Unit 7 Coordinate Geometry

Students' Learning Outcomes

At the end of the unit, the students will be able to:

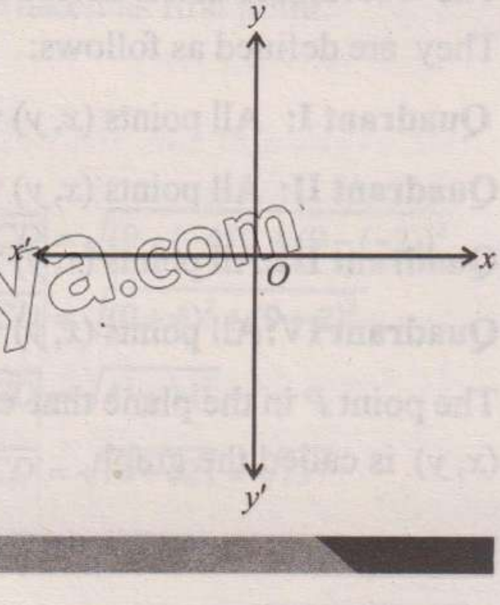
- Derive distance formula by locating the position of two points in coordinate plane.
- Calculate the midpoint of a line segment.
- Find the gradient of a straight line when coordinates of two points are given.
- Find the equation of a straight line in the form $y = mx + c$.
- Find the gradient of parallel and perpendicular lines.
- Apply distance and midpoint formulas to solve real-life situations such as physical measurements or distances between locations.
- Apply concepts from coordinate geometry to real world problems (such as, aviation and navigation, landscaping, map reading, longitude and latitude).
- Derive equation of a straight line in:
  - slope- intercept form
  - point-slope form
  - two-point form
  - intercepts form
  - symmetric form
  - normal form
- Show that a linear equation in two variables represents a straight line and reduce the general form of the equation of a straight line to the other standard forms.

INTRODUCTION

Geometry is one of the most ancient branches of mathematics. The Greeks systematically studied it about four centuries B.C. Most of the geometry taught in schools is due to Euclid who expounded thirteen books on the subject (300 B.C.). A French philosopher and mathematician Rene Descartes (1596-1650 A.D.) introduced algebraic methods in geometry which gave birth to analytic geometry (or coordinate geometry). Our aim is to present fundamentals of the subject in this book.

7.1 Coordinate Plane

Draw in a plane two mutually perpendicular number lines $x'x$ and $y'y$, one horizontal and the other vertical. Let O be their point of intersection, called origin and the real number 0 of both the lines is represented by O. The two lines are called **coordinate axes.** The horizontal line $x'Ox$ is called the ***x*-axis** and the vertical line $y'Oy$ is called the ***y*-axis.**

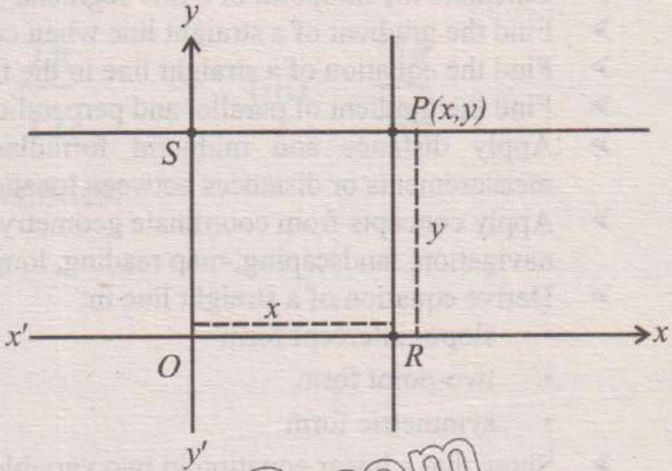


Important information:

The Cartesian coordinate system or the rectangular coordinate system was invented by French mathematician René Descartes, when he tried to describe the path of a fly crawling along criss-cross beams on the ceiling while he lay on his bed. The Cartesian coordinate system created a link between algebra and geometry. Geometric shapes could now be described algebraically using the coordinates of the points that make up the shapes.

The points lying on Ox are $+ve$ and on Ox' are $-ve$.

The points lying on Oy are $+ve$ and Oy' are $-ve$.



Suppose P is any point in the plane. Then P can be located by using an ordered pair of real numbers. Through P draw lines parallel to the coordinates axes meeting x-axis at R and y-axis at S. Let the directed distance $\overline{OR} = x$ and the directed distance $\overline{OS} = y$.

The ordered pair (x, y) gives us enough information to locate the point P. Thus, P has coordinates (x, y). The first component of the ordered pair (x, y) is called x-coordinate or **abscissa** and the second component is called y-coordinate or **ordinate** of P. The reverse of this technique also provides a method for associating exactly one point in the plane with any ordered pair (x, y) of real numbers. This method of pairing off in a one-to-one fashion the points in a plane with ordered pairs of real numbers is called the two dimensional rectangular (or Cartesian) coordinate system.

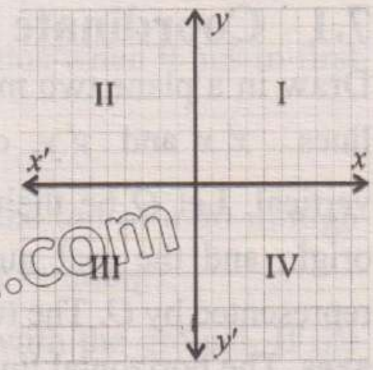
The coordinate axes divide the plane into four equal parts called quadrants. They are defined as follows:

Quadrant I: All points (x, y) with $x > 0, y > 0$

Quadrant II: All points (x, y) with $x < 0, y > 0$

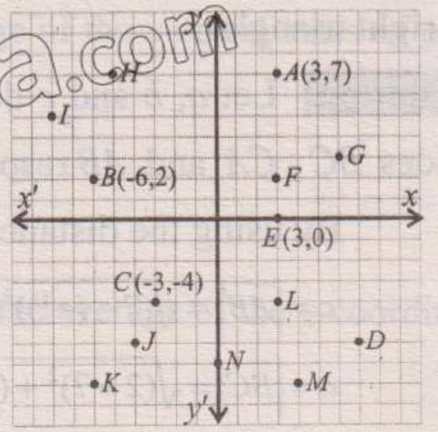
Quadrant III: All points (x, y) with $x < 0, y < 0$

Quadrant IV: All points (x, y) with $x > 0, y < 0$



The point P in the plane that corresponds to an ordered pair (x, y) is called the graph.

Thus, given a set of ordered pairs of real numbers, the graph of the set is the aggregate of all points in the plane that correspond to ordered pairs of the set.



Need to know!

The points on *x-axis* are of the form $(a, 0)$ and the points on *y-axis* are of the form $(0, b)$.

Challenges!

(i) Write down the coordinates of the points if not mentioned in the adjacent figure.

(ii) Locate $(0, -1)$, $(2, 2)$, $(-4, 7)$ and $(-3, -3)$

7.1.1 The Distance Formula

Let $A\ (x_1, y_1)$ and $B\ (x_2, y_2)$ be two points in the plane. To find the distance $d = |\overline{AB}|$, we draw a horizontal line from A to a point C lies directly below B, forming a right triangle ABC.

Note: $|\overline{AB}|$ stands for $m\overline{AB}$

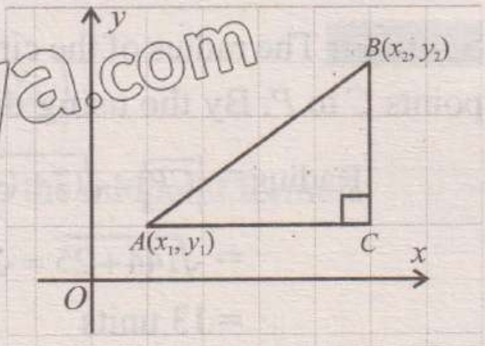
So that $|\overline{AC}| = |x_2 - x_1|$ and $|\overline{BC}| = |y_2 - y_1|$

By using Pythagoras Theorem, we have

$$d^2 = |\overline{AB}|^2 = |\overline{AC}|^2 + |\overline{BC}|^2$$

$$= (x_2 - x_1)^2 + (y_2 - y_1)^2$$

or $$\boxed{d = |\overline{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}} \qquad \ldots\text{(i)}$$



The distance is always taken to be non-negative. It is not a directed distance from A to B.

If A and B lie on a line parallel to one of the coordinate axes, then by the formula (i), the distance $\overline{AB}$ is absolute value of the directed distance $\overline{AB}$.

The formula (i) shows that any of the two points can be taken as first point.

Example 1: Find the distance between the points:

(i) $A(5, 6), B(5, -2)$ (ii) $C(-4, -2), D(0, 9)$

Solution: By the distance formula, we have

(i) $d = |\overline{AB}| = \sqrt{(5-5)^2 + (-2-6)^2}$

$d = |\overline{AB}| = \sqrt{(0)^2 + (-8)^2}$

$d = |\overline{AB}| = \sqrt{0 + 64} = 8$

(ii) $d = |\overline{CD}| = \sqrt{(0-(-4))^2 + (9-(-2))^2}$

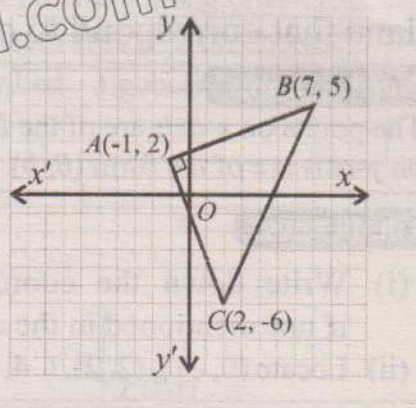
$d = |\overline{CD}| = \sqrt{(0+4)^2 + (9+2)^2}$

$d = |\overline{CD}| = \sqrt{4^2 + 11^2}$

$d = |\overline{CD}| = \sqrt{16 + 121} = \sqrt{137}$

Example 2: Show that the points $A(-1, 2)$, $B(7, 5)$ and $C(2, -6)$ are vertices of a right triangle.

Solution: Let a, b and c denote the lengths of the sides BC, CA and AB respectively.



By using the distance formula, we have

$$c = \left|\overline{AB}\right| = \sqrt{(7-(-1))^2 + (5-2)^2} = \sqrt{73}$$

$$a = \left|\overline{BC}\right| = \sqrt{(2-7)^2 + (-6-5)^2} = \sqrt{146}$$

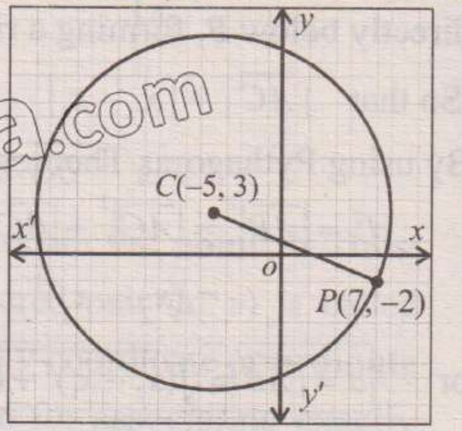
$$b = \left|\overline{CA}\right| = \sqrt{(2-(-1))^2 + (-6-2)^2} = \sqrt{73}$$

Clearly: $a^2 = b^2 + c^2$

Therefore, ABC is a right triangle with right angle at A.

Example 3: The point $C(-5, 3)$ is the centre of a circle and $P(7, -2)$ lies on the circle. What is the radius of the circle?

Solution: The radius of the circle is the distance from the points C to P. By the using distance formula, we have



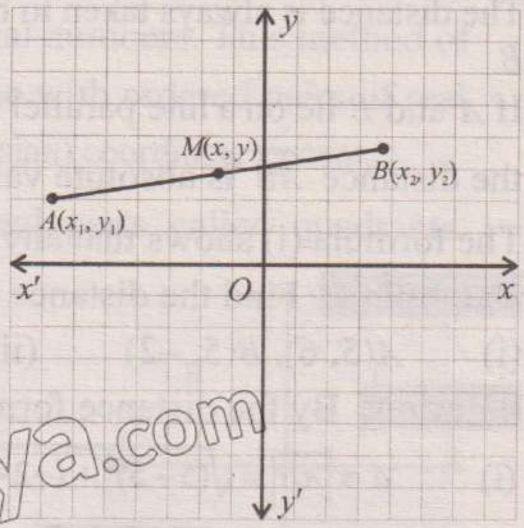
$$\text{Radius} = \left|\overline{CP}\right| = \sqrt{(7-(-5))^2 + (-2-3)^2}$$

$$= \sqrt{144 + 25} = \sqrt{169}$$

$$= 13 \text{ units}$$

7.1.2 Mid Point Formula

The midpoint formula is used in geometry to find centeral point between two given points in a coordinate plane. This formula is particularly useful when you need to divide a line segment into two equal halves or parts.



Derivation of the Midpoint Formula

Consider two points $A(x_1, y_1)$ and $B(x_2, y_2)$ on a two-dimensional plane. The line segment joining these two points has a midpoint $M(x, y)$, where x and y are the coordinates of the midpoint.

To derive the formula for $M(x, y)$ we need to find the average of the x-coordinates and y-coordinates of points A and B separately.

1. *x*-Coordinate of the Midpoint

The x-coordinate of the midpoint is the average of the *x*-coordinates of points *A* and *B*.

i.e., $x = \dfrac{x_1 + x_2}{2}$

2. *y*-Coordinate of the Midpoint

Similarly, the y-coordinate of the midpoint is the average of the *y*-coordinates of points *A* and *B*.

i.e., $y = \dfrac{y_1 + y_2}{2}$

Thus, the coordinates of the midpoint $M(x, y)$ are:

$$M(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Example 4: Find the midpoint of the line segment joining the points $A(2,3)$ and $B(8,7)$.

Solution: Using the midpoint formula:

$$M(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Substitute $x_1 = 2, y_1 = 3, x_2 = 8$ and $y_2 = 7$, into the midpoint formula

$$M(x, y) = \left(\frac{2+8}{2}, \frac{3+7}{2}\right)$$

$$M(x, y) = \left(\frac{10}{2}, \frac{10}{2}\right) = (5, 5)$$

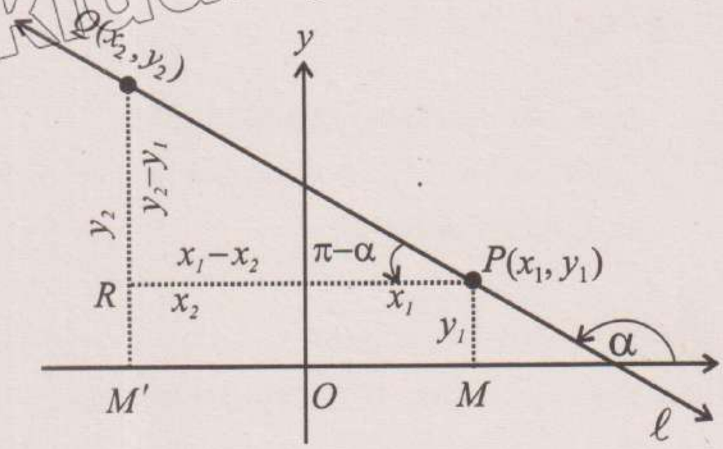
EXERCISE 7.1

1. Describe the location in the plane of the point $P(x, y)$, for which
   (i) $x > 0$ (ii) $x > 0$ and $y > 0$ (iii) $x = 0$ (iv) $y = 0$
   (v) $x > 0$ and $y \leq 0$ (vi) $y = 0, x = 0$ (vii) $x = y$
   (viii) $x \geq 3$ (ix) $y > 0$ (x) *x* and *y* have opposite signs.
2. Find the distance between the points:
   (i) $A(6, 7), B(0, -2)$ (ii) $C(-5, -2), D(3, 2)$
   (iii) $L(0, 3), M(-2, -4)$ (iv) $P(-8, -7), Q(0, 0)$
3. Find in each of the following:
   (i) The distance between the two given points

Case (i). When $0 < \alpha < \frac{\pi}{2}$

In the right triangle PRQ, we have

$$m = \tan\alpha = \frac{y_2 - y_1}{x_2 - x_1}$$



Case (ii). When $\frac{\pi}{2} < \alpha < \pi$

In the right triangle PRQ,

$$\tan(\pi - \alpha) = \frac{y_2 - y_1}{x_1 - x_2}$$

or $$-\tan\alpha = \frac{y_2 - y_1}{x_1 - x_2}$$

or $$\tan\alpha = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{or} \quad m = \frac{y_2 - y_1}{x_2 - x_1}$$

Thus if $P(x_1, y_1)$ and $Q(x_2, y_2)$ are two points on a line, then slope of $\overleftrightarrow{PQ}$ is given by:

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{or} \quad m = \frac{y_1 - y_2}{x_1 - x_2}$$

Note:
(i) $m \neq \frac{y_2 - y_1}{x_1 - x_2}$ and $m \neq \frac{y_1 - y_2}{x_2 - x_1}$
(ii) ℓ is horizontal iff $m = 0$ $(\because \alpha = 0^\circ)$
(iii) ℓ is vertical iff m is not defined $(\because \alpha = 90^\circ)$
(iv) If slope of $\overline{AB}$ = slope of $\overline{BC}$, then the points A, B and C are collinear.

Theorem 2: The two lines ℓ_1 and ℓ_2 with slopes m_1 and m_2 respectively are:

(i) parallel iff $m_1 = m_2$

(ii) perpendicular iff $m_1 = \frac{-1}{m_2}$

or $m_1 m_2 = -1$

Remember !
The symbol:
(i) $\parallel$ stands for "parallel".
(ii) $\nparallel$ stands for "not parallel".
(iii) $\perp$ stands for "perpendicular"

Example 5: Show that the points $A(-3, 6)$, $B(3, 2)$ and $C(6, 0)$ are collinear.

Solution: We know that the points A, B and C are collinear if the line AB and BC have the same slopes.

Here slope of $\overline{AB} = \frac{2-6}{3-(-3)} = \frac{-4}{3+3} = \frac{-4}{6} = \frac{-2}{3}$ and slope of $\overline{BC} = \frac{0-2}{6-3} = \frac{-2}{3}$

$\because$ Slope of AB = Slope of BC

Thus A, B and C are collinear.

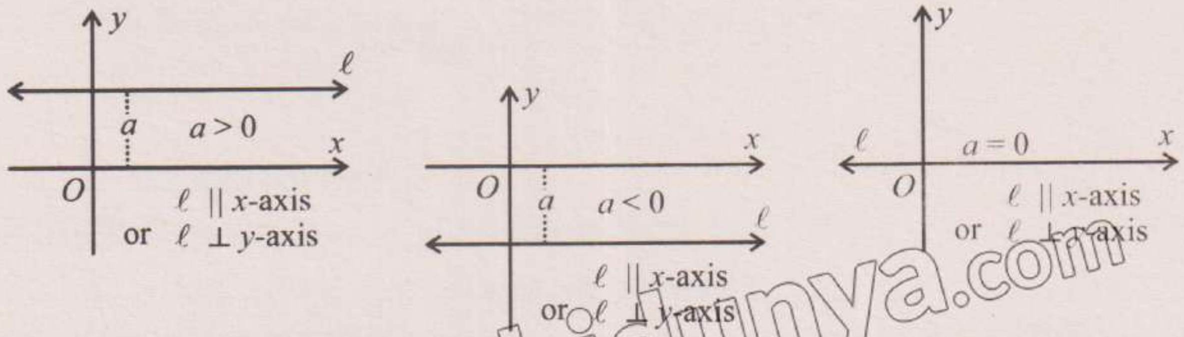
Example 6: Show that the triangle with vertices $A(1,1)$, $B(4,5)$ and $C(12,-1)$ is a right triangle.

Solution: Slope of $\overline{AB} = m_1 = \frac{5-1}{4-1} = \frac{4}{3}$ and slope of $\overline{BC} = m_2 = \frac{-1-5}{12-4} = \frac{-6}{8} = \frac{-3}{4}$

Since $m_1 \cdot m_2 = \left(\frac{4}{3}\right)\left(-\frac{3}{4}\right) = -1$, therefore, $\overline{AB} \perp \overline{BC}$

So ΔABC is a right triangle.

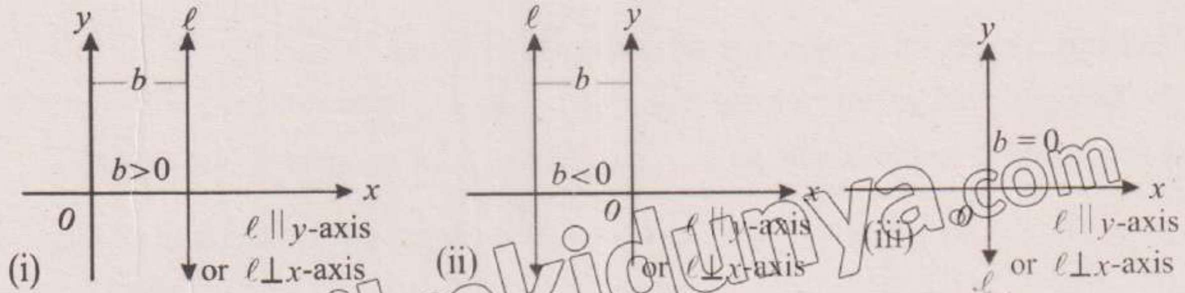
7.2.2 Equation of a Straight Line Parallel to the x-axis (or perpendicular to the y-axis)



All the points on the line ℓ parallel to x-axis remain at a constant distance (say a) from x-axis. Therefore, each point on the line has its distance from x-axis equal to a, which is its y-coordinate (ordinate). So, all the points on this line satisfy the equation: $\boxed{y = a}$

> **Note:**
> If $a > 0$, then the line ℓ is above the x-axis.
> If $a < 0$, then the line ℓ is below the x-axis.
> If $a = 0$, then the line ℓ becomes the x-axis.
> Thus the equation of x-axis is $y = 0$

7.2.3 Equation of a straight Line Parallel to the y-axis (or perpendicular to the x-axis)

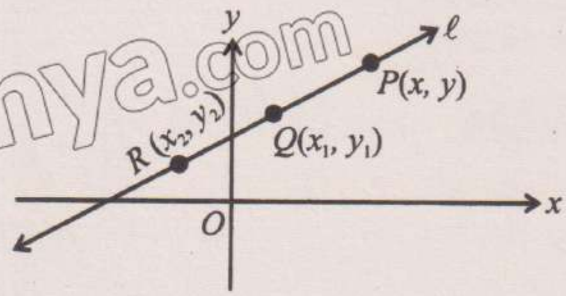


All the points on the line ℓ parallel to y-axis remain at a constant distance (say b) from y-axis. Each point on the line has its distance from y-axis equal to b, which is its x-coordinate (abscissa). So, all the points on this line satisfy the equation: $\boxed{x = b}$

4. Two-point Form of Equation of a Straight Line

Theorem 5: Equation of a non-vertical straight line passing through two points $Q(x_1, y_1)$ and $R(x_2, y_2)$ is:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$
or
$$y - y_2 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_2)$$



Proof: Let $P(x, y)$ be an arbitrary point of the line passing through $Q(x_1, y_1)$ and $R(x_2, y_2)$.

So, $\frac{y - y_1}{x - x_1} = \frac{y - y_2}{x - x_2} = \frac{y_2 - y_1}{x_2 - x_1}$ (P, Q and R are collinear points)

We take

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

or $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$, the required equation of the line PQ.

or $(y_2 - y_1)x - (x_2 - x_1)y + (x_1 y_2 - x_2 y_1) = 0$

We may write this equation in determinant form as: $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$

$y - y_2 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_2)$ can be derived similarly.

Example 9: Find an equation of line through the points (–2,1) and (6,–4).

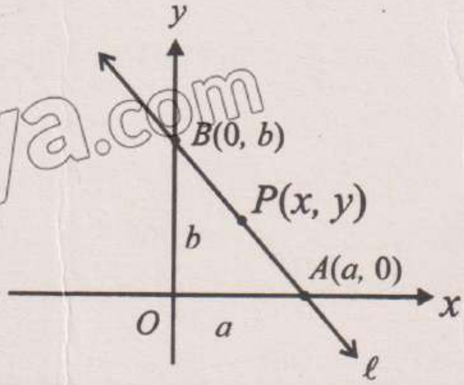
Solution: Using two-points form of the equation of straight line, the required equation is:

$$y - 1 = \frac{-4-1}{6-(-2)}[x - (-2)] \quad \text{or} \quad y - 1 = \frac{-5}{8}(x+2) \text{ or } 5x + 8y + 2 = 0$$

5. Intercept Form of Equation of a Straight Line

Theorem 6: Equation of a line whose non-zero x and y-intercepts are a and b respectively is:

$$\frac{x}{a} + \frac{y}{b} = 1$$



Proof: Let $P(x, y)$ be an arbitrary point of the line whose non-zero x and y-intercepts are a and b respectively. Obviously, the points $A(a, 0)$ and $B(0, b)$ lie on the required line. So, by the two-point form of the equation of line, we have

$$y-0=\frac{b-0}{0-a}(x-a) \qquad (P, A \text{ and } B \text{ are collinear})$$

or $\quad -ay=b(x-a) \quad$ or $\quad bx+ay=ab$

or $\quad \frac{x}{a}+\frac{y}{b}=1 \quad$ (dividing by ab)

Hence the result.

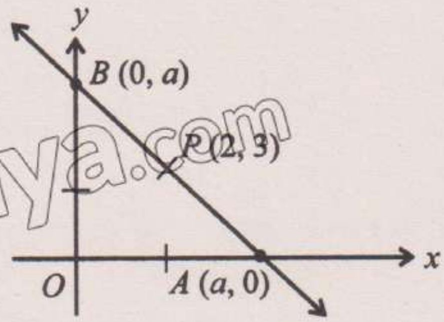
Example 10: Write down an equation of the line which cuts the x-axis at $(2, 0)$ and y-axis at $(0, -4)$.

Solution: As 2 and -4 are respectively x and y-intercepts of the required line, so by two-intercepts form of equation of a straight line, we have

$$\frac{x}{2}+\frac{y}{-4}=1 \qquad \text{or} \qquad 2x-y-4=0$$

Which is the required equation.

Example 11: Find an equation of the line through the point $P(2, 3)$ which forms an isosceles triangle with the coordinate axes in the first quadrant.



Solution: Let OAB be an isosceles triangle so that the line AB passes through $A(a, 0)$ and $B(0, a)$, where a is some positive real number.

Slope of $AB=\frac{a-0}{0-a}=-1$. But AB passes through P (2, 3).

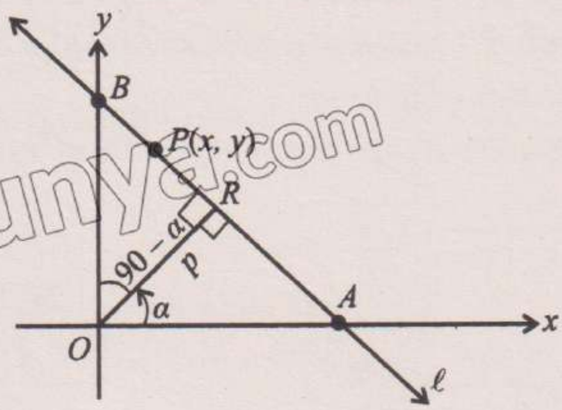
Equation of the line through $P(2, 3)$ with slope -1 is

$$y-3=-1(x-2) \qquad \text{or} \qquad x+y-5=0$$

6. Normal Form of Equation of a Straight Line

Theorem 7: An equation of a non-vertical straight-line ℓ, such that length of the perpendicular from the origin to ℓ is p and α is the inclination of this perpendicular, is

$$\boxed{x\cos\alpha + y\sin\alpha = p}$$



Proof: Let the line ℓ meet the x-axis and y-axis at the points A and B respectively. Let $P(x, y)$ be an arbitrary point of line AB and let $\overline{OR}$ be perpendicular to the line ℓ. Then $|\overline{OR}|=p$

iii. Symmetric Form

$$m = \tan\alpha = \frac{-a}{b},\ \sin\alpha = \frac{a}{\pm\sqrt{a^2+b^2}},\cos\alpha\frac{b}{\pm\sqrt{a^2+b^2}}$$

A point on $ax + by + c = 0$ is $\left(\frac{-c}{a},0\right)$

Equation of the line symmetric form becomes

$$\frac{x-\left(-\frac{c}{a}\right)}{b/\pm\sqrt{a^2+b^2}} = \frac{y-0}{a/\pm\sqrt{a^2+b^2}} = r\,(say)$$

is the required transformed equation. Sign of the radical to be chosen properly.

iv. Two -Point Form

We choose two arbitrary points on $ax + by + c = 0$. Two such points are $\left(\frac{-c}{a},0\right)$ and $\left(0,\frac{-c}{b}\right)$. Equation of the line through these points is:

$$\frac{y-0}{0+\frac{c}{b}} = \frac{x+\frac{c}{a}}{-\frac{c}{a}-0} \qquad \text{i.e., } y-0 = \frac{-a}{b}\left(x+\frac{c}{a}\right)$$

v. Intercept Form

$$ax+by=-c \quad \text{or} \quad \frac{ax}{-c}+\frac{by}{-c}=1 \quad \text{i.e.,} \quad \frac{x}{-c/a}+\frac{y}{-c/b}=1$$

which is an equation in two intercepts form.

vi. Normal Form

The equation: $ax + by + c = 0$ …(i)

can be written in the normal form as:

$$\frac{ax+by}{\pm\sqrt{a^2+b^2}} = \frac{-c}{\pm\sqrt{a^2+b^2}} \qquad \ldots\text{(ii)}$$

The sign of the radical to be such that the right hand side of (ii) is positive.

Proof. We know that an equation of a line in normal form is

$x\cos\alpha + y\sin\alpha = p$ ... (iii)

If (i) and (iii) are identical, we must have

$$\frac{a}{\cos\alpha} = \frac{b}{\sin\alpha} = \frac{-c}{p}$$

i.e., $\frac{p}{-c} = \frac{\cos\alpha}{a} = \frac{\sin\alpha}{b} = \frac{\sqrt{\cos^2\alpha+\sin^2\alpha}}{\pm\sqrt{a^2+b^2}} = \frac{1}{\pm\sqrt{a^2+b^2}}$

Hence, $\cos\alpha = \dfrac{a}{\pm\sqrt{a^2+b^2}}$, $\sin\alpha = \dfrac{b}{\pm\sqrt{a^2+b^2}}$ and $p = \dfrac{-c}{\pm\sqrt{a^2+b^2}}$

Substituting for cosα , sinα and p into (iii), we have

$$\frac{ax+by}{\pm\sqrt{a^2+b^2}} = \frac{-c}{\pm\sqrt{a^2+b^2}}$$

Thus (i) can be reduced to the form (ii) by dividing it by $\pm\sqrt{a^2+b^2}$. The sign of the radical to be chosen so that the right hand side of (ii) is positive.

Example 13: Transform the equation $5x - 12y + 39 = 0$ into

(i) Slope intercept form (ii) Two-intercept form

(iii) Normal form (iv) Point-slope form

(v) Two-point form (vi) Symmetric form

Solution:

(i) We have $12y = 5x + 39$ *or* $y = \frac{5}{12}x + \frac{39}{12}$, $m = \frac{5}{12}$, y-intercept $c = \frac{39}{12}$

(ii) $5x - 12y = -39$ or $\frac{5x}{-39} + \frac{12y}{39} = 1$ *or* $\frac{x}{-39/5} + \frac{y}{39/12} = 1$ is the required equation.

(iii) $5x - 12y = -39$. Divide both sides by $\pm\sqrt{5^2+12^2} = \pm 13$. Since R.H.S is to be positive, we have to take negative sign.

Hence $\frac{5x}{-13} + \frac{12y}{13} = 3$ is the normal form of the equation.

(iv) A point on the line is $\left(\frac{-39}{5}, 0\right)$ and its slope is $\frac{5}{12}$.

Equation of the line can be written as: $y - 0 = \frac{5}{12}\left(x + \frac{39}{5}\right)$

(v) Another point on the line is $\left(0, \frac{39}{12}\right)$. Line through $\left(\frac{-39}{5}, 0\right)$ and $\left(0, \frac{39}{12}\right)$ is

$$\frac{y-0}{0-\frac{39}{12}} = \frac{x+\frac{39}{5}}{\frac{-39}{5}-0}$$

Example 16: An Engineer is building a bridge between two points on a riverbank. Suppose the coordinates of the two points where the bridge will start and end are (2, 5) and (8, 9). Find the coordinates of the midpoint, which will represent the centre of the bridge.

Solution: We apply the midpoint formula:

$$M = \left(\frac{2+8}{2}, \frac{5+9}{2}\right)$$

$$M = \left(\frac{10}{2}, \frac{14}{2}\right) = (5, 7)$$

Thus, the centre of the bridge is at the point (5, 7)

Example 17: A landscaper is designing a triangular garden with corners at points $A(2, 3)$, $B(5, 7)$, and C(6, 2). Calculate the lengths of the sides of the triangular gardern.

Solution: Use the distance formula to find the length of each side:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|\overline{AB}| = \sqrt{(5-2)^2 + (7-3)^2}$$

$$|\overline{AB}| = \sqrt{(3)^2 + (4)^2}$$

$$|\overline{AB}| = \sqrt{9+16} = \sqrt{25} = 5 \text{ units}$$

$$|\overline{BC}| = \sqrt{(6-5)^2 + (2-7)^2}$$

$$|\overline{BC}| = \sqrt{(1)^2 + (-5)^2}$$

$$|\overline{BC}| = \sqrt{1+25} = \sqrt{26} = 5.10 \text{ units}$$

$$|\overline{AC}| = \sqrt{(6-2)^2 + (2-3)^2}$$

$$|\overline{AC}| = \sqrt{(4)^2 + (-1)^2}$$

$$|\overline{AC}| = \sqrt{16+1} = \sqrt{17} = 4.12 \text{ units}$$

Thus, the lengths of the sides of the triangular garden are:

$m\overline{AB} = 5$ units , $m\overline{BC} \approx 5.10$ units , $m\overline{AC} \approx 4.12$ units

Example 18: A pilot needs to travel from city $A(50, 60)$ to city $B(120, 150)$. Determine the heading angle the plane should take relative to the east direction.

Solution: The heading angle can be calculated using the slope:

$$m = \frac{150-60}{120-50} = \frac{90}{70} = \frac{9}{7}$$

Let θ be the required angle, then

$$\tan\theta = m = \frac{9}{7}$$

$$\theta = \tan^{-1}\left(\frac{9}{7}\right)$$

$$\theta = \tan^{-1}(1.2857)$$

$$\theta \approx 52.13^\circ$$

> **Do you know?**
> **Aviation** is the operation and flight of aircraft, including airplanes, helicopters and drones.
> **Navigation** is the process of determining and controlling the route of a vehicle, such as an aircraft, from one place to another.

Thus, the plane should take a heading angle of 52.13° north of east.

Latitude measures how far a location is from the equator. It ranges from 0° at the equator to 90° north (at the North Pole) or 90° south (at the South Pole).
Longitude measures how far a location is from the Prime Meridian (which runs through Greenwich, London). It ranges from 0° at the Prime Meridian to 180° east and 180° west.

Example 19: Abdul Hadi is traveling from point *A* (Latitude 10° N, Longitude 50° E) to point *B* (Latitude 20° N, Longitude 60° E). Find the midpoint of his journey in terms of latitude and longitude.

Solution:

Given that

Point A (Latitude 10° N, Longitude 50° E)

Point B (Latitude 20° N, Longitude 60° E)

$$\text{Midpoint latitude} = \frac{10^\circ + 20^\circ}{2} = 15^\circ N$$

$$\text{Midpoint longitude} = \frac{50^\circ + 60^\circ}{2} = 55^\circ E$$

Thus, the midpoint of Abdul Hadi's journey would be at Latitude 15° N, Longitude 55° E.

Example 20: A landscaper is designing a straight pathway from P(2, 3) to Q(8, 9). What is the length of the pathway?

Solution:

The length of the straight pathway can be found using the distance formula:

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(8-2)^2 + (9-3)^2}$$

$$= \sqrt{(6)^2 + (6)^2}$$

$$= \sqrt{36+36}$$

$$= 6\sqrt{2}$$

So, the length of the pathway is approximately $6\sqrt{2}$ units.

Exercise 7.3

1. If the houses of two friends are represented by coordinates (2, 6) and (9, 12) on a grid. Find the straight line distance between their houses if the grid units represent kilometres?
2. Consider a straight trail (represented by coordinate plane) that starts at point (5, 7) and ends at point (15, 3). What are the coordinates of the midpoint?
3. An architect is designing a park with two buildings located at (10, 8) and (4, 3) on the grid. Calculate the straight-line distance between the buildings. Assume the coordinates are in metres.
4. A delivery driver needs to calculate the distance between two delivery locations. One location is at (7, 2) and the other is at (12, 10) on the city grid map, where each unit represents kilometres. What is the distance between the two locations?
5. The start and end points of a race track are given by coordinates (3, 9) and (9, 13). What is the midpoint of the track?
6. The coordinates of two points on a road are $A(3, 4)$ and $B(7, 10)$. Find the midpoint of the road.
7. A ship is navigating from port A located at (12° N, 65° W) to port B at (20 ° N, 45 ° W). If the ship travels along the shortest path on the surface of the Earth, calculate the straight line distance between the points.
8. Farah is fencing around a rectangular field with corners at (0,0), (0,5), (8, 5) and (8, 0). How much fencing material will she need to cover the entire perimeter of the field?

9. An airplane is flying from city X at (40° N, 100° W) to city Y at (50° N, 80° W). Use coordinate geometry, calculate the shortest distance between these two cities.

10. A land surveyor is marking out a rectangular plot of land with corners at (3, 1), (3, 6), (8, 6), and (8, 1). Calculate the perimeter.

11. A landscaper needs to install a fence around a rectangular garden. The garden has its corners at the coordinates: $A(0, 0)$, $B(5, 0)$, $C(5, 3)$, and $D(0, 3)$. How much fencing is required?

REVIEW EXERCISE 7

1. Four options are given against each statement. Encircle the correct option.

(i) The equation of a straight line in the slope-intercept form is written as:

(a) $y = m(x + c)$ (b) $y - y_1 = m(x - x_1)$
(c) $y = c + mx$ (d) $ax + by + c = 0$

(ii) The gradients of two parallel lines are:

(a) equal (b) zero
(c) negative reciprocals of each other (d) always undefined

(iii) If the product of the gradients of two lines is –1, then the lines are:

(a) Parallel (b) perpendicular
(c) Collinear (d) coincident

(iv) Distance between two points $P(1, 2)$ and $Q(4, 6)$ is:

(a) 5 (b) 6 (c) $\sqrt{13}$ (d) 4

(v) The midpoint of a line segment with endpoints (–2, 4) and (6, –2) is:

(a) (4, 2) (b) (2, 1) (c) (1, 1) (d) (0, 0)

(vi) A line passing through points (1, 2) and (4, 5) is:

(a) $y = x + 1$ (b) $y = 2x + 3$
(c) $y = 3x - 2$ (d) $y = x + 2$

(vii) The equation of a line in point-slope form is:

(a) $y = m(x + c)$ (b) $y - y_1 = m(x - x_1)$
(c) $y = c + mx$ (d) $ax + by + c = 0$

(viii) $2x + 3y - 6 = 0$ in the slope-intercept form is:

(a) $y = \frac{-2}{3}x + 2$ (b) $y = \frac{2}{3}x - 2$
(c) $y = \frac{2}{3}x + 1$ (d) $y = \frac{-2}{3}x - 2$

(ix) The equation of a line in symmetric form is:

(a) $\frac{x}{a}+\frac{y}{b}=1$ (b) $\frac{x-x_1}{l}+\frac{y-y_1}{m}=\frac{z-z_1}{1}$

(c) $\frac{x-x_1}{\cos\alpha}=\frac{y-y_1}{\sin\alpha}=r$ (d) $y-y_1=m(x-x_1)$

(x) The equation of a line in normal form is:

(a) $y=mx+c$ (b) $\frac{x}{a}+\frac{y}{b}=1$

(c) $\frac{x-x_1}{\cos\alpha}=\frac{y-y_1}{\sin\alpha}$ (d) $x\cos\alpha+y\sin\alpha=p$

2. Find the distance between two points $A(2, 3)$ and $B(7, 8)$ on a coordinate plane.
3. Find the midpoint of the line segment joining the points (4, –2) and (–6, 3).
4. Calculate the gradient (slope) of the line passing through the points (1, 2) and (4, 6).
5. Find the equation of the line in the form $y = mx + c$ that passes through the points (3, 7) and (5, 11).
6. If two lines are parallel and one line has a gradient of $\frac{2}{3}$, what is the gradient of the other line?
7. An airplane needs to fly from city A to coordinates (12, 5) to city B at coordinates (8, –4). Calculate the straight-line distance between these two cities.
8. In a landscaping project, the path starts at (2, 3) and ends at (10, 7). Find the midpoint.
9. A drone is flying from point (2, 3) to point (10, 15) on the grid. Calculate the gradient of the line along which the drone is flying and the total distance travelled.
10. For a line with a gradient of –3 and a y-intercept of 2, write the equation of the line in:
    (a) Slope-intercept form
    (b) Point-slope form using the point (1, 2)
    (c) Two-point form using the points (1, 2) and (4, –7)
    (d) Intercepts form
    (e) Symmetric form
    (f) Normal form