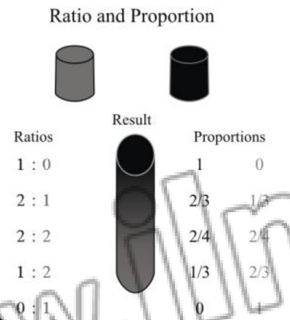


UNIT 14
RATIO AND PROPORTION

Ratio **(GRW 2013, 15, 17, SWL 2014, 16, FSD 2013, 16, D.G.K 2014, 15, 17, MTN 2014, 15)** **(K.B)**

Ratio is the comparison of two alike quantities. For any two quantities a and b it is expressed as $a:b=\frac{a}{b}, b\neq 0$, where a and b are called the elements (terms) of ratio. (Elements must be expressed in the same units.)

For example

Ratio between 100 m and 250 m is $100:250=2:5$

Proportion **(K.B)**

(GRW 2016, SWL 2017, FSD 2014, 16, 17, MTN 2013, 14, 16, 17, SGD 2016, 17)

Equality of two ratios is called proportion.
That is if $a:b=c:d$, then a,b,c and d are said to be in proportion.
Where *a* and *d* are called extremes and *b* and *c* are called means.

Note **(K.B + U.B)**

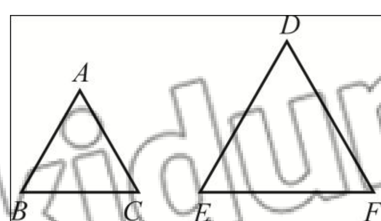
Knowledge of a ratio and proportion is necessary requirement of many occupations like food service occupation, medications in health, preparing maps for land survey and construction works, profit to east ratio etc.

Similar Triangles **(U.B)**

(LHR 2013, 15, GRW 2016, 17, SWL 2014, 15, 16, SGD 2013, MTN 2013, 14, BWP 2017, D.G.K 2015, RWP 2016, FSD 2015)

Two triangles are called similar if corresponding angles are congruent. In similar triangles corresponding sides are proportional.

In $\Delta ABC \leftrightarrow \Delta DEF$



If $\angle A \cong \angle D$ $\angle B \cong \angle E$ $\angle C \cong \angle F$ and $\frac{m\overline{AB}}{m\overline{DE}}=\frac{m\overline{BC}}{m\overline{EF}}=\frac{m\overline{CA}}{m\overline{FD}}$

then ΔABC and ΔDEF are called similar triangles, which is symbolically written as $\Delta ABC \sim \Delta DEF$.

Note **(U.B)**

Congruent triangles are also similar. But two similar triangles are not necessarily congruent, as congruence of their corresponding sides is not necessary.

Real Life Example of Similar Shapes (K.B)

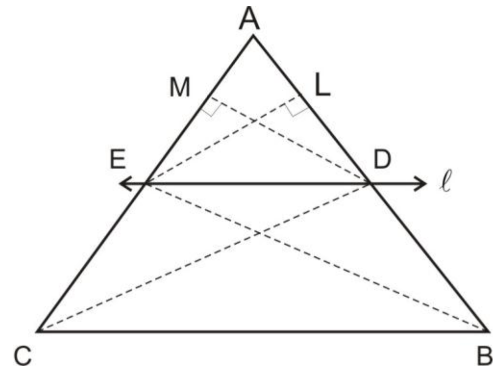
A photographer can develop prints of different sizes from the same negative. In spite of the difference in sizes, these pictures like each other. One photograph is simply an enlargement of another. They are said to be similar in shape.

Theorem 14.1.1 (K.B)

A line parallel to one side of a triangle and intersecting the other two sides divides them proportionally.

Given:

In ΔABC, the line ℓ is intersecting the sides $\overline{AC}$ and $\overline{AB}$ at points E and D respectively such that $\overline{ED} \parallel \overline{CB}$



To Prove

$m\overline{AD} : \overline{DB} = m\overline{AE} : m\overline{EC}$

Construction:

Join B to E and C to D .From D draw $\overline{DM} \perp \overline{AC}$ and from E draw $\overline{EL} \perp \overline{AB}$

Proof

Statements	Reasons
In triangles BED and AED, EL is the common perpendicular	
$\therefore$ Area of $\Delta BED = \frac{1}{2} \times m\overline{BD} \times m\overline{EL}$.......(i)	Area of a $\Delta = \frac{1}{2}$ (base)(height)
and Area of $\Delta AED = \frac{1}{2} \times m\overline{AD} \times m\overline{EL}$.......(ii)	
Thus Area of $\frac{\Delta BED}{\Delta AED} = \frac{m\overline{DB}}{m\overline{AD}}$........(iii)	Dividing (i) by (ii)
Similarly $\frac{\text{Area of } \Delta CDE}{\text{Area of } \Delta ADE} = \frac{m\overline{EC}}{m\overline{AE}}$........(iv)	
But $\Delta BED \cong \Delta CDE$	(Areas of triangles with common base and same altitudes are equal. Given that $\overline{ED} \parallel \overline{CB}$, so altitudes are equal).
$\therefore$ From (iii) and (iv) We have $\frac{m\overline{DB}}{m\overline{AD}} = \frac{m\overline{EC}}{m\overline{AE}}$ or	
$\frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EC}}$	Taking reciprocal of both sides.
Hence $m\overline{AD} : m\overline{DB} = m\overline{AE} : m\overline{EC}$	

Corollaries:

From the above theorem we also have

(i) $\frac{m\overline{BD}}{m\overline{AB}} = \frac{m\overline{CE}}{m\overline{AC}}$ *and* $\frac{m\overline{AD}}{m\overline{AB}} = \frac{m\overline{AE}}{m\overline{AC}}$

(ii) **(a)** if $\frac{m\overline{AD}}{m\overline{AB}} = \frac{m\overline{AE}}{m\overline{AC}}$, *then* $\overline{DE} \parallel \overline{BC}$ **(b)** *if* $\frac{m\overline{AB}}{m\overline{DB}} = \frac{m\overline{AC}}{m\overline{EC}}$, *then* $\overline{DE} \parallel \overline{BC}$

Note

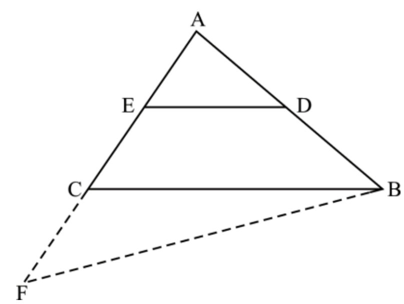
(i) Two point determine a line and three non-collinear points determine a plane.
(ii) A line segment has exactly one midpoint.
(iii) If two intersection lines form equal adjacent angle, the lines are perpendicular.

Theorem: 14.1.2 Converse of Theorem 14.1.1 (K.B)

If a line segment intersects the two sides of a triangle in the same ratio, then it is parallel to the third side.

Given

In ΔABC, $\overline{ED}$ intersect $\overline{AB}$ and $\overline{AC}$ such that
$m\overline{AD} : \overline{DB} = m\overline{AE} : m\overline{EC}$



To Prove

$\overline{ED} \parallel \overline{CB}$

Construction

If $\overline{ED} \nparallel \overline{CB}$ then draw $\overline{BF} \parallel \overline{DE}$ to meet $\overline{AC}$ produced at F.

Proof

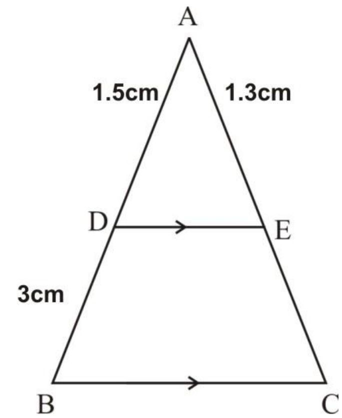
Statements	Reasons
In ΔABF	
$\overline{DE} \parallel \overline{BF}$	Construction
$\therefore \frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EF}}$ ....(i)	(A line parallel to one side of a triangle divides the other two sides proportionally Theorem 14.1.1)
But $\frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EC}}$ ....(ii)	Given
$\therefore \frac{m\overline{AE}}{m\overline{EF}} = \frac{m\overline{AE}}{m\overline{EC}}$	From (i) and (ii)
or $m\overline{EF} = m\overline{EC}$,	
This is possible only if point F is coincident with C.	(Property of real numbers)
$\therefore$ Our supposition is wrong	
Hence $\overline{ED} \parallel \overline{CB}$	

Exercise 14.1

Q.1 In ΔABC **(FSD 2017, MTN 2013, D.G.K 2017)** **(A.B)**

$\overline{DE} \parallel \overline{BC}$

(i) **If** $\overline{AD} = 1.5cm$ $\overline{BD} = 3cm$



$\overline{AE} = 1.3cm$, then find $\overline{CE}$

$$\frac{\overline{AD}}{\overline{BD}} = \frac{\overline{AE}}{\overline{EC}}$$

By substituting the values of $\overline{AD}, \overline{BD}$ and $\overline{AE}$

So

$$\frac{1.5}{3} = \frac{1.3}{EC}$$

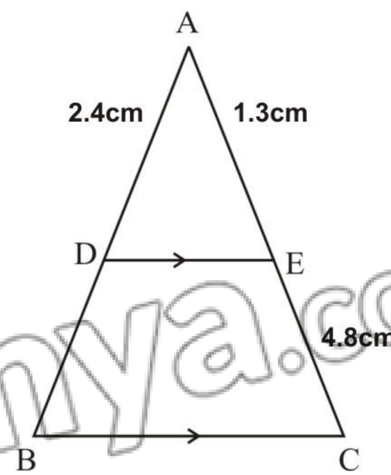
$$\overline{EC}(1.5) = 1.3 \times 3$$

$$\overline{EC} = \frac{1.3 \times 3}{1.5}$$

$$\overline{EC} = \frac{3.9}{1.5}$$

$$\overline{EC} = 2.6cm$$

(ii) **If AD = 2.4cm** $\overline{AE} = 3.2cm$ **(A.B)**



$\overline{EC} = 4.8cm$ find AB

$$\frac{AD}{AB} = \frac{AE}{AC}$$

$$\overline{AC} = AE + EC$$

$$\overline{AC} = 3.2 + 4.8$$

$$\overline{AC} = 8cm$$

$$\therefore \frac{\overline{AD}}{\overline{AB}} = \frac{\overline{AE}}{\overline{AC}}$$

$$\frac{2.4}{AB} = \frac{3.2}{8}$$

$$2.4 \times 8 = (3.2)\overline{AB}$$

$$\frac{19.2}{3.2} = \overline{AB}$$

$$\overline{AB} = 6cm$$

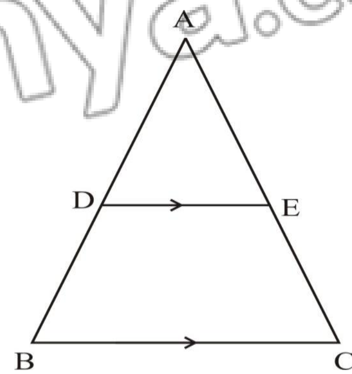
(iii) **If** $\frac{\overline{AD}}{\overline{BD}} = \frac{3}{5}$, $\overline{AC} = 4.8cm$ **find** $\overline{AE}$ **(SWL 2017, BWP 2016, MTN 2015)** **(A.B)**

$$\overline{AC} = \overline{AE} + \overline{EC}$$

$$\overline{AC} = \overline{EC} + \overline{AE}$$

$$\overline{AE} = 4.8 - \overline{EC}$$

By theorem 14.1, we have



$$\frac{\overline{AD}}{\overline{BD}}=\frac{\overline{AE}}{\overline{EC}}$$
$$\frac{\overline{AD}}{\overline{BD}}=\frac{\overline{AC}-\overline{EC}}{\overline{EC}}$$
$$\frac{3}{5}=\frac{4.8-\overline{EC}}{\overline{EC}}$$
$$3\left(\overline{EC}\right) = 5\left(4.8 - \overline{EC}\right)$$
$$3\left(\overline{EC}\right) = 24 - 5\left(\overline{EC}\right)$$
$$3\left(\overline{EC}\right) + 5\left(\overline{EC}\right) = 24$$
$$8\left(\overline{EC}\right) = 24$$
$$\left(\overline{EC}\right) = \frac{\cancel{24}^{3}}{\cancel{8}}$$
$$\overline{EC} = 3cm$$
$$\overline{AE}=\overline{AC}-\overline{EC}$$
$$=4.8-3$$
$$=1.8cm$$

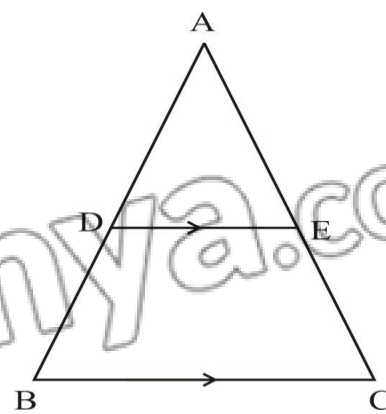
(iv) **If** $\overline{AD}=2.4cm\ \overline{AE}=3.2cm\ \overline{DE}=2cm\ \overline{BC}=5cm.$ **Find** $\overline{AB}, \overline{DB}, \overline{AC}, \overline{CE}.$ **(A.B)**

Solution:

Since $\triangle ABC \sim \triangle ADE$

$$\frac{\overline{AD}}{\overline{AB}}=\frac{\overline{AE}}{\overline{AC}}=\frac{\overline{DE}}{\overline{BC}}$$

(in similar triangles corresponding sides are proportional)



$$\frac{2.4}{AB}=\frac{3.2}{AC}=\frac{2}{5}$$

$\frac{2.4}{AB}=\frac{2}{5}$ and $\frac{3.2}{AC}=\frac{2}{5}$

(2.4)5 = 2(AB)   16.0 = 2(AC)

$\frac{12.0}{2}=AB$ $\frac{\cancel{16}^{8}}{\cancel{2}}=AC$

$\overline{AB}=6\,cm$ $\overline{AC}=8cm$

Now

$$\overline{DB}=\overline{AB}-\overline{AD}$$
$$\overline{DB}=6-2.4$$
$$\overline{BD}=3.6\,cm$$

And

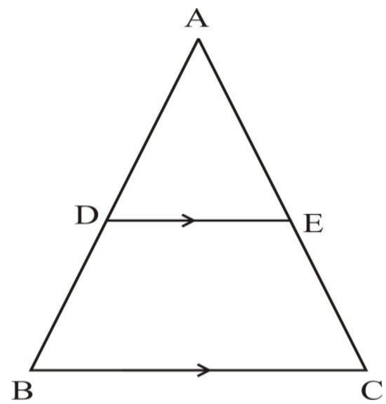
$$\overline{CE}=\overline{AC}-\overline{AE}$$

$\overline{CE} = 8 - 3.2$

$\overline{CE} = 4.8cm$

(v) **If** $\overline{AD} = 4x - 3$ $\overline{AE} = 8x - 7$ **(A.B)**

$\overline{BD} = 3x - 1$ and $\overline{CE} = 5x - 3$ Find the value of x



By theorem 14.1, we have

$$\frac{\overline{AD}}{\overline{BD}} = \frac{\overline{AE}}{\overline{EC}}$$

By putting the value of $\overline{AD}, \overline{AE}, \overline{BD}$ and $\overline{CE}$

$$\frac{4x - 3}{3x - 1} = \frac{8x - 7}{5x - 3}$$

By cross multiplying

$(4x - 3)(5x - 3) = (8x - 7)(3x - 1)$

$20x^2 - 12x - 15x + 9 = 24x^2 - 8x - 21x + 7$

$20x^2 - 27x + 9 = 24x^2 - 29x + 7$

$0 = 24x^2 - 20x^2 - 29x + 27x + 7 - 9$

$4x^2 - 2x - 2 = 0$

$2(2x^2 - x - 1) = 0$

$2x^2 - 2x + 1x - 1 = \frac{0}{2}$

$2x(x - 1) + 1(x - 1) = 0$

$(x - 1)(2x + 1) = 0$

$x - 1 = 0$ $\quad\quad$ $2x + 1 = 0$

$x = 1$ $\quad\quad$ $2x = -1$

$$x = -\frac{1}{2}$$

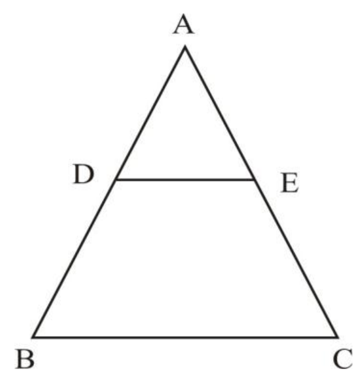
Distance is not taken in negative it is always in positive so the value of x = 1.

Q.2 **In ΔABC is an isosceles triangle $\angle$A is vertex angle and $\overline{DE}$ intersects the sides $\overline{AB}$ and $\overline{AC}$ as shown in the figure so that** **(A.B + U.B)**

$m\overline{AD} : m\overline{DB} = m\overline{AE} : m\overline{EC}$

Prove that ΔADE is also an isosceles triangle.

Given:



ΔABC is an isosceles triangle, $\angle$A is vertex and $\overline{DE}$ intersects the sides $\overline{AB}$ and $\overline{AC}$.

$$\frac{m\overline{AD}}{m\overline{BD}} = \frac{m\overline{AE}}{m\overline{EC}}$$

To Prove

$m\overline{AD} = m\overline{AE}$

Proof

$$\frac{\overline{AD}}{\overline{BD}}=\frac{\overline{AE}}{\overline{EC}}$$

Or $\frac{\overline{BD}}{\overline{AD}}=\frac{\overline{EC}}{\overline{AE}}$

Or $\frac{\overline{AD}+\overline{BD}}{\overline{AD}}=\frac{\overline{AE}+\overline{EC}}{\overline{EC}}$ (by componendo-dividendo theorem)

As we know

$\overline{AB}=\overline{AD}+\overline{BD}$

$\overline{AC}=\overline{AE}+\overline{EC}$

$$\frac{\overline{AB}}{\overline{AD}}=\frac{\overline{AC}}{\overline{AE}}$$

From this

$$\frac{\overline{AB}}{\overline{AD}}=\frac{\overline{AC}}{\overline{AE}}$$

$\overline{AD}=\overline{AE}$

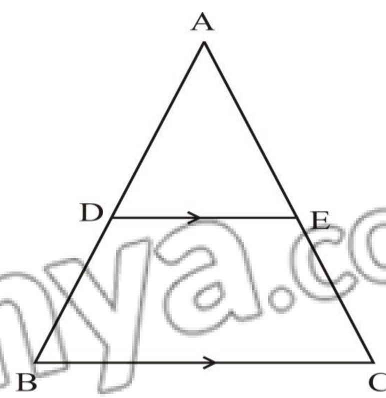
$\overline{AB}=\overline{AC}$

Q.3 In an equilateral triangle ABC shown in the figure $m\overline{AE}:m\overline{AC}=m\overline{AD}:m\overline{AB}$ find all the three angles of ΔADE and name it also. (A.B + U.B)

(FSD 2017, SWL 2014, RWP 2016, SGD 2017, RWP 2015, D.G.K 2014, 15)

Given

ΔABC is equilateral triangle



To prove

To find the angles of ΔADE

Solution:

$$\frac{m\overline{AE}}{m\overline{AC}}=\frac{m\overline{AD}}{m\overline{AB}}$$

All angles are equal. Each angle of equilateral triangle is equal to 60^o.

$m\angle A = m\angle B = m\angle C$

$m\overline{BC} \| m\overline{DE}$

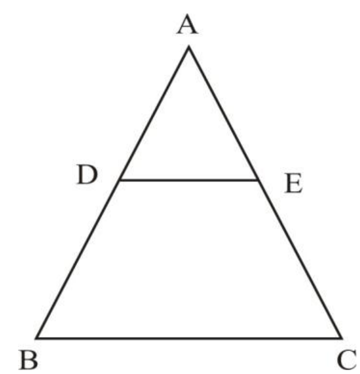
$m\angle ADE = m\angle ABC = 60^o$ (corresponding angles of // lines)

$m\angle AED = m\angle ACB = 60^o$ (corresponding angles of // lines)

$m\angle A = 60^o$ (given)

ΔADE is an equilateral triangle.

Q.4 Prove that line segment drawn through the midpoint of one side of a triangle and parallel to another side bisect the third side (A.B + K.B)



Given

$\overline{AD}=\overline{BD}$

$\overline{DE} \| \overline{BC}$

To Prove

$\overline{AE} = \overline{EC}$

In ΔABC

$\overline{DE} || \overline{BC}$

In theorem it is already discussed that

$\frac{\overline{AD}}{\overline{BD}} = \frac{\overline{AE}}{\overline{EC}}$

As we know $\overline{AD} = \overline{BD}$ or $\overline{BD} = \overline{AD}$

$\frac{\cancel{\overline{AD}}}{\cancel{\overline{AD}}} = \frac{\overline{AE}}{\overline{EC}}$

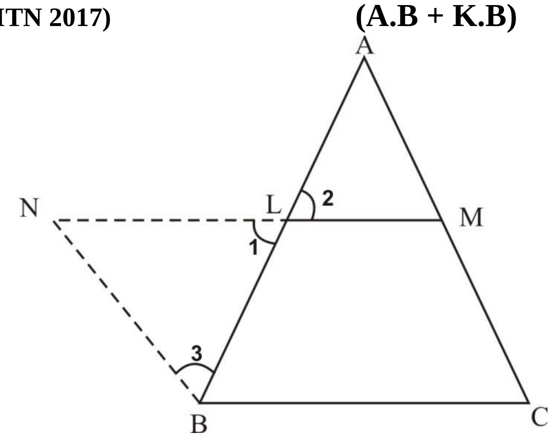
$1 = \frac{\overline{AE}}{\overline{EC}}$

$\overline{EC} = \overline{AE}$

Q.5 Prove that the line segment joining the midpoint of any two sides of a triangle is parallel to the third side **(MTN 2017)** **(A.B + K.B)**

Given

ΔABC the midpoint of $\overline{AB}$ and $\overline{AC}$ are L and M respectively



To Prove

$\overline{LM} || \overline{BC}$ and $m\overline{LM} = \frac{1}{2}\overline{BC}$

Construction

Join M to L and produce $\overline{ML}$ to N such that $\overline{ML} \cong \overline{LN}$

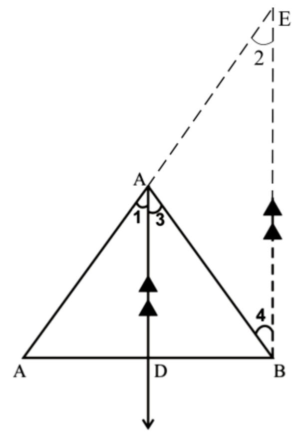
Join N to B and in the figure name the angles $\angle 1$, $\angle 2$, and $\angle 3$

Proof

Statements	Reasons
$\Delta BLN \leftrightarrow \Delta ALM$	
$\overline{BL} \cong AL$	Given
$\angle 2 = \angle 1$ or $\angle 1 = \angle 2$	Vertical angles
$\overline{NL} = \overline{ML}$	Construction
$\therefore\ \Delta BLN \cong \Delta ALM$	Corresponding angles of congruent triangles (Given)
$\therefore \angle A = \angle 3$	
And $\overline{NB} \cong \overline{AM}$	Corresponding sides of congruent triangles
$\overline{NB} \| \overline{AM}$	
$\overline{ML} = \overline{AM}$	Given
$\overline{NB} \cong \overline{ML}$	
$\overline{BC}\,\overline{MN}$ is parallelogram	
$\therefore\ \overline{BC} \| \overline{LM}$ or $\overline{BC} \| \overline{NL}$	
$\overline{BC} \cong \overline{NM}$	(Opposite side of parallelogram BCMN)

$m\overline{LM} = \frac{1}{2} m\overline{NM}$ Hence $m\overline{LM} = \frac{1}{2} m\overline{BC}$	(Opposite side of parallelogram)

Theorem 14.1.3 **(K.B)**



The internal bisector of an angle of a triangle divides the sides opposite to it in the ratio of the lengths of the sides containing the angle.

Given

In ΔABC internal angle bisector of $\angle A$ meets $\overline{CB}$ at the points D.

To prove

$m\overline{BD} : m\overline{DC} = m\overline{AB} : m\overline{AC}$

Construction

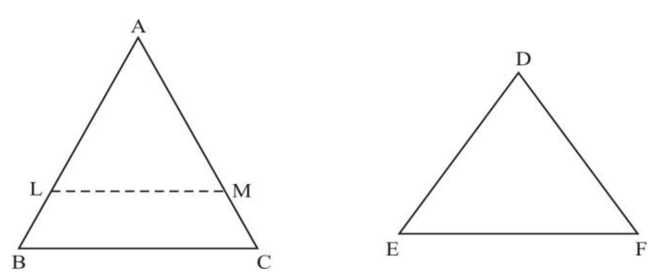
Draw a line segment $\overline{BE} \parallel \overline{DA}$ to meet $\overline{CA}$ Produced at E

Proof

Statements	Reasons
$\because \overline{AD} \parallel \overline{EB}$ and $\overline{EC}$ intersect them	Construction
$m\angle 1 = m\angle 2$..........(i)	Corresponding angles
Again $\overline{AD} \parallel \overline{EB}$ and $\overline{AB}$ intersects them	
$\therefore$ $m\angle 3 = m\angle 4$ ....(ii)	Alternate angles
But $m\angle 1 = m\angle 3$	Given
$\therefore$ $m\angle 2 = m\angle 4$	From (i) and (ii)
And $\overline{AB} \cong \overline{AE}$ or $\overline{AE} \cong \overline{AB}$	In a Δ, the sides opposite to congruent angles are also congruent
Now $\overline{AD} \parallel \overline{EB}$	Construction
$\therefore \frac{m\overline{BD}}{m\overline{DC}} = \frac{m\overline{EA}}{m\overline{AC}}$	A line parallel to one side of a triangle and intersecting the other two sides divides them proportionally.
or $\frac{m\overline{BD}}{m\overline{DC}} = \frac{m\overline{AB}}{m\overline{AC}}$	$m\overline{EA} = m\overline{AB}$ (proved)
Thus $m\overline{BD} : m\overline{DC} = m\overline{AB} : \overline{AC}$	

Theorem 14.1.4 **(K.B)**

If two triangles are similar, then the measures of their corresponding sides are proportional



Given

$\Delta ABC \sim \Delta DEF$

i.e $\angle A \cong \angle D$, $\angle B \cong \angle E$ and $\angle C \cong \angle F$

To Prove

$\dfrac{m\overline{AB}}{m\overline{DE}} = \dfrac{m\overline{AC}}{m\overline{DF}} = \dfrac{m\overline{BC}}{m\overline{EF}}$

Construction

(I) Suppose that $mAB > m\overline{DE}$

(II) $m\overline{AB} \leq m\overline{DE}$

On $\overline{AB}$ take a point L such that $m\overline{AL} = m\overline{DE}$

On $\overline{AC}$ take a point M such that $m\overline{AM} = m\overline{DF}$

Join L and M by the line segment LM

Proof

Statements	Reasons
In $\Delta ALM \leftrightarrow \Delta DEF$	
$\angle A \cong \angle D$	Given
$\overline{AL} \cong \overline{DE}$	Construction
$\overline{AM} \cong \overline{DF}$	Construction
Thus $\Delta ALM \cong \Delta DEF$	S.A.S Postulate
And $\angle L \cong \angle E$, $\angle M \cong \angle F$	(Corresponding angles of congruent triangles)
Now $\angle E \cong \angle B$ and $\angle F \cong \angle C$	Given
$\therefore$ $\angle L \cong \angle B$, $\angle M \cong \angle C$	Transitivity of congruence
Thus $\overline{LM} \parallel \overline{BC}$	Corresponding angles are equal
Hence $\dfrac{m\overline{AL}}{m\overline{AB}} = \dfrac{m\overline{AM}}{m\overline{AC}}$	A line parallel to one side of a triangle and intersecting the other two sides divides them proportionally.
Or $\dfrac{m\overline{DE}}{m\overline{AB}} = \dfrac{m\overline{DF}}{m\overline{AC}}$ ....(i)	$m\overline{AL} = m\overline{DE}$ and $m\overline{AM} = m\overline{DF}$ (Construction)
Similarly by intercepting segments on $\overline{BA}$ and $\overline{BC}$, we can prove that	
$\dfrac{m\overline{DE}}{m\overline{AB}} = \dfrac{m\overline{EF}}{m\overline{BC}}$ ....(ii)	
Thus $\dfrac{m\overline{DE}}{m\overline{AB}} = \dfrac{m\overline{DF}}{m\overline{AC}} = \dfrac{m\overline{EF}}{m\overline{BC}}$	By (i) and (ii)
Or $\dfrac{m\overline{AB}}{m\overline{DE}} = \dfrac{m\overline{AC}}{m\overline{DF}} = \dfrac{m\overline{BC}}{m\overline{EF}}$	By taking reciprocals
If $m\overline{AB} = m\overline{DE}$	
Then in $\Delta ABC \leftrightarrow \Delta DEF$	

(II) If $m\overline{AB} < m\overline{DE}$, it can similarly be proved by taking intercepts on the sides of ΔDEF	
$\angle A \cong \angle D$	
$\angle B \cong \angle E$	
And $\overline{AB} \cong \overline{DE}$	
So $\Delta ABC \cong \Delta DEF$	A.S.A ≅ A.S.A
Thus $\frac{m\overline{AB}}{m\overline{DE}} = \frac{m\overline{AC}}{m\overline{DF}} = \frac{m\overline{BC}}{m\overline{EF}} = 1$	$\overline{AC} \cong \overline{DF}$, $\overline{BC} \cong \overline{EF}$
Hence the result is true for all the cases.	

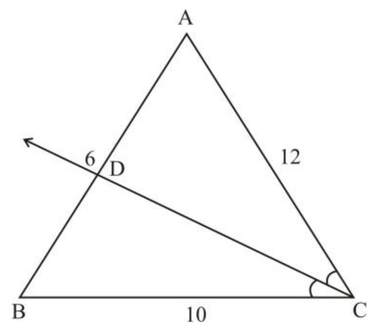
Exercise 14.2

Q.1 In ΔABC as shown in the figure $\overrightarrow{CD}$ bisects ∠C and meets $\overline{AB}$ at $D.m\overline{BD}$ **is equal to**

(a) 5 **(b)** 16 **(c)** 10 **(d)** 18 **(A.B + U.B)**

Solution:

By Theorem 14.3, we have



$$\frac{m\overline{BD}}{m\overline{DA}} = \frac{m\overline{BC}}{m\overline{CA}}$$

$$\frac{\overline{BD}}{6} = \frac{10}{12}$$

$$\overline{BD} = \frac{10 \times 6}{12} = \frac{\cancel{60}^{5}}{\cancel{12}}$$

$$\overline{BD} = 5$$

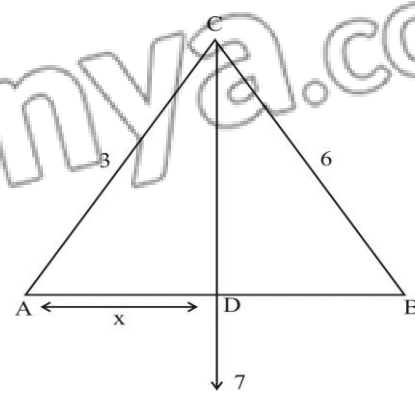
Q.2 In ΔABC shown in the figure $\overrightarrow{CD}$ bisects ∠C. If $m\overline{AC} = 3$, $\overline{CB} = 6$ and $m\overline{AB} = 7$ **then find** $m\overline{AD}$ and $\overline{DB}$ **(A.B)**

Solution:

Let $\overline{AD} = x$

Then, $\overline{BD} = 7 - x$

By theorem 14.3, we have



$$\frac{m\overline{AD}}{m\overline{BD}} = \frac{m\overline{AC}}{m\overline{CB}}$$

$$\frac{x}{7 - x} = \frac{\cancel{3}^{1}}{\cancel{6}_{2}}$$

$$\frac{x}{7 - x} = \frac{1}{2}$$

2x = 7–x

2x + x = 7

3x = 7

$$x = \frac{7}{3} \quad \text{or} \quad \overline{AD} = \frac{7}{3}$$

$$\overline{AB} = \overline{AD} + \overline{BD}$$

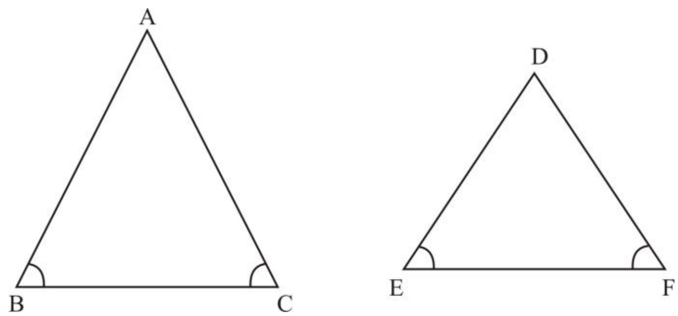
$7 = \frac{7}{3} + \overline{BD}$

$7 - \frac{7}{3} = \overline{BD}$

$\frac{21-7}{3} = \overline{BD}$

$\overline{BD} = \frac{14}{3}$

Q.3 Show that in any corresponding of two triangles if two angles of one triangle are congruent to the corresponding angles of the other, then the triangle are similar (A.B + U.B)



Given

ΔABC and Δ DEF

$\angle B \cong \angle E$

$\angle C \cong \angle F$

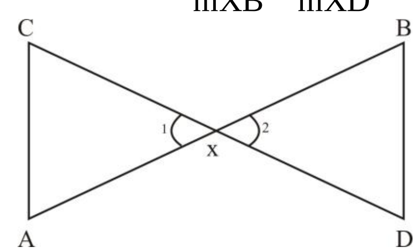
To Prove

$\Delta ABC \cong \Delta DEF$

Proof

Statements	Reasons
$\angle A + \angle B + \angle C = 180^o$	Sum of three angles of a triangle = 180°
$\angle D + \angle E + \angle F = 180$	
$\angle A \cong \angle D$	
$\angle B = \angle E$	
$\angle C = \angle F$	
Hence Δ ABC ~ ΔDEF	

Q.4 If line segment $\overline{AB}$ and $\overline{CD}$ are intersecting at point *X* and $\frac{m\overline{AX}}{m\overline{XB}} = \frac{m\overline{CX}}{m\overline{XD}}$ then show that ΔAXC and ΔBXD are similar



Given

Line segment $\overline{AB}$ and $\overline{CD}$ intersect at X

$\frac{m\overline{AX}}{m\overline{XB}} = \frac{m\overline{CX}}{m\overline{XD}}$

To Prove
ΔCXA and ΔDXB are similar
Proof

Statements	Reasons
$\frac{\overline{AX}}{\overline{XB}} = \frac{\overline{CX}}{\overline{XD}}$	Given
$\angle 1 \cong \angle 2$	
$\overline{AC} \parallel \overline{BD}$	Vertical angles
$\angle A = m\angle B$	
$m\angle C = m\angle D$	Alternate angles
Hence proved the triangle are similar	

Review Exercise 14

Q.1 Which of the following are true which are false? (K.B + U.B)

(i) Congruent triangles are of same size and shape. (True)

(ii) Similar triangles are of same shape but different sizes. (True)

(iii) Symbol used for congruent is '~' (False)

(iv) Symbol used for similarity is '≅' (False)

(v) Congruent triangle are similar (True)

(vi) Similar triangles are congruent (False)

(vii) A line segment has only one midpoint (True)

(viii) One and only one line can be drawn through two points (True)

(ix) Proportion is non equality of two ratio (False)

(x) Ratio has no unit (True)

Q.2 Define the following

(i) Ratio (K.B)

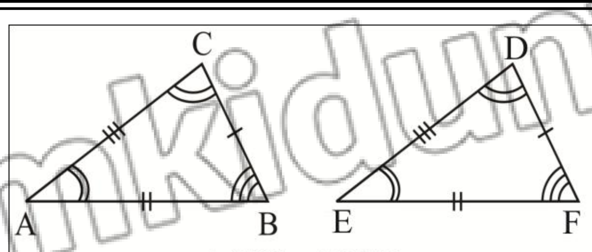
Ans: see definition

(ii) Proportion (K.B)

Ans: see definition

(iii) Congruent Triangles (K.B)

Two triangles are said to be congruent (symbols ≅) if there exists correspondence between them such that all the corresponding sides and angles are congruent.



$\Delta ABC \cong \Delta DEF$

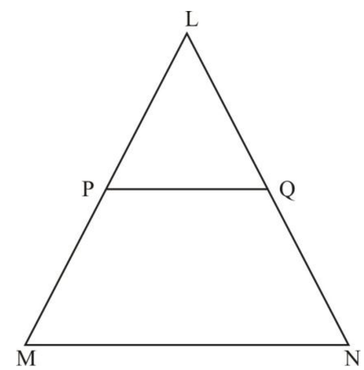
(iv) Similar Triangles **(A.B)**

Ans: see definition

Q.3 In ΔLMN shown in the figure $\overline{MN} \parallel \overline{PQ}$ **(K.B)**

(i) If mLM = 5cm, $m\overline{LP} = 2.5\text{cm}$

mLQ = 2.3 cm then find LN



$$\frac{m\overline{LP}}{m\overline{LM}} = \frac{m\overline{LQ}}{m\overline{LN}}$$

$$\frac{2.5}{5} = \frac{2.3}{\overline{LN}}$$

$$(2.5)\ \overline{LN} = 5 \times 2.3$$

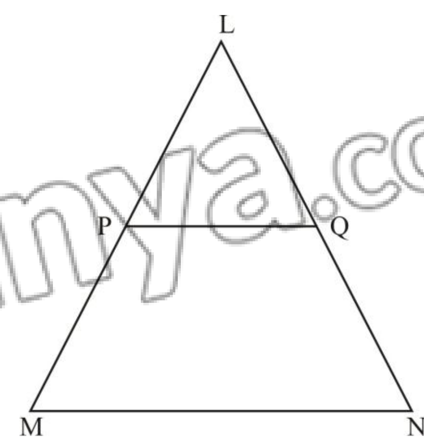
$$\overline{LN} = \frac{11.5}{2.5}$$

$$\overline{LN} = 4.6\text{cm}$$

(ii) If mLM = 6cm, mLQ = 2.5cm **(A.B + U.B)**

mQN = 5cm then find

mLP



$$\frac{m\overline{LP}}{m\overline{LM}} = \frac{m\overline{LQ}}{m\overline{LN}}$$

$$\frac{LP}{6} = \frac{2.5}{LN}$$

$$\overline{LN} = \overline{LQ} + \overline{QN}$$

$$\overline{LN} = 2.5 + 5$$

$$\overline{LN} = 7.5\text{cm}$$

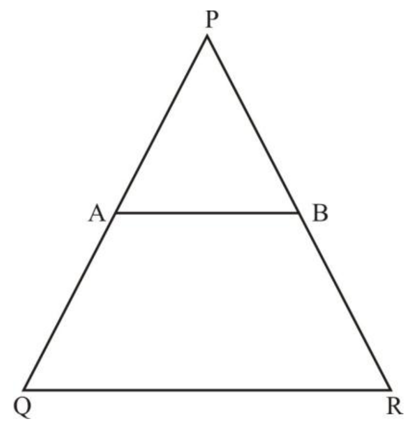
$$\frac{\overline{LP}}{6} = \frac{2.5}{7.5}$$

$$\overline{LP} = \frac{2.5 \times 6}{7.5}$$

$$\overline{LP} = \frac{15}{7.5}$$

$$\overline{LP} = 2\text{cm}$$

Q.4 **In the show figure let mPA = 8x – 7 mPB = 4x – 3** $m\overline{AQ}=5x-3$ $m\overline{BR}=3x-1$ **find the value of x if** $\overline{AB}\parallel\overline{QR}$ **(A.B + U.B)**



$$\frac{mPA}{mAQ}=\frac{mBP}{mBR}$$

$$\frac{8x-7}{5x-3}=\frac{4x-3}{3x-1}$$

By cross multiplying

$$(8x-7)(3x-1)=(4x-3)(5x-3)$$

$$24x^2-8x-21x+7=20x^2-12x-15x+9$$

$$24x^2-29x+7=20x^2-27x+9$$

$$24x^2-20x^2-29x+27x+7-9=0$$

$$4x^2-2x-2=0$$

$$4x^2-4x+2x-2=0$$

$$4x(x-1)+2(x-1)=0$$

$$(x-1)(4x+2)=0$$

$$x-1=0$$

$$x=1$$

$$4x+2=0$$

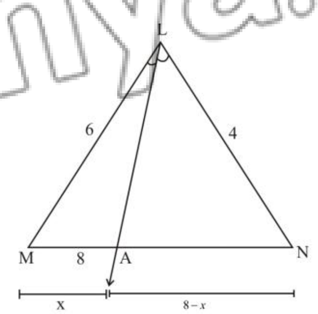
$$4x=-2$$

$$x=\frac{-\cancel{2}^{1}}{\cancel{4}_{2}}$$

$$x=\frac{-1}{2}$$

Length is always taken as positive not negative so value of x = 1

Q.5 **In ΔLMN Shown in figure** $\overrightarrow{LA}$ **bisects ∠L. If** $m\overline{LN}=4m$ $m\overline{LM}=6cm$ $m\overline{MN}=8$ **then find** $m\overline{MA}$ and $m\overline{AN}$ **(A.B + K.B)**



$$\frac{m\overline{MA}}{m\overline{AN}}=\frac{m\overline{LM}}{m\overline{LN}}$$

$$\overline{MA}=x$$

$$\overline{AN}=8-x$$

$$\frac{x}{8-x}=\frac{6}{4}$$

$$4x=6(8-x)$$

$$4x=48-6x$$

$$4x+6x=48$$

$$10x=48$$

$x = \frac{48}{10}$

$x = 4.8cm$

$m\overline{MA} = 4.8cm$

$\overline{MN} = \overline{MA} + \overline{AN}$

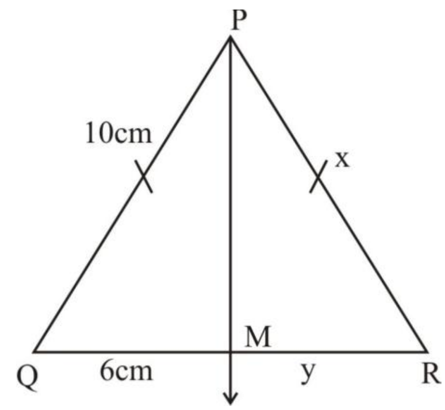
$8 = 4.8 + \overline{AN}$

$8 - 4.8 = \overline{AN}$

$\overline{AN} = 3.2cm$

Q.6 In Isosceles ΔPQR Shown in the figure, find the value of x and y

As we know that it is isosceles triangle **(A.B + K.B)**



So

$\overline{PQ} = \overline{RP}$

10= x

Or

x = 10cm

$\overline{PM} \perp \overline{QR}$

So it bisects the side and bisects the angle also

SO $\overline{QM} = \overline{MR}$

6 = y

Or

y = 6cm