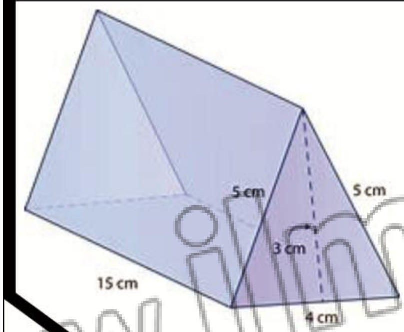


UNIT 16

THEOREMS RELATED WITH AREA

Introduction (K.B)

In this unit we will state and prove some important theorems related with and of parallelograms and triangles along with corollaries. We shall apply them to solve appropriate problems and to prove some useful results.

Area of a Figure (K.B)

The region enclosed by the bounding lines of a closed figure is called area of figure.

Unit of Area of Figure:

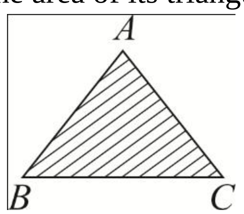
The area of a closed region is expressed in square units (say, sq. m or m^2) i.e., positive real number.

Triangular Region (K.B)

The interior of a triangle is the part of the plane enclosed by the triangle.

A triangular region is the union of a triangle and its interior i.e., the three line segments forming the Triangle and its interior.

By area of a triangle we mean the area of its triangular region.



Congruent Area Axiom (K.B)

If two figures are congruent then their areas are also congruent.

For example:

If $\Delta ABC \cong \Delta PQR$. Then area of (region ΔABC) = Area of (region ΔPQR)

Rectangular Region (K.B)

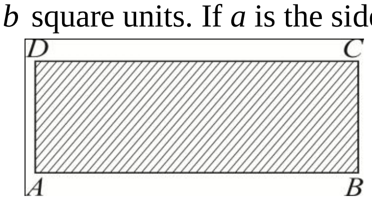
The interior of a rectangle is the part of the plane enclosed by the rectangle.

A rectangle region is the union of a rectangle and its interior.

A rectangular region can be divided into two or more than two triangular regions in many ways.

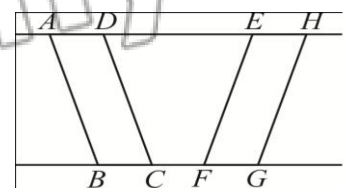
Unit of Rectangular Region (K.B)

If the length and width of rectangle are a units and b units respectively, then the area of the rectangle is equal to $a \times b$ square units. If a is the side of a square, its area $= a^2$ square units.

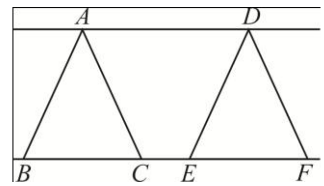


Between the same parallels **(K.B)**

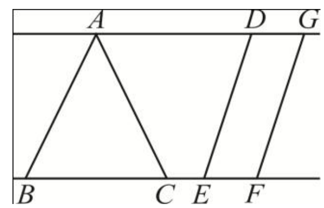
Two parallelograms are said to be between the same parallels, when their bases are in the same straight line and their sides opposite to these bases are also in a straight line; as the parallelograms $ABCD,\ EFGH$ is the given figure.



Two triangles are said to be between the same parallels, when their bases are in the same straight line and the line joining their vertices is parallel to their bases; as the $\Delta s\ ABC,\ DEF$ in the given figure.



A triangle and a parallelogram are said to be between the same parallels, when their bases are in the same straight line, and the side of the parallelogram opposite the base, produced if necessary, passes through the vertex of the triangle as are the ΔABC and the parallelogram $DEFG$ in the given figure.



Definition **(K.B + U.B)**

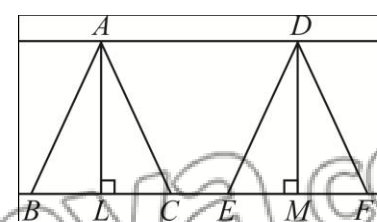
If one side of a parallelogram is taken as its base, the perpendicular distance between that side and the side parallel to it, is called the Altitude or Height of the parallelogram.

Definition **(K.B + U.B)**

If one is a triangle is taken as its base, the perpendicular to that side, form the opposite vertex is called Altitude or Height of the triangle.

Useful result **(K.B + U.B)**

"Triangles or parallelogram having the same or equal altitudes can be placed between the same parallels, and conversely".



Side of it, and spouse $\overline{AL},\overline{DM}$ are the equal altitudes.

We have to show $\overline{AD}$ is parallel to $BCEF$.

Proof

$\overline{AL}$ and $\overline{DM}$ are parallel, they are both perpendicular to $\overline{BF}$. Also $m\overline{AL}=m\overline{DL}$.(given)

$\therefore\ \overline{AD}$ is parallel to $\overline{LM}$.

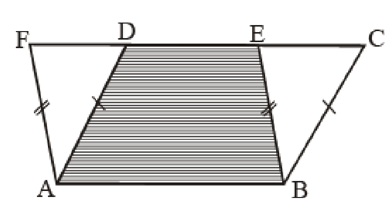
A similar proof may be given in the case of parallelograms.

Useful result **(K.B + U.B)**

A diagonals of a parallelogram divides it into two congruent triangles $(S.S.S.)$ and hence of equal area.

Theorem 16.1.1 **(K.B + U.B)**

Parallelograms on the same base and between the same parallel lines (or of the same altitude) are equal in area



Given

Two parallelograms ABCD and ABEF having the same base $\overline{AB}$ between the same parallel lines $\overline{AB}$ and $\overline{DE}$

To prove

Area of parallelogram ABCD=area of parallelogram ABEF

Proof

Statements	Reasons
Area of (parallelogram ABCD) = Area of (Quad. ABED) + Area of (Δ CBE) … (1)	[Area addition axiom]
Area of (parallelogram ABEF) = Area of (Quad. ABED) + Area of (Δ DAF) … (2)	[Area addition axiom]
In Δ s CBE and DAF	
$m\overline{CB} = m\overline{DA}$	[opposite sides of a Parallelogram]
$m\overline{BE} = m\overline{AF}$	[opposite sides of a Parallelogram]
$m\angle CBE = m\angle DAF$	$\left[\because \overline{BC} \parallel \overline{AD}, \overline{BE} \parallel \overline{AF}\right]$
$\Delta CBE \cong \Delta DAF$	[S.A.S Cong.axiom]
Area of (ΔCBE) = area of (ΔDAF) … (3)	[Cong. Area axiom]
Hence area of (Parallelogram ABCD) = area of (parallelogram ABEF)	From (1), (2) and (3)

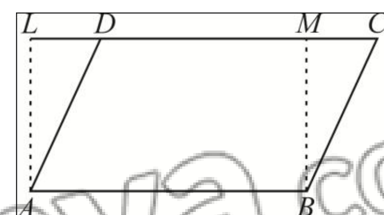
Corollaries: **(K.B + U.B)**

(i) Triangle on equal bases and between the same parallels are equal in area. The same parallels are equal in area.

(ii) Triangle in the same straight line, area equal in area.

Proof:

Let $ABCD$ be a parallelogram $\overline{AL}$ is an altitude corresponding to side $\overline{AB}$



(i) Since parallelogram $ABCD$ and rectangle $ALMB$ are on the same base $\overline{AB}$ and between the same parallels,

$\therefore$ by above theorem it follows that

Area of (parallelogram $ABCD$) = Area of (rectangle $ALMB$)

(ii) But area of rectangle $ALMB$) $= AB \times AL$

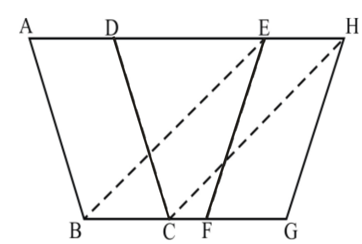
Hence area of (parallelogram ABCD) $= AB \times AL$

Theorem 16.1.2 **(K.B + U.B + A.B)**

Parallelograms on equal bases and having the same (or equal) altitude area equal in area.

Given:

Parallelogram ABCD, EFGH are on equal base $\overline{BC}$, $\overline{FG}$ having equal altitudes



To prove

Area of (Parallelogram ABCD) = area of (parallelogram EFGH)

Construction:

Place the parallelogram ABCD and EFGH So that their equal bases $\overline{BC}, \overline{FG}$ are in the straight line BCFG. Join $\overline{BE}$ and $\overline{CH}$

Proof

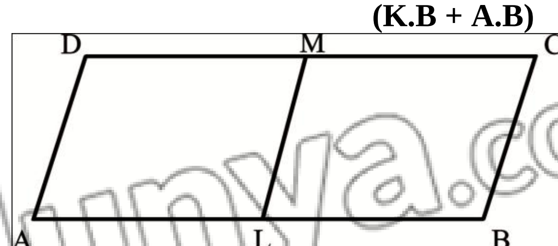
Statements	Reasons
The give 11^{mg} ABCD and EFGH are between the same parallels	
Hence ADEH is a straight line $\parallel$ to $\overline{BC}$	Their altitudes are equal (given)
$\therefore$ m$\overline{BC}$ = m$\overline{FG}$ =m$\overline{EH}$	
Now m$\overline{BC}$ =m$\overline{EH}$ and they are $\parallel$	Given
$\therefore \overline{BE}$ and $\overline{CH}$ are both equal and $\parallel$	EFGH is a parallelogram
Hence EBCH is a Parallelogram	A quadrilateral with two opposite side congruent and parallel is a parallelogram
Now $\parallel^{gm}$ ABCD = $\parallel^{gm}$ EBCH –(i)	Being on the same base $\overline{BC}$ and between the same parallels
But $\parallel^{gm}$ EBCH = $\parallel^{gm}$ EFGH – (ii)	Being on the same base $\overline{EH}$ and between the same parallels
Hence area $\parallel^{gm}$ (ABCD)= Area $\parallel^{gm}$ (EFGH)	From (i) and (ii)

Exercise 16.1

Q.1 Show that the line segment joining the mid point of opposite sides of a purallogram divides it into two equal parallelograms. **(K.B + A.B)**

Given

ABCD is a parallelogram. L is the midpoint of $\overline{AB}$ and M is the midpoint of $\overline{DC}$



To prove

Area of parallelogram ALMD = area of parallelogram LBCM.

Proof

Statements	Reasons
$\overline{AB} \parallel \overline{DC}$	Opposite sides of parallelogram ABCD.
$\overline{AL} \cong \overline{LB}$ …(i)	L is midpoint of $\overline{AB}$
The parallelograms ALMD and LBCM are on equal bases and between the same parallel lines $\overline{AB}$ and $\overline{DC}$	From equation (i)
Hence area of parallelogram ALMD= area of parallelogram LBCM.	They have equal areas

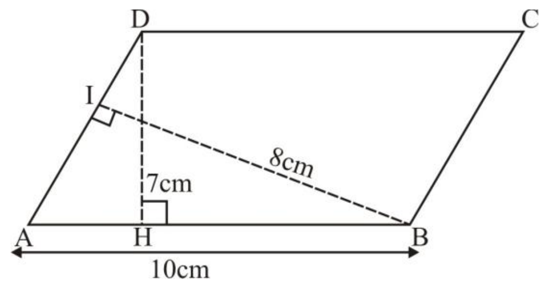
Q.2 In a parallelogram ABCD, m $\overline{AB}$=10cm the altitudes Corresponding to Sides AB and AD are respectively 7cm and 8cm Find $\overline{AD}$ (K.B + A.B)

$\overline{AB}$ = 10 cm

$\overline{DH}$ = 7cm

$\overline{MB}$ = 8cm

$\overline{AD}$ =?



Formula

Area of parallelogram = base x altitude

$\overline{AB} \times \overline{DH} = \overline{AD} \times \overline{IB}$

$10 \times 7 = \overline{AD} \times 8$

$\frac{70^{35}}{8^{4}} = \overline{AD}$

$\frac{35}{4} = \overline{AD}$

$\overline{AD} = \frac{35}{4}$

Or

$\overline{AD}$ = 8.75cm

Q.3 If two parallelograms of equal areas have the same or equal bases, their altitude are equal (K.B + A.B)

In parallelogram opposite side and opponents angles are Congruent.

Given

Parallelogram ABCD and parallelogram MNOP

OD is altitude of parallelogram ABCD

PQ is altitude of parallelogram MNOP

Area of ABCD $\|^{gm}$ $\cong$ Area of MNOP $\|^{gm}$

To prove

$m\overline{OD} \cong m\overline{PQ}$

Proof

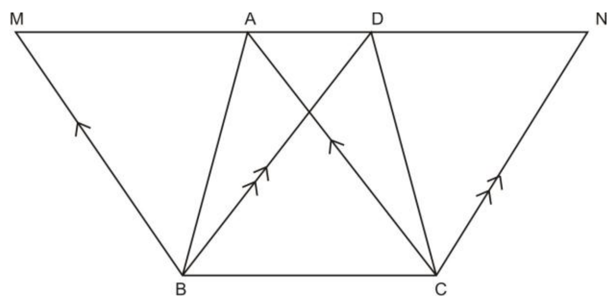
Statements	Reasons
Area of parallelogram ABCD= Area of parallelogram MNOP	Given
Area of parallelogram= base × height	Given
$\overline{AB}\times\overline{OD} = \overline{MN}\times\overline{PQ}$	
We know that	
$\overline{AB} = \overline{MN}$	
So	
$\frac{\cancel{AB}}{\cancel{AB}}\times\overline{OD} = \overline{PQ}$	Proved
$\overline{OD} = \overline{PQ}$	

Theorem 16.1.3 **(A.B + U.B)**

Triangle on the same base and of the same (i.e…equal) altitudes are equal in area

Given

Δ's ABC, DBC on the same base $\overline{BC}$ and having equal altitudes



To prove

Area of (ΔABC) = area of (ΔDBC)

Construction:

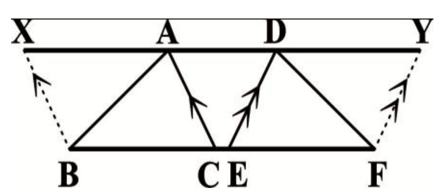
Draw $\overline{BM}\parallel$ to $\overline{CA}$, $\overline{CN}\parallel$ to $\overline{BD}$ meeting $\overline{AD}$ produced in M.N.

Proof

Statements	Reasons
ΔABC and ΔDBC are between the same $\parallel^{s}$	Their altitudes are equal
Hence MADN is parallel to $\overline{BC}$	
$\therefore$ Area $\parallel^{gm}$ (BCAM)= Area $\parallel^{gm}$ (BCND)	These $\parallel^{gm}$ are on the same base $\overline{BC}$ and between the same $\parallel^{s}$
But ΔABC= $\frac{1}{2}\parallel^{gm}$ (BCAM)-------(ii)	
And ΔDBC= $\frac{1}{2}\parallel^{gm}$ (BCND)------(iii)	Each diagonal of a $\parallel^{gm}$
Hence area (ΔABC) = Area(ΔDBC)	Bisects it into two congruent triangles From (i) (ii) and (iii)

Theorem 16.1.4 **(K.B + U.B + A.B)**

Triangles on equal bases and of equal altitudes are equal in area.



Given

Δ ABC, DEF on equal bases $\overline{BC}$, $\overline{EF}$ and having altitudes equal

To prove
Area (ΔABC) = Area (ΔDEF)
Construction:
Place the Δs ABC and DEF so that their equal bases $\overline{BC}$ and $\overline{EF}$ are in the same straight line BCEF and their vertices on the same side of it .Draw $\overline{BX} \parallel \overline{CA}$ and $\overline{FY} \parallel \overline{ED}$ meeting $\overline{AD}$ produced in X, Y respectively
Proof

Statements	Reasons
ΔABC, ΔDEF are between the same parallels $\therefore$ XADY is $\parallel^{gm}$ to BCEF	Their altitudes are equal (given)
$\therefore$ area $\parallel^{gm}$ (BCAX) = A area $\parallel^{gm}$ (EFYD)----(i)	These $\parallel^{gm}$ are on equal bases and between the same parallels
But $\Delta ABC = \frac{1}{2} \parallel^{gm}$ (BCAX)----(ii)	Diagonal of a $\parallel^{gm}$ bisect it
And area of $\Delta DEF = \frac{1}{2}$ area of $\parallel^{gm}$ (EFYD)__ (iii)	
$\therefore$ area (ΔABC) = area (ΔDEF)	From (i), (ii) and (iii)

Corollaries: **(K.B + A.B)**

(i) Triangles on equal bases and between the same parallels are equal in area.
(ii) Triangles having a common vertex and equal bases in the same straight line, area equal in area.

Exercise 16.2

Q.1

Show that **(K.B + U.B)**

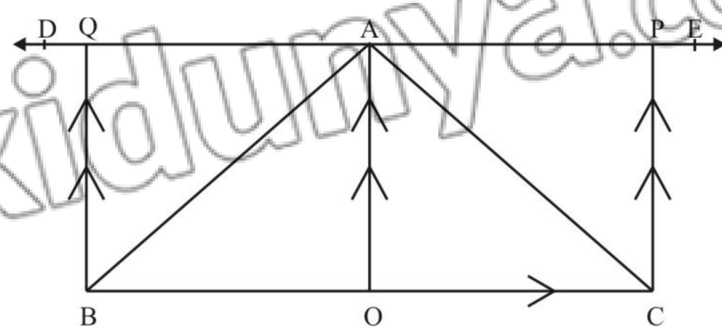
Given

ΔABC, O is the mid point of $\overline{BC}$

$\overline{OB} \cong \overline{OC}$

To prove

Area ΔABO = area ΔACO



Construction

Draw $\overline{DE} \parallel \overline{BC}$

$\overline{CP} \parallel \overline{OA}$

$\overline{BQ} \parallel \overline{OA}$

Proof

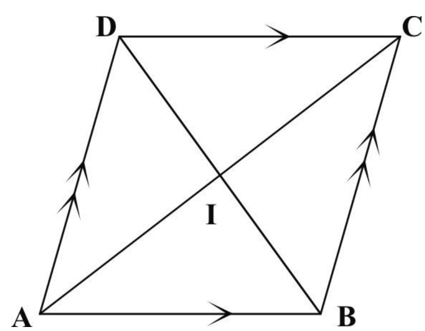
Statements	Reasons
$\overline{BQ} \parallel \overline{OA}$	Construction
$\overline{OB} \parallel \overline{AQ}$	Construction
$\parallel^{gm}$ BOAQ	Base of same
$\parallel^{gm}$ COAP	Parallel line of $\overleftrightarrow{DE}$
$\overline{OB} \cong \overline{OC}$	O is the mid point of $\overline{BC}$
Area of $\parallel^{gm}$ BOAQ= Area of $\parallel^{gm}$ COAP … (i)	
Area of $\Delta ABO = \frac{1}{2}$ Area of $\parallel^{gm}$ BOAQ	
Area of Δ ACO $= \frac{1}{2}$ Area of $\parallel^{gm}$ COAP	
Area of Δ ABO = Area of Δ ACO	Dividing equation (i) both side by (ii)

So median of a triangle divides it into two triangles of equal area.

Q.2 Prove that a parallelogram is divided by its diagonals into four triangles of equal area.

Given: **(K.B + U.B)**

In parallelogram ABCD, $\overline{AC}$ and $\overline{BD}$ are its diagonals, which meet at I



To prove:

Triangles ABI, BCI CDI and ADI have equal areas.

Proof:

Triangles ABC and ABD have the same base $\overline{AB}$ and are between the same parallel lines $\overline{AB}$ and $\overline{DC}$ $\therefore$ they have equal areas.

Or area of ΔABC =area of Δ ABD

Or area of Δ ABI + area of Δ BCI= area of Δ ABI+ area of Δ ADI

$\Rightarrow$ Area of Δ BCI = area of Δ ADI ... (i)

Similarly area of Δ ABC = area of Δ BCD

$\Rightarrow$ Area of Δ ABI +area of Δ BCI = area of Δ BCI + area of Δ CDI

$\Rightarrow$ Area of Δ ABI = area of Δ CDI… (ii)

As diagonals of a parallelogram bisect each other I is the midpoint of $\overline{AC}$ so $\overline{BI}$ is a median of Δ ABC

$\therefore$ Area of Δ ABI = area of Δ BCI… (iii)

$\Delta CDI \cong \Delta AOI$

$\overline{BI} \cong \overline{DI}$

Area of Δ ABI = area of Δ BCI = area of Δ CDI= area of Δ ADI

Q.3 Divide a triangle into six equal triangular parts **(K.B + U.B)**

Given

Δ ABC

To prove

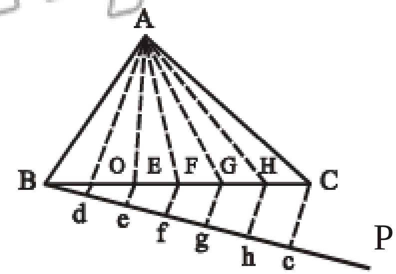
To divide Δ ABC into six equal part triangular parts

Construction

Take $\overrightarrow{BP}$ any ray making an acute angle with $\overline{BC}$ draw six arcs of the same radius on $\overrightarrow{BP}$ i.e $m\overline{Bd} = mde = mef = mfg = mgh = mhc$

Join c to C and parallel line segments as

$\overline{cC} \parallel \overline{hH} \parallel \overline{gG} \parallel \overline{fF} \parallel \overline{eE} \parallel \overline{do}$



Join A to O,E,F,G,H

Proof

Base $\overline{BC}$ of ΔABC has been divided to six equal parts.
We get six triangles having equal base and same altitude
Their area is equal
Hence $\Delta BOA = \Delta OEA = \Delta EFA = \Delta FGA = \Delta GHA = \Delta HCA$

Review Exercise 16

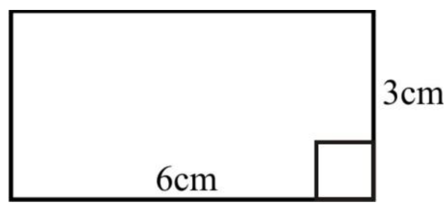
Q.1 Which of the following are true and which are false? (K.B + U.B + A.B)

(i) Area of a figure means region enclosed by bounding lines of closed figures. (True)
(ii) Similar figures have same area. (False)
(iii) Congruent figures have same area. (True)
(iv) A diagonal of a parallelogram divides it into two non-congruent triangles. (False)
(v) Altitude of a triangle means perpendicular from vertex to the opposite side (base). (True)
(vi) Area of a parallelogram is equal to the product of base and height. (True)

Q.2 Find the area of the following. (A.B + U.B)

(i) **(GRW 2016, FSD 2017, MTN 2013, BWP 2017, SGD 2015)**

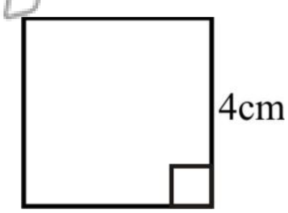
Given
Length of rectangle = ℓ = 3cm
Width of rectangle = w = 6cm



Required:
Area of rectangle =?
Solution:
Area of rectangle = length × width
= 3cm × 6cm
$\Rightarrow$ Area of rectangle = 18 cm^2

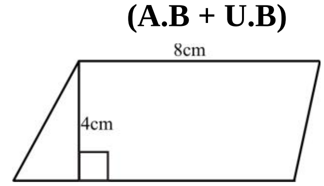
(ii) **(LHR 2013, GRW 2017, RWP 2017)**
(A.B + U.B)

Given
Length of square = ℓ = 4cm
Required:
Area of square =?
Solution:
Area of square = $\ell \times \ell$
= ℓ^2
= (4cm)^2
$\Rightarrow$ Area of square = 16cm^2



(iii)

Given **(A.B + U.B)**
Height of parallelogram = 4cm
Base of parallelogram = 8cm
Required:
Area of parallelogram = ?



Solution:

Area of parallelogram = b × h

= 8cm × 4cm

⇒ area of parallelogram = 32 cm^2

(iv) **(LHR 2017, MTN 2016)**

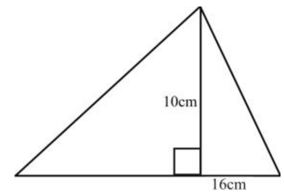
Given: **(K.B + U.B)**

Height of triangle = h = 10 m

Base of triangle = b = 16cm

Required:

Area of triangle =?



Solution:

Area of triangle = $\frac{1}{2} \times b \times h$

$= \frac{1}{\cancel{2}} \times {}^{8}\cancel{16}\ cm \times 10cm$

= 8cm × 10 cm

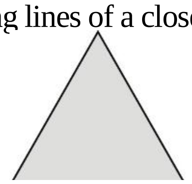
=80cm^2

Q.3 Define the following

(i) Area of a figure **(K.B + U.B)**

(LHR 2013, 14, GRW 2015, 17, FSD 2013, 14, BWP 2017, SGD 2017, D.G.K 2016)

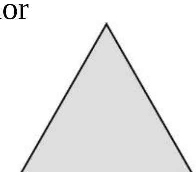
The region enclosed by the bounding lines of a closed figure is known as area of the figure.



(ii) Triangular Region **(K.B + A.B)**

(LHR 2015, FSD 2017, MTN 2015, 17, SWL 2016, SGD 2017, D.G.K 2014, 15)

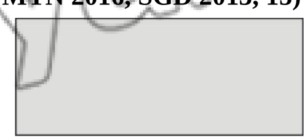
A triangular region is the union of a triangle and its interior i-e three line segments forming the triangle and its interior



(iii) Rectangular Region **(K.B + U.B)**

(LHR 2016, GRW 2013, 14, MTN 2016, SGD 2013, 15)

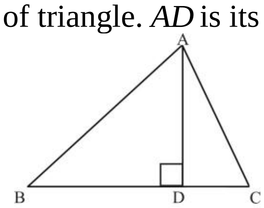
A rectangular region is the union of a rectangle and its interior. A rectangular region can be divided into two or more than two triangular regions in many ways.



(iv) Altitude or Height **(K.B + A.B)**

(SWL 2015, 17, BWP 2016, SGD 2015, MTN 2015)

If one side of a triangle is taken as its base, the perpendicular distance form one vertex opposite side is called altitude of triangle. $\overline{AD}$ is its altitude.



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SELF TEST

Time: 40 min **Marks: 25**

Q.1 Mark the Correct multiple choice question. **(7×1=7)**

1 The area of a closed region is expressed in _________ units.

(A) Square (B) Cubic
(C) Degree 1 (D) Degree 4

2 The _________ of a triangle is the part of the plane enclosed by the triangle.

(A) Exterior (B) Altitude
(C) Interior (D) Perpendicular

3 If $\Delta ABC \equiv \Delta PQR$ and $\Delta LMN \equiv \Delta PQR$ then area of ΔABC is equal to area of:

(A) ΔXYZ (B) ΔDEF
(C) ΔLMN (D) None of these

4 If length of rectangle is a units and width is b units, then area of rectangle is

(A) a + b (B) a – b
(C) a × b (D) a ÷ b

5 Parallelogram is divided by its one diagonal into _________ triangles of equal Area.

(A) Six (B) Four
(C) Two (D) Infinite

6 Similar figures have________ area.

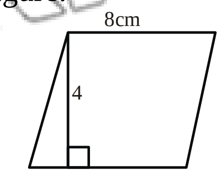
(A) Equal (B) Same
(C) Unequal (D) May be equal or may not be equal

7 ________of a triangle means perpendicular distance to base from its opposite vertex.

(A) Hypotenuse (B) Altitude
(C) Base (D) Acute angle

Q.2 Give Short Answers to following Questions. **(5×2=10)**

(i) Define rectangular region.
(ii) What is meant by two triangles between same parallel lines?
(iii) State congruent area Axiom.
(iv) The area of a parallelogram is equal to that of rectangle on the same base and having same altitude.
(v) Find the area of the following figure.



Q.3 Answer the following Questions in detail. **(8)**

Parallelogram on equal bases and having the same altitudes are equal in area.

Note:

Parents or guardians can conduct this test in their supervision in order to check the skill of students.