposite angles are equal
$m\angle C = 140°$

Theorem 11.1.2 **(K.B + A.B)**

Statement:

If two opposite sides of quadrilateral are congruent and parallel, it is a parallelogram.



Given

In a quadrilateral *ABCD*,

$\overline{AB} \cong \overline{DC}$ and $\overline{AB} \parallel \overline{DC}$

To prove

ABCD is a parallelogram

Construction

Join the point B to D and in the figure name the angles as indicated $\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$.

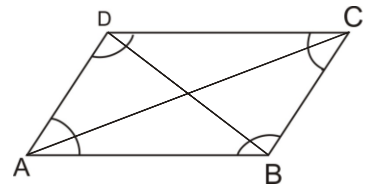
Proof

Statements	Reasons
In $\Delta ABD \leftrightarrow \Delta CDB$	
$\overline{AB} \cong \overline{DC}$	Given
$\angle 2 \cong \angle 1$	Alternate angles
$\overline{BD} \cong \overline{BD}$	Common
$\therefore \Delta ABD \cong \Delta CDB$	SAS postulate
Now $\angle 4 \cong \angle 3$............(i)	(Corresponding angles of congruent triangles)
$\therefore \overline{AD} \parallel \overline{BC}$...............(ii)	from (i)
and $\overline{AD} = \overline{BC}$...........(iii)	corresponding of sides of congruent triangles
Also $\overline{AB} \parallel \overline{DC}$...........(iv)	Given
Hence ABCD is a parallelogram	From (ii)-(iv)

Exercise 11.2

Q.1 Prove that a quadrilateral is a parallelogram if its **(K.B + U.B)**
(a) Opposite angles are congruent
(b) Diagonals bisects each other

(a) Given



In quadrilateral ABCD
$m\angle A = m\angle C, m\angle B = m\angle D$

To Prove
ABCD is a parallelogram

Statements	Reasons
$m\angle A = m\angle C \ldots$(i)	Given
$m\angle B = m\angle D \ldots$(ii)	Given
$m\angle A + m\angle B + m\angle C + m\angle D = 360°$	Sum of Angles of quadrilateral $= 360°$
$m\angle A + m\angle B = 180°$	Sum of angles of same sides $180°$
$m\angle C + m\angle D = 180°$	
$\overline{AD} \parallel \overline{BC}$	
Similarity it can be proved that $\overline{AB} \parallel \overline{DC}$	
Hence ABCD is a parallelogram	

(b) Given **(K.B + U.B)**

In quadrilateral ABCD, diagonals $\overline{AC}$ and $\overline{BD}$ bisect each other.
i.e. $\overline{OA} = \overline{OC}, \overline{OB} = \overline{OD}$

To Prove
ABCD is a parallelogram

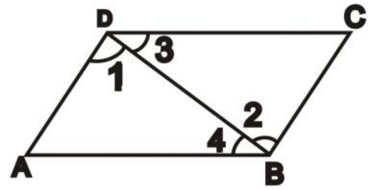
Proof

Statements	Reasons
In ΔABO ↔ ΔCDO	
$\overline{OA} \cong \overline{OC}$	Given
$\overline{OB} \cong \overline{OD}$	Given
$\angle AOB \cong \angle COD$	Vertical opposite angles
$\therefore \angle 1 \cong \angle 2$	Corresponding angles of congruent triangles
$\Delta ABO \cong \angle CDO$	$S.A.S \cong S.A.S$
Hence, $\overline{AB} \parallel \overline{CD} \ldots$(i)	$\angle 1 \cong \angle 2$
By taking BOC and is ΔAOD it can be prove that	
$\overline{AD} \parallel \overline{BC}$...(ii)	From (i) and (ii)
Hence ABCD is a parallelogram	

Q.2 Prove that a quadrilateral is a parallelogram if its opposite sides are congruent
(K.B + U.B)

Given

In quadrilateral *ABCD*



(i) $\overline{AB} \cong \overline{DC}$

(ii) $\overline{AD} \cong \overline{BC}$

To prove

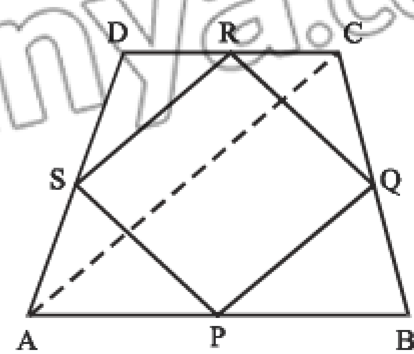
ABCD is a parallelogram i.e. $\overline{AD} \| \overline{BC}$

Prove

Statements	Reasons
$\Delta CDB \leftrightarrow \Delta ABD$	
$\overline{AB} \cong \overline{DC}$	Given
$\overline{AD} \cong \overline{BC}$	Given
$\overline{BD} \cong \overline{BD}$	Common
$\Delta ABD \cong \Delta CDB$	$S.S.S \cong S.S.S$
Thus, $\angle 1 \cong \angle 2$	Corresponding angles of congruent triangles
$\angle 4 \cong \angle 3$	Corresponding angles of congruent triangles
(i) $\overline{AD}$ P $\overline{BC}$	Alternate angles are congruent
$\overline{AB} \| \overline{DC}$	Alternate angles are congruent
$\therefore$ ABCD is a parallelogram	

Example **(A.B + U.B)**

The line segments, joining the mid-points of the sides of a quadrilateral, taken in order, form a parallelogram.



Given

A quadrilateral *ABCD*, in which *P* is the mid-point of $\overline{AB}$ *Q* is the mid-point of $\overline{BC}$, *R* is the mid-point of $\overline{CD}$, *S* is the mid-point of $\overline{DA}$

P is joined to *Q*, *Q* is joined to *R*.

R is joined to *S* and *S* is joined to *P*.

To prove

PQRS is a parallelogram.

Construction

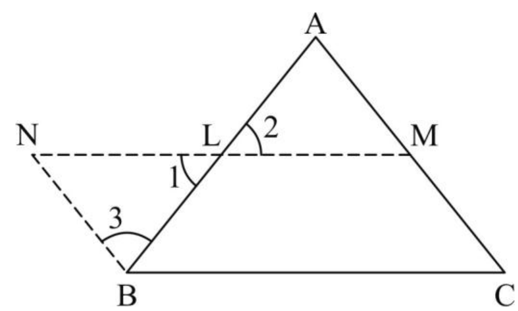
Join *A* to *C*.

Proof

Statements	Reasons
In ΔDAC,	
$\overline{SR} \parallel \overline{AC}$	S is the midpoint of $\overline{DA}$
$m\overline{SR} = \frac{1}{2} m\overline{AC}$	R is the midpoint of $\overline{CD}$
In ΔBAC,	
$\overline{PQ} \parallel \overline{AC}$	P is the midpoint of $\overline{AB}$
$m\overline{PQ} = \frac{1}{2} m\overline{AC}$	Q is the midpoint of $\overline{BC}$
$\overline{SR} \parallel \overline{PQ}$	Each $\parallel \overline{AC}$
$m\overline{SR} = m\overline{PQ}$	Each$= \frac{1}{2}, m\overline{AC}$
Thus $PQRS$ is a parallelogram	$\overline{SR} \parallel \overline{PQ}, m\overline{SR} = m\overline{PQ}$ (proved)

Theorem 11.1.3 **(A.B + U.B)**

The line segment, joining the midpoints of two sides of a triangle, is parallel to the third side and is equal to one half of its length.



Given

In ΔABC, the mid-points of $\overline{AB}$ and $\overline{AC}$ are L and M respectively

To prove

$\overline{LM} \parallel \overline{BC}$ and $m\overline{LM} = \frac{1}{2} m\overline{BC}$

Construction

Join M to L and produce $\overline{ML}$ to N such that $\overline{ML} \cong \overline{LN}$

Join N to B and in the figure, name the angles $\angle 1$, $\angle 2$ and $\angle 3$ as shown.

Proof

Statements	Reasons
In $\Delta BLN \leftrightarrow \Delta ALM$	
$\overline{BL} \cong \overline{AL}$	Given
$\angle 1 \cong \angle 2$	Vertical angles
$\overline{NL} \cong \overline{ML}$	Construction
$\therefore \Delta BLN \cong \Delta ALM$	S.A.S postulate

$\therefore \angle A \cong \angle 3$...(i)	(Corresponding angles of congruent triangles)
And $\overline{NB} \cong \overline{AM}$...(ii)	(Corresponding sides of congruent triangles)
But $\overline{NB} \parallel \overline{AM}$	from (i), alternative <s
Thus	
$\overline{NB} \parallel \overline{MC}$ ………………(iii)	(M is a point of $\overline{AC}$)
$\overline{MC} \cong AM$ …………….(iv)	Given
$\overline{NB} \cong \overline{MC}$ …………….(v)	{from (ii) and (iv)}
$BC\,MN$ is a parallelogram	From (iii) and (v)
$\therefore \overline{BC} \parallel \overline{LM}$ or $\overline{BC} \parallel \overline{NL}$	(Opposite sides of a parallelogram BCMN)
$\overline{BC} \cong \overline{NM}$ …………….(vi)	(Opposite sides of a parallelogram)
$m\overline{LM} = \frac{1}{2} m\overline{NM}$ ……………(vii)	Construction.
Hence, $m\overline{LM} = \frac{1}{2} m\overline{BC}$	{from (vi) and (vii)}

Important note

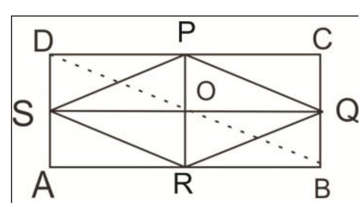
That instead of producing $\overline{ML}$ to N, we can take N on $\overline{LM}$ produced.

Exercise 11.3

Q.1 Prove that the line segments joining the midpoint of the opposite side of a quadrilateral bisect each other. (K.B + U.B)

Given

ABCD is quadrilaterals point *QRSP* are the mid point of the sides $\overline{RP}$ and $\overline{SQ}$ are joined they meet at *O*.



To Prove

$\overline{OP} \cong \overline{OR}$ $\overline{OQ} \cong \overline{OS}$

Construction

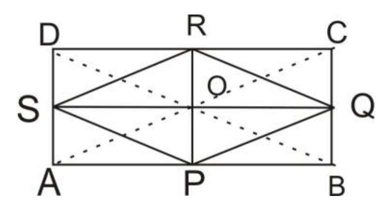
Join *P*,*Q*,*R* and S in order. Also join *C* to *A*.

Proof

Statements	Reasons
$SP \parallel AC$ … (i)	In $\Delta ADC, S, P$ are mid points of AD, DC
$m\overline{SP} = \frac{1}{2} m\overline{AC}$...(ii)	
$\overline{AC} \parallel \overline{RQ}$...(iii)	In ΔABC, Q,R are midpoints of $\overline{BC}$, $\overline{AB}$
$m\overline{RQ} = \frac{1}{2}\overline{AC}$...(iv)	
$m\overline{SP} \parallel \overline{RQ}$...(v)	
and $\overline{RQ} = \overline{SP}$...(vi)	From (ii) and (iv)
Now $\overline{RP}$ *and* $\overline{QS}$ diagonals of parallelogram PQRS intersect at O.	
$\therefore\ \overline{OP} \cong \overline{OR}$ $\overline{OS} \cong \overline{OQ}$	Diagonals of a parallelogram bisects each other.

Q.2 Prove that the line segments joining the midpoint of the opposite sides of a rectangle are the right bisectors of each other. (K.B + U.B)

[Hint: Diagonals of a rectangle are congruent]



Given

(i) $ABCD$ is a rectangle

(ii) $P,Q.R.S$ are the midpoints of $\overline{AB}$, $\overline{CD}$ and $\overline{DA}$

(iii) $\overline{SQ}$ and $\overline{RP}$ cut each other at point O.

To Prove

$\overline{OS} \cong \overline{OQ}$

$\overline{OP} \cong \overline{OR}$

$\overline{PR} \perp \overline{QS}$

Construction

Join P to Q and Q to R and R to S and S to P

Join A to C and B to D

Proof

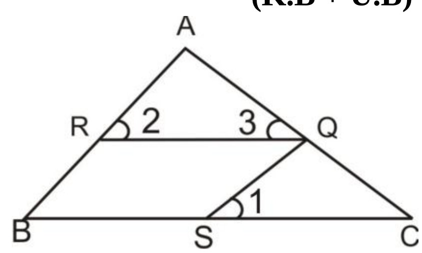
Statements	Reasons
Midpoint of $\overline{BC}$ is Q	Given
Midpoint of $\overline{AB}$ is P	Given
$\therefore \overline{AC} \parallel \overline{PQ}$.................(i)	
$\frac{1}{2}\overline{AC} = \overline{PQ}$.............(ii)	

In ΔADC	
$\overline{AC} \parallel \overline{SR}$................(iii)	
$\frac{1}{2}\overline{AC} = \overline{SR}$............(iv)	
$\overline{PQ} = \overline{SR}$	From equation (i) and (ii) each are parallel to
$\overline{SP} = \overline{RQ}$	$\overline{AC}$ each are half of $\overline{DB}$
By joined B to D we can prove	
$\overline{RQ} \parallel \overline{SP}$	
$m\overline{SR} \parallel m\overline{PQ}$	Each of them = $\frac{1}{2}\overline{AC}$
$m\overline{AC} \parallel m\overline{BD}$	
PQRS is a parallelogram all it sides are equal	
$\overline{OP} \cong \overline{OR}$	
$\overline{OS} \cong \overline{OQ}$	
$\Delta OQR \leftrightarrow \Delta OQP$	
$\overline{OR} \cong \overline{OP}$	Proved
$\overline{OQ} \cong \overline{OQ}$	Common
$\overline{RQ} \cong \overline{PQ}$	Adjacent
$\therefore \Delta OQR \cong \Delta OQP$	
$\angle ROQ \cong \angle POQ$...............(vii)	
$\angle ROQ + \angle POQ = 180$........(viii)	Supplementary angle
$\angle ROQ = \angle POQ = 90°$	From (vii) and (viii)
Thus $\overline{PR} \perp \overline{QS}$	

Q.3 Prove that line segment passing the midpoint of one side and parallel to other side of a triangle also bisects the third side. (K.B + U.B)

Given

In $\Delta ABC, R$ is the midpoint of $\overline{AB}, \overline{RQ} \parallel \overline{BC}$



To prove

$\overline{AQ} = \overline{QC}$

Construction

Draw $\overline{QS} \parallel \overline{AB}$

Proof

Statements	Reasons
$\overline{RQ} \parallel \overline{BS}$	Given
$\overline{QS} \parallel \overline{BR}$	Construction
$RBSQ$ is a parallelogram	
$\overline{QS} \cong \overline{BR}$...(i)	Opposite side
$\overline{AR} \cong \overline{RB}$...(ii)	Given
$\overline{QS} \cong \overline{AR}$...(iii)	From (i) and (ii)
$\angle 1 \cong \angle B$ and	
$\angle 1 \cong \angle 2$...(iv)	
$\Delta ARQ \leftrightarrow \Delta QSC$	
$\angle 2 \cong \angle 1$ $\angle 3 \cong \angle C$	From (iv)
$\overline{AR} \cong SQ$	From (iii)
Hence, $\Delta ARQ \cong \Delta QSC$	$A.A.S \cong A.A.S$
$\overline{AQ} \cong \overline{QC}$	Corresponding sides

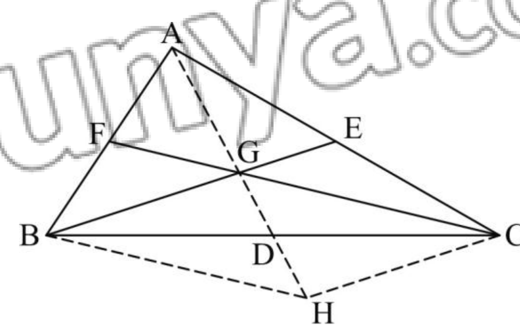
Theorem: 11.1.4 **(A.B + U.B)**

Statement: **The median of triangle are concurrent and their point of concurrency is the point of trisection of each median.**

Given ΔABC

To prove

The medians of the ΔABC are concurrent and the point of concurrency is the point of trisection of each median



Construction

Draw two medians $\overline{BE}$ and $\overline{CF}$ of the ΔABC which intersect each other at point G. Join A to G and produce it to the point H such that AG □ $\overline{GH}$ Join H to the points B and C

$\overline{AH}$ Intersects $\overline{BC}$ at the point D.

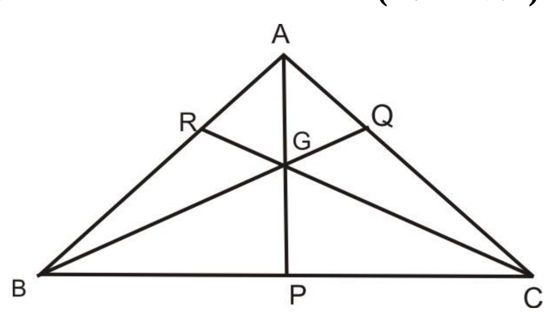
Proof

Statements	Reasons
In Δ ACH,	
$\overline{GE} \parallel \overline{HC}$	G and E are mid-points of sides $\overline{AH}$ and $\overline{AC}$ respectively
Or $\overline{BE} \parallel \overline{HC}$ ....(i)	G is point of $\overline{BE}$
Similarly $\overline{CF} \parallel \overline{HB}$...(ii)	
$\therefore$ BHCG is a parallelogram	From (i) and (ii)
And	
$m\overline{GD} = \frac{1}{2} m\overline{GH}$...(iii)	Diagonals $\overline{BC}$ and $\overline{GH}$ of a parallelogram $BHCG$ intersect each other at point D.
$\overline{BD} \cong \overline{CD}$	
$\overline{AD}$ is a median of ΔABC	
medians $\overline{AD}, \overline{BE}$ and $\overline{CF}$ pass through the point G	G is the intersecting point of $\overline{BE}$, $\overline{CF}$ and $\overline{AD}$ pass through it.
Now $\overline{GH} \cong \overline{AG}$...(iv)	Construction
$m\overline{GD} = \frac{1}{2} m\overline{AG}$	From (iii) and (iv)
and G is the point of trisection of $\overline{AD}$...(v)	
similarly it can be proved that G is also the point of trisection of $\overline{CF}$ and $\overline{BE}$	

Exercise 11.4

Q.1 The distance of the point of concurrency of the medians of a triangle from its vertices are respectively 1.2 cm. 1.4 cm and 1.6 cm. Find the length of its medians.
(K.B + U.B)

Let ΔABC with the point of concurrency of medians at G



$\overline{AG} = 1.2cm$, $\overline{BG} = 1.4cm$ and $\overline{CG} = 1.6cm$

$$\overline{AP} = \frac{3}{2}\overline{AG} = \frac{3}{2} \times 1.2 = 1.8cm$$

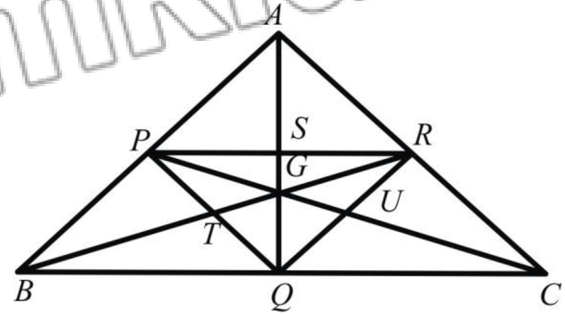
$$\overline{BQ} = \frac{3}{2}\overline{BG} = \frac{3}{2} \times 1.4 = 2.1cm$$

$$\overline{CR} = \frac{3}{2}\overline{CG} = \frac{3}{2} \times 1.6 = 2.4cm$$

Q.2 Prove that the point of concurrency of the medians of a triangle and the triangle which is made by joining the midpoint of its sides to the same. (K.B + U.B)

Given

In ΔABC, AQ, CP, BR are medians which meet at G.



To prove

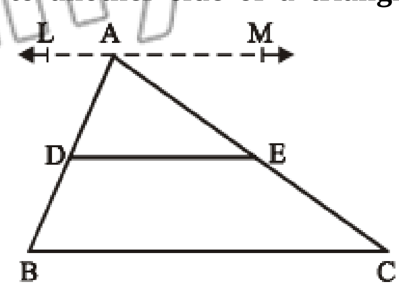
G is the point of concurrency of the medians of ΔABC and ΔPQR

Proof

Statements	Reasons
$\overline{PR} \parallel \overline{BC}$	P, R are midpoint of $\overline{AB}, \overline{AC}$
$\overline{BQ} \parallel \overline{PR}$	
Similarly $\overline{QR} \parallel \overline{BP}$	
$\therefore PBQR$ is a parallelogram it diagonals $\overline{BR}$ and $\overline{PQ}$ bisector each other at T	
Similarly U is the midpoint of QR and S is midpoint of $\overline{PR}$	
$\therefore \overline{PU}, \overline{QS}, \overline{RT}$ are medians of ΔPQR	
(i) $\overline{AQ}$ and $\overline{SQ}$ pass through G	
(ii) $\overline{BR}$ and $\overline{TR}$ pass through G	
(iii) $\overline{UP}$ and $\overline{CP}$ pass through G	
Hence G is point of concurrency of medians of ΔPQR and ΔABC	

Example

A line, through the mid-point of one side, parallel to another side of a triangle, bisects the third side. (K.B + A.B)



Given

In $\Delta ABC, D$ is the mid-point of $\overline{AB}$.

$\overline{DE} \parallel \overline{BC}$ which cuts $\overline{AC}$ at E.

To prove

$\overline{AE} \cong \overline{BC}$

Construction

Through A, draw $\overleftrightarrow{LM} \parallel \overline{BC}$.

Proof

Statements	Reasons
Intercepts cut by $\overleftrightarrow{LM}, \overline{DE}, \overline{BC}$ on $\overline{AC}$ are congruent. i.e., $\overline{AE} \cong \overline{EC}$.	Intercepts cut by parallels $\overleftrightarrow{LM}, \overline{DE}$. $\overline{BC}$ on $\overline{AB}$ are congruent (given)

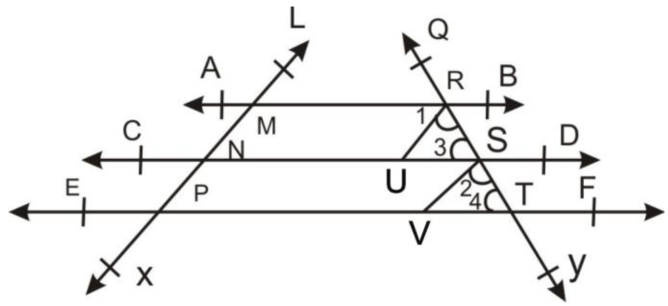
Theorem 11.1.5 **(K.B + A.B)**

Statement: In three or more parallel lines make congruent segments on a transversal they also intercept congruent segments on any other line that cuts them.

Given

$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$

The transversal $\overleftrightarrow{LX}$ intersects $\overleftrightarrow{AB}, \overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$ at the points *M*, *N* and *P* respectively, such that $\overline{MN} \cong \overline{NP}$. The transversal $\overleftrightarrow{QY}$ intersects them at point *R*, *S* and *T* respectively.



Prove

$\overline{RS} \cong \overline{ST}$

Construction

From R, draw $\overline{RU} \parallel \overline{LX}$, which meets $\overline{CD}$ at U, from S draw $\overline{SV} \parallel \overline{LX}$ which meets $\overline{EF}$ at V. as shown in the figure let the angles be labeled as $\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$.

Proof

Statements	Reasons
MNUR is parallelogram	$\overline{RU} \parallel \overline{LX}$ (Construction) $\overline{AB} \parallel \overline{CD}$ (given)
$\therefore \overline{MN} \cong \overline{RU}$ (*i*)	(Opposite side of parallelogram).
Similarly.	
$\overline{NP} \cong \overline{SV}$ (*ii*)	
But $\overline{MN} \cong \overline{NP}$ (*iii*)	Given
$\therefore \overline{RU} \cong \overline{SV}$	{from (i) (ii) and (iii)} each is $\parallel$ $\overline{LX}$ (construction)
Also $\overline{RU} \parallel \overline{SV}$	Each is $\parallel \overline{LX}$ (construction)
$\therefore \angle 1 \cong \angle 2$	Corresponding angles
and $\angle 3 \cong \angle 4$	Corresponding angles
In $\Delta RUS \leftrightarrow \Delta SVT$	
$\overline{RU} \cong \overline{SV}$	Proved
$\angle 1 \cong \angle 2$	Proved
$\angle 3 \cong \angle 4$	Proved
$\therefore \Delta RUS \cong \Delta SVT$	$S.A.A \cong S.A.A$
Hence $\overline{RS} \cong \overline{ST}$	(Corresponding sides of congruent triangles)

Important Point **(K.B + U.B + A.B)**

This theorem helps us in dividing line segment into parts of equal lengths. It is also used in the division of a line segment into proportional parts.

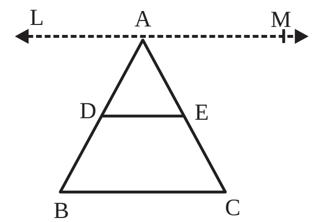
Corollaries **(K.B + A.B)**

A line, through the mid-point of one side, parallel to another side of a triangle, bisects the third side.

Given

In ΔABC, D is the mid-point of $\overline{AB}$.

$\overline{DE} \parallel \overline{BC}$ which cuts $\overline{AC}$ at E.



To Prove

$\overline{AE} \cong \overline{EC}$

Construction

Through A, draw $\overleftrightarrow{LM} \parallel \overline{BC}$.

Proof

Statements	Reasons
Intercepts cut by $\overleftrightarrow{LM}$, $\overline{DE}$, $\overline{BC}$ on $\overline{AC}$ are congruent. i.e., $\overline{AE} \cong \overline{EC}$	Intercepts cut by parallels $\overleftrightarrow{LM}$, $\overline{DE}$, $\overline{BC}$ on $\overline{AB}$ are congruent (given)

(ii) The parallel line form the mid-point of one non-parallel side of a trapezium to the parallel sides bisects the other non-parallel side.

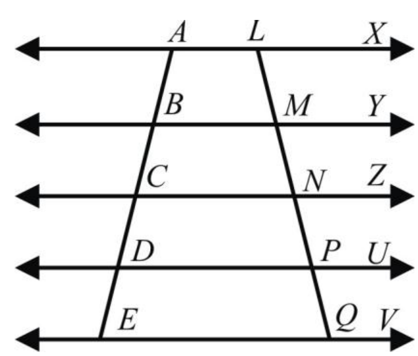
(iii) If one side of a triangle is divided into congruent segments, the line drawn from the point of division parallel to the other side will make congruent segments on third side.

Exercise 11.5

Q.1 In the given figure **(K.B + U.B)**

$\overleftrightarrow{AX} \parallel \overleftrightarrow{BY} \parallel \overleftrightarrow{CZ} \parallel \overleftrightarrow{DU} \parallel \overleftrightarrow{EV}$ and $\overline{AB} = \overline{BC} = \overline{CD} = \overline{DE}$

If $\overline{MN} = 1cm$ then find the length of $\overline{LN}$ and $\overline{LQ}$



$\therefore \overline{PQ} \cong \overline{NP} \cong \overline{MN} \cong \overline{LM}$

$MN = 1cm$

Given

$\overline{AP} \cong \overline{PQ} \cong \overline{QR} \cong \overline{RS} \cong \overline{ST}$

Therefore, $\overline{LN} = \overline{LM} + \overline{MN}$

$\overline{LM} = \overline{MN}$

so, $\overline{LN} = \overline{MN} + \overline{MN}$

$\overline{LN} = 1+1$

$\overline{LN} = 2cm$

$\overline{LM} = \overline{NP} = \overline{PQ} = \overline{MN} = 1cm$

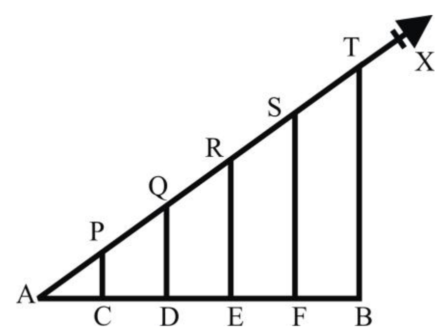
$So, \overline{LM} = 1cm, \overline{NP} = 1cm, \overline{PQ} = 1cm$

$LQ = \overline{LM} + \overline{MN} + \overline{NP} + \overline{PQ}$

$LQ = 1+1+1+1$

$LQ = 4cm$

Q.2 Take a line segment of length $5.5cm$ **and divide it into five congruent parts [Hint: draw an acute angle** $\angle BAX$**. On** $\overline{AX}$ *take* $\overline{AP} \cong \overline{PQ} \cong \overline{RS} \cong \overline{ST}$ **join *T* to *B* draw lines parallel to** $\overline{TB}$ **from the point *P*,*Q* *R* and *S*.** **(K.B + U.B)**



Proof

Construction:

(i) Take a line segment $AB = 5.5cm$

(ii) Draw any acute angle $\angle BAX$

(iii) Draw arcs on $\overrightarrow{AX}$ which are

$\overline{AP} \cong \overline{PQ} \cong \overline{QR} \cong \overline{RS} \cong \overline{ST}$

(iv) Join T to B

(v) Draw lines $\overline{SF}, \overline{RE}, \overline{QD}, \& \overline{PC}$ Parallel to $\overline{TB}$.

Result:

Line segment $\overline{AB}$ is divided into congruent line segments $\overline{AC} \cong \overline{CD} \cong \overline{DE} \cong \overline{EF} \cong \overline{FB}$.

Review Exercise 11

Q.1 Fill in the blanks **(K.B + U.B)**

(i) In a parallelogram opposite side are ……

Ans: Congruent

(ii) In a parallelogram opposite angles are …….

Ans: Congruent

(iii) Diagonals of a parallelogram …….. each other at a point.

Ans: Bisects

(iv) Medians of a triangle are ……….

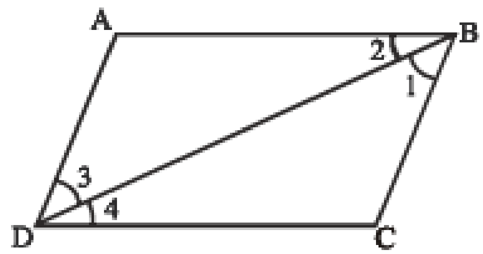
Ans: Concurrent

(v) Diagonals of a parallelogram divide the parallelogram into two ……. Triangles

Ans: Congruent

Q.2 In parallelogram *ABCD*

(i) $m\overline{AB} = ...$ **(K.B + A.B)**



Ans: $m\overline{AB} = m\overline{DC}$

(ii) $m\overline{BC}............$

Ans: $m\overline{BC} = m\overline{AD}$

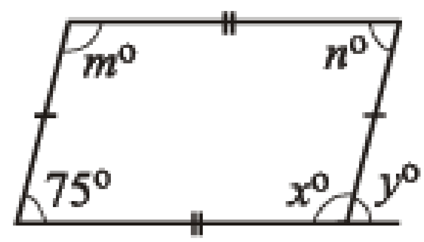
(iii) $m\angle 1 \cong ....$

Ans: $m\angle 1 = m\angle 3$

(iv) $m\angle 2 = ...$

Ans: $m\angle 2 = m\angle 4$

Q.3 Find the unknown in the figure given **(GRW 2014, MTN 2016, D.G.K 16) (K.B + A.B)**



Solution:

$n° = 75$

$y° = n°$

Substituting the value of $n°$

$y° = 75°$

$x° + 75 = 180$ Adjacent supplementary angles

$x° = 180 – 75$

$x° = 105°$

$m° = x°$

$m° = 105°$

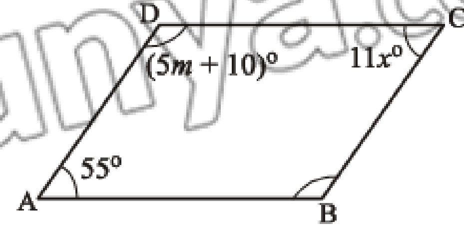
Q.4 If the given figure ABCD is a parallelogram then find *x*, *m* **(K.B + A.B)**
(GRW 2016, MTN 2017, D.G.K 2017)

Solution:

$11x° = 55°$

$x° = \frac{\cancel{55}^{5}}{\cancel{11}}$

$x° = 5°$



$\angle A + \angle B = 180°$

$\angle B = 180° - \angle A$

$\angle B = 180° - 55 = 130°$

$\angle B = 130°$

$\angle D + \angle C = 180°$

$5m + 10° + 55° = 180°$

$5m + 65° = 180°$

$5m = 180° - 65°$

$5m = 115°$

$m = \frac{115°}{5°}$

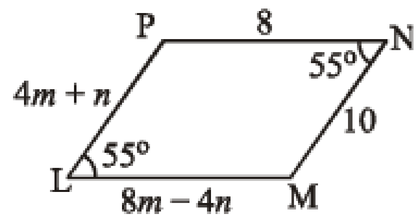
$m = 23°$

Q.5 The given figure ∠MNP is a parallelogram finds the value of m, n
(BWP 2015) (K.B + A.B)

$4m + n = 10$………. (i) (In parallelogram opposite sides are congruent)

$8m – 4n = 8$ … (ii)

Multiply 4 with equation



$4(4m + n) = 4 \times 10$

$16m + 4n = 40$ … (iii)

Adding equation (ii) and (iv)

$8m - \cancel{4n} = 8$

$16m + \cancel{4n} = 40$

$24m = 48$

$m = \frac{48}{24}$

$m = 2$

Putting the value of m in equation (i) $4(2) + n = 10$

$8 + n = 10$

$n = 10 – 8$

$n = 2$

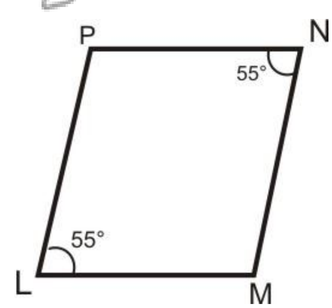
Q.6 In the equation 5, sum of the opposite angles of the parallelogram in 110°
(K.B + A.B)

$\angle L + \angle M = 180$

$55° + \angle M = 180°$

$\angle M = 180° - 55°$

$\angle M = 125°$



$\angle P = \angle M$ opposite angles are congruent in parallelogram

$\angle P = 125°$