

A line is called a right bisector of a line segment if it is perpendicular to the line segment and passes through its mid-point.



In the above ℓ is called right bisector of line segment AB, if $\ell \perp \overline{AB}$ and $m\overline{AC} = m\overline{CB}$.

Bisector of angle

 $(\mathbf{K}.\mathbf{B} + \mathbf{A}.\mathbf{B})$

A ray BP is called the bisector of $m \angle ABC$ if P is a point in the interior of the angle and $m \angle ABP = m \angle PBC$

Or

A ray is called bisector of an angle if it divides the angle into two equal parts.



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	Join <i>P</i> to the points <i>A</i> and <i>B</i>	JULUL CIOR	
Proof		G[[U]]UUUU	1
In Δ \overline{AC}	$ACP \leftrightarrow \Delta BCP$ $\cong \overline{BC}$	Given	
	$CP \cong \angle BCP$	Given $\overline{PC} \perp \overline{AB}$, so that each \angle at $C = 90^{\circ}$	
\overline{PC}	$\cong \overline{PC}$	Common	
$\therefore \Delta$	$ACP \cong \Delta BCP$	S.A.S Postulate	
Hen	ce $\overline{PA} \cong \overline{PB}$	(Corresponding sides of congruent triangles)	
	$\{Converse of Theorem 12.1.1\}$ Any point equidistant from the en segment is on the right bisector of it.Given \overline{AB} is a line segment. Point P is such th To proveThe point P is the on the right bisector of ConstructionJoin P to C, the midpoint of \overline{AB} ProofStatements	d points of a line at $\overline{PA} \cong \overline{PB}$ of \overline{AB} A C B	
$ \begin{array}{c} In \ \square \\ \overline{PA} \\ \overline{PC} \\ \overline{AC} \\ \square \\ \square \\ \square \\ \square \\ \square \\ In \ \square \\ \overline{PC} \\ \overline{AC} \\ \square $	Statements $\Delta ACP \leftrightarrow \Delta BCP$ $\cong \overline{PB}$ $\Xi = \overline{PC}$ $\Xi = \overline{BC}$ $ACP \cong \Delta BCP$ $CP \cong \angle BCP$ $CP \cong \angle BCP$ $CP \cong \angle BCP$ $CP \equiv \angle BCP$ $CP \equiv \angle BCP = 180^{\circ}$ (i)	ReasonsGivenCommonConstructionS.S.S \cong S.S.SCorresponding angles of congruent trianglesSupplementary anglesFrom (i) and (ii) $m \angle ACP = 90^{\circ}$ (Proved)Constructionfrom (iii) and (iv)	DWU

\mathbf{U}_{nit} – 12



Unit – 12



Proof

Statements	Reasons
$\overline{OQ} \cong \overline{OR}$ (i)	O is on the right bisector of \overline{QR}
$\overline{OP} \cong \overline{OQ}$ (ii)	O is on the right bisector of \overline{PQ}
$\overline{OP} \cong \overline{OQ} \cong \overline{OR} _ (iii)$	From (i) and (ii)
$\therefore O$ is equidistant from P,Q and R	From (iii)
Hence, point O is equidistant from three	
villages.	
Theorem 12.1.3 (K	$\mathbf{B} + \mathbf{A} \cdot \mathbf{B}$ A
The right bisectors of the sides of a tr	iangle are
concurrent.	
Given	T
ΔABC	B

To prove

The right bisectors of \overline{AB} , \overline{BC} and \overline{CA} are concurrent

Construction

Draw the right bisectors of \overline{AB} and \overline{BC} which meet each other at the point O. Join O to A, B and C.

Proof

N	Statements	Reasons
NN	10 -	(Each point on right bisector of a
\bigcirc	$\overline{OA} \cong \overline{OB} \longrightarrow (i)$	segment is equidistant from its end
		points)
	$\overline{OB} \cong \overline{OC} \longrightarrow (ii)$	As in (i)
	$\overline{OA} \cong \overline{OC}$	from (i) and (ii)
	\therefore Point O is on the right bisector of $\overline{CA} \rightarrow (iv)$	(O is equidistant from A and C)
	But point O is on the right bisector of \overline{AB} and of $\overline{BC} \longrightarrow (v)$	Construction
	Hence the right bisectors of the three sides of triangle are concurrent at O	{from (iv) and (v)}
	Note	(K.B)

Note

Hence $\overline{PQ} \cong \overline{PR}$

- (a) The right bisectors of the sides of an acute triangle intersect each other inside the triangle.
- (b) The right bisectors of the sides of a right triangle intersect each other on the hypotenuse.
- (c) The right bisectors of the sides of an obtuse triangle intersect each other outside the triangle.



(Corresponding sides of congruent triangles)

Unit – 12 Theorem 12.1.5 (Converse of Th



Q.2 The bisectors of $\angle A, \angle B$ and $\angle C$ of a quadrilateral ABCP meet each other at point O. Prove that the bisector of $\angle P$ will also pass through the point O. (K.B + A.B)



Given

ABCP is quadrilateral. \overline{AO} , \overline{BO} , \overline{CO} are bisectors of $\angle A$, $\angle B$ and $\angle C$ meet at point O.

To prove

PO is bisector of $\angle P$

Construction:

Join P to O.

Draw $\overline{OQ} \perp \overline{AP}$, $\overline{ON} \perp \overline{PC}$ and $\overline{OL} \perp \overline{AB}$, $\overline{OM} \perp \overline{BC}$

Proof:

Statements	Reasons
$\overline{OM} \cong \overline{ON}$ (i)	O is on the bisector of $\angle C$
$\overline{OL} \cong \overline{OM}$ (ii)	O is on the bisector of $\angle B$
$\overline{OL} \cong \overline{OQ}$ (iii)	O is on the bisector of $\angle A$
$\overline{OQ} \cong \overline{ON}$ (iv)	From (i), (ii), (iii)
Point O lines on the bisector of $\angle P$	From (iv)
$\therefore \overline{OP}$ is the bisector of angle P	COUL
altitude are concurrent. Given ΔABC $\overline{AB} \cong \overline{AC}$ due to isosceles triangle \overline{PM} is right of \overline{AB} \overline{QN} is right bisector of \overline{AC} \overline{PM} and \overline{QN} intersect each other at point O To Prove The altitude of ΔABC lies at point O Lair A to O and extend it to get \overline{BC} at D	$(\mathbf{K},\mathbf{B} + \mathbf{A},\mathbf{B})$ (\mathbf{H},\mathbf{B}) $(\mathbf{H},\mathbf{A},\mathbf{B})$ $(\mathbf{H},\mathbf{A},\mathbf{A},\mathbf{B})$ $(\mathbf{H},\mathbf{A},\mathbf{A},\mathbf{A},\mathbf{A},\mathbf{A},\mathbf{A},\mathbf{A},A$
Join A to O and extend it to cut <i>BC</i> at D.	

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	Proof	Nannal Veloge
	Statements In $\Box ABC$, $m\overline{AB} \cong m\overline{AC}$	Given
N	$\frac{1}{2}m\overline{AB} = \frac{1}{2}m\overline{AC}$	Dividing both side by 2
Mg /	$\overline{AQ} \cong \overline{AP}$ In $\Delta AQO \leftrightarrow \Delta APO$	$\overline{PM} \perp \overline{AB}, \overline{QN} \perp \overline{AC}$
	$\angle APO \cong \angle AQO$	Each 90° (Given)
	$\overline{AQ} \cong \overline{AP}$	Already Proved
	$\overline{AO} \cong \overline{AO}$	Common
	$\Delta APO \cong \Delta AQO$	$H.S \cong H.S$
	$\angle PAO \cong \angle QAO$ (i)	Corresponding angles of congruent triangles
	$\Delta BAD \leftrightarrow \Delta CAD$	
	$\overline{AB} \cong \overline{AC}$	Given
	$\overline{AD} \cong \overline{AD}$	Common
	$\angle BAD \cong \angle CAD$	Proved from (i)
	$\Delta BAD \cong \Delta CAD$	$S.A.S \cong S.A.S$
	$\angle ODB \cong \angle ODC \rightarrow (i)$	Each angle is 90° (Given)
	$m \angle ODM + m \angle ODC = 180^{\circ} \rightarrow (ii)$	Supplementary angle
	$m \angle ODM \cong m \angle ODC = 90^\circ \rightarrow (iii)$	From (i) and (ii)
	$\therefore \overline{AD} \perp \overline{BC}$	From (iii)
	Point 0 lies on altitude \overline{AD}	ALANNICIO
	Q.4 Prove that the altitudes of a triangle :	are concurrent. (K.B + A.B)
MN	Given In \triangle ABC AD,BE,CF are its altitudes	
0	1.e AD \perp BC,BE \perp AC,CF \perp AB	
	To Prove $\overline{AD} \overline{D} \overline{D} \overline{D}$	\bigvee
	AD, BE and CF are concurrent	₽ P

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Passing through A, B, C take $\overline{RQ} \ \overline{BC}, \overline{RP} \ \overline{AC} \text{ and } \overline{QP} \ \overline{AB} \text{ respectively forming a } \Delta PQR$ Proof Proof $\overline{BC} \ \overline{AQ}$ Construction $\overline{AB} \ \overline{QC}$ Construction $\overline{AB} \ \overline{QC}$ Construction $\therefore ABCQ \text{ is a } P^{\text{sm}}$ Hence $\overline{AQ} \not\approx \overline{BC}$ Similarly $\overline{AB} \not\ll \overline{QC}$ Hence point A is midpoint RQ And $\overline{AD} \perp \overline{BC}$ Given $\overline{BC} \ \overline{RQ}$ Opposite sides of parallelogram $\overline{AD} \ \overline{RQ}$ Thus $\overline{AD} \perp \text{ is right bisector of } \overline{RQ}$ and	
$\overline{RQ} \ \overline{BC}, \overline{RP} \ \overline{AC} \text{ and } \overline{QP} \ \overline{AB} \text{ respectively forming a } \Delta PQR$ Proof Reasons $\overline{BC} \ \overline{AQ}$ Construction $\overline{AB} \ \overline{QC}$ Construction $\overline{AB} \ \overline{QC}$ Construction $\overline{AB} \ \overline{QC}$ Construction $\overline{AB} \ \overline{QC}$ Given $\overline{AB} \ \oplus \overline{AB} \ \oplus \overline{AB}$	
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$\overline{AD} \ \overline{RQ}$ Thus $\overline{AD} \perp$ is right bisector of \overline{RQ} similarly \overline{BE} is a right bisector of \overline{RP} and	n ABCQ
Thus $\overline{AD} \perp$ is right bisector of \overline{RQ} similarly \overline{BE} is a right bisector of \overline{RP} and	
similarly \overline{BE} is a right bisector of \overline{RP} and	
\overline{CF} is right bisector of PQ	
$\therefore \perp^{s} \overline{AD}, \overline{BE}, \overline{CF}$ are right bisector of sides	- 50
of ΔPQR	I COU
Consument	7.00
The bisectors of the angles of a triangle are concurrent	
Given	\backslash
To Prove	\backslash
The bisector of $\angle A$, $\angle B$, and $\angle C$ are concurrent $F \not \downarrow$	$\sum E$
Draw the bisectors of $\angle B$ and $\angle C$ which intersect at point I.	- \
$\overrightarrow{\text{IE}} + \overrightarrow{\text{AB}} \cdot \overrightarrow{\text{ID}} + \overrightarrow{\text{BC}} \text{ and } \overrightarrow{\text{IE}} + \overrightarrow{\text{CA}}$	

Unit – 12



Note

 $(\mathbf{K}.\mathbf{B} + \mathbf{A}.\mathbf{B})$

In practical geometry also, by constructing angle bisectors of a triangles, we shall verify that they are concurrent.

Exercise 12.3

Q.1 Prove that the bisectors of the angles of base of an isosceles triangle intersect each other on its altitude. (K.B + A.B)



Construction

Draw $\overline{AD} \perp \overline{BC}$, also draw angle bisectors of $\angle B$ and $\angle C$ which meet each other at point O. Label the angles $\angle 1, \angle 2, \angle 3, \angle 4, \angle 5, \angle 6, \angle 7, \angle 8$ as shown in the figure.



Prove that the bisectors of two exterior and third interior angle of a triangle are concurrent (K.B + A.B)





- Which of the following are true and which are false? 0.1 $(\mathbf{K}.\mathbf{B} + \mathbf{U}.\mathbf{B})$
- **Bisection means to divide into two equal parts** (i)
- (True) Right bisection of line segment means to draw perpendicular which passes through **(ii)** the midpoint of line segment (True)
- (iii) Any point on the right bisector of a line segment is not equidistant from its end points (False)
- (iv) Any point equidistant from the end points of a line segment is on the right bisector of it (True)
- The right bisectors of the sides of a triangle are not concurrent (False) **(v)** (True)
- The bisectors of the angles of a triangle are concurrent (vi)
- Any point on the bisector of an angle is not equidistant from its arms (vii) (False)
- Any point inside an angle equidistant from its arms, is on the bisector of it (viii) (True)
- If CD is right bisector of line segment AB, then 0.2 (i) $m\overline{OA} =$ (ii) $m\overline{AQ} =$ $(\mathbf{K}.\mathbf{B} + \mathbf{U}.\mathbf{B})$

Solution:

(i)

- (i) $m\overline{OA} = m\overline{OB}$
- (ii) $m\overline{AQ} = m\overline{BQ}$

Q.3 **Define the following**

 $(\mathbf{K}.\mathbf{B} + \mathbf{U}.\mathbf{B})$

Right Bisector of a Line Segment (LHR 2017, GRW 2016, SWL 2014, 16, 17, BWP 2015, 16, SGD 2013, 15, 16, 17, RWP 2013, FSD 2013, D.G.K 2015)



A line *l* is called a right bisector of a line segment AB if *l* is perpendicular to the line segment and passes through its midpoint.

Bisector of an Angle (ii)

(GRW 2014, 16, 17, FSD 2013, 16, MTN 2013, 14, 15, SGD 2013, 15) A ray BP is called the bisector of $m \angle ABC$, if P is a point in the interior of the angle and $m \angle ABP = m \angle PBC$.

The given triangle ABC is equilateral triangle and AD is bisector of angle A, then

find, the values of unknown x° , v° and z° . $(\mathbf{K}.\mathbf{B} + \mathbf{U}.\mathbf{B})$ (MTN 2013) **Solution**

In equilateral triangle all side are equal to each and each angle of the triangle equal to 60°. i.e. $m \angle A = m \angle B = m \angle C = 60^{\circ}$ So

B

۶B



Q.5 In the given congruent triangle LMO and LNO find the unknowns *x* and m given



Uni	it – 12	Line Bisectors and Angle Bisectors
Time: Q.1	SELF 1 : 40 min Mark the Correct multiple choice questi	EST Marks: 25 (7×1=7)
1.	Each angle in an equilateral triangle is -	in measure
M	(A) 30°	(B) 45°
90	(C) 60°	(D) 50°
2.	If two adjacent angles are supplementar	y then their uncommon arms are
	(A) Collinear	(B) Concurrent
	(C) Congruent	(D) None of these
3.	A Ray has end point	
	(A) One	(B) Two
	(C) Three	(D) None of these
	(A) 30°	x° N (B) 60°
	(C) 80°	(D) 90°
5.	 (c) so Each diagonal of a parallelogram bisects (A) Four (C) Two 	 it into congruent triangles. (B) Six (D) One
6. The line segment, joining the mid-points of two sides of a triangle is parallel to the		s of two sides of a triangle is parallel to the
0	third side and is equal to of (A) Half	(B) Double
N	(C) $\frac{1}{4}$ th	(D) $\frac{1}{3}$
7.	Any point on the bisector of an angle is e	equidistant from its
	(A) Arms	(B) Vertex

7

7

 $(5 \times 2 = 10)$

Q.2 Give Short Answers to following Questions.

(i) Find the value of unknown for the given congruent triangles.

- (ii) Define congruent triangles.
- (iii) State S.A.S postulate.
- (iv) One angle of a parallelogram is 130. Find the measure of its remaining angles.
- (v) The given triangle ABC is equilateral triangle and \overline{AD} is bisector of angle A, then find the values of unknowns, x^o , y^o , y^o .

of students.