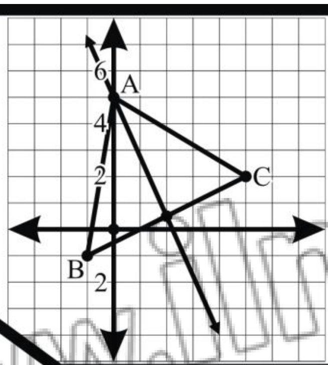


UNIT 9

INTRODUCTION TO COORDINATE GEOMETRY

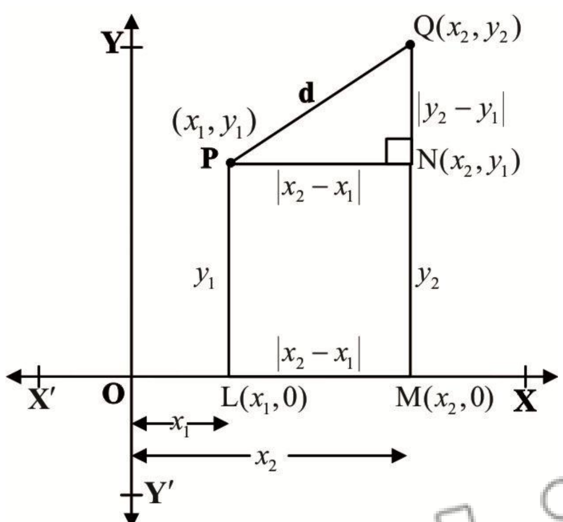
Coordinate Geometry **(K.B)**

(GRW 2013, FSD 2015, SWL 2015)

The study of geometrical shapes in a plane is called plane geometry. Coordinate geometry is the study of geometrical shapes in the Cartesian plane (Coordinate plane).

Distance Formula

1. Finding distance between two points.



Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be two points in the coordinate plane where d is the length of the line segment PQ i.e, $|PQ|$ = d

The line segments MQ and LP parallel to y-axis meet x-axis at point M and L respectively with coordinates M$(x_2, 0)$ and L $(x_1, 0)$.

The line segment PN is parallel to x-axis

In the right triangle PNQ

$\left|\overline{NQ}\right| = |y_2 - y_1|$ and $\left|\overline{PN}\right| = |x_2 - x_1|$

Using Pythagoras theorem

$$\left(\overline{PQ}\right)^2 = \left(\overline{PN}\right)^2 + \left(\overline{QN}\right)^2$$

$$d^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2$$

Taking square root on both sides

$$\sqrt{d^2} = \pm\sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

Since d > 0 always

Use of Distance Formula **(K.B)**

Example # 1

Using distance formula, find the distance between the points $P(1,2)$ *and* $Q(0,3)$.

$$\left|\overline{PQ}\right| = \sqrt{(0-1)^2 + (3-2)^2}$$

$$= \sqrt{(-1)^2 + (1)^2}$$

$$= \sqrt{1+1}$$

$$= \sqrt{2}$$

Exercise 9.1

Q.1 Find the distance between the following pairs of points

Solution:

(a) $A(9,2), B(7,2)$

(LHR 2016, GRW 2014, 17, FSD 2014, 15, SWL 2016, 17, D.G.K 2013)

Distance $= \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$$|AB| = \sqrt{|7-9|^2 + |2-2|^2}$$

$$|AB|=\sqrt{(-2)^2+(0)^2}$$
$$|AB|=\sqrt{4}$$
$$|AB|=2$$

(b) $A(2,-6),B(3,-6)$ **(K.B + A.B + U.B)**

(LHR 2017, MTN 2016, SWL 2014, RWP 2016, D.G.K 2014, 15, 16)

Distance $=\sqrt{|x_2-x_1|^2+|y_2-y_1|^2}$
$$|AB|=\sqrt{|3-2|^2+|-6-(-6)|^2}$$
$$|AB|=\sqrt{(1)^2+(-6+6)^2}$$
$$|AB|=\sqrt{1+(0)^2}$$
$$|AB|=\sqrt{1}$$
$$|AB|=1$$

(c) $A(-8,1),B(6,1)$ **(K.B + A.B + U.B)**

(LHR 2015, 17, GRW 2015, FSD 2013, 15, MTN 2013, 17, BWP 2013, SGD 2016, 17, D.G.K 2014)

Distance $=\sqrt{|x_2-x_1|^2+|y_2-y_1|^2}$
$$|AB|=\sqrt{|6-(-8)|^2+|1-1|^2}$$
$$|AB|=\sqrt{(6+8)^2+(0)^2}$$
$$|AB|=\sqrt{(14)^2}$$
$$|AB|=14$$

(d) $A(-4,\sqrt{2}),B(-4,-3)$ **(K.B+A.B+U.B)**

(LHR 2013, 15, SWL 2015, MTN 2013, BWP 2014, D.G.K 2017, SGD 2017)

$$d=\sqrt{|x_2-x_1|^2+|y_2-y_1|^2}$$
$$|AB|=\sqrt{|-4-(-4)|^2+|-3-\sqrt{2}|^2}$$
$$|AB|=\sqrt{(-4+4)^2+\left(-(3+\sqrt{2})\right)^2}$$
$$|AB|=\sqrt{(0)^2+(3+\sqrt{2})^2}$$
$$|AB|=\sqrt{(3+\sqrt{2})^2}$$
$$|AB|=3+\sqrt{2}$$

(e) $A(3,-11),B(3,-4)$ **(K.B + A.B + U.B)**

(LHR 2014, GRW 2013, SGD 2015, MTN 2014, 15, SWL 2015, BWP 2017)

$$d=\sqrt{|x_2-x_1|^2+|y_2-y_1|^2}$$
$$|AB|=\sqrt{|3-3|^2+|-4-(-11)|^2}$$
$$|AB|=\sqrt{(0)^2+(-4+11)^2}$$
$$|AB|=\sqrt{(7)^2}$$
$$|AB|=7$$

(f) $A(0,0),B(0,-5)$ **(K.B + A.B + U.B)**

(LHR 2013, GRW 2013, SGD 2017, D.G.K 2015)

$$d=\sqrt{|x_2-x_1|^2+|y_2-y_1|^2}$$
$$|AB|=\sqrt{|0-0|^2+|-5-0|^2}$$
$$|AB|=\sqrt{(-5)^2}$$
$$|AB|=\sqrt{25}$$
$$|AB|=5$$

Q.2 Let P be the print on *x-axis* with *x*-coordinate P and Q be the point on y-axis with y-coordinate b as given below. Find the distance between P and Q.

Solution:

(i) $a=9, b=7$ **(K.B + A.B + U.B)**

P (9, 0) and Q (0, 7)

$$d=\sqrt{|x_2-x_1|^2+|y_2-y_1|^2}$$
$$|PQ|=\sqrt{|0-9|^2+|7-0|^2}$$
$$|PQ|=\sqrt{(-9)^2+(7)^2}$$
$$|PQ|=\sqrt{81+49}$$
$$|PQ|=\sqrt{130}$$

(ii) $a=2, b=3$ **(K.B + A.B + U.B)**

$P(2,0), Q(0,3)$

$$d=\sqrt{|x_2-x_1|^2+|y_2-y_1|^2}$$

$$|PQ|=\sqrt{|0-2|^2+|3-0|^2}$$

$$|PQ|=\sqrt{(-2)^2+(3)^2}$$

$$|PQ|=\sqrt{4+9}$$

$$|PQ|=\sqrt{13}$$

(iii) $a=-8, b=6$ **(K.B + A.B + U.B)**

$P(-8,0), Q(0,6)$

$$|d|=\sqrt{|x_2-x_1|^2+|y_2-y_1|^2}$$

$$|PQ|=\sqrt{|0-(-8)|^2+|6-0|^2}$$

$$|PQ|=\sqrt{(8)^2+(6)^2}$$

$$|PQ|=\sqrt{64+36}$$

$$|PQ|=\sqrt{100}$$

$$|PQ|=10$$

(iv) $a=-2, b=-3$ **(K.B + A.B + U.B)**

$P(-2\ ,0)\ ,Q(0,-3)$

$$|d|=\sqrt{|x_2-x_1|^2+|y_2-y_1|^2}$$

$$d=\sqrt{|0-(-2)|^2+|-3-0|^2}$$

$$d=\sqrt{(2)^2+(-3)^2}$$

$$d=\sqrt{4+9}$$

$$d=\sqrt{13}$$

(v) $a=\sqrt{2}, b=1$ **(K.B + A.B + U.B)**

$P\left(\sqrt{2},0\right), Q(0,1)$

$$d=\sqrt{|x_2-x_1|^2+|y_2-y_1|^2}$$

$$d=\sqrt{\left|0-\sqrt{2}\right|^2+|1-0|^2}$$

$$d=\sqrt{\left(-\sqrt{2}\right)^2+(1)^2}$$

$$d=\sqrt{2+1}$$

$$d=\sqrt{3}$$

(vi) $a=-9, b=-4$ **(K.B + A.B + U.B)**

$P(-9,0), Q(0,-4)$

$$d=\sqrt{|x_2-x_1|^2+|y_2-y_1|^2}$$

$$|PQ|=\sqrt{|0-(-9)|^2+|-4-0|^2}$$

$$|PQ|=\sqrt{(9)^2+(-4)^2}$$

$$|PQ|=\sqrt{81+16}$$

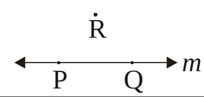
$$|PQ|=\sqrt{97}$$

Collinear or Non-Collinear Points in the Plane **(K.B)**

Two or more than two points which lie on the same straight line are called **collinear points** with respect to that line otherwise they are called **non-collinear.**

Let PQ be a line, then all the points on line *m* are collinear.

In the given figure the points P and Q are collinear with respect to the line m and the points P and R are not collinear with respect to it.



Use of Distance Formula to show the co linearity of three or more Points in the Plane **(K.B)**

Let P, Q and R be three points in the plane. They are called collinear if $|PQ|+|QR|=|PR|$ otherwise they are non-collinear

Example

Using distance formula shows that the points,

(i) $P(-2,-1), Q(0,3)$ and $R(1,5)$ are collinear.

Solution:

By using the distance formula, we find

$$|PQ|=\sqrt{(0+2)^2+(3+1)^2}$$

$$=\sqrt{4+16}=\sqrt{20}=2\sqrt{5}$$

$|QR|=\sqrt{(1-0)^2+(5-3)^2}=\sqrt{1+4}=\sqrt{5}$

And

$|PR|=\sqrt{(1+2)^2+(5+1)^2}$

$=\sqrt{9+36}=\sqrt{45}=3\sqrt{5}$

Since, $|PQ|+|QR|=2\sqrt{5}+\sqrt{5}=3\sqrt{5}=|PR|$

Therefore, the points P, Q and R are collinear

(ii) $P(-2,-1), Q(0,3), R(1,5)$ and $S(1,-1)$ are collinear.

$|PS|=\sqrt{(-2-1)^2+(-1+1)^2}=\sqrt{(-3)^2+0}=3$

$|PQ|=\sqrt{(1-0)^2+(-1-1)^2}=\sqrt{1+4}=\sqrt{5},$

$|QS|=\sqrt{(1-0)^2+(-1-3)^2}=\sqrt{1+16}=\sqrt{17},$

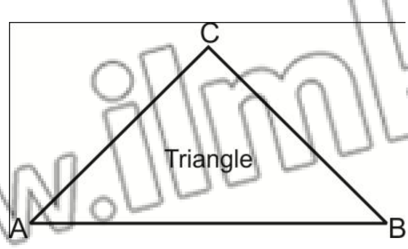
Since

$|PQ|+|QS|\neq|PS|,$

Therefore, the points P, Q and S are not collinear and hence, the points P, Q, R and S are also not collinear.

Triangle (MTN 2016) (K.B)

A closed figure in a plane obtained by joining three non-collinear points is called a triangle



In triangle ABC the non-collinear points A, B and C are three vertices of the triangle ABC. The line segments AB, BC, and CA are called sides of triangle.

Types of Triangle (K.B)

The idea of triangle to its different kinds depending on the length of three sides of the triangle as:

(i) Equilateral triangle

(ii) Isosceles triangle

(iii) Right angled triangle

(iv) Scalene triangle

Equilateral Triangle (K.B)

If the lengths of all the three sides of a triangle are same then the triangle is called an equilateral triangle.

Example (A.B)

Check whether ΔOPQ is an equilateral triangle, where $O(0,0), P\left(\frac{1}{\sqrt{2}},0\right)$ and $Q\left(\frac{1}{2\sqrt{2}},\frac{\sqrt{3}}{2\sqrt{2}}\right)$.

(LHR 2015, MTN 2014, RWP 2016, D.G.K 2016)

Solution:

$|OP|=\sqrt{\left(\frac{1}{\sqrt{2}}-0\right)^2+(0-0)^2}$

$=\sqrt{\left(\frac{1}{\sqrt{2}}\right)^2+0}=\sqrt{\left(\frac{1}{\sqrt{2}}\right)^2}=\frac{1}{\sqrt{2}}$

$|QO|=\sqrt{\left(O-\frac{1}{2\sqrt{2}}\right)^2+\left(0-\frac{\sqrt{3}}{2\sqrt{2}}\right)}$

$=\sqrt{\frac{1}{8}+\frac{3}{8}}=\sqrt{\frac{4}{8}}=\sqrt{\frac{1}{2}}=\frac{1}{\sqrt{2}}$

$$|PQ| = \sqrt{\left(\frac{1}{2\sqrt{2}} - \frac{1}{\sqrt{2}}\right)^2 + \left(\frac{\sqrt{3}}{2\sqrt{2}} - 0\right)^2}$$

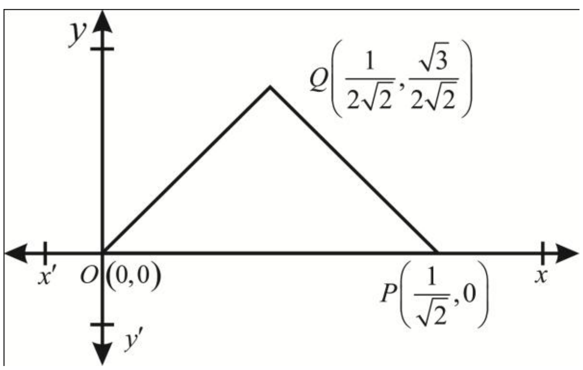
$$= \sqrt{\left(\frac{1-2}{2\sqrt{2}}\right)^2 + \left(\frac{\sqrt{3}}{2\sqrt{2}}\right)^2}$$

$$= \sqrt{\frac{1}{8} + \frac{3}{8}} = \sqrt{\frac{4}{8}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

i.e., $|OP| = |QO| = |PQ| = \frac{1}{\sqrt{2}}$

Therefore points $O(0,0)$, $Q\left(\frac{1}{2\sqrt{2}}, \frac{\sqrt{3}}{2\sqrt{2}}\right)$ and $P\left(\frac{1}{\sqrt{2}}, 0\right)$ are not collinear.

Hence the triangle OPQ is equilateral.



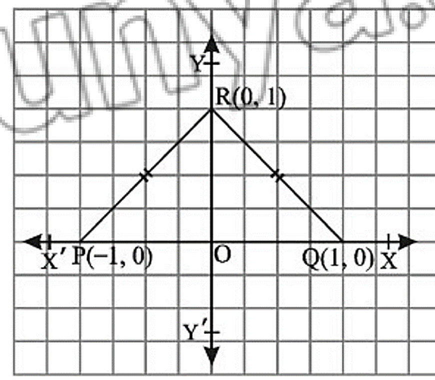
Isosceles Triangle **(K.B)**

(GRW 2017, SWL 2015, MTN 2016)

An isosceles triangle, PQR is a triangle which has two of its sides with equal length while the third side has a different length

Example **(A.B)**

The ΔPQR is an isosceles triangle as for the non-collinear points $P(-1,0), Q(1,0)$ and $R(0,1)$ shown in the figure.



$$|PQ| = \sqrt{(1-(-1))^2 + (0-0)^2}$$

$$= \sqrt{(1+1)^2 + 0} = \sqrt{4} = 2$$

$$|QR| = \sqrt{(0-1)^2 + (1-0)^2}$$

$$= \sqrt{(-1)^2 + 1^2} = \sqrt{1+1} = \sqrt{2}$$

$$|PR| = \sqrt{0-(1)^2 + (1-0)^2} = \sqrt{1+1} = \sqrt{2}$$

Since $|QR| = |PR| = \sqrt{2}$ and $|PQ| = 2 \neq \sqrt{2}$ So the non-collinear points P, Q, R form an isosceles ΔPQR.

Right Angle Triangle **(K.B)**

(FSD 2013, SWL 2014, MTN 2013, 16, SGD 2015)

A triangle in which one of the angles has measure equal to $90°$ is called a right angle triangle

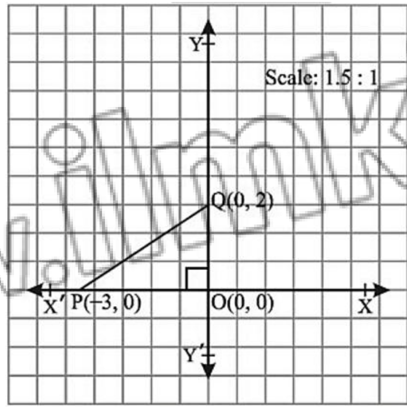
Example **(A.B)**

Let $O(0,0)$, $P(-3,0)$ and $Q(0,2)$ be three non-collinear points. Verify that ΔOPQ is right angled

$$|OQ| = \sqrt{(0-0)^2 + (2-0)^2} = \sqrt{2^2} = 2$$

$$|OP| = \sqrt{(-3)^2 + (0)^2} = \sqrt{9} = 3$$

$$|PQ| = \sqrt{(-3)^2 + (-2)^2} = \sqrt{9+4} = \sqrt{13}$$



Now $|OQ|^2+|OP|^2=(2)^2+(3)^2=13$

and $|PQ|^2=13$

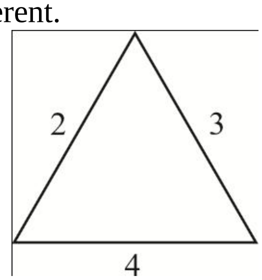
Since $|OQ|^2+|OP|^2=|PQ|^2$, therefore $\angle POR=90°$

Hence the given non collinear points form a right triangle

Scalene Triangle (K.B)

(LHR 2013, 14, 16, SGD 2016, 17, BWP 2017, D.G.K 2013)

A triangle is called a scalene triangle if measures of all the three sides are different.



Example **(A.B)**

Show that the points $P(1,2)$, $Q(-2,1)$ and $R(2,1)$ in the plane form a scalene triangle.

Solution:

$$|PQ|=\sqrt{(-2-1)^2+(1-2)^2}$$

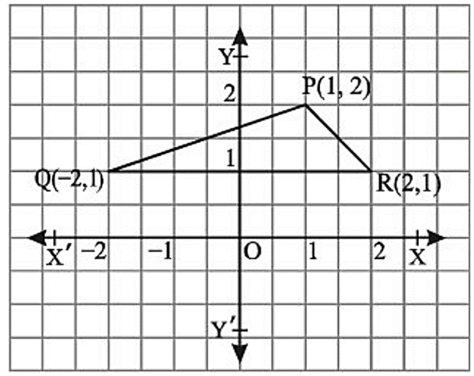
$$=\sqrt{(-3)^2+(-1)^2}=\sqrt{9+1}=\sqrt{10}$$

$$|QR|=\sqrt{(2+2)^2+(1-1)^2}=\sqrt{4^2+0^2}=\sqrt{4^2}=4$$

and

$$|PR|=\sqrt{(2-1)^2+(1-2)^2}$$

$$=\sqrt{1^2+(-1)^2}=\sqrt{1+1}=\sqrt{2}$$



Hence $|PQ|=\sqrt{10}$, $|QR|=4$ and $|PR|=\sqrt{2}$

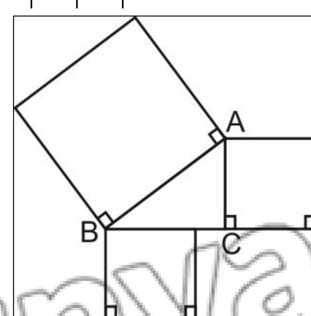
The points P,Q and R are non collinear since, $|PQ|+|QR|>|PR|$

Thus the given points form a scalene triangle.

Visual Proof of Pythagoras's Theorem (K.B)

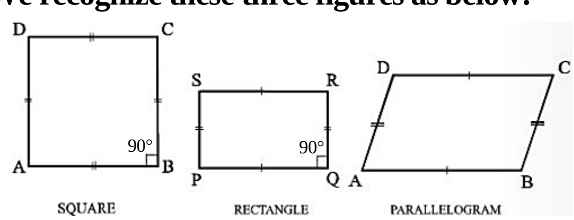
In right angle triangle ABC,

$|AB|^2=|BC|^2+|CA|^2$



Using distance formula to show that given four non-collinear points form a square, a rectangle and a parallelogram (U.B)

We recognize these three figures as below:



Square (GRW 2013) **(K.B)**

A square is a closed figure in the plane formed by four non collinear points such that lengths of all sides are equal and measure of each angle is 90°.

Example **(A.B)**

If $A(2,2), B(2,-2), C(-2,-2)$ and $D(-2,2)$ be four no collinear points in the plane then verify that they form a square ABCD

Solution:

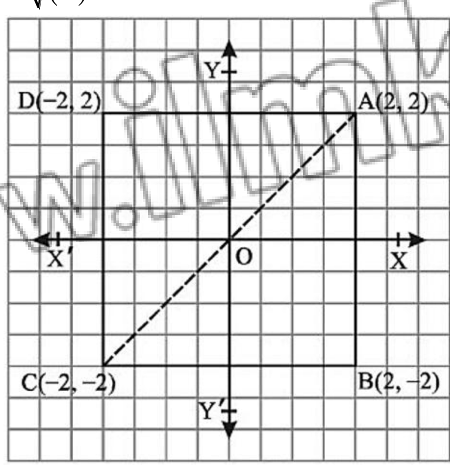
Since

$$|AB| = \sqrt{(2-2)^2 + (-2-2)^2}$$
$$= \sqrt{0^2 + (-4)^2} = \sqrt{16} = 4$$

$$|BC| = \sqrt{(-2-2)^2 + (-2+2)^2}$$
$$= \sqrt{(-4)^2 + (0)^2} = \sqrt{16} = 4$$

$$|CD| = \sqrt{(-2-(-2)) + (2-(-2))^2}$$
$$= \sqrt{(-2+2)^2 + (2+2)^2}$$
$$= \sqrt{0+16} = \sqrt{16} = 4$$

$$|DA| = \sqrt{(2+2)^2 + (2-2)^2}$$
$$= \sqrt{(4)^2 + 0} = \sqrt{16} = 4$$



Hence $|AB| = |BC| = |CD| = |DA| = 4$

Also

$$|AC| = \sqrt{(-2-2)^2 + (-2-2)^2}$$
$$= \sqrt{(-4)^2 + (-4)^2}$$
$$= \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$$

Now $|AB|^2 + |BC|^2 = (4)^2 + (4)^2 = 32$

and $|AC|^2 = \left(4\sqrt{2}\right)^2 = 32$

Since $|AB|^2 + |BC|^2 = |AC|^2$, therefore

$\angle ABC = 90°$

Hence the given four non-collinear points form a square,

Rectangle (SWL 2016, SGD 2017) **(U.B)**

A figure formed in the plane by four non-collinear points is called a rectangle

If

(i) its opposite sides are equal in length

(ii) the angle at each vertex is of measure 90°

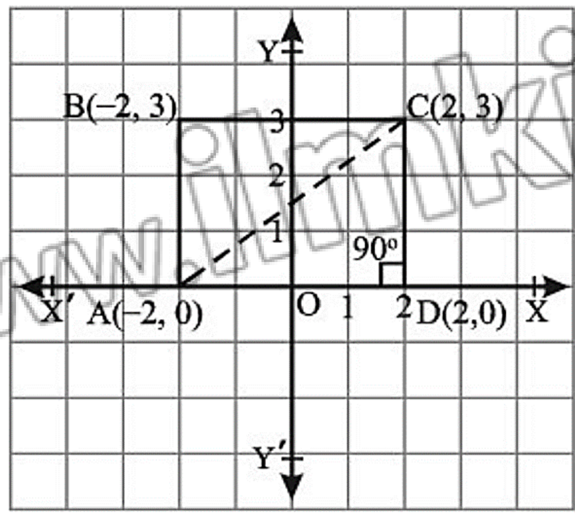
Example **(A.B)**

Show that the points $A(-2,0), B(-2,3), C(2,3)$ and $D(2,0)$ form a rectangle.

Solution:

Use distance formula

$$|AB| = \sqrt{(-2+2)^2 + (3-0)^2} = \sqrt{0+9} = \sqrt{9} = 3$$
$$|DC| = \sqrt{(2-2)^2 + (3-0)^2} = \sqrt{0+9} = \sqrt{9} = 3$$
$$|AD| = \sqrt{(2+2)^2 + (0-0)^2} = \sqrt{16+0} = 4$$
$$|BC| = \sqrt{(2+2)^2 + (3-3)^2} = \sqrt{16+0} = \sqrt{16} = 4$$



Since $|AB| = |DC| = 3$ and $|AD| = |BC| = 4$

Therefore, opposite sides are equal.

Also

$|AC| = \sqrt{(2+2)^2 + (3-0)^2} = \sqrt{16+9} = \sqrt{25} = 5$

Now $|AD|^2 + |DC|^2 = (4)^2 + (3)^2 = 25$ and

$|AC|^2 = (5)^2 = 25$

Since $|AD|^2 + |DC|^2 = |AC|^2$

Therefore $m\angle ADC = 90°$

Hence the given points form a rectangle

Parallelogram **(K.B)**

(SGD 2017, FSD 2013, D.G.K 2013)

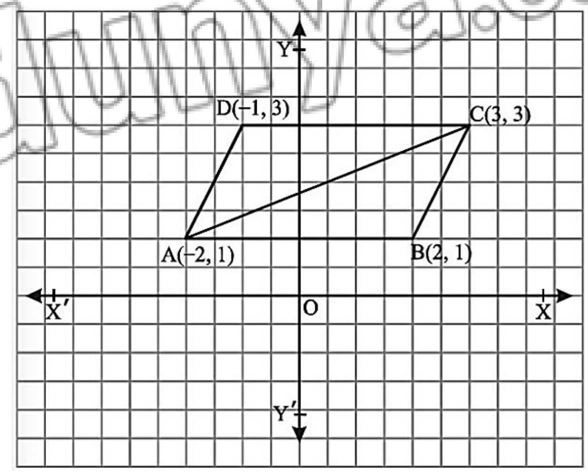
A figure formed by four non-collinear points in the plane is called a parallelogram if

(i) Its opposite sides are of equal length.

(ii) Its opposite sides are parallel.

Example: **(A.B)**

Show that the points $A(-2,1), B(2,1),$ $C(3,3)$ and $D(-1,3)$ form a parallelogram.



By distance formula

$|AB| = \sqrt{(2+2)^2 + (1-1)^2} = \sqrt{4^2 + 0} = \sqrt{16} = 4$

$|CD| = \sqrt{(3+1)^2 + (3-3)^2} = \sqrt{4^2 + 0} = \sqrt{16} = 4$

$|AD| = \sqrt{(-1+2)^2 + (3-1)^2}$

$= \sqrt{1^2 + 2^2} = \sqrt{1+4} = \sqrt{5}$

$|BC| = \sqrt{(3-2)^2 + (3-1)^2} = \sqrt{1^2 + 2^2} = \sqrt{5}$

Since $|AB| = |CD| = 4$ and ع

Hence the given points form a parallelogram.

Exercise 9.2

Q.1 Show whether the points with vertices (5,-2),(5,4) and (-4,1) are the vertices of an equilateral triangle or an isosceles triangle $P(5,-2), Q(5,4), R(-4,1)$ **(A.B)**

Solution:

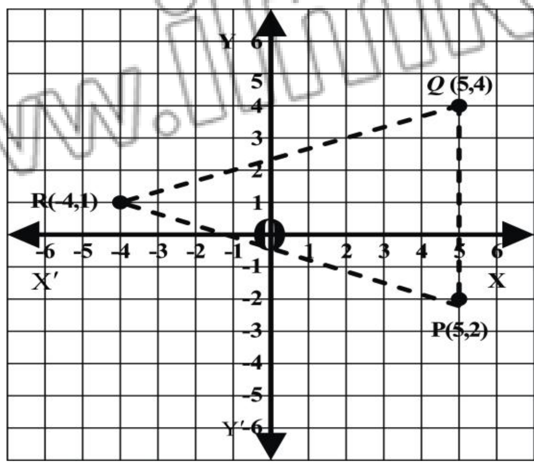
We know that the distance formula is

$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

We have $P(5,-2), Q(5,4)$

$|PQ| = \sqrt{|5-5|^2 + |4-(-2)|^2}$

$|PQ| = \sqrt{(0)^2 + (4+2)^2}$



$|PQ| = \sqrt{(6)^2}$

$|PQ| = 6$

$|QR| = \sqrt{|-4-5|^2 + |1-4|^2}$

$|QR| = \sqrt{(-9)^2 + (-3)^2}$

$|QR| = \sqrt{81+9}$

$|QR| = \sqrt{90}$

$|QR| = \sqrt{9\times 10} = 3\sqrt{10}$

$|RP| = \sqrt{|5-(-4)|^2 + |-2-1|^2}$

$|RP| = \sqrt{(5+4)^2 + (-3)^2}$

$|RP| = \sqrt{(9)^2 + 9}$

$|RP| = \sqrt{81+9}$

$|RP| = \sqrt{90}$

$|RP| = \sqrt{9\times 10} = 3\sqrt{10}$

As $|QR| = |PR|$

Lengths of two sides of triangle are equal, so it is an isosceles triangle.

Q.2 Show whether or not the points with vertices $(-1,1),(5,4),(2,-2)$ and $(-4,1)$ form a Square? (A.B)

Solution:

$P(-1,1)\,Q(5,4)\,R(2,-2)\,S(-4,1)$

Distance $= \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$|PQ| = \sqrt{\left|5-(-1)^2\right| + |4-1|^2}$

$|PQ| = \sqrt{|5+1|^2 + |3|^2}$

$|PQ| = \sqrt{6^2 + 9}$

$|PQ| = \sqrt{36+9}$

$|PQ| = \sqrt{45}$

$|PQ| = \sqrt{9\times 5}$

$|PQ| = 3\sqrt{5}$

$|QR| = \sqrt{|2-5|^2 + |-2-4|^2}$

$|QR| = \sqrt{(-3)^2 + (-6)^2}$

$|QR| = \sqrt{9+36}$

$|QR| = \sqrt{45}$

$|QR| = \sqrt{9\times 5}$

$|QR| = 3\sqrt{5}$

$|RS| = \sqrt{|-4-2|^2 + |1-(-2)|^2}$

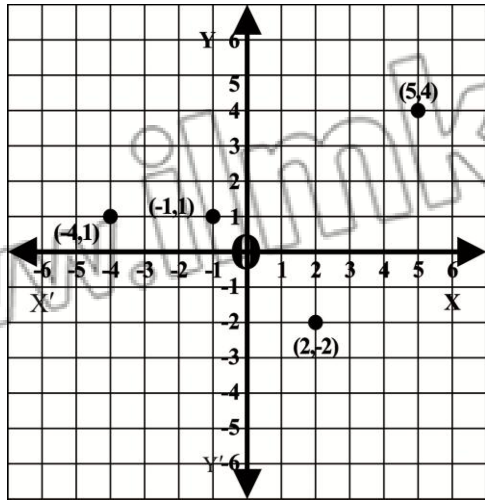
$|RS| = \sqrt{(-6)^2 + (1+2)^2} = \sqrt{36+(3)^2}$

$|RS| = \sqrt{36+9}$

$|RS| = \sqrt{45}$

$|RS| = \sqrt{9\times 5}$

$|RS| = 3\sqrt{5}$



$$|S\ P|=\sqrt{|-4-(-1)|^2+|1-1|^2}$$

$$|S\ P|=\sqrt{(-4+1)^2+(0)^2}$$

$$|S\ P|=\sqrt{(-3)^2}$$

$$|S\ P|=\sqrt{9}$$

$$|S\ P|=3$$

If all the lengths are same then it will be a Square, all the lengths are not equal so it is not square.

$$|P\ Q|=|Q\ R|=|R\ S|\neq|S\ P|$$

Q.3 Show whether or not the points with coordinates $(1,3),(4,2)$ and $(-2,6)$ are vertices of a right triangle? (A.B)

Solution:

$$A(1,3),B(4,2),C(-2,6)$$

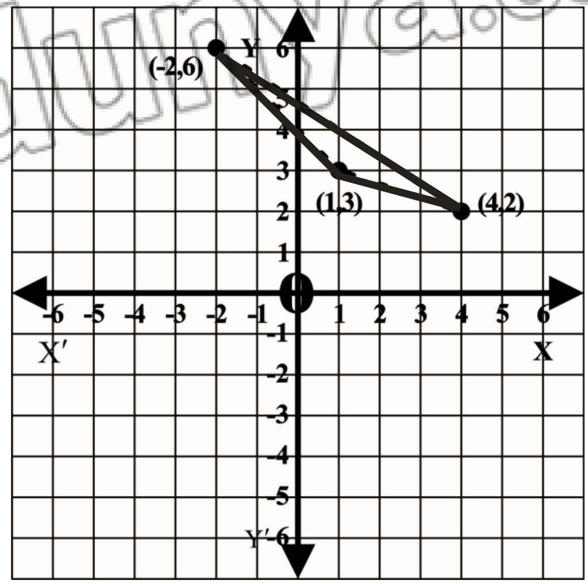
$$d=\sqrt{|x_2-x_1|^2+|y_2-y_1|^2}$$

$$|A\ B|=\sqrt{|4-1|^2+|2-3|^2}$$

$$|A\ B|=\sqrt{(3)^2+(-1)^2}$$

$$|A\ B|=\sqrt{9+1}$$

$$|A\ B|=\sqrt{10}$$



$$|B\ C|=\sqrt{|-2-4|^2+|6-2|^2}$$

$$|B\ C|=\sqrt{(-6)^2+(4)^2}$$

$$|B\ C|=\sqrt{36+16}$$

$$|B\ C|=\sqrt{52}$$

$$|C\ A|=\sqrt{|-2-1|^2+|6-3|^2}=\sqrt{(-3)^2+(3)^2}$$

$$|C\ A|=\sqrt{9+9}$$

$$|C\ A|=\sqrt{18}$$

By Pythagoras theorem

$$(\text{Hyp})^2=(\text{Base})^2+(\text{Perp})^2$$

$$\left(\sqrt{52}\right)^2=\left(\sqrt{18}\right)^2+\left(\sqrt{10}\right)^2$$

$$52=18+10$$

$52=28$

Since $52\neq28$

So it is not right angle triangle.

Q.4 Use distance formula to prove whether or not the points $(1,1),(-2,-8)$ and $(4,10)$ lie on a straight line? (A.B)

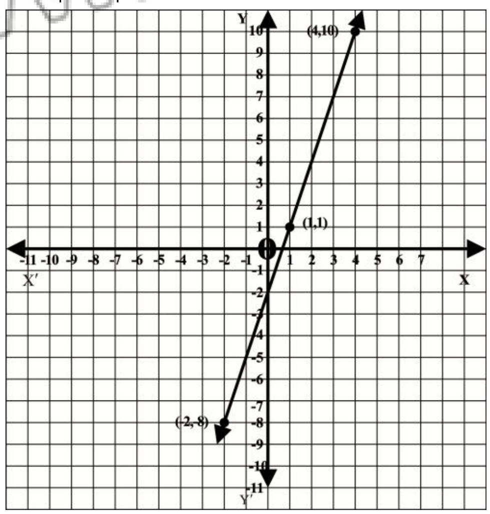
Solution:

$$A(1,1),B(-2,-8),C(4,10)$$

$$d=\sqrt{|x_2-x_1|^2+|y_2-y_1|^2}$$

$$|A\ B|=\sqrt{|-2-1|^2+|-8-1|^2}$$

$$|AB| = \sqrt{(-3)^2 + (-9)^2}$$
$$|AB| = \sqrt{9+81}$$
$$|AB| = \sqrt{90}$$
$$|AB| = \sqrt{9\times10}$$
$$|AB| = 3\sqrt{10}$$



$$|BC| = \sqrt{|4-(-2)|^2 + |10-(-8)|^2}$$
$$|BC| = \sqrt{(4+2)^2 + (10+8)^2}$$
$$|BC| = \sqrt{(6)^2 + (18)^2}$$
$$|BC| = \sqrt{36+324}$$
$$|BC| = \sqrt{360}$$
$$|BC| = \sqrt{36\times10}$$
$$|BC| = 6\sqrt{10}$$
$$|AC| = \sqrt{|4-1|^2 + |10-1|^2}$$
$$|AC| = \sqrt{(3)^2 + (9)^2}$$
$$|AC| = \sqrt{9+81}$$
$$|AC| = \sqrt{90}$$
$$|AC| = \sqrt{9\times10}$$
$$|AC| = 3\sqrt{10}$$
$$|AC| + |AB| = |BC|$$
$$3\sqrt{10} + 3\sqrt{10} = 6\sqrt{10}$$
$$6\sqrt{10} = 6\sqrt{10}$$

It means that they lie on same line so they are collinear.

Q.5 Find K given that point (2, K) is equidistance from (3, 7) and (9,1)

Solution:

$$M(2,K), A(3,7) \text{ and } B(9,1)$$

(3,7) (2,K) (9,1)
A M B

$$|\overline{AM}| = |\overline{BM}|$$
$$\sqrt{|2-3|^2 + |K-7|^2} = \sqrt{|9-2|^2 + |1-K|^2}$$
$$\sqrt{(-1)^2 + (K-7)^2} = \sqrt{(7)^2 + (1-K)^2}$$

Taking square on both Sides

$$\left(\sqrt{1+K^2+49-14K}\right)^2 = \left(\sqrt{49+1+K^2-2K}\right)^2$$
$$K^2 - 14K + 50 = 50 + K^2 - 2K$$
$$\cancel{K^2} - 14K \cancel{+50} \cancel{-50} \cancel{-K^2} + 2K = 0$$
$$-12K = 0$$
$$K = \frac{0}{-12}$$
$$K = 0$$

Q.6 Use distance formula to verify that the points $A(0,7), B(3,-5), C(-2,15)$ **are Collinear. (A.B)**

Solution:

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$
$$|AB| = \sqrt{|3-0|^2 + |-5-7|^2}$$
$$|AB| = \sqrt{(3)^2 + (-12)^2}$$
$$|AB| = \sqrt{9+144}$$
$$|AB| = \sqrt{153}$$
$$|AB| = \sqrt{9\times17}$$
$$|AB| = 3\sqrt{17}$$
$$|BC| = \sqrt{|-2-3|^2 + |15-(-5)|^2}$$
$$|BC| = \sqrt{(-5)^2 + (15+5)^2}$$

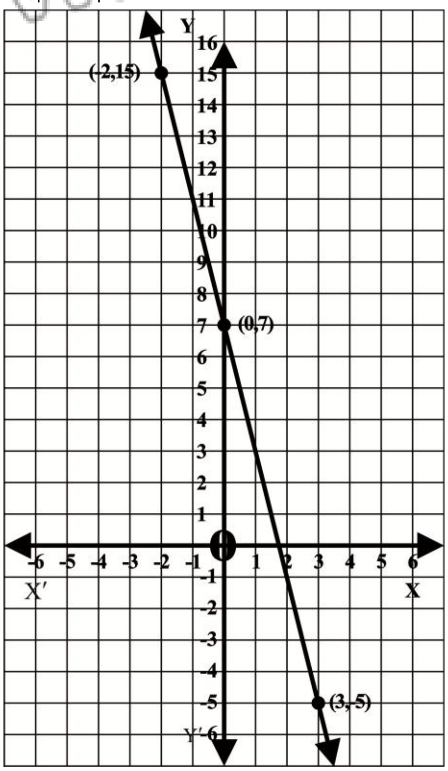
$|BC| = \sqrt{25 + (20)^2}$

$|BC| = \sqrt{25 + 400}$

$|BC| = \sqrt{425}$

$|BC| = \sqrt{25 \times 17}$

$|BC| = 5\sqrt{17}$



$|AC| = \sqrt{|-2-0|^2 + |15-7|^2}$

$|AC| = \sqrt{(-2)^2 + (8)^2}$

$|AC| = \sqrt{4 + 64}$

$|AC| = \sqrt{68}$

$|AC| = \sqrt{4 \times 17}$

$|AC| = 2\sqrt{17}$

$|AB| + |AC| = |BC|$

$3\sqrt{17} + 2\sqrt{17} = 5\sqrt{17}$

$5\sqrt{17} = 5\sqrt{17}$

L.H.S = R.H.S

So

They lie on same line and they are collinear.

Q.7 Verify whether or not the points $O(0,0)$, $A(\sqrt{3},1)$, $B(\sqrt{3},-1)$ are the vertices of an equilateral triangle. (A.B)

Solution:

$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$|OA| = \sqrt{|\sqrt{3} - 0|^2 + |1 - 0|^2}$

$|OA| = \sqrt{(\sqrt{3})^2 + (1)^2}$

$|OA| = \sqrt{3 + 1}$

$|OA| = \sqrt{4}$

$|OA| = 2$

$|OB| = \sqrt{|\sqrt{3} - 0|^2 + |-1 - 0|^2}$

$|OB| = \sqrt{(\sqrt{3})^2 + (-1)^2}$

$|OB| = \sqrt{3 + 1}$

$|OB| = \sqrt{4}$

$|OB| = 2$

$|AB| = \sqrt{|\cancel{\sqrt{3}} - \cancel{\sqrt{3}}|^2 + |-1 - 1|^2}$

$|AB| = \sqrt{0 + (-2)^2}$

$|AB| = \sqrt{4}$

$|AB| = 2$

All the sides are same in length so it is equilateral triangle.

Q.8 **Show that the points $A(-6,-5)$, $B(5,-5)$, $C(5,-8)$ and $D(-6,-8)$ are the vertices of a rectangle. Find the lengths of its diagonals. Are they equal? (K.B + A.B)**

Solution:

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$|AB| = \sqrt{|5-(-6)|^2 + |-5-(-5)|^2}$$

$$|AB| = \sqrt{(5+6)^2 + (-5+5)^2}$$

$$|AB| = \sqrt{(11)^2 + (0)^2} = \sqrt{121}$$

$$|AB| = 11$$

$$|BC| = \sqrt{|5-5|^2 + |-8-(-5)^2|}$$

$$|BC| = \sqrt{(0)^2 + (-8+5)^2} = \sqrt{(-3)^2}$$

$$|BC| = \sqrt{(-3)^2} = \sqrt{9}$$

$$|BC| = 3$$

$$|DC| = \sqrt{|-6-5|^2 + |-8-(-8)|^2}$$

$$|DC| = \sqrt{(-11)^2 + (-8+8)^2}$$

$$|DC| = \sqrt{121+0} = \sqrt{121}$$

$$|DC| = 11$$

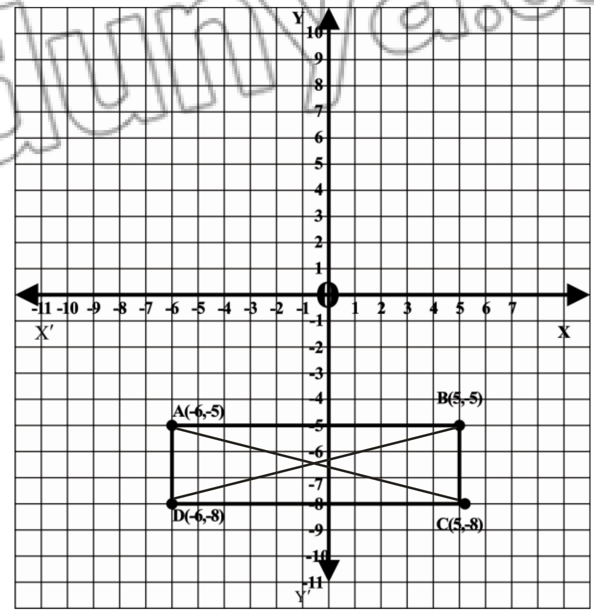
$$|DA| = \sqrt{|-6-(-6)|^2 + |-5-(-8)|^2}$$

$$|DA| = \sqrt{(-6+6)^2 + (-5+8)^2}$$

$$|DA| = \sqrt{(0)^2 + (3)^2} = \sqrt{0+9}$$

$$|DA| = \sqrt{9}$$

$$|DA| = 3$$



Diagonal distance of $|AC|$ and $|BD|$

$$|AC| = \sqrt{|5-(-6)|^2 + |-8-(-5)|^2}$$

$$|AC| = \sqrt{(5+6)^2 + (-8+5)^2}$$

$$|AC| = \sqrt{(11)^2 + (-3)^2}$$

$$|AC| = \sqrt{121+9}$$

$$|AC| = \sqrt{130}$$

$$|BD| = \sqrt{|-6-5|^2 + |-8-(-5)|^2}$$

$$|BD| = \sqrt{(-11)^2 + (-8+5)^2}$$

$$|BD| = \sqrt{121+(-3)^2} = \sqrt{121+9}$$

$$|BD| = \sqrt{130}$$

By Pythagoras theorem

$$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$$

$$(AC)^2 = (DC)^2 + (AD)^2$$

$$\left(\sqrt{130}\right)^2 = (11)^2 + (3)^2$$

130 = 121 + 9

130 = 130

L.H.S = R.H.S

Lengths of both diagonals are same it is a rectangle.

Q.9 Show that the point $M(-1,4), N(-5,3), P(1,-3)$ and $Q(5,-2)$ are vertices of a parallelogram. (K.B + A.B)

Solution:

$$d=\sqrt{|x_2-x_1|^2+|y_2-y_1|^2}$$

$$|M\ N|=\sqrt{|-5-(-1)|^2+|3-4|^2}$$

$$|M\ N|=\sqrt{(-5+1)^2+(-1)^2}$$

$$|M\ N|=\sqrt{(-4)^2+1}=\sqrt{16+1}$$

$$|M\ N|=\sqrt{17}$$

$$|N\ P|=\sqrt{|1-(-5)|^2+|-3-3|^2}$$

$$|N\ P|=\sqrt{(1+5)^2+(-6)^2}$$

$$|N\ P|=\sqrt{(6)^2+(6)^2}=\sqrt{36+36}$$

$$|N\ P|=\sqrt{72}$$

$$|N\ P|=\sqrt{36\times2}$$

$$|N\ P|=6\sqrt{2}$$

$$|P\ Q|=\sqrt{|5-1|^2+|-2-(-3)|^2}$$

$$|P\ Q|=\sqrt{(4)^2+(-2+3)^2}$$

$$|P\ Q|=\sqrt{16+(1)^2}$$

$$|P\ Q|=\sqrt{16+1}$$

$$|P\ Q|=\sqrt{17}$$

$$|M\ Q|=\sqrt{|5-(-1)|^2+|-2-4|^2}$$

$$|M\ Q|=\sqrt{(5+1)^2+(-6)^2}$$

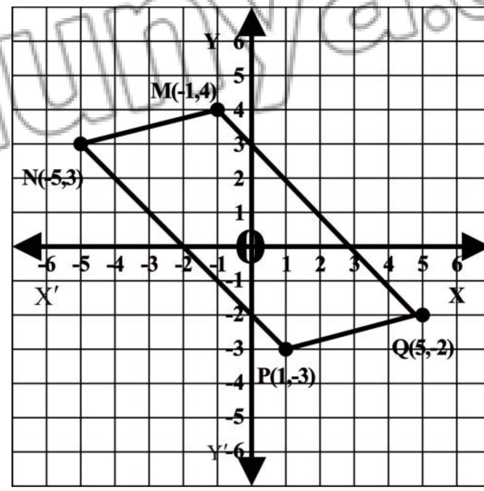
$$|M\ Q|=\sqrt{(6)^2+36}$$

$$|M\ Q|=\sqrt{36+36}$$

$$|M\ Q|=\sqrt{72}$$

$$|M\ Q|=\sqrt{36\times2}$$

$$|M\ Q|=6\sqrt{2}$$



Length of a diagonal

$$|N\ Q|=\sqrt{|5-(-5)|^2+|-2-3|^2}$$

$$|N\ Q|=\sqrt{(5+5)^2+(-5)^2}$$

$$|N\ Q|=\sqrt{(10)^2+(25)}$$

$$|N\ Q|=\sqrt{100+25}=\sqrt{125}$$

$m\angle N=90^o$, if

$$(QM)^2+(MN)^2=(QN)^2$$

$$\left(6\sqrt{2}\right)^2+\left(\sqrt{17}\right)^2=\left(\sqrt{125}\right)^2$$

$$36\times2+17=125$$

72 + 17 = 125

89 = 125

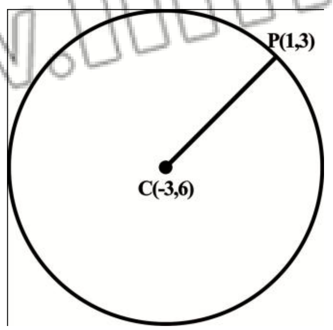
They are not equal, so it is not right angle.

But $|M\ N|=|P\ Q|$ and $|N\ P|=|M\ Q|$

Opposite side are equal so it is a Parallelogram.

Q.10 Find the length of the diameter of the circle having centre at $C(-3,6)$ and passing through P (1, 3).
(K.B + A.B)

Solution:



CP is the radius of a circle
So

$|CP| = \sqrt{|-3-1|^2 + |6-3|^2}$

$|CP| = \sqrt{(-4)^2 + (3)^2}$

$|CP| = \sqrt{16+9}$

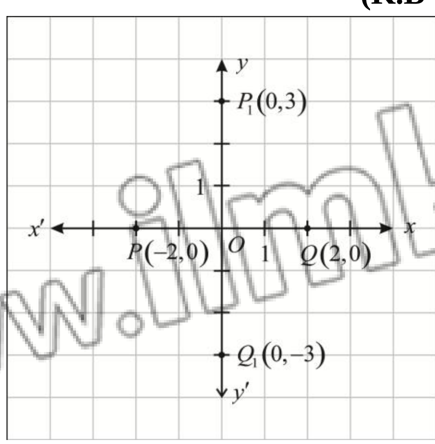
$|CP| = \sqrt{25}$

$|CP| = 5$

Diameter = 2 radius
Diameter = 2(CP)
Diameter = 2(5)
Diameter = 10 units.

Recognition of Midpoint

(K.B + A.B)



Let P(–2, 0) and Q(2, 0) be two points on the x-axis. Then the origin O(0, 0) is the mid-point of P an Q, since

$|OP| = 2 = |OQ|$ and the points P, O, and Q are collinear.

Similarly the origin is the mid-point of the points $P_1(0,3)$ *and* $Q_1(0,-3)$ since

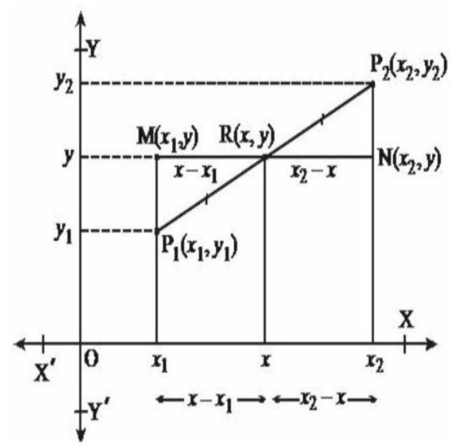
$|OP_1| = 3 = |OQ_1|$ and the points

P_1, O *and* Q_1 are collinear,

Mid-Point Formula

(K.B + U.B)

Let $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ be any two points in the plane and $R(x, y)$ be midpoint of points P_1 and P_2 on the line segment P_1P_2 as shown in the figure below



If line segment *MN* parallel to *x*-axis has its midpoint $R(x, y)$ then $x_2 - x = x - x_1$

$\Rightarrow 2x = x_1 + x_2 \quad \Rightarrow x = \dfrac{x_1 + x_2}{2}$

Similarly $y = \dfrac{y_1 + y_2}{2}$

Thus the point $R(x, y) = R\left[\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right]$

is the midpoint of the points $P_1(x_1, y_1)$ and

$P_2(x_2, y_2)$.

Verification of the midpoint formula

(K.B + A.B)

$$|P_1R| = \sqrt{\left(\frac{x_1+x_2}{2}-x_1\right)^2+\left(\frac{y_1+y_2}{2}-y_1\right)^2}$$

$$= \sqrt{\left(\frac{x_2-x_1}{2}\right)^2+\left(\frac{y_2-y_1}{2}\right)^2}$$

$$= \frac{1}{2}\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \frac{1}{2}|P_1P_2|$$

And

$$|P_2R| = \sqrt{\left(\frac{x_1+x_2}{2}-x_2\right)^2+\left(\frac{y_1+y_2}{2}-y_2\right)^2}$$

$$= \sqrt{\left(\frac{x_1+x_2-2x_2}{2}\right)^2+\left(\frac{y+y_2-2y_2}{2}\right)^2}$$

$$= \sqrt{\left(\frac{x_1-x_2}{2}\right)^2+\left(\frac{y_1-y_2}{2}\right)^2}$$

$$= \sqrt{\left(\frac{x_2-x_1}{2}\right)^2+\left(\frac{y_2-y_1}{2}\right)^2}$$

$$= \frac{1}{2}\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$\Rightarrow |P_2R| = |P_1R| = \frac{1}{2}|P_1P_2|$$

Thus it verifies that $R\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$ is the mid point of the line segment P_1RP_2 which lies on the line segment since

$|P_1R|+|P_2R| = |P_1P_2|$

Example # 3 **(K.B + A.B)**

Let ABC be a triangle as shown below. If M_1, M_2 and M_3 are the middle points of the line segments AB, BC and CA respectively, find the coordinates of M_1, M_2 and M_3. Also determine the type of the triangle $M_1M_2M_3$.

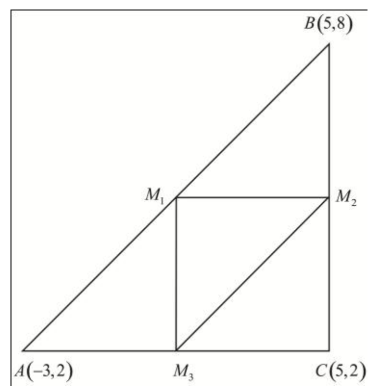
Solution:

Mid-point of

$$AB = M_1\left[\frac{-3+5}{2},\frac{2+8}{2}\right] = M_1(1,5)$$

Mid-point of

$$BC = M_2\left[\frac{5+5}{2},\frac{8+2}{2}\right] = M_2(5,5)$$



And mid-point of

$$AC = M_3\left[\frac{5-3}{2},\frac{2+2}{2}\right] = M_3(1,2)$$

The triangle $M_1M_2M_3$ has sides with length

$$|M_1M_2| = \sqrt{(5-1)^2+(5-5)^2} = \sqrt{4^2+0} = \sqrt{16} = 4$$

$$|M_2M_3| = \sqrt{(1-5)^2+(2-5)^2} = \sqrt{(-4)^2+(-3)^2}$$

$$= \sqrt{16+9} = \sqrt{25} = 5$$

And

$$|M_1M_3| = \sqrt{(1-1)^2+(2-5)^2} = \sqrt{0^2+(-3)^2} = 3$$

All the lengths of three sides are different. Hence the triangle $M_1M_2M_3$ is a scalene triangle.

Exercise 9.3

Q.1 Find the midpoint of the line Segments joining each of the following pairs of points
(K.B + A.B)

Solution:

(a) $A(9,2), B(7,2)$

(MTN 2016, 17, SGD 2017, D.G.K 2016)

Let $M(x,y)$ be the midpoint of AB,
Then by midpoint formula

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$M(x,y)=M\left(\frac{9+7}{2},\frac{2+2}{2}\right)$$

$$=M\left(\frac{\cancel{16}^{8}}{\cancel{2}},\frac{\cancel{4}^{2}}{\cancel{2}}\right)$$

$$=M(8,2)$$

(b) $A(2,-6), B(3,-6)$

(LHR 2017, MTN 2014, 15, 17, SWL 2015, SGD 2013, D.G.K 2014)

Let $M(x,y)$ be the midpoint of AB then by Midpoint formula

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$M(x,y)=M\left(\frac{2+3}{2},\frac{-6-6}{2}\right)$$

$$M(x,y)=M\left(\frac{5}{2},\frac{-\cancel{12}^{6}}{\cancel{2}}\right)$$

$$M(x,y)=M(2.5,-6)$$

(c) $A(-8,1), B(6\ ,1)$

(LHR 2017, GRW 2017)

Let $M(x,y)$ be the midpoint of AB then by Midpoint formula

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$M(x,y)=M\left(\frac{-8+6}{2},\frac{1+1}{2}\right)$$

$$M(x,y)=M\left(\frac{-\cancel{2}}{\cancel{2}},\frac{\cancel{2}}{\cancel{2}}\right)$$

$$M(x,y)=M(-1,1)$$

(d) A (-4 , 9) , B (-4 , -3)

Let $M(x,y)$ be the midpoint of AB then by Midpoint formula

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$M(x,y)=M\left(\frac{-4-4}{2},\frac{9-3}{2}\right)$$

$$M(x,y)=M\left(\frac{-\cancel{8}^{4}}{\cancel{2}},\frac{\cancel{6}^{3}}{\cancel{2}}\right)$$

$$M(x,y)=M(-4,3)$$

(e) $A(3,-11), B(3,-4)$

(LHR 2017, GRW 2014, MTN 2014, 15, SGD 2015, SWL 2015, 16, FSD 2017, BWP 2016)

Let $M(x,y)$ is the midpoint of AB

$$M(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$M(x,y)=M\left(\frac{3+3}{2},\frac{-11-4}{2}\right)$$

$$M(x,y)=M\left(\frac{6}{2},\frac{-15}{2}\right)$$

$$M(x,y)=M(3,-7.5)$$

(f) A (0, 0) , B (0, -5)

(LHR 2017, GRW 2016, FSD 2016, SWL 2014, D.G.K 2015)

Let $M(x,y)$ is the midpoint of AB

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

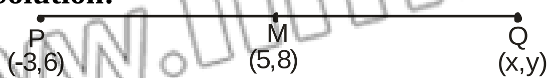
$$M(x,y)=M\left(\frac{0+0}{2},\frac{0-5}{2}\right)$$

$$M(x,y)=M\left(\frac{0}{2},\frac{-5}{2}\right)$$

$$=M(0,-2.5)$$

Q.2 The end point P of a line segment PQ is $(-3,6)$ and its midpoint is $(5,8)$ find the coordinates of the end point Q. (FSD 2017) (K.B + U.B)

Solution:



Let Q be the point $(x,y), M(5,8)$ is the midpoint of PQ

$$M(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$x=\frac{x_1+x_2}{2}$$

$$5=\frac{-3+x}{2}$$

$$5\times 2=-3+x$$

$$10+3=x$$

$$x=13$$

$$y=\frac{y_1+y_2}{2}$$

$$8=\frac{6+y}{2}$$

$$2\times 8=6+y$$

$$16-6=y$$

$$y=10$$

Hence point Q is $(13,10)$

Q.3 Prove that midpoint of the hypotenuse of a right triangle is equidistant from it three vertices $P(-2,5), Q(1,3)$ and $R(-1,0)$. (K.B + A.B + U.B)

Solution:

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$d=\sqrt{|x_2-x_1|^2+|y_2-y_1|^2}$$

$$|P\ Q|=\sqrt{|-2-1|^2+|5-3|^2}$$

$$|P\ Q|=\sqrt{(-3)^2+(2)^2}$$

$$|P\ Q|=\sqrt{9+4}$$

$$|P\ Q|=\sqrt{13}$$

$$|Q\ R|=\sqrt{|1-(-1)|^2+|3-0|^2}$$

$$|Q\ R|=\sqrt{(1+1)^2+(3)^2}$$

$$|Q\ R|=\sqrt{(2)^2+9}=\sqrt{4+9}$$

$$|Q\ R|=\sqrt{13}$$

$$|P\ R|=\sqrt{|-2-(-1)|^2+|5-0|^2}$$

$$|P\ R|=\sqrt{|-2+1|^2+|5|^2}$$

$$|P\ R|=\sqrt{(-1)^2+(5)^2}=\sqrt{1+25}$$

$$|P\ R|=\sqrt{26}$$

whether it is right angle triangle or not, we use the Pythagoras theorem

$$(PR)^2=(PQ)^2+(QR)^2$$

$$\left(\sqrt{26}\right)^2=\left(\sqrt{13}\right)^2+\left(\sqrt{13}\right)^2$$

26 = 13 + 13

26 = 26 (Satisfied)

It is a right angle triangle and PR is hypotenuse.

Midpoint of PR

$$M(x,y)=\left(\frac{-2-1}{2},\frac{5+0}{2}\right)$$

$$M(x,y)=\left(\frac{-3}{2},\frac{5}{2}\right)$$

$$|MP|=|MR|$$

(i)

$$|MP|=\sqrt{\left|\frac{-3}{2}-(-2)\right|^2+\left|\frac{5}{2}-5\right|^2}$$

$$=\sqrt{\left(\frac{-3}{2}+2\right)^2+\left(\frac{5-10}{2}\right)^2}$$

$$|MP|=\sqrt{\left(\frac{-3+4}{2}\right)^2+\left(\frac{-5}{2}\right)^2}$$

$$=\sqrt{\left(\frac{1}{2}\right)^2+\frac{25}{4}}$$

$$|MP|=\sqrt{\frac{1}{4}+\frac{25}{4}}=\sqrt{\frac{1+25}{4}}$$

$$|MP|=\sqrt{\frac{26}{4}}$$

$$|MP|=\frac{\sqrt{26}}{2}$$

(ii)

$$|M\ R|=\sqrt{\left|\frac{-3}{2}-(-1)\right|^2+\left|\frac{5}{2}-0\right|^2}$$

$$|M\ R|=\sqrt{\left(\frac{-3}{2}+1\right)^2+\left(\frac{5}{2}\right)^2}$$

$$|M\ R|=\sqrt{\left(\frac{-3+2}{2}\right)^2+\frac{25}{4}}$$

$$=\sqrt{\left(\frac{-1}{2}\right)^2+\frac{25}{4}}$$

$$|M\ R|=\sqrt{\frac{1}{4}+\frac{25}{4}}$$

$$|M\ R|=\sqrt{\frac{1+25}{4}}=\sqrt{\frac{26}{4}}$$

$$|M\ R|=\frac{\sqrt{26}}{2}$$

(iii) $M\left(\frac{-3}{2},\frac{5}{2}\right),\ Q(1,3)$

$$|MQ|=\sqrt{\left(\frac{-3}{2}-1\right)^2+\left(\frac{5}{2}-3\right)^2}$$

$$=\sqrt{\left(\frac{-3-2}{2}\right)^2+\left(\frac{5-6}{2}\right)^2}$$

$$=\sqrt{\left(\frac{-5}{2}\right)^2+\left(\frac{-1}{2}\right)^2}$$

$$=\sqrt{\frac{25}{4}+\frac{1}{4}}=\sqrt{\frac{26}{4}}$$

Hence proved

Q.4 If $O(0,0)$, $A(3,0)$ and $B(3,5)$ are three points in the plane find M_1 and M_2 as midpoint of the line segments AB and OB respectively find $|M_1M_2|$. (K.B + U.B)

Solution:

M_1 is the midpoint of AB

$$M_1(x,y)=M_1\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$=M_1\left(\frac{3+3}{2},\frac{0+5}{2}\right)$$

$$=M_1\left(\frac{6}{2},\frac{5}{2}\right)$$

$$=M_1\left(3,\frac{5}{2}\right)$$

M_2 is the midpoint of OB

$$M_2\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$=M_2\left(\frac{0+3}{2},\frac{0+5}{2}\right)$$

$$=M_2\left(\frac{3}{2},\frac{5}{2}\right)$$

Now

$$|M_1M_2|=\sqrt{\left|\frac{3}{2}-3\right|^2+\left|\frac{5}{2}-\frac{5}{2}\right|^2}$$

$$|M_1M_2|=\sqrt{\left(\frac{3-6}{2}\right)^2+(0)^2}$$

$$=\sqrt{\left(\frac{-3}{2}\right)^2+0}$$

$$|M_1M_2|=\sqrt{\frac{9}{4}}$$

$$|M_1M_2|=\frac{3}{2}$$

Q.5 Show that the diagonals of the parallelogram having vertices $A(1,2), B(4,2), C(-1,-3)$ and $D(-4,-3)$ bisect each other.

(K.B + A.B + U.B)

Solution:

$ABCD$ is parallelogram with vertices $A(1,2), B(4,2), C(-1,-3), D(-4,-3)$.

Let $\overline{BD}$ and $\overline{AC}$ the diagonals of parallelogram intersect at point *M*.

Finding midpoint of *AC*

Using midpoint formula

$$M_1(x,y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$M_1(x,y) = M_1\left(\frac{1-1}{2}, \frac{2-3}{2}\right)$$

$$M_1(x,y) = M_1\left(\frac{0}{2}, \frac{-1}{2}\right) = \left(0, \frac{-1}{2}\right)$$

Midpoint of BD,

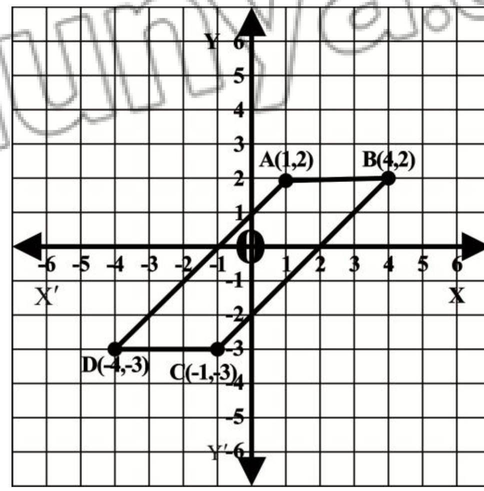
$$M_2(x,y) = M_2\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$M_2(x,\ y) = M_2\left(\frac{4-4}{2}, \frac{2-3}{2}\right)$$

$$M_2(x,y) = M_2\left(\frac{0}{2}, \frac{-1}{2}\right)$$

$$M_2(x,y) = M_2\left(0, \frac{-1}{2}\right)$$

As M_1 and M_2 Coincide. So the diagonals of the parallelogram bisect each other.



Q.6 The vertices of a triangle are $P(4,6), Q(-2,-4)$ and $R(-8,2)$.
Show that the length of the line segment joining the midpoints of the line segments $\overline{PR}, \overline{QR}$ is $\frac{1}{2}\overline{PQ}$

(K.B + A.B + U.B)

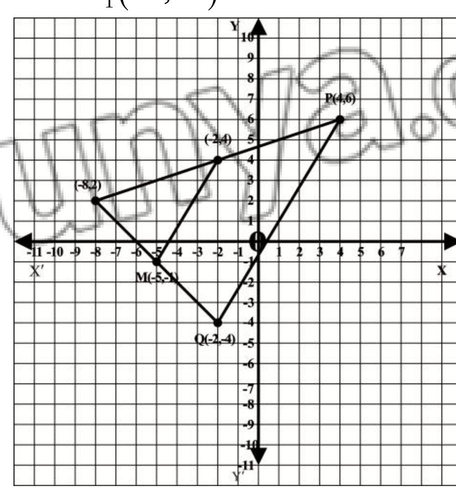
Solution:

M_1 the midpoint of QR is

$$M_1(x,y) = M_1\left(\frac{-2-8}{2}, \frac{-4+2}{2}\right)$$

$$= M_1\left(\frac{-10}{2}, \frac{-2}{2}\right)$$

$$= M_1(-5,-1)$$



M_2 the midpoint of PR is

$$M_2(x,y)=M\left(\frac{4-8}{2},\frac{6+2}{2}\right)$$
$$M_2(x,y)=M_2\left(\frac{-4}{2},\frac{8}{2}\right)$$
$$M_2(x,y)=M_2(-2,4)$$
Now
$$|M_1M_2|=\sqrt{|-5+2|^2+|4+1|^2}$$
$$|M_1M_2|=\sqrt{(-3)^2+(5)^2}$$
$$|M_1M_2|=\sqrt{9+25}$$
$$|M_1M_2|=\sqrt{34}$$
$$|P\ Q|=\sqrt{|4+2|^2+|6+4|^2}$$
$$|P\ Q|=\sqrt{(6)^2+(10)^2}=\sqrt{36+100}$$
$$|P\ Q|=\sqrt{136}$$
$$|P\ Q|=\sqrt{4\times 34}$$
$$|P\ Q|=2\sqrt{34}$$
$$\frac{|P\ Q|}{2}=\sqrt{34}$$
OR
$$\frac{1}{2}|PQ|=\sqrt{34}$$
Hence we proved that
$$|M_1M_2|=\frac{1}{2}|PQ|$$

Review Exercise 9

Q.1 Choose the Correct answer

(i) Distance between points (0, 0) and (1, 1) is **(K.B + U.B)**
(LHR 2017, GRW 2013, 16, SWL 2013, 15, FSD 2017, SGD 2016, 17, MTN 2013)
(a) 0 **(b)** 1
(c) 2 **(d)** $\sqrt{2}$

(ii) Distance between the points (1, 0) and (0 ,1) is **(K.B + U.B)**
(LHR 2016, FSD 2013, SWL 2013, SGD 2013, BWP 2013, 14, MTN 2016, 17)
(a) 0 **(b)** 1
(c) $\sqrt{2}$ **(d)** 2

(iii) Midpoint of the points (2, 2) and (0, 0) is **(K.B + U.B)**
(a) (1, 1) **(b)** (1, 0)
(c) (0, 1) **(d)** (-1, -1)

(iv) Midpoint of the points (2, -2) and (-2, 2) is **(K.B + U.B)**
(a) (2, 2) **(b)** (-2, -2)
(c) (0, 0) **(d)** (1, 1)

(v) A triangle having all sides equal is called **(K.B + U.B)**
(LHR 2013, GRW 2014, SWL 2014, 15)
(a) Isosceles **(b)** Scalene
(c) Equilateral **(d)** None of these

(vi) A triangle having all sides different is called **(K.B + U.B)**
(a) Isosceles **(b)** Scalene
(c) Equilateral **(d)** None of these

ANSWER KEYS

i	ii	iii	iv	v	vi
d	c	a	c	c	b

Q.2 Answer the following which is true and which is false

(i)	**A line has two end points**	**(K.B + U.B)**	**(False)**
(ii)	**A line segment has one end point**	**(K.B + U.B)**	**(False)**
(iii)	**A triangle is formed by the three collinear points**	**(K.B + U.B)**	**(False)**
(iv)	**Each side of triangle has two collinear vertices.**	**(K.B + U.B)**	**(True)**
(v)	**The end points of each side of a rectangle are Collinear**	**(K.B + U.B)**	**(True)**
(vi)	**All the points that lie on the x-axis are Collinear**	**(K.B + U.B)**	**(True)**
(vii)	**Origin is the only point Collinear with the points of both axis separately**		**(True)**

Q.3 Find the distance between the following pairs of points

Solution:

(i) $(6,3)(3,-3)$

$A(6,3), B(3,-3)$

$$d=\sqrt{|x_2-x_1|^2+|y_2-y_1|^2}$$

$$|AB|=\sqrt{|3-6|^2+|-3-3|^2}$$

$$|AB|=\sqrt{(-3)^2+(-6)^2}$$

$$|AB|=\sqrt{9+36}$$

$$|AB|=\sqrt{45}$$

$$|AB|=\sqrt{9\times5}$$

$$|AB|=3\sqrt{5}$$

(ii) $(7,5),(1,-1)$

$A(7,5), B(1,-1)$

$$d=\sqrt{|x_2-x_1|^2+|y_2-y_1|^2}$$

$$|AB|=\sqrt{|7-1|^2+|5-(-1)|^2}$$

$$|AB|=\sqrt{(6)^2+(5+1)^2}$$

$$|AB|=\sqrt{36+(6)^2}=\sqrt{36+36}$$

$$|AB|=\sqrt{72}=\sqrt{36\times2}$$

$$|AB|=6\sqrt{2}$$

(iii) $(0,0),(-4,-3)$

$A(0,0), B(-4,-3)$

$$d=\sqrt{|x_2-x_1|^2+|y_2-y_1|^2}$$

$$|AB|=\sqrt{|0-(-4)|^2+|0-(-3)|^2}$$

$$|AB|=\sqrt{(4)^2+(3)^2}$$

$$|AB|=\sqrt{16+9}$$

$$|AB|=\sqrt{25}$$

$$|AB|=5$$

Q.4 Find the midpoint between following pairs of points

Solution:

(i) $(6,6),(4,-2)$ **(SWL 2017, SGD 2017)**

$$M(x,y)=M\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$M(x,y)=M\left(\frac{6+4}{2},\frac{6-2}{2}\right)$$

$$M(x,y)=M\left(\frac{10}{2},\frac{4}{2}\right)$$

$$M(x,y)=M(5,2)$$

(ii) $(-5,-7),(-7,-5)$ **(BWP 2014)**

$$M(x,y)=M\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$M(x,y)=M\left(\frac{-5-7}{2},\frac{-7-5}{2}\right)$$

$$M(x,y)=M\left(\frac{-12}{2},\frac{-12}{2}\right)$$

$$M(x,y)=M(-6,-6)$$

(iii) $(8,0),(0,-12)$ **(LHR 2016, SGD 2016)**

$$M(x,y)=M\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$M(x,y)=M\left(\frac{8+0}{2},\frac{0-12}{2}\right)$$

$$M(x,y)=M\left(\frac{8}{2},\frac{-12}{2}\right)$$

$$M(x,y)=M(4,-6)$$

Q.5 Define the following

Solution:

(i) Co-ordinate Geometry (K.B)

Co-ordinate geometry is the study of geometrical shapes in the Cartesian plane (or coordinate plane)

(ii) Collinear Points (K.B)

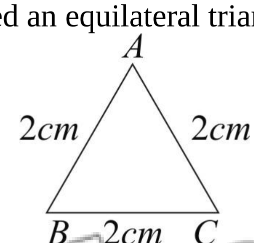
Two or more than two points which lie on the same straight line are called collinear points with respect to that line.

(iii) Non- Collinear Points (K.B)

The points which do not lie on the same straight line are called non-collinear points.

(iv) Equilateral Triangle (K.B)

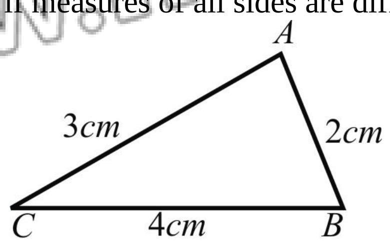
If the lengths of all three sides of a triangle are same then the triangle is called an equilateral triangle.



ΔABC is an equilateral triangle.

(v) Scalene Triangle (K.B)

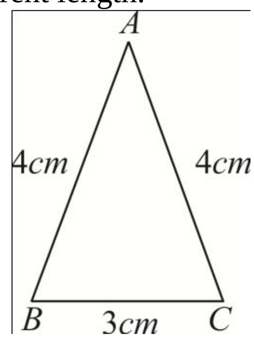
A triangle is called a scalene triangle if measures of all sides are different.



ΔABC is a Scalene triangle.

(vi) Isosceles Triangle (K.B)

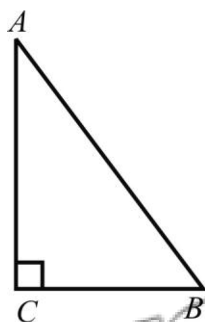
An isosceles triangle is a triangle which has two of its sides with equal length while the third side has different length.



ΔABC is an isosceles triangle

(vii) Right Triangle (K.B)

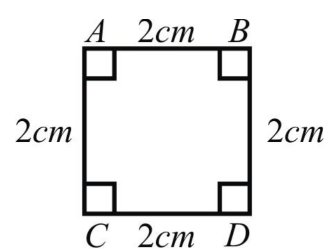
A triangle in which one of the angles has measure equal to 90° is called a right triangle.



ΔABC is a right angled triangle.

(viii) A Square is a closed figure formed by four non- collinear points such that lengths of all sides are equal and measure of each angle is 90°.

(K.B)



$ABCD$ is a square.

CUT HERE

SELF TEST

Time: 40 min **Marks: 25**

Q.1 Four possible answers (A), (B), (C) & (D) to each question are given, mark the correct answer. **(7×1=7)**

1 The point (−3, 1) lies in ___________ Quadrant.

(A) 1^{st} **(B)** 2^{nd}

(C) 3^{rd} **(D)** 4^{th}

2 The x-coordinate of the point is called __________

(A) x-axis **(B)** Origin

(C) Abscissa **(D)** Ordinate

3 Which ordered pair satisfies the equation $y=-2x+1$?

(A) $(-1, 4)$ **(B)** $(-1, 3)$

(C) $(0, 3)$ **(D)** $(1, -5)$

4 If $(x-1, y+1)=(0,0)$ **then** (x, y) **is:**

(A) $(1,-1)$ **(B)** $(-1,1)$

(C) $(1,1)$ **(D)** $(-1,-1)$

5 The three points P, Q and R form triangle if and only if they are non-collinear and

(A) $|PQ|+|QR|=|PR|$ **(B)** $|PQ|+|QR|<|PR|$

(C) $|PQ|+|QR|>|PR|$ **(D)** $|PQ|+|QR|\leq|PR|$

6 Distance between $S(-1,3)$ **and** $R(3,-2)$ **is:**

(A) $|SR|=41$ **(B)** $|SR|=\sqrt{41}$

(C) $|SR|=36$ **(D)** $|SR|=45$

7 Midpoint of $(3,2)$ **and** $(4,-3)$ **is:**

(A) $\left(\frac{7}{2},\frac{-1}{2}\right)$ **(B)** $\left(\frac{3}{2},\frac{1}{2}\right)$

(C) $\left(\frac{9}{2},2\right)$ **(D)** None of these

Q.2 Give Short Answers to following Questions. **(5×2=10)**

(i) Draw the graph of the relation one mile = 1.6 *km*.

(ii) Find the value of m and c of the line by expressing them in the form $y = mx + c$, when $5x + 6y = 7$.

(iii) Check whether the points $\mathrm{P}(-2,-1)$, $\mathrm{Q}(0,3)$ and $\mathrm{R}(1,5)$ are collinear or not?

(iv) The end point P of a line segment PQ is $(-3,6)$ and its mid point is $(5,8)$. Find the coordinates of the end point Q.

(v) Draw a triangle where $P(-3,2), Q(4,3)$ and $R(4,4)$

Q.3 Answer the following Questions. **(4+4=8)**

(a) Find k, given that the point (2, k) is equidistant from (3, 7) and (9, 1).

(b) Solve the following pair of equations in x and y graphically. $x = 3y$; $2x - 3y = -6$

NOTE: Parents or guardians can conduct this test in their supervision in order to check the skills of the students.