

Q.14 $x^2 + \frac{1}{x^2} - 2$

Sol: $= x^2 - 2 + \frac{1}{x^2}$
 $= (x)^2 - 2(x)\left(\frac{1}{x}\right) + \left(\frac{1}{x}\right)^2$
 $= \left(x - \frac{1}{x}\right)^2$

Q.15 $5x^3 - 30x^2 + 45x$

Sol: $= 5x[x^2 - 6x + 9]$
 $= 5x[(x)^2 - 2(x)(3) + (3)^2]$
 $= 5x(x - 3)^2$

Q.16 $a^2 + b^2 + 2ab + 2bc + 2ac$

Sol: $= (a^2 + 2ab + b^2) + 2ac + 2bc$
 $= [(a)^2 + 2(a)(b) + (b)^2] + 2c(a + b)$
 $= (a + b)^2 + 2c(a + b)$
 $= (a + b)[(a + b) + 2c]$
 $= (a + b)(a + b + 2c)$

Exercise 2.2

Resolve into factors.

Q.1 $x^2 + 2xy + y^2 - a^2$

Sol: $= (x^2 + 2xy + y^2) - a^2$
 $= [(x)^2 + 2(x)(y) + (y)^2] - (a)^2$
 $= (x + y)^2 - (a)^2$
 $= (x + y + a)(x + y - a)$

Q.2 $4a^2 - 4ab + b^2 - 9c^2$

Sol: $= (4a^2 - 4ab + b^2) - 9c^2$
 $= [(2a)^2 - 2(2a)(b) + (b)^2] - (3c)^2$

$$= (2a - b)^2 - (3c)^2$$

$$= (2a - b + 3c)(2a - b - 3c)$$

Q.3 $x^2 + 6ax + 9a^2 - 16b^2$

Sol: $= (x^2 + 6ax + 9a^2) - 16b^2$

$$= [(x)^2 + 2(x)(3a) + (3a)^2] - (4b)^2$$

$$= (x + 3a)^2 - (4b)^2$$

$$= (x + 3a + 4b)(x + 3a - 4b)$$

Q.4 $y^2 - c^2 + 2cx - x^2$

Sol: $= y^2 - (c^2 - 2cx + x^2)$

$$= (y)^2 - [(c)^2 - 2(c)(x) + (x)^2]$$

$$= (y)^2 - (c - x)^2 \quad \text{Formula: } [a^2 - b^2 = (a + b)(a - b)]$$

$$= [y + (c - x)][y - (c - x)]$$

$$= (y + c - x)(y - c + x)$$

$$= (y - x + c)(y + x - c)$$

Q.5 $x^2 + y^2 + 2xy - 4x^2y^2$

Sol: $= (x^2 + 2xy + y^2) - 4x^2y^2$

$$= [(x)^2 + 2(x)(y) + (y)^2] - (2xy)^2$$

$$= (x + y)^2 - (2xy)^2$$

$$= (x + y + 2xy)(x + y - 2xy)$$

Q.6 $a^2 - 4ab + 4b^2 - 9a^2c^2$

Sol: $= (a^2 - 4ab + 4b^2) - (9a^2c^2)$

$$= [(a)^2 - 2(a)(2b) + (2b)^2] - [3ac]^2$$

$$= (a - 2b)^2 - (3ac)^2$$

$$= (a - 2b + 3ac)(a - 2b - 3ac)$$

Q.7 $x^2 - 2xy + y^2 - a^2 + 2ab - b^2$

Sol: $= (x^2 - 2xy + y^2) - (a^2 - 2ab + b^2)$

$$= [(x)^2 - 2(x)(y) + (y)^2] [(a)^2 - 2(a)(b) + (b)^2]$$

$$= (x - y)^2 - (a - b)^2$$

$$= [(x - y) + (a - b)][(x - y) - (a - b)]$$

$$= (x - y + a - b)(x - y - a + b)$$

Q.8 $y^4 + 4$

Sol: $= (y^2)^2 + (2)^2 + (2)(y^2)(2) - 2(y^2)(2)(\text{completing square root})$

$$\begin{aligned}
 &= [(y^2)^2 + (2)^2 + 4y^2] - 4y^2 \\
 &= (y^2 + 2)^2 - (2y)^2 \\
 &= (y^2 + 2 + 2y)(y^2 + 2 - 2y) \\
 &= (y^2 + 2y + 2)(y^2 - 2y + 2)
 \end{aligned}$$

Q.9 $z^4 + 64y^4$

Sol: $= (z^2)^2 + (8y^2)^2 + 2(z^2)(8y^2) - 2(z^2)(8y^2)$ (completing square root)

$$\begin{aligned}
 &= (z^2 + 8y^2)^2 - 16z^2y^2 \\
 &= (z^2 + 8y^2)^2 - (4zy)^2 \\
 &= (z^2 + 8y^2 - 4zy)(z^2 + 8y^2 + 4zy)
 \end{aligned}$$

Q.10 $x^4 + 324$

Sol: $= (x^2)^2 + (18)^2 + 2(x^2)(18) - 2(x^2)(18)$ (completing square root)

$$\begin{aligned}
 &= (x^2 + 18)^2 - 36x^2 \\
 &= (x^2 + 18)^2 - (6x)^2 \\
 &= (x^2 + 18 + 6x)(x^2 + 18 - 6x) \\
 &= (x^2 + 6x + 18)(x^2 - 6x + 18)
 \end{aligned}$$

Q.11 $z^4 - z^2 + 16$

Sol: $= (z^2)^2 + (4)^2 + 2(z^2)(4) - 9z^2$ (completing square root)

$$\begin{aligned}
 &= (z^2 + 4)^2 - (3z)^2 \\
 &= (z^2 + 4 - 3z)(z^2 + 4 + 3z) \\
 &= (z^2 - 3z + 4)(z^2 + 3z + 4)
 \end{aligned}$$

Q.12 $4x^4 - 5x^2y^2 + y^4$

Sol: $= (2x^2)^2 - 5x^2y^2 + (y^2)^2$ (completing square root)

$$\begin{aligned}
 &= (2x^2)^2 + 2(2x^2)(y^2) + (y^2)^2 - 9x^2y^2 \text{ (completing square root)} \\
 &= (2x^2 + y^2)^2 - (3xy)^2 \\
 &= (2x^2 + y^2 - 3xy)(2x^2 + y^2 + 3xy)
 \end{aligned}$$

Exercise 2.3

Factorize:

Q.1 $x^2 + 9x + 20$

Sol: $= x^2 + 4x + 5x + 20$

$$= (x^2 + 4x) + (5x + 20)$$