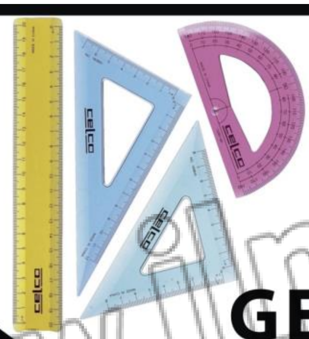


# UNIT 17
# PRACTICAL GEOMETRY - TRIANGLES



## Use of Geometrical Figures **(K.B + A.B)**

The knowledge of construction of triangles, rectangles, squares etc. is very useful in everyday life. Especially in the occupations of wood-working, graphic art and metal trade etc. Intermixing of geometrical figures is used to create artistic look. The geometrical constructions are usually made with the help of a pair of compasses, set squares divider and a straight edge.

## Note **(K.B + U.B)**

If the given line segments are too big or too small, a suitable scale may be taken for constructing the figure.

## Elements of a Triangle **(K.B + A.B)**

A triangle has six elements, three sides and three angles.

## Cases of Triangles **(K.B + U.B)**

There are six cases of triangles:

When,

**(i)** Three sides are given.

**(ii)** Two sides and including angle is given.

**(iii)** Two sides and non-including angle is given.

**(iv)** Two angles and including side is given.

**(v)** Two angles and including side is given.

**(vi)** Three angles are given.

## Note **(K.B + A.B)**

**(i)** In case (i), (ii), (iv), (v) unique triangle is formed.
**(ii)** In case (iii) (ambiguous case) two triangles are possible.
**(iii)** In case (v) infinite number of triangles is possible.



## Construction of Triangles **(K.B + U.B)**

**(a) To construct a triangle, having given two sides and the included angle.**

**Given**

Two sides, say

$m\overline{AB} = 4.6cm$ and $m\overline{AC} = 4cm$ and the included angle, $\angle A = 60°$

**Required**

To construct the $\Delta ABC$ using given information of sides and the included construction angle $\angle 60^\circ$.

**Steps of Construction**

**(i)** Draw a line segment $m\overline{AB} = 4.6cm$

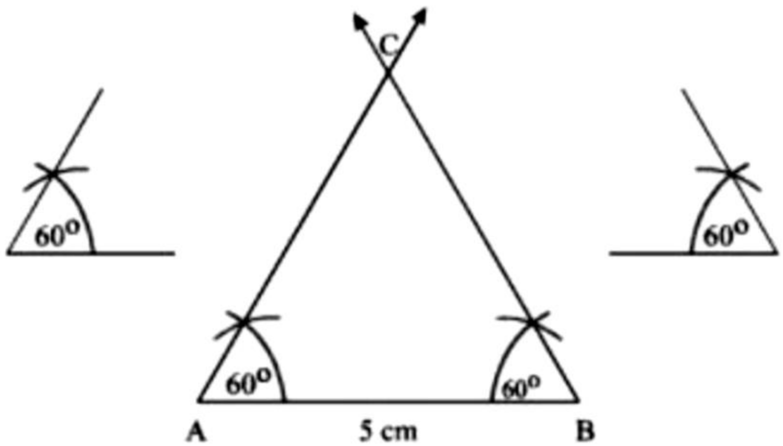
**(ii)** At the end $A$ of $\overline{AB}$ make $m\angle BAC = 60^\circ$

**(iii)** Cut of $m\overline{AC} = 4cm$ from the terminal side of $\angle 60^\circ$

**(iv)** Join $\overline{BC}$

**(v)** Then $ABC$ is required triangle.

**(b) To construct a triangle, having given one side and two of the angles.** **(A.B)**




**Given:**

The side $m\overline{AB} = 5cm$ say and two of the angle say, $m\angle A = 60^\circ$ and $m\angle B = 60^\circ$.

**Required:**

**To construct a $\Delta ABC$.**

**Steps of Construction**

**(i)** Draw the line segment $m\overline{AB} = 5cm$

**(ii)** At the end point $A$ of $\overline{AB}$ make $\angle BAC = \angle 60^\circ$

**(iii)** At the end point B of $\overline{BA}$ make $m\angle ABC = \angle 60^\circ$

**(iv)** The terminal sides of these two angles meet at $C$.

**(v)** The $ABC$ in the required $\Delta$.

**Note** **(A.B + U.B)**

When two angles of a triangle are given, the third angle can be found from the fact that the sum of three angles of triangle is 180°. Thus two angles being know, all the three are known, and we can take any two of these three angles as the base angles with given side as base.

**Ambiguous Case** **(K.B)**

A case of triangle in which non-including angle is acute and facing the shorter side. In this case, numbers of triangles are not conformed. There may be no triangle.

**(c) To construct a triangle having given two of its sides and the angel opposite to one of them.**

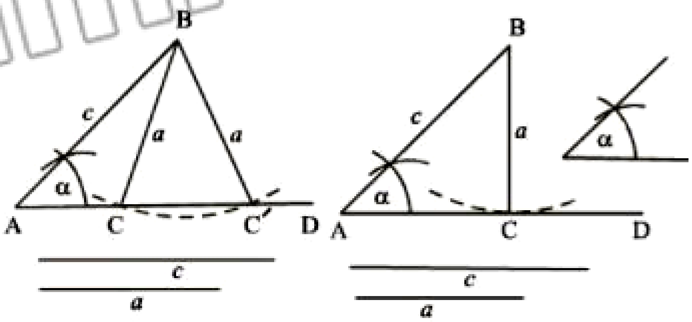



Figure (a) Figure (b)

**Given**

Two sides $a, c$ and $m\angle A = \alpha$ opposite to one of them, say $a$.

**Required**

To construct a triangle.

**Construction**

**(i)** Draw a line segment $AD$ of any length

**(ii)** At A make $m\angle DAB = m\angle A = \alpha$

**(iii)** Cut off $AB = C$

**(iv)** With centre B and radius equal to a, draw an arc.

**Three Cases Arise** **(K.B)**

**Case I:**

When the arc with radius $a$ cuts $\overline{AD}$ in two distant point $c$ and $c'$ as in Figures (a) joint $\overline{BC}$ and $\overline{BC}$

Then both the triangles $ABC$ and $ABC'$ have the given parts and are the required triangles.

**Case II:**

When the arc with radius a only touches $\overline{AD}$ at $C$, as figure (b).

Join $\overline{BC}$

Then $\Delta ABC$ is the required triangle right angled at $C$.

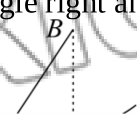



**Figure (c)**

**Case III:**

When the arc with radius a neither cuts nor touches $\overline{AD}$ as Figure (c).

There will no triangle in this case

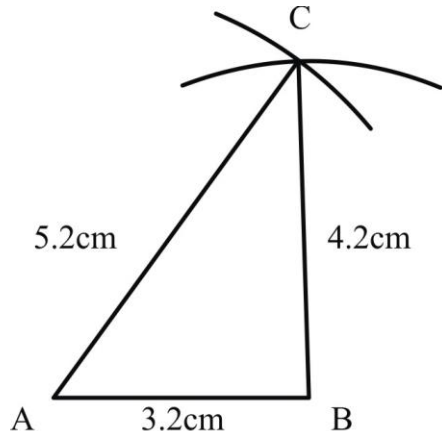
## Exercise 17.1

**Q.1 Construct a $\Delta$ ABC in which**

**(i)** $m\overline{AB}=3.2cm$ $m\overline{BC}=4.2cm$ $m\overline{CA}=5.2cm$

**(K.B + A.B)**

**(LHR 2013, 14, GRW 2013, 14, BWP 2017)**




i. Draw a line segment $m\overline{AB}=3.2cm$

ii. Taking A as centre draw an arc of radius 5.2cm.

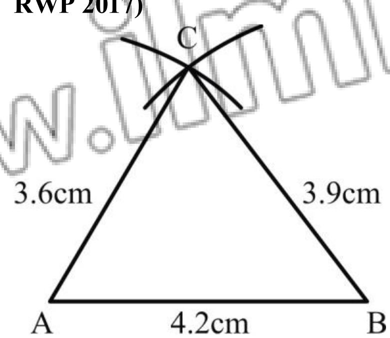
iii. Taking B as centre draw an arc of radius 4.2cm to cut at point C.

iv. Join C to A and C to B.
Thus $\Delta ABC$ is the required triangle.

**(ii)** $m\overline{AB}=4.2cm$ $m\overline{BC}=3.9cm$ $m\overline{CA}=3.6cm$

**(K.B + A.B)**

**(GRW 2013, FSD 2015, 17, MTN 2016, 17, RWP 2017)**




i. Draw a line segment $m\overline{AB}=4.2cm$

**(K.B + U.B)**

ii. Taking A as centre draw an arc of radius 3.6cm.

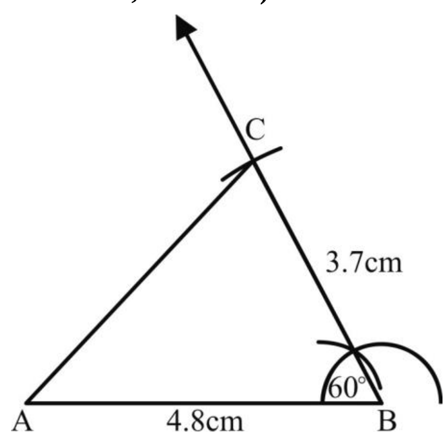
iii. Taking B as centre draw an arc of radius 3.9cm to cut at point C.

iv. Join C to A and C to B.
Thus $\Delta ABC$ is the required triangle.

**(iii)** $m\overline{AB}=4.8cm$ $m\overline{BC}=3.7cm$ $m\angle B=60^{o}$

**(K.B + U.B)**

**(LHR 2013, 15, 16, 17, GRW 2013, 17, SWL 2013, SGD 2016)**




i. Draw a line segment $m\overline{AB}=4.8cm$.

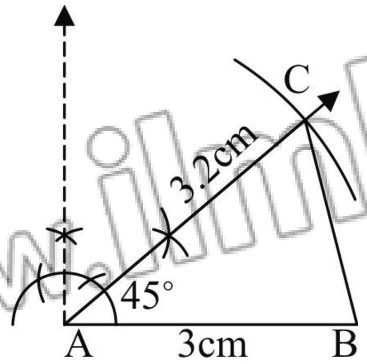
ii. Taking B as centre draw an angle of 60°.

iii. Taking B as centre draw an arc of radius 3.7cm cutting terminal side of 60° at C.

iv. Join C to A.
Thus $\Delta ABC$ is the required triangle.

**(iv)** $m\overline{AB}=3cm$ $m\overline{AC}=3.2cm$ $m\angle A=45^{o}$

**(U.B + A.B)**




**i.** Draw a line segment $m\overline{AB} = 3cm$.

**ii.** Taking A as centre draw an angle of 45°.

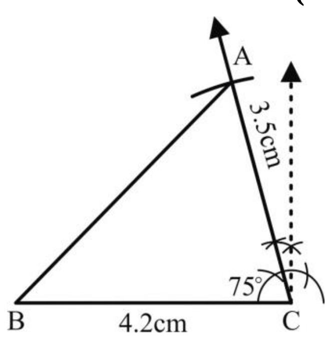
**iii.** Taking A as centre draw an arc of radius 3.2cm to cut the terminal side of angle at C.

**iv.** Join C to B.

Thus $\Delta ABC$ is the required triangle.

**(v)** $m\overline{BC} = 4.2cm$ $m\overline{CA} = 3.5cm$ $m\angle C = 75^o$

**(U.B + A.B)**




**i.** Draw a line segment $m\overline{BC} = 4.2cm$.

**ii.** Taking C as centre draw an angle of 75°.

**iii.** Taking C as centre draw an arc of radius 3.5cm.

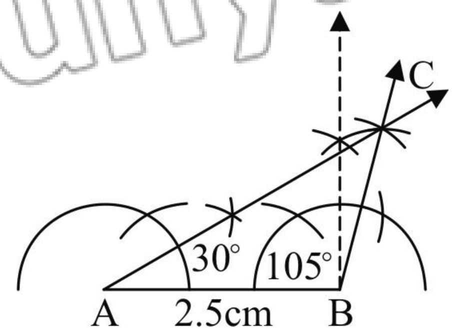
**iv.** Cutting the terminal side of angle at A.

**v.** Join A to B.

Thus $\Delta ABC$ is the required triangle.

**(vi)** $m\overline{AB} = 2.5cm$ $m\angle A = 30^o$ $m\angle B = 105^o$

**(U.B + A.B)**




**i.** Draw a line segment $m\overline{AB} = 2.5cm$.

**ii.** Taking A as centre draw an angle of 30°.

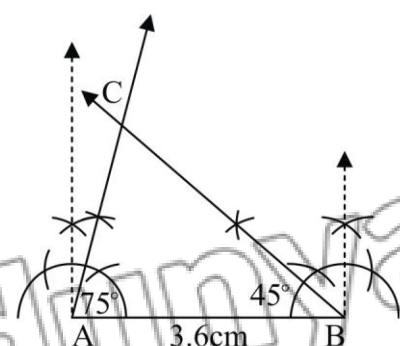
**iii.** Taking B as centre draw an angle of 105°.

**iv.** Terminal sides of these two angles meet at C.

Thus $\Delta ABC$ is the required triangle.

**(vii)** $m\overline{AB} = 3.6cm$ $m\angle A = 75^o$ $m\angle B = 45^o$

**(U.B + K.B)**




**i.** Draw a line segment $m\overline{AB} = 3.6cm$.

**ii.** Taking A as centre draw an angle of 75°.

**iii.** Taking B as centre draw an angle of 45°.

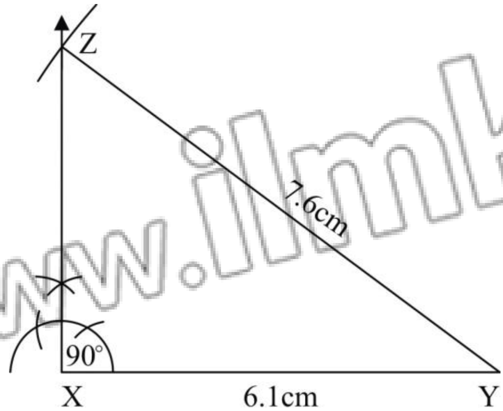
**iv.** Terminal sides of these two angles meet at point C.

Thus $\Delta ABC$ is the required triangle.

**Q.2 Construct a $\Delta XYZ$ in which**

**(i)** $m\overline{YZ} = 7.6cm$ $m\overline{XY} = 6.1cm$ $m\angle X = 90^o$

**(K.B + U.B)**




**i.** Draw a line segment $m\overline{XY} = 6.1cm$.

**ii.** Taking X as Centre draw an angle of 90°.

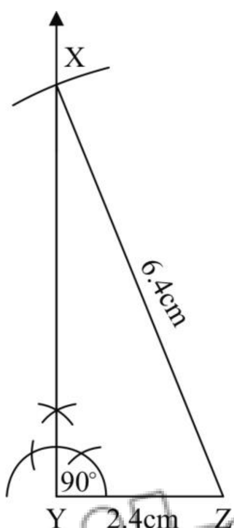
**iii.** Taking Y as Centre draw an arc of radius 7.6cm to cut terminal sides of angle at Z.

**iv.** Join Y to Z.

Thus $\Delta XYZ$ is the required triangle.

**(ii)** $m\overline{ZX} = 6.4cm \quad m\overline{YZ} = 2.4cm \quad m\angle Y = 90^o$

**(U.B + A.B)**




**(A.B)**

**i.** Draw a line segment $m\overline{YZ} = 2.4cm$.

**ii.** Taking Y as centre draw an angle of 90°.

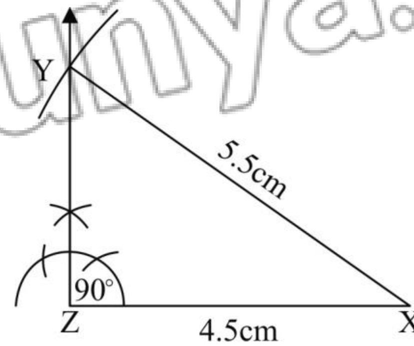
**iii.** Taking Z as centre draw an arc of radius 6.4cm. Which cuts the terminal side of angle at X.

**iv.** Join X and Z.

Thus $\Delta XYZ$ is the required triangle.

**(iii)** $m\overline{XY} = 5.5cm \quad m\overline{ZX} = 4.5cm \quad m\angle Z = 90^o$

**(U.B + A.B)**




**i.** Draw a line segment 4.5cm.

**ii.** Taking Z as centre draw an angle of 90°.

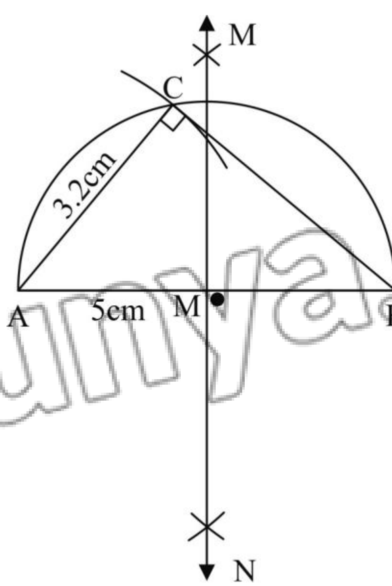
**iii.** Taking X as centre draw an arc of radius 5.5cm. Which cut the terminal side angle at Y.

**iv.** Join Y to X.

Thus $\Delta XYZ$ is the required triangle.

**Q.3 Construct a right angled Δ measure of whose hypotenuse is 5cm and one side is 3.2 cm**

**(A.B)**




**Construction:**

**i.** Draw a line segment $m\overline{AB}$=5cm.

**ii.** Bisect $\overline{AB}$ at M.

**iii.** Taking M as centre take a radius $\overline{AM}$ or $\overline{BM}$ and draw a semicircle.

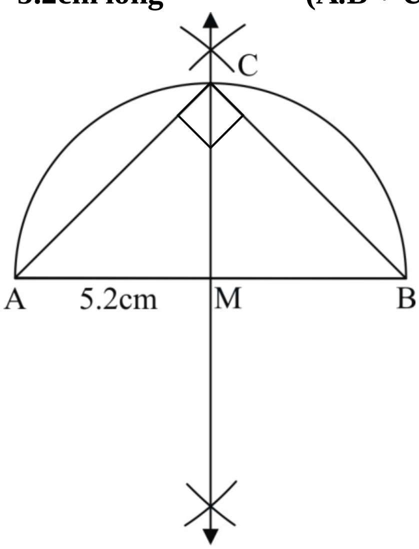
**iv.** Taking A as centre draw an arc of radius 3.2cm cutting semicircle at C.

**v.** Join C to A and C to B.

Thus $\Delta ABC$ is the required right angled triangle.

**Q.4 Construct right angled isosceles triangle whose hypotenuse is**

**(i) 5.2cm long (A.B + U.B)**




**Construction:**

**i.** Draw a line segment $m\overline{AB} = 5.2cm$.

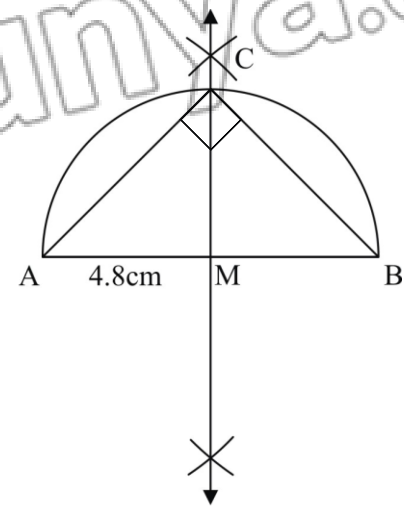
**ii.** Bisect $\overline{AB}$ at point M.

**iii.** With M as centre draw a semi circle of radius $\overline{AM}$ or $\overline{BM}$ which intersects the right bisector at C.

**iv.** Join A to C and B to C.

$\Delta$ABC is the required right angled isosceles triangle with $m\angle C = 90^{o}$.

**(ii) 4.8cm long (A.B + K.B)**
**(FSD 2015)**




**i.** Take a line segment $m\overline{AB} = 4.8cm$.

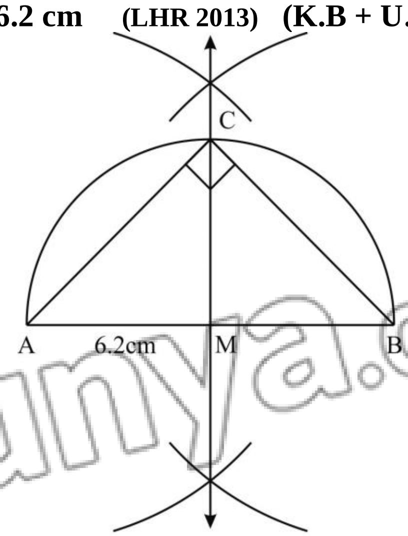
**ii.** Bisect $\overline{AB}$ at point M.

**iii.** Taking M as centre draw a semi circle of radius $\overline{AM}$ or $\overline{MB}$ which intersects the right bisector at C.

**iv.** Join A to C and B to C.

Thus ABC is the right angled isosceles triangle with $\angle C = 90^{o}$.

**(iii) 6.2 cm (LHR 2013) (K.B + U.B)**




**i.** Take a line segment $m\overline{AB} = 6.2cm$.

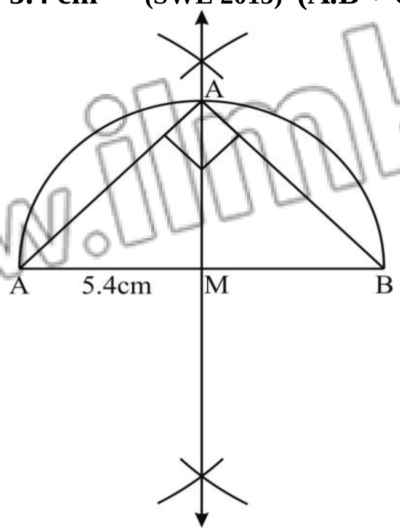
**ii.** Bisect $\overline{AB}$ at point M.

**iii.** Taking M as a centre draw a semi circle of radius $\overline{AM}$ or $\overline{BM}$ which intersects the right bisector at C.

**iv.** Join A to C and B to C.

Thus $\Delta$ABC is the right angled isosceles triangle with $\angle C = 90^{o}$.

**(iv)** **5.4 cm** **(SWL 2015)** **(A.B + U.B)**




**Construction:**

**i.** Take a line segment $m\overline{AB} = 5.4cm$.

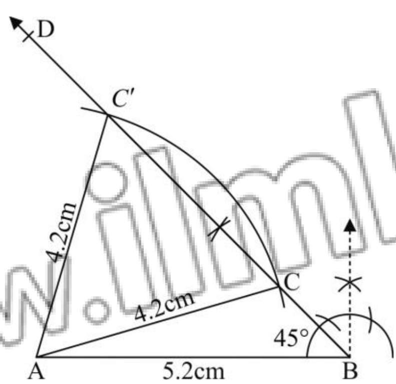
**ii.** Bisect $\overline{AB}$ at point M.

**iii.** Taking M as a centre draw a semi circle of radius $\overline{AM}$ or $\overline{BM}$ which intersects the right bisector at C.

**iv.** Join A to C and B to C.
Thus ΔABC is the right angled isosceles triangle with $\angle C = 90^o$.

**Q.5** **(Ambiguous case) Construct a Δ ABC in which (A.B + U.B + K.B)**

**(i)** $m\overline{AC} = 4.2cm$ $m\overline{AB} = 5.2cm$ $m\angle B = 45^o$




**Construction:**

**i.** Draw a line segment $m\overline{AB} = 5.2cm$.

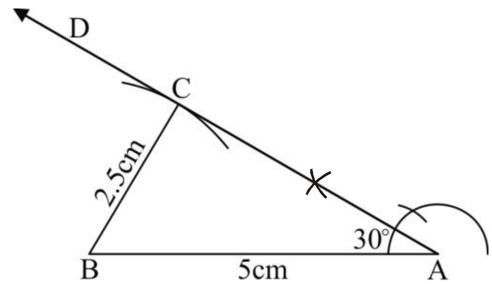
**ii.** At the end point B of $\overline{BA}$ make $\angle B = 45^o$.

**iii.** With centre at A and radius 4.2cm draw an arc which cuts $\overrightarrow{BD}$ in two distinct points C and C`.

**iv.** Draw $\overline{AC}$ and $\overline{AC'}$.

∴ ΔABC and ΔABC are required triangles.

**(ii)** $m\overline{BC} = 2.5cm$ $m\overline{AB} = 5cm$ $m\angle A = 30^o$




**Construction:**

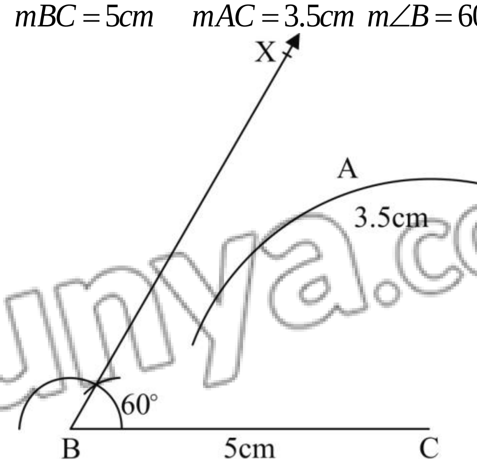
**i.** Take a line segment $m\overline{AB} = 5cm$.

**ii.** At the end point A of $\overline{AB}$ make $m\angle A = 30^o$.

**iii.** Taking B as centre draw an arc of radius 2.5cm which touch as $\overrightarrow{AD}$ at point C.

**iv.** Join B to C.
∴ ΔABC is required triangle.

**(iii)** $m\overline{BC} = 5cm$ $m\overline{AC} = 3.5cm$ $m\angle B = 60^o$




**Construction:**

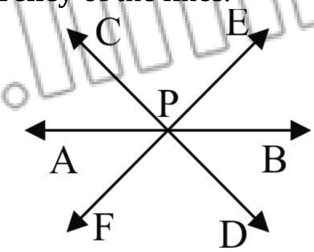
**i.** Take a line segment $m\overline{BC} = 5cm$.

**ii.** At the end point B of $\overline{BC}$ make an angle of $\angle B = 60^o$.

**iii.** Taking C as centre draw an arc of radius 3.5cm which does not touches or intersects $\overrightarrow{BX}$ at any point.
∴ ΔABC is not possible.

**Concurrent Lines:** **(A.B + U.B)**

Three or more then three lines are said to be concurrent, if they all pass through the same point. The common point is called the **point of concurrency** of the lines.



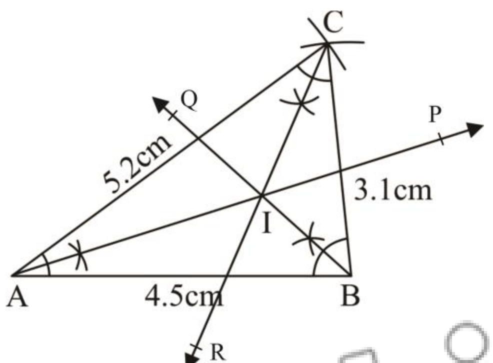

In the given figure lines AB, CD and EF are concurrent lines and point P is point of concurrency.

## In centre of the Triangle (A.B + K.B)

**(LHR 2016, GRW 2016, MTN 2015, SWL 2016, 17, D.G.K 2016, 17)**

The internal bisectors of the angles of a triangle meet at a point called the incentre of the triangle.

It is denoted by I.



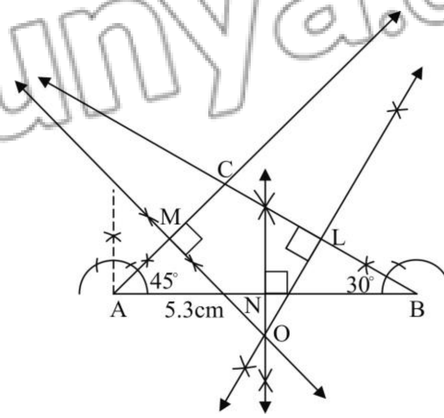

## Circumcentre of the Triangle:

**(K.B + U.B)**

**(LHR 2016, 17, GRW 2015, 16, 17, SWL 2016, FSD 2017, MTN 2017, RWP 2017, SGD 2016, 17, D.G.K 2017)**

The point of concurrency of the three perpendicular bisectors of the side of triangle is called the circumcentre of the triangle.

It is denoted by O.

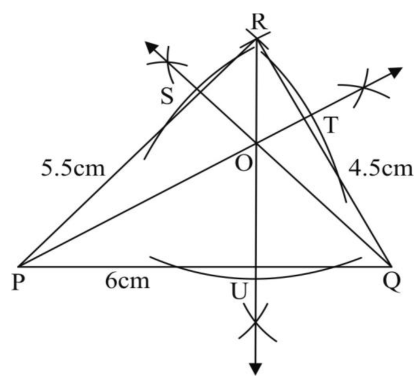



## Orthocentre of the Triangle:

**(K.B + A.B)**

The point of concurrency of three altitudes of a triangle meet in called its orthocentre.

It is denoted by O.

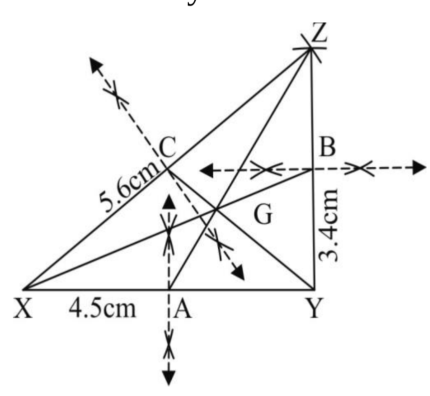



## Centroid of the Triangle:

**(LHR 2013, 17, RWP 2016) (A.B)**

The point where the three medians of a triangle meet is called the centroid of the triangle.

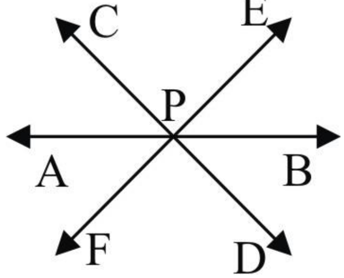
It is denoted by G.



**(i) Point of concurrency**

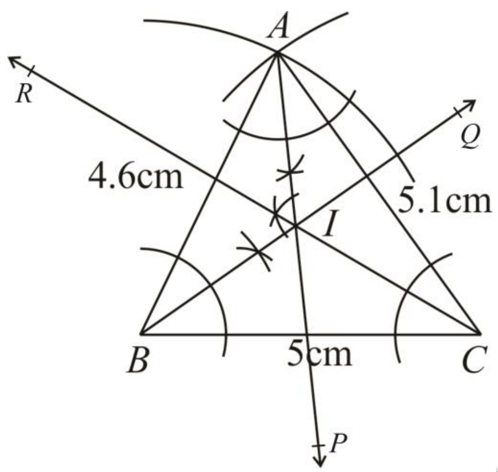
**(LHR 2014, SGD 2015, 16) (A.B + U.B)**

Three are more than three lines are said to be concurrent if these lines pass through the same point and that point is called the point of concurrency. In the figure, $P$ is the point of concurrency.




**(a) Draw Angle Bisects of a given Triangle** **(A.B + K.B)**

**Example:**




**(i)** Construct a $\Delta ABC$ having given $m\overline{AB} = 4.6cm$, $m\overline{BC} = 5cm$ and $m\overline{CA} = 5.1cm$.

**(ii)** Draw its angle bisectors and verify that they are concurrent

**Given**

The sides $m\overline{AB} = 4.6cm, m\overline{BC} = 5cm$ and $m\overline{CA} = 5.1cm$ of a $\Delta ABC$

**Required**

**(i)** To construct $\Delta ABC$

**(ii)** To draw its angle bisectors and verify their concurrency.

**Construction:**

**(i)** Take $m\overline{BC} = 5cm$

**(ii)** With $B$ as centre and radius $m\overline{BA} = 4.6cm$ draw an arc.

**(iii)** With $C$ as centre and radius $m\overline{CA} = 5.1cm$ draw an arc draw another arc which intersects the first arc at $A$.

**(iv)** Join $\overline{BA}$ and $\overline{CA}$ to complete the $\Delta ABC$.

**(v)** Draw bisectors of $\angle B$ and $\angle C$ meeting each other in the point I.

**(vi)** Now draw bisector of the third $\angle A$

**(vii)** We observe that the third angle bisector also passes through point I.

**(viii)** Hence the angle bisectors of the $\Delta ABC$ are concurrent at I, which lies with in the triangle.

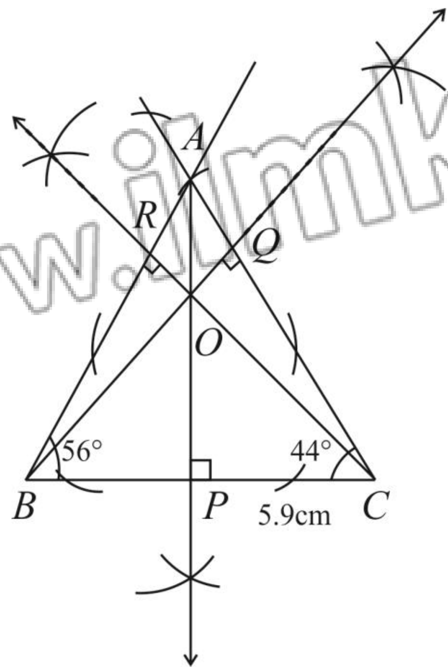
**(b) Draw Altitude of a given Triangle and Verify their Concurrency**

**(A.B + U.B + K.B)**

**Example:**

**(i)** Construct a $\Delta ABC$ in which $m\overline{BC} = 5.9cm$, $m\angle B = 56°$ and $m\angle C = 44°$

**(ii)** Draw the altitudes of the triangle and verify that they are concurrent.




**Given**

The side Required $m\overline{BC} = 5.9cm$ and $m\angle B = 56°$, $m\angle C = 44°$

**(i)** To construct the $\Delta ABC$

**(ii)** To draw its altitudes and verify their concurrency.

**Construction:**

**(i)** Take $m\overline{BC} = 5.9cm$

**(ii)** Using protector draw $m\angle CBA = 56°$ and $m\angle BCA = 44°$ to complete the $\Delta ABC$.

**(iii)** From the vertex $A$ drop $\overline{AP} \perp \overline{BC}$.

**(iv)** From the vertex $B$ drop $\overline{BQ} \perp \overline{CA}$. these two altitudes meet in the point $O$ inside the $\Delta ABC$

**(v)** Now from the vertex C drop. $\overline{CR} \perp \overline{AB}$

**(vi)** We observe that this third altitude also passes through the point of intersection $O$ of the first two altitude also passes through the point of intersection $O$ of the first two altitudes (vii) Hence the three altitudes of $\Delta ABC$ are concurrent at O.

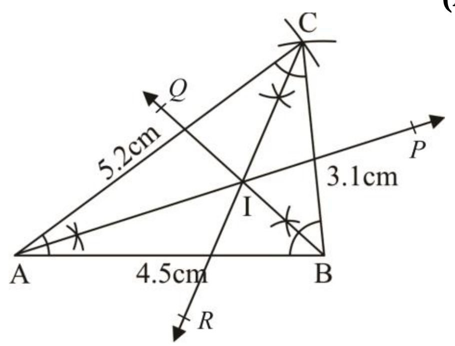
## Exercise 17.2

**Q.1 Construct the following $\Delta$'s ABC. Draw the Bisector of their angle and verify their Concurrency.**

**(A.B + U.B + K.B)**

**(i)** $mAB = 4.5cm$ $m\overline{BC} = 3.1cm$ $m\overline{CA} = 5.2cm$

**(A.B)**




**Steps of Constructions:**

**i.** Construct triangle ABC with given information.

**ii.** Draw $\overrightarrow{AL}$ bisector of $\angle A$.

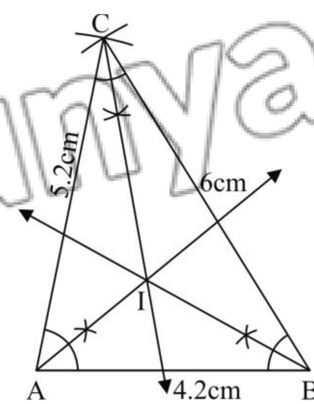
**iii.** Draw $\overrightarrow{BM}$ bisector of $\angle B$.

**iv.** Draw $\overrightarrow{CN}$ bisector of $\angle C$.

**v.** Bisectors of $\angle A, \angle B$ and $\angle C$ intersect each other at point I. Hence, angle bisectors of $\Delta ABC$ are concurrent.

**(ii)** $mAB = 4.2cm$ $m\overline{BC} = 6cm$ $m\overline{CA} = 5.2cm$

**(K.B)**




**Steps of Constructions:**

**i.** Construct triangle ABC with given information.

**ii.** Draw $\overrightarrow{AL}$ bisector of $\angle A$.

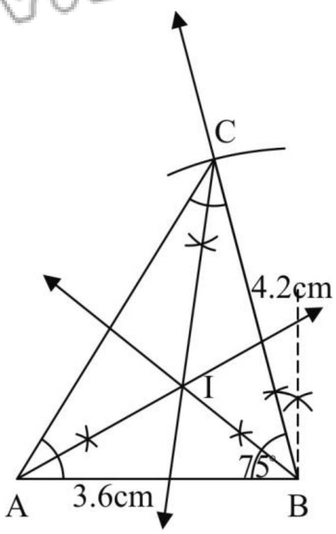
**iii.** Draw $\overrightarrow{BM}$ bisector of $\angle B$.

**iv.** Draw $\overrightarrow{CN}$ bisector of $\angle C$.

**v.** Bisectors of $\angle A, \angle B$ and $\angle C$ intersect each other at point I.
Hence, angle bisectors of $\Delta ABC$ are concurrent.

**(iii)** $mAB = 3.6cm$ $m\overline{BC} = 4.2cm$ $m\angle B = 75^{o}$

**(U.B)**




**Steps of Constructions:**

**i.** Construct triangle ABC with given information.

**ii.** Draw $\overrightarrow{AL}$ bisector of $\angle A$.

**iii.** Draw $\overrightarrow{BM}$ bisector of $\angle B$.

**iv.** Draw $\overrightarrow{CN}$ bisector of $\angle C$.

**v.** Bisectors of $\angle A, \angle B$ and $\angle C$ intersect each other at point I.
Hence, angle bisectors of $\Delta ABC$ are concurrent.

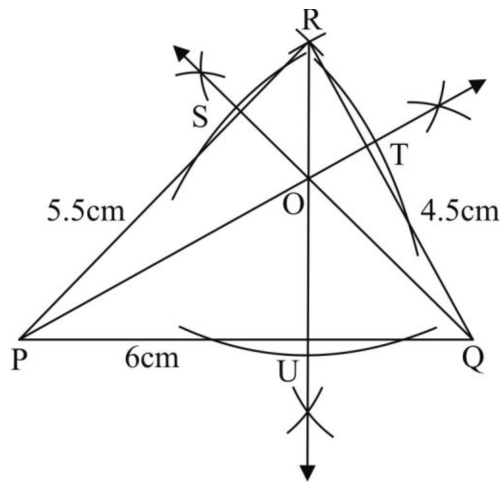
**Note**

- Angle bisectors of **all types** of triangle intersect each other **inside** the triangle.

**Q.2** **Construct the following triangles PQR. Draw their altitudes and show that they are concurrent.**

**(i)** $m\overline{PQ} = 6cm, m\overline{QR} = 4.5cm$ and $m\overline{PR} = 5.5cm$

**(A.B)**




**Steps of Constructions:**

**i.** Construct triangle PQR with given information.

**ii.** From point P draw $\overline{PT} \perp \overline{QR}$.

**iii.** From point Q draw $\overline{QS} \perp \overline{PR}$.

**iv.** From point R draw $\overline{RU} \perp \overline{PQ}$.

**v.** Altitudes intersect each other at point O.
Hence, altitudes of $\Delta PQR$ are concurrent.

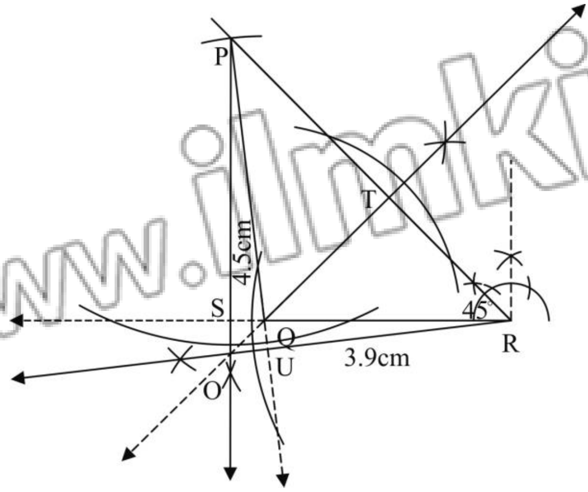
**(ii)** $m\overline{PQ} = 4.5cm$ $m\overline{QR} = 3.9cm$ $m\angle R = 45^{o}$

**(K.B)**

**Required:**

**i.** To construct ΔPQR.

**ii.** To draw altitudes and verify their concurrency.




**Steps of Constructions:**

**i.** Construct triangle PQR with given information.

**ii.** From point P draw $\overline{PS} \perp \overline{QR}$.

**iii.** From point Q draw $\overline{QT} \perp \overline{PR}$.

**iv.** From point R draw $\overline{RU} \perp \overline{PQ}$.

**v.** Altitudes intersect each other at point O.

Hence, altitudes of $\Delta PQR$ are concurrent.

**(iii)** $m\overline{RP} = 3.6cm$ $m\angle Q = 30°$ $m\angle P = 105°$

**(U.B)**

Sum of three angles in a triangle is 180° so,

$\angle P + \angle Q + \angle R = 180°$

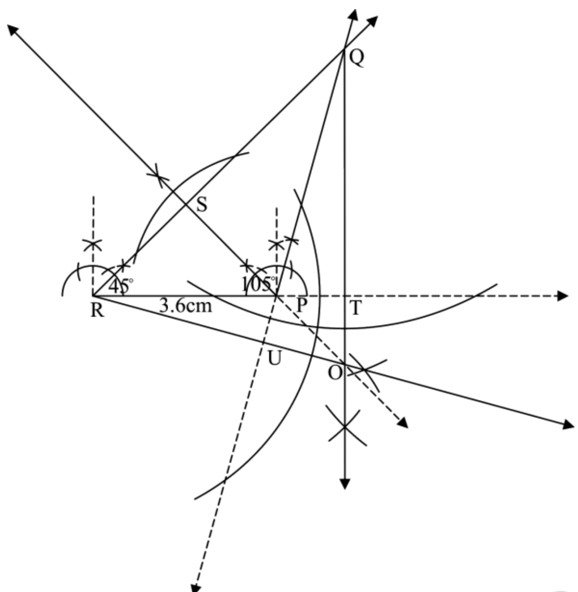
$105 + 30 + \angle R = 180°$

$135 + \angle R = 180°$

$\angle R = 180° - 135°$

$\angle R = 45°$

So




**Steps of Constructions:**

**i.** Construct triangle PQR with given information.

**ii.** From point P draw $\overline{PS} \perp \overline{QR}$.

**iii.** From point Q draw $\overline{QT} \perp \overline{PR}$.

**iv.** From point R draw $\overline{RU} \perp \overline{PQ}$.

**v.** Altitudes intersect each other at point O.

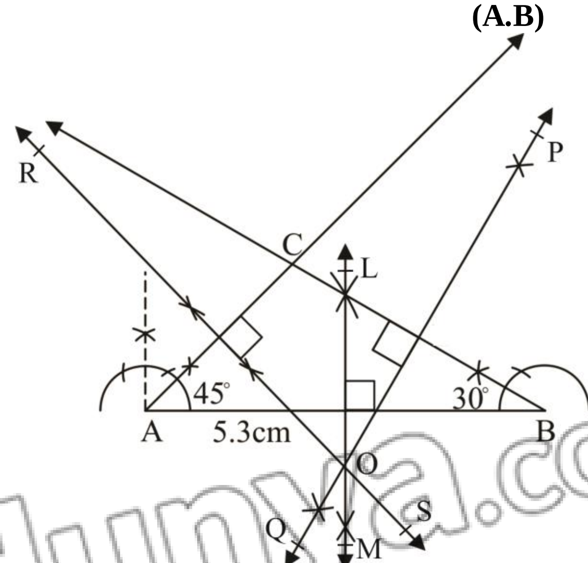
Hence, altitudes of $\Delta PQR$ are concurrent.

**Note** **(U.B)**

- Altitudes of **acute** angled triangle intersect each other **inside** the triangle.
- Altitudes of **obtuse** angled triangle intersect each other **outside** the triangle.
- Altitudes of **right** angled triangle intersect each other **at vertex** of right angle.

**Q.3 Contract the following triangles ABC draw the perpendicular bisector of three sides and verify their concurrency. Do they meet inside the triangle?**

**(i)** $\overline{AB} = 5.3cm$ $m\angle A = 45°$ $m\angle B = 30°$ **(A.B)**




**Steps of Constructions:**

**i.** Construct triangle ABC with given information.

**ii.** Draw $\overleftrightarrow{LM} \perp \overline{AB}$, such that it bisect $\overline{AB}$.

**iii.** Draw $\overleftrightarrow{PQ} \perp \overline{BC}$, such that it bisect $\overline{BC}$.

iv. Draw $\overleftrightarrow{RS} \perp \overline{CA}$, such that it bisect $\overline{AC}$.

v. Perpendicular bisectors of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ intersect each other at point O.

Hence, perpendicular bisectors of sides of a $\Delta$ are concurrent.

**(ii)** $m\overline{BC} = 2.9cm$ $m\angle A = 30°$ $m\angle B = 60°$

The sum of three angles in a triangle is 180° then

$\angle A + \angle B + \angle C = 180°$

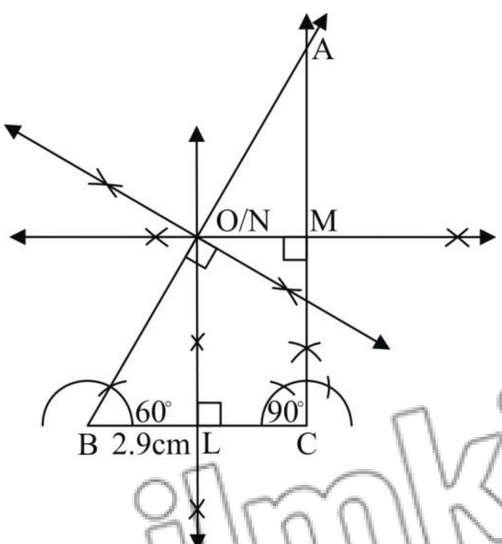
$30 + 60 + \angle C = 180°$

$90 + \angle C = 180°$

$\angle C = 180° - 90°$

$\angle C = 90°$

**(A.B)**




**Steps of Constructions:**

i. Construct triangle ABC with given information.

ii. Draw $\overleftrightarrow{LM} \perp \overline{AB}$, such that it bisect $\overline{AB}$.

iii. Draw $\overleftrightarrow{PQ} \perp \overline{BC}$, such that it bisect $\overline{BC}$.

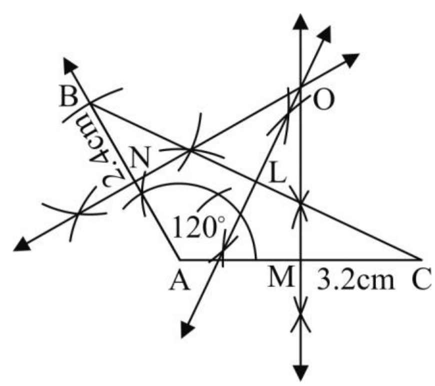
iv. Draw $\overleftrightarrow{RS} \perp \overline{CA}$, such that it bisect $\overline{AC}$.

v. Perpendicular bisectors of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ intersect each other at point O.

Hence, perpendicular bisectors of sides of a $\Delta$ are concurrent.

**(iii)** $m\overline{AB} = 2.4cm$ $m\overline{AC} = 3.2cm$ $m\angle A = 120°$

**(K.B)**




**Steps of Constructions:**

i. Construct triangle ABC with given information.

ii. Draw $\overleftrightarrow{LM} \perp \overline{AB}$, such that it bisect $\overline{AB}$.

iii. Draw $\overleftrightarrow{PQ} \perp \overline{BC}$, such that it bisect $\overline{BC}$.

iv. Draw $\overleftrightarrow{RS} \perp \overline{CA}$, such that it bisect $\overline{AC}$.

v. Perpendicular bisectors of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ intersect each other at point O.

Hence, perpendicular bisectors of sides of a $\Delta$ are concurrent.

**Note** **(K.B)**

- Right bisectors of **acute** angled triangle intersect each other **inside** the triangle.
- Right bisectors of **obtuse** angled triangle intersect each other **outside** the triangle.
- Right bisectors of **right** angled triangle intersect each other **mid of hypotenuse** the triangle.

**Q.4 Construct the following $\Delta s\ XYZ$. Draw their three medians and show that they are concurrent.**

**(i)** $m\overline{YZ} = 4.1cm \quad m\angle Y = 60° \quad m\angle X = 75°$ **(A.B)**

Sum of three angles in a triangle is 180° then

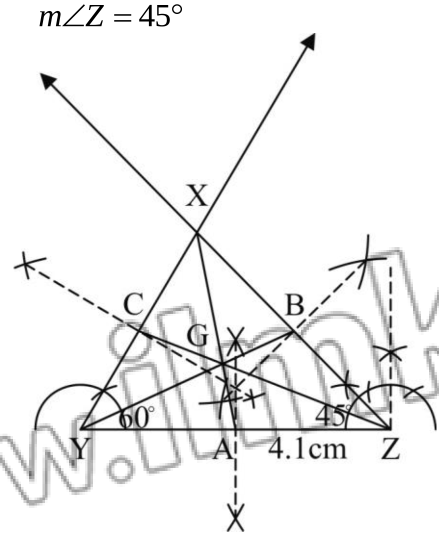
$m\angle X + m\angle Y + m\angle Z = 180°$

$75 + 60 + m\angle Z = 180°$

$135 + m\angle Z = 180°$

$m\angle Z = 180° - 135°$

$m\angle Z = 45°$




**Steps of Constructions:**

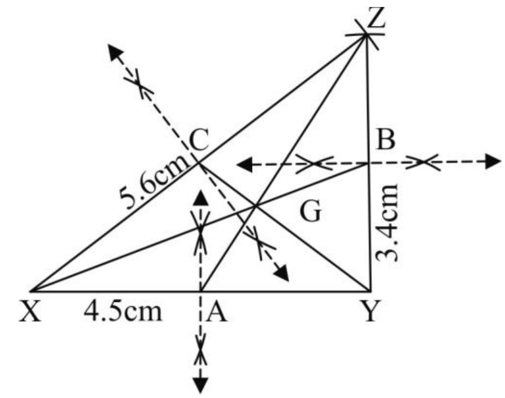
**i.** Construct triangle XYZ with given information.

**ii.** Find midpoints A, B and C of YZ, ZX and XY respectively.

**iii.** Join A to X, B to Y and C to Z.

**iv.** Thus, required medians are formed. They intersect each other at point G. Hence, medians a $\Delta$ are concurrent.

**(ii)** $m\overline{XY} = 4.5cm \quad m\overline{YZ} = 3.4cm \quad m\overline{ZX} = 5.6cm$ **(K.B)**




**Steps of Constructions:**

**i.** Construct triangle XYZ with given information.

**ii.** Find midpoints A, B and C of YZ, ZX and XY respectively.

**iii.** Join A to X, B to Y and C to Z.

**iv.** Thus, required medians are formed. They intersect each other at point G. Hence, medians a $\Delta$ are concurrent.

**(iii)** $m\overline{ZX} = 4.3cm \quad m\angle X = 75°$ and $m\angle Y = 45°$

Sum of three angles in a triangle is 180° then

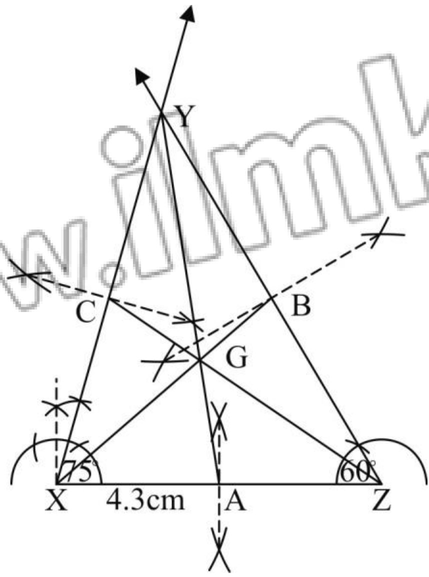
$m\angle X + m\angle Y + m\angle Z = 180°$

$75 + 45 + m\angle Z = 180°$

$120° + m\angle Z = 180°$

$m\angle Z = 180° - 120°$

$m\angle Z = 60°$




**Steps of Constructions:**

**i.** Construct triangle XYZ with given information.

**ii.** Find midpoints A, B and C of YZ, ZX and XY respectively.

**iii.** Join A to X, B to Y and C to Z.

**iv.** Thus, required medians are formed. They intersect each other at point G. Hence, medians a $\Delta$ are concurrent.

**Note** **(K.B)**

Medians of **all types** of triangle intersect each other **inside** the triangle.

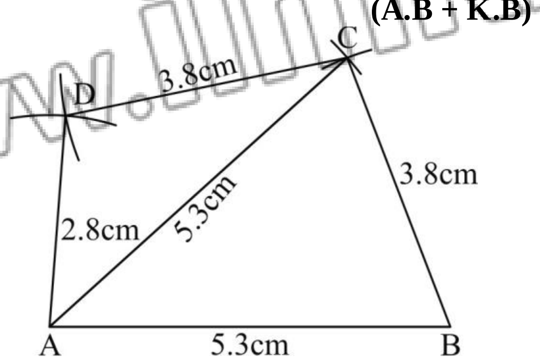
## Exercise 17.3

**Q.1**

**(i) Construction a quadrilateral ABCD, having**

$m\overline{AB} = \overline{AC} = 5.3cm \quad m\overline{BC} = m\overline{CD} = 3.8cm$ and $m\overline{AD} = 2.8cm$.

**(A.B + K.B)**




**Construction:**

**i.** Draw a line segment $\overline{AB} = 5.3cm$.

**ii.** Taking B as centre draw an arc of radius $\overline{BC} = 3.8cm$.

**iii.** Taking A as centre draw an arc of radius $\overline{AC} = 5.3cm$ to cut at C.

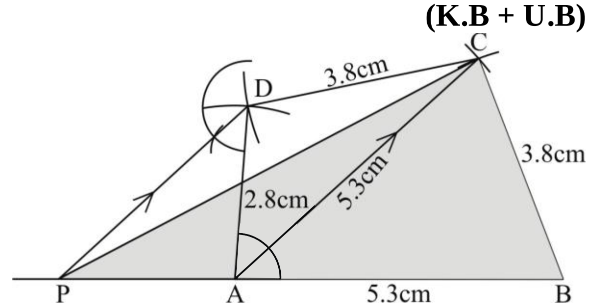
**iv.** Taking C as centre draw an arc of radius $\overline{CD} = 3.8cm$.

**v.** Taking A as centre draw an arc of radius $\overline{AB} = 2.8cm$ to cut at D.

**vi.** Join B to C, C to D, A to C and A to D. ABCD is the required quadrilateral.

**(ii) On the side $\overline{BC}$ construct a $\Delta$equal in area to the quadrilateral ABCD.**

**(K.B + U.B)**




**Construction:**

**i.** Join A to C.

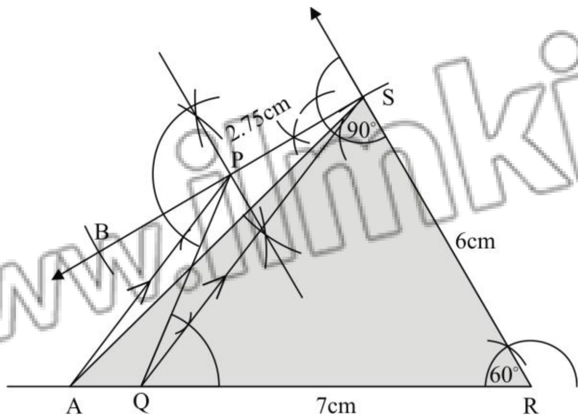
**ii.** Through D draw $\overline{DP} \parallel \overline{CA}$ meeting $\overline{BA}$ produced at P.

**iii.** Join $\overline{PC}$.

**iv.** Then PBC is required triangle.
$\Delta s\ APC, ADC$ stand on the same base AC and same parallels AC and PD.
Hence
$\Delta APC = \Delta ADC$
$\Delta APC + \Delta ABC = \Delta ADC + \Delta ABC$
or $\Delta PBC =$quadrilateral ABCD

**Q.2 Construct a $\Delta$ equal to the quadrilateral PQRS, having**
$m\overline{QR} = 7cm \quad m\overline{RS} = 6cm$
$m\overline{SP} = 2.75cm \quad m\angle QRS = 60°$ **and** $m\angle RSP = 90°$.

**(U.B + K.B)**




**Construction:**

**i.** Draw a line segment $\overline{QR} = 7cm$.

**ii.** At point R draw an angle of 60°.

**iii.** Taking R as center draw an arc of radius of 6cm to cut at S.

**iv.** At point S draw an angle 90°.

**v.** Taking S as centre draw an arc of radius of 5.5cm, cutting the terminal side of 90° at point B.

**vi.** Find the mid point of $m\overline{SB}$ at point P.

**vii.** Join P to Q.

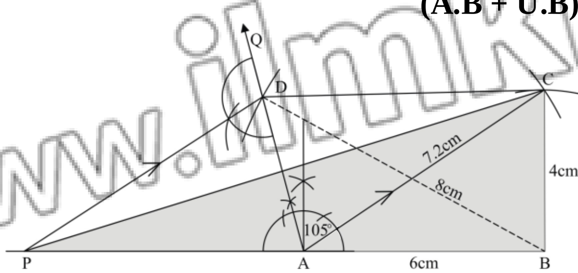
**viii.** Draw $\overline{PA}$ parallel to $\overline{SQ}$

**ix.** Join A to S.

**x.** ΔARS is required triangle equal in area to quadrilateral PQRS.

**Q.3 Construct a Δequal in area to quadrilateral ABCD having** $m\overline{AB} = 6cm$ $m\overline{BC} = 4cm,$ $\overline{AC} = 7.2cm$ $m\angle BAD = 105^{o}$ **and** $m\overline{BD} = 8cm$**.**

**(A.B + U.B)**




**Construction:**

**i.** Draw a line segment $\overline{AB} = 6cm$.

**ii.** Taking A as centre draw an arc of radius 7.2cm.

**iii.** Taking B as centre draw an arc of radius $4cm$ to cut at C. Join C to A and C to B.

**iv.** Taking A as centre make an angle $\angle QAB = 105^{o}$.

**v.** Taking B as centre make an arc of radius 8cm to cut at D point.

**vi.** Join D to C to complete the ABCD quadrilateral.

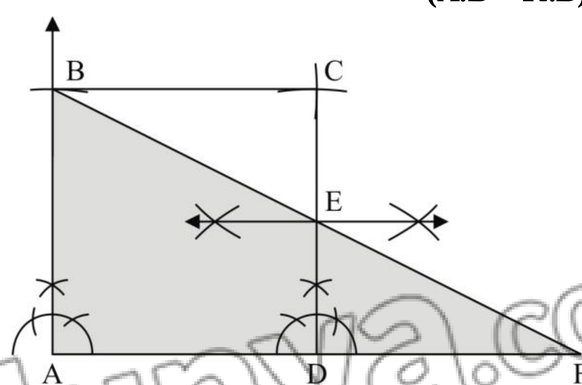
**vii.** Draw $\overline{DP} \parallel \overline{CA}$ o meet $\overrightarrow{BA}$ produced at P.

**viii.** Join C to P.

Thus $\Delta PBC$ is the required triangle.

**Q.4 Construct a right angled triangle equal in area to given square.**

**(A.B + K.B)**




**Construction:**

Let measurement of each side of square is 3.8cm.

**i.** Construct a square ABCD with each side 3.8cm long.

**ii.** Bisect $\overline{CD}$ at E.

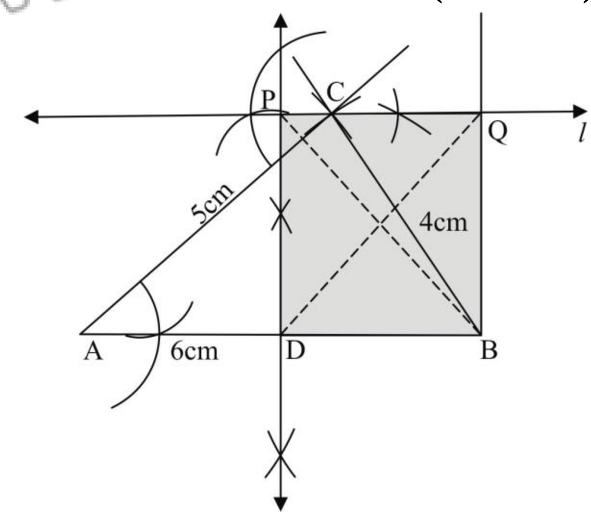
**iii.** Join B to E and produced it to meet $\overline{AD}$ produced in F.

ΔABF is required triangle equal in area to square ABCD.

## Exercise 17.4

**Q.1 Construct a Δ with sides 4cm, 5cm and 6cm and construct a rectangle having its area equal to that of the Δ measure its diagonals. Are they equal**

**(A.B + K.B)**




**Construction:**

**i.** Draw a line segment $\overline{AB} = 6cm$.

**ii.** Taking A as centre draw an arc of radius 5cm.

**iii.** Taking B as centre draw an arc of radius 4cm to cut at C. Join A to C and B to C.

**iv.** ABC is the requiredΔ.

**v.** Draw a line $l$ through C parallel to $\overline{AB}$.

**vi.** Draw the ⊥ bisector of $\overline{AB}$ in D and cutting the line at P.

**vii.** On the line $l$, cut $\overline{PQ}$ equal to $\overline{DB}$.

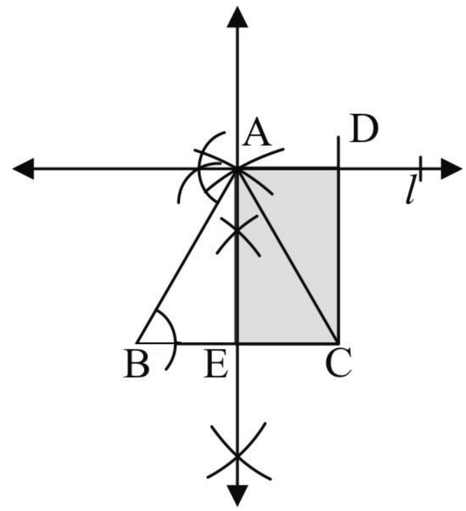
**viii.** Join B to Q.

**ix.** PQBD is the required rectangle.

**x.** The length of each diagonal measured to be 4.5cm.

**xi.** The length of each diagonal is same.

**Q.2 Transform an isosceles Δ into a rectangle. (U.B + A.B)**




**Construction:**

**i.** Draw a line segment $\overline{BC}$.

**ii.** With B as centre draw in arc of suitable radius.

**iii.** With C as centre draw another are of same radius which cuts the first arc at point A.

**iv.** Join A to B and A to C.

**v.** ΔABC is the isosceles Δ with $m\overline{AB} = m\overline{AC}$.

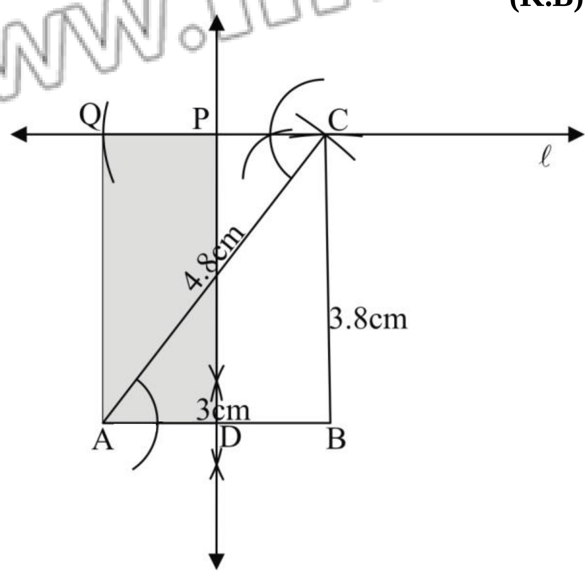
**vi.** Draw the perpendicular bisector of $\overline{BC}$ passing through point A.

**vii.** Through A draw a line $l \parallel \overrightarrow{BC}$.

**viii.** On $l$ cut $\overline{AD}$ equal to $\overline{EC}$ and the Join C with D.

**ix.** CDAE is the required rectangle equal in area to ΔABC.

**Q.3** **Construct a ABC such that $m\overline{AB} = 3cm$, $m\overline{BC} = 3.8cm$ and $m\overline{AC} = 4.8cm$. Construct a rectangle equal in area to the ΔABC, and measure its sides.** **(K.B)**




**Construction:**

**i.** Draw a line segment $\overline{AB} = 3cm$.

**ii.** Taking B as centre draw an arc of radius $\overline{BC} = 3.8cm$.

**iii.** Taking A as centre draw an arc of radius $\overline{AC} = 4.8cm$ to cut at C.

**iv.** Join C to A and C to B.

**v.** ABC is the requiredΔ.

**vi.** Through C draw a line *l* parallel $\overline{AB}$.

**vii.** Draw the ⊥ bisector of $\overline{AB}$ cutting the line *l* in P.

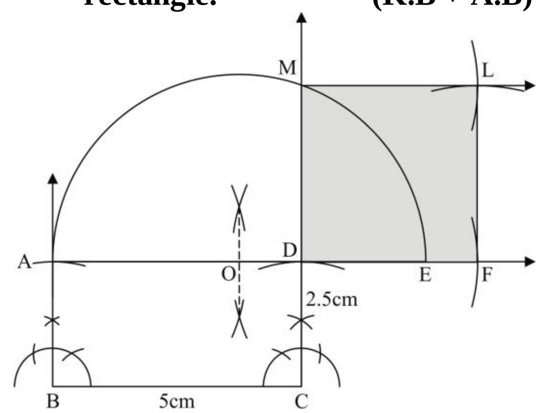
**viii.** On ℓ cut $\overline{PQ} \cong \overline{DA}$.

**ix.** PQAD is the required rectangle measure of sides of rectangle PQAD

$m\overline{PD}$ = 3.8cm $m\overline{AD}$ = 1.5cm

## Exercise 17.5

**Q.1** **Construct a rectangle whose adjacent sides are 2.5cm and 5cm respectively. Construct a square having area equal to the given rectangle.** **(K.B + A.B)**




**Construction:**

**i.** Make the rectangle ABCD with given lengths of sides.

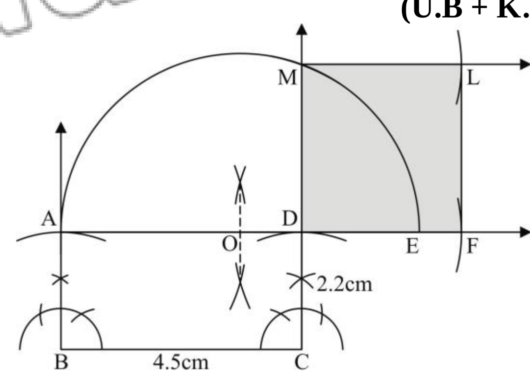
**ii.** Produce AD to point E such that $m\overline{DE} = m\overline{DC}$.

**iii.** Bisect $\overline{AE}$ at O.

**iv.** With O as centre and $\overline{OA}$ radius draw a semicircle cutting $\overline{CD}$ produced in M.

**v.** With $\overline{DM}$ as side complete the square $DFLM$.

**Q.2** **Construct a square equal in area to a rectangle whose adjacent sides are 4.5cm and 2.2cm respectively. Measure the sides of the square and find its area and compare with the area of the rectangle.** **(U.B + K.B)**



**Construction:**

**i.** Make the rectangle ABCD with given sides.

**ii.** Produce AD and cut $m\overline{DE} = m\overline{DC}$.

**iii.** Bisect $\overline{AE}$ at O.

**iv.** With O as centre and $\overline{OA}$ radius draw a semicircle cutting $\overline{CD}$ produced in M.

**v.** With $\overline{DM}$ as side complete the square $DF\angle M$.

**vi.** Side of the square (average) = 3.15cm

Area $= 3.15 \times 3.15 = 9.9cm^2$

Area of rectangle $= 2.2 \times 4.5 = 9.9cm^2$

Area of rectangle = Area of square

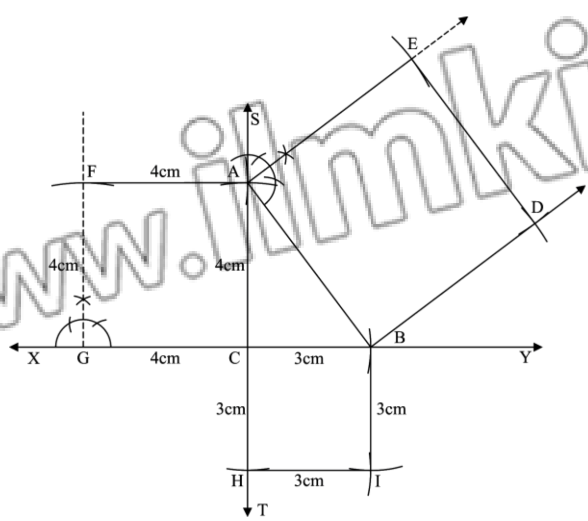
**Q.3 In Q2 above verify by measurement that the perimeter of the square is less then that of the rectangle. (A.B + U.B)**

Perimeter of rectangle = 2 [length + briclth]

= 2 [4.5 + 2.2]
= 2 [6.7]
= 13.4 cm

Perimeter of square = 4 × $l$
= 4 × 3.2
= 12.8 cm

**Q.4 Construct a square equal in area to the sum of two squares having sides 3cm and 4cm respectively. (K.B)**




**Construction:**

**i.** Draw a line segment $\overleftrightarrow{XY}$.

**ii.** Draw a line perpendicular $\overrightarrow{ST}$ at point C.

**iii.** Cut of $\overline{CB} = 3cm$ and $\overline{CG} = 4cm$.

**iv.** $\overline{CG}$ is the side of square complete the square ACGF.

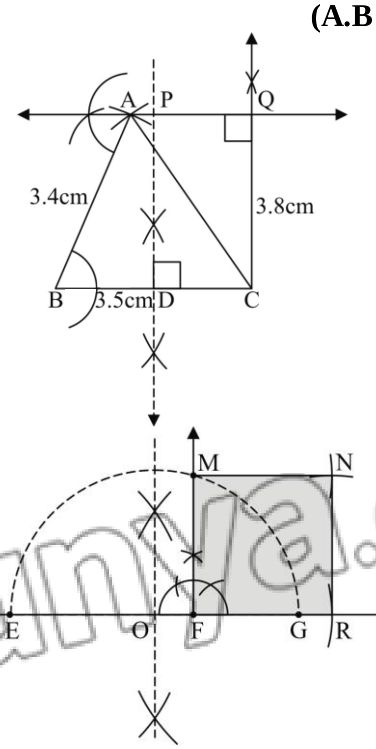
**v.** $\overline{CB}$ is the side of square complete the square CBIH.

**vi.** Join B to A.

**vii.** $\overline{AB}$ is the side of square so, complete the square ABDE.

**viii.** ABDE is the required square.
Using Pythagoras theorem to prove.

**Q.5 Construct a Δ having base 3.5cm and other two sides equal to 3.4cm and 3.8cm respectively. Transform it into a square of equal area (A.B + U.B)**




**Construction:**

**i.** Draw $\overleftrightarrow{PAQ} \parallel \overline{BC}$

**ii.** Draw perpendicular bisector of $\overline{BC}$, bisector it at D and meeting $\overleftrightarrow{PAQ}$ at P.

**iii.** Draw $\overline{CQ} \perp \overline{PQ}$ meeting it in Q.

**iv.** Take a line EFG and cut radius $\overline{EF} = \overline{DP}$ and $\overline{FG} = \overline{DC}$.

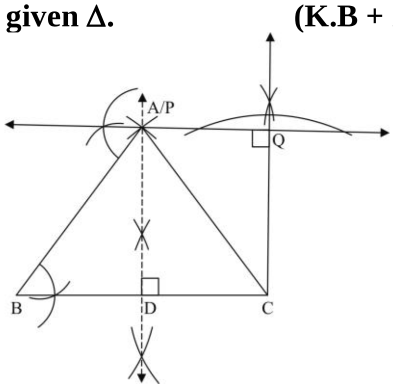
**v.** Bisect $\overline{EG}$ at O.

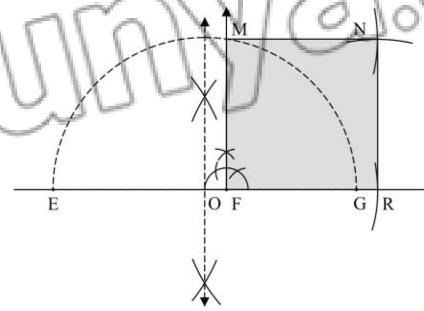
**vi.** With O as centre and radius $= \overline{OE}$ draw a semi-circle.

**vii.** At F draw $\overline{FM} \perp \overline{EG}$ meeting the semi-circle at M.

**viii.** With $\overline{MF}$ as a side, complete the required square FMNR.

**Q.6 Construct a $\Delta$ having base 5 and other sides equal to 5cm and 6cm construct a square equal in area to given $\Delta$. (K.B + A.B)**




**Construction:**

**i.** Draw $\overleftrightarrow{PAQ} \parallel \overline{BC}$

**ii.** Draw perpendicular bisector of $\overline{BC}$, bisector it at D and meeting $\overleftrightarrow{PAQ}$ at P.

**iii.** Draw $\overline{CQ} \perp \overline{PQ}$ meeting it in Q.

**iv.** Take a line EFG and cut radius $\overline{EF} = \overline{DP}$ and $\overline{FG} = \overline{DC}$.

**v.** Bisect $\overline{EG}$ at O.

**vi.** With O as centre and radius $= \overline{OE}$ draw a semi-circle.

**vii.** At F draw $\overline{FM} \perp \overline{EG}$ meeting the semi-circle at M.

**viii.** With $\overline{MF}$ as a side, complete the required square FMNR.

## Revised Exercise 17

**Q.1 Fill in the blanks to make the statements true:**

**(i)** The side of right angled triangle opposite to 90° is called _________.

**(ii)** The line segment joining a vertex of a triangle which is to the mid point of its opposite side is called a ____________.

**(iii)** A line drawn from a vertex of a triangle which is ________ to its opposite side is called an altitude of the triangle.

**(iv)** The bisectors of the three angles of a triangle are_________.

**(v)** The point of concurrency of right bisectors of the three sides of the triangle is __________ from its vertices.

**(vi)** Two or more triangles are said to be similar if they are equiangular and measures of their corresponding sides are_________.

**(vii)** The altitudes of a right triangle are concurrent at the ______of the right angle.

# ANSWER KEY

**(Fill in the Blank)**

| | | | |
|---|---|---|---|
| i | Hypotenuse | v | Equidistant |
| ii | Median | vi | Proportional |
| iii | Perpendicular | vii | Vertex |
| Iv | Concurrent | | |

**Q.2 Multiple Choice Questions. (Choose the correct answer).**

**(i) The triangle having two sides congruent is called**

**(a)** Scalene **(b)** Right angled

**(c)** Equilateral **(d)** Isosceles

**(ii) A quadrilateral having each angle equal to 90° is called**

**(a)** Parallelogram **(b)** Rectangle

**(c)** Trapezium **(d)** Rhombus

**(iii) The right bisectors of the three sides of a triangle are**

**(a)** Congruent **(b)** Collinear

**(c)** Concurrent **(d)** Parallel

**(iv) The __________ altitudes of an isosceles triangle are congruent.**

**(a)** Two **(b)** Three

**(c)** Four **(d)** None of these

**(v) A point equidistant from the end points of a line – segment is on its__________.**

**(a)** Bisector **(b)** Right - bisector

**(c)** Perpendicular **(d)** Median

**(vi) __________ congruent triangles can be made by joining the mid-points of the sides of a triangle.**

**(a)** Three **(b)** Four

**(c)** Five **(d)** Two

**(vii) The diagonals of parallelogram __________ each other.**

**(a)** Bisect **(b)** Trisect

**(c)** Bisect at right angle **(d)** None of these

**(viii) The medians of a triangle cut each other in the ration__________.**

**(a)** 4:1 **(b)** 3:1

**(c)** 2:1 **(d)** 1:1

**(ix)** **One angle on the base of an isosceles triangle is 30°. What is the measure of its vertical angle ________.**

(a) 30° (b) 60°

(c) 90° (d) 120°

**(x)** **If the three altitudes of a triangle are congruent then, the triangle will be____.**

(a) Isosceles (b) Equilateral

(c) Right angled (d) Acute angled

**(xi)** **If two medians of a triangle are congruent then the triangle will be_________.**

(a) Isosceles (b) Equilateral

(c) Right angled (d) Acute angled

**ANSWER KEY**

**(MCQ'S)**

| | | | |
|---|---|---|---|
| i | d | vii | a |
| ii | b | viii | c |
| iii | c | ix | d |
| iv | a | x | a |
| v | b | xi | a |
| vi | b | | |

## SELF TEST

**Time: 40 min** **Marks: 25**

**Q.1 Mark the Correct multiple choice question.** **(7×1=7)**

**1 The side of a right angled triangle opposite to 90° is called:**

**(A)** Base **(B)** Perpendicular
**(C)** Altitude **(D)** Hypotenuse

**2 The line segment joining a vertex of a triangle to the mid-point of its opposite side is called a __________**

**(A)** Angle **(B)** Altitude
**(C)** Median **(D)** Perpendicular bisector

**3 The medians of a triangle cut each other in the ratio**

**(A)** 4 : 1 **(B)** 3 : 1
**(C)** 2 : 1 **(D)** 1 : 1

**4 If two altitudes of a triangle are congruent then the triangle will be**

**(A)** Isosceles **(B)** Equilateral
**(C)** Right angled **(D)** Acute angled

**5 The bisectors of the three angles of a triangle are _________**

**(A)** Congruent **(B)** Equal
**(C)** Concurrent **(D)** Parallel

**6 One angle on the base of an isosceles triangle is 30°. What is measure of its vertical angle**

**(A)** 30° **(B)** 60°
**(C)** 90° **(D)** 120°

**7 ___________ congruent triangles can be made by joining the mid-points of the sides of a an equilateral triangle**

**(A)** Three **(B)** Four
**(C)** Five **(D)** Two

**Q.2 Give Short Answers to following Questions.** **(5×2=10)**

**(i)** Construct a ΔABC in which $mAB = 3.6\ cm,\ m\angle A = 75^o,\ m\angle B = 45^o$

**(ii)** Construct a right-angled Δ measure of whose hypotenuse is 5 cm and one side is 3.2 cm.

**(iii)** Define orthocenter of a triangle.

**(iv)** Write the names of any four polygons.

**(v)** Transform an isosceles triangle to a rectangle.

**Q.3 Answer the following Questions in detail.** **(4+4=8)**

**(a)** Construct the triangle XYZ. Draw their medians and show that they are concurrent, when $mYZ = 3.6$ cm, $m\angle Y = 75^o$, $m\angle X = 45^o$.

**(b)** Construct a triangle equal in area to the quadrilateral PQRS, having $mQR = 7cm,\ mRS = 6cm,\ mSP = 2.75cm,\ m\angle QRS = 60^o$, $m\angle RSP = 90^o$.

**Note:**

Parents or guardians can conduct this test in their supervision in order to check the skill of students.