PYTHAGORAS THEOREM

Work of Pythagoras on Right-Angled Triangles

(LHR 2016, GRW 2014, 17, BWP 2017, SWL 2015, 16, 17, MTN 2015, D.G.K 2014, 15, 17) Pythagoras, a Greek philosopher and mathematician, discovered the simple but important relationship between the sides of a right-angled triangle. He formulated this relationship in the form of a theorem called Pythagoras' theorem after his name.

Note

B

Birth of Pythagoras 580 BC – 572 BC Death of Pythagoras 500 BC – 490 BC

Theorem 15.1.1

 $(\mathbf{U}.\mathbf{B} + \mathbf{K}.\mathbf{B})$

(U.B + K.B)

 $(\mathbf{U}.\mathbf{B} + \mathbf{K}.\mathbf{B})$

In a right angled triangle, the square of the length of hypotenuse is equal to the sum of the squares of the lengths of the other two sides.



Given

 \triangle ACB is a right angled triangle in which m $\angle C = 90^{\circ}$ and m $\overline{BC} = a$, m $\overline{AC} = b$ and

m AB = c

To prove $c^2 = a^2 + b^2$

Construction

Draw \overline{CD} perpendicular from C on \overline{AB}

Let $\overline{\text{MCD}} = h$, $\overline{\text{MAD}} = x$ and $\overline{\text{MBD}} = y$. Line segment CD splits $\triangle \text{ABC}$ into two \triangle s ADC and BDC which are separately shown in the figures (ii) –a and (ii) –b respectively.

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	Proof	O JULIU VELO	
	$\frac{\text{Statements}}{\text{In } \Delta \text{ ADC} \leftrightarrow \Delta \text{ ACB}}$	Reasons Refer to figure (ii)-a and (i)	
	∠A≅∠A	Common – Self Congruent	
	$\angle ADC \cong \angle ACB$	Construction- given, each angle = 90°	
MA	$\angle C \cong \angle B$	$\angle C$ and $\angle B$ complements of $\angle A$	
AA r	$\therefore \Delta ADC \sim \Delta ACB$	Congruency of three angles	
	. x _ b	(Measures of corresponding sides of similar triangles	
	$\frac{1}{b} = \frac{1}{c}$	are proportional)	
	or $x = \frac{b^2}{c}$ (i)		
	Again in $\triangle BDC \leftrightarrow \triangle BCA$	Refer to figure (ii)-b and (i)	
	$\angle B \cong \angle B$	Common – self Congruent	
	$\angle BDC \cong \angle BCA$	Construction – given each angle = 90°	
	$\angle C \cong \angle A$	$\angle C$ and $\angle A$ complements of $\angle B$	
	$\therefore \Delta BDC : \Delta BCA$	Congruency of three angles	
	y a	(Corresponding sides of similar triangles are	
	$\frac{1}{a} = \frac{1}{c}$	proportional)	
	or $y = \frac{a^2}{c}$ (ii)		ก
	But $y + x = c$	Supposition	10
	$\therefore \frac{a^2}{c} + \frac{b^2}{c} = c$	By (i) and (ii)	
	or $a^2 + b^2 = c^2$	Multiplying both side by c	
-	i.e. $c^2 = a^2 + b^2$		
NN	Corollary:	$(\mathbf{U}.\mathbf{B} + \mathbf{K}.\mathbf{B})$	
00	In a right angle $\triangle ABC$, the righ	t angle at A.	
	(1) $mAB = mBC - mCA$ (ii) $m\overline{AC}^2 = m\overline{BC}^2 - \overline{AB}^2$		
	(11) $mAC = mBC - mAB$	$A \qquad B$	

Note

 $(\mathbf{U}.\mathbf{B} + \mathbf{K}.\mathbf{B})$ Pythagoras' theorem has many proofs. The one we have given is based on the proportionality of the sides of two similar triangles. For convenience $\Delta s ADC$ and CDB have been shown separately. Otherwise, the theorem is usually proved using figure (i) only. Theorem 15.1.2 Converse of Pythagoras Theorem 15.1.1 $(\mathbf{U}.\mathbf{B} + \mathbf{K}.\mathbf{B})$ If the Square of one side of a triangle is equal to the sum of the square of the other two sides, then the triangle is a right angled triangle. Given In a $\triangle ABC$, $m\overline{AB} = c, m\overline{BC} = a, m\overline{AC} = b$ Such that $a^2 + b^2 = c^2$. To prove a $\Delta A \overline{C} B$ is a right angled triangle. Construction Draw CD perpendicular to BC Such that $CD \cong CA$. Join the points B and D. b Proof Statements Reasons ΔDCB is a right angled triangle. Construction $\therefore \left(m\overline{BD}\right)^2 = a^2 + b^2$ Pythagoras theorem But $a^2 + b^2 = c^2$ Given ∴ (mBD Taking Square root on both sides or mBD = c Now in $\triangle DCB \leftrightarrow \triangle ACB$ Construction CD≅ CA $\overline{BC} \cong \overline{BC}$ Common Each side = c $DB \cong AB$

 $\therefore \Delta \text{ DCB} \cong \Delta \text{ ACB}$ $\therefore \angle DCB \cong \angle ACB$

But m \angle DCB = 90° \therefore m \angle ACB = 90° Hence the \triangle ACB is a Right angled triangle.

(U.B + K.B + A.P)

Corollary Let c be the longest of the sides a, b and c of a triangle.

If $a^2 + b^2 = c^2$, then the triangle is right.

If sum of the squares of two sides is equal to third side, then triangle is right angled triangle.

 $S.S.S \cong S.S.S$

Construction

triangles

Corresponding angles of congruent

If $a^2 + b^2 > c^2$, then the triangle is acute.

If sum of the squares of two sides is greater than the third side, the triangle is acute angled triangle.

If $a^2 + b^2 < c^2$, then the triangle is obtuse.

If sum of the squares of two sides is smaller than the third side, the triangle is obtuse angled triangle.

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Q.1 Verify that the ∆s having the following measures of sides are right-angled. (A.B)

 Δ s are right angled, if (U.B) (Hypotenuse)² = (base)² + (Perpendicular)² (LHR 2016, GRW 2013, FSD 2015, 17, MTN 2013, SWL 2014, 15, 17, SGD 2013, 17) a = 5cm ⇒ a² = 25cm² b = 12cm ⇒ b² = 144cm²

 $c = 13cm \implies c^{2} = 169cm^{2}$ Larger Size is Hypotenuse, So $c^{2} = a^{2} + b^{2}$ Putting the values 169 = 25 + 144169 = 169

Satisfied So, given measures form a right angled triangle.

(ii) (GRW 2016, FSD 2017, MTN 2013, 16, SWL 2015, SGD 2017) $a = 1.5 \text{cm} \implies a^2 = 2.25 \text{cm}^2$ $b = 2 \text{cm} \implies b^2 = 4 \text{cm}^2$ $c = 2.5 \text{cm} \implies c^2 = 6.25 \text{ cm}^2$ Larger Size is Hypotenuse, So

$$c^2 = a^2 + b^2$$

Putting the values (U.B)

6.25 = 2.25 + 4

6.25 = 6.25Satisfied

So, given measures form a right angled triangle.

(iii) (LHR 2013, 14, 15, 16, GRW 2015, FSD 2014, SWL 2017, SGD 2015, RWP 2017) $a = 9 \text{cm} \Rightarrow a^2 = 81 \text{cm}^2$ $b = 12 \text{cm} \Rightarrow b^2 = 144 \text{cm}^2$ $c = 15 \text{cm} \Rightarrow c^2 = 225 \text{cm}^2$ Larger Size is Hypotenuse, So $c^2 = a^2 + b^2$ Putting the values (U.B)

 $225 \text{cm}^2 = 8.1 \text{cm} + 144 \text{cm}$ $225 \text{cm}^2 = 225 \text{cm}^2$ Satisfied. So, given measures form a right

angled triangle.

 $a^2 = 256 cm^2$ a = 16cm(iv) b = 30cm $b^2 = 900 \text{ cm}^2$ $c^2 = 1156 cm^2$ c = 34cmLarger Size is Hypotenuse, So $c^2 = a^2 + b^2$ Putting the values 1156 = 256 + 9001156 = 1156(A.B) Satisfied So, given measures form a right angled triangle. Verify that $a^2 + b^2$, $a^2 - b^2$ and 2ab

Q.2 Verify that $a^2 + b^2$, $a^2 - b^2$ and 2ab are the measures of the sides of a right angled triangle where a and b are any two real numbers (a >b).

(LHR 2017) (U.B + A.B)

Solution:

Let
$$a = 2$$
 and $b = 1$
 $a^{2} + b^{2} = (2)^{2} + (1)^{2} = 4 + 1 = 5$
 $a^{2} - b^{2} = (2)^{2} - (1)^{2} = 4 - 1 = 3$
 $2ab = 2(2)(1) = 4$

Since $a^2 + b^2$ is the largest side so $a^2 + b^2$ will be hypotenuse.



So

Triangle is right angled, if

$$(a^{2}+b^{2})^{2} = (2ab)^{2} + (a^{2}-b^{2})^{2}$$

$$a^{4}+b^{4}+2a^{2}b^{2} = 4a^{2}b^{2} + a^{4} + b^{4} - 2a^{2}b^{2}$$

$$a^{4}+b^{4}+2a^{2}b^{2} = a^{4}+b^{4}+2a^{2}b^{2}$$

L.H.S = R.H.S

It is proved that it is a right angled triangle



MATHEMATICS-9







×	Uni	t – 15		Pythagoras Theorem	
CUT HER	te	2		MG7ºGe	
	Time	40 min	36111111	Marks: 25	
i	01	Mark the Correct multiple choic	e question	(7×1=7)	
i	Q.1	wark the correct multiple clock	e question.	(//1-/)	
I	1	Which of the following is trichot	omy property of real numb	ber:	
-	N	(A) $m\overline{AB} = m\overline{BC}$	(B) $m\overline{AB} > m\overline{BC}$		
ANA	NN V	(C) $\overline{\text{mAB}} < \overline{\text{mBC}}$	(D) None of these	•	
YU	2	Which of following set of lengths	can be the lengths of the s	ides of a triangle.	
I		(A) $2 \text{ cm}, 3 \text{ cm}, 5 \text{ cm}$	(B) $3 \text{ cm}, 4 \text{ cm}, 5$	cm	
I	3	(C) 2 cm, 4 cm, 7 cm Two sides of a triangle measure	$(\mathbf{D}) \neq \operatorname{CIII}, \supset \operatorname{CIII}, /$ 10 cm and 15 cm which o	CIII f the following measure is	
I	3	The following measure is			
I		(A) 5 cm	(B) 20 cm		
1		(C) 25 cm	(D) 30 cm		
	4 In the figure, P is any point lying away from the line AB. Then mPL will be				
		shortest distance if	۴P		
1					
		<u>د ب</u>	• •		
		(A) $m \angle PLA = 80^{\circ}$	$(\mathbf{B}) \mathbf{m} \angle \mathbf{PLB} = 10$	0^{o}	
		(C) m \angle PLA = 90°	(D) $m \angle PLA = 70$	0	
i	5	If a line segment intersects the	two sides of a triangle in	the same ratio then it is	
i		to the third side			
i		(A) Perpendicular	(B) Parallel		
Ī	6	In AABC as shown in the figure $$	\overline{D} bisasts (C and mosts \overline{A}	$\overline{\mathbf{P}}$ at \mathbf{D} , m $\overline{\mathbf{PD}}$ is accurate to \mathbf{P}	
Ī	0	In ZABC as shown in the lighte, C	A	s at D. IIIBD is equal to:	
I		-	$\langle \rangle$		
I				NV Ceroe	
I		\Box	87541111		
1			10 C	D	
I		(\mathbf{A}) 5 (\mathbf{C}) 10	(B) 16		
I	7	(C) 10 If $a^2 \pm b^2 > c^2$ then triangle is call	61 (U)		
I	200	(A) Acute	(B) Obtuse		
ANA	1/1/1	(C) Scalene	(D) Right		
MM.	8	In a right angled triangle the gre	atest angle is of		
0		(A) 60°	(B) 90°		
I	0	(C) 120° If hypotennics of an issue last in the	(D) 180°	of other side is of low oth	
l	У	(A) 2	triangles is 12 cm then each ($(\mathbf{R}) \Delta$	or other side is of length	
I		(\mathbf{C}) 1	(D) 4./2		
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 $(5 \times 2 = 10)$

Q.2 Give Short Answers to following Questions.

- (i) If 13 cm, 12 cm, and 5 cm are the lengths of a triangle, then verify that difference of measure of any two sides of a triangle is less then measure of the third side.
- (ii) In a triangle ABC, $m \angle B = 70^{\circ}$ and $m \angle C = 45^{\circ}$, which of the sides of the triangle is longest and which is the shortest?

(iii) The three sides of a triangle are of measure 8, *x*, and 17 respectively for what value of *x* will it become base of a right angled triangle?

(iv) In $\triangle ABC$ shown in the figure, \overrightarrow{CD} bisects $\angle C$. If $\overline{mAC} = 3$, $\overline{mCB} = 6$ and $\overline{mAB} = 7$, then find \overline{mAD} and \overline{mDB}



(v) Find the value of x in the shown figure If AD = 2.4 cm, $\overline{AE} = 3.2$ cm, $\overline{DE} = 2$ cm, BC = 5 cm find \overline{AB} , \overline{DB} , \overline{AC} , \overline{CE} .

Q.3 Answer the following Questions.

(4+4=8)

(a) Define similar triangles.



(b) A ladder 17m long rests against a vertical wall. The foot of the ladder is 8m away from the base of the wall. How high up the wall will the ladder reach?Note:

Parents or guardians can conduct this test in their supervision in order to check the skills of the student.