Q.20 $4\sqrt{3}x^2 + 5x - 2\sqrt{3}$

Sol: $4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3}$

$(4\sqrt{3}x^2 + 8x) - (3x + 2\sqrt{3})$

$4x(\sqrt{3}x + 2) - \sqrt{3}(\sqrt{3}x + 2)$

$= (\sqrt{3}x + 2)(4x - \sqrt{3})$

Formulae

(i) $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

(ii) $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

(iii) $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

(iv) $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Exercise 2.4

Factorize:

Q.1 $8x^3 - y^3$

Sol: $(2x)^3 - (y)^3$

$(2x - y)[(2x)^2 + (2x)(y) + (y)^2]$

$(2x - y)(4x^2 + 2xy + y^2)$

Q.2 $27x^3 + 1$

Sol: $= (3x)^3 + (1)^3$

$= (3x + 1)[(3x)^2 - (3x)(1) + (1)^2]$

$= (3x + 1)(9x^2 - 3x + 1)$

Q.3 $1 - 343x^3$

Sol: $= (1)^3 - (7x)^3$

$= (1 - 7x)[(1)^2 + 1(7x) + (7x)^2]$

$= (1 - 7x)(1 + 7x + 49x^2)$

Q.4 $a^3b^3 + 512$

Sol: $= (ab)^3 + (8)^3$

$(ab + 8)[(ab)^2 - (ab)(8) + (8)^2]$

$= (ab + 8)(a^2b^2 - 8ab + 64)$

Q.5 $\mathbf{27 - 1000y^3}$

Sol: $= (3)^3 - (10y)^3$

$= (3 - 10y)[(3)^2 + (3)(10y) + (10y)^2]$

$= (3 - 10y)(9 + 30y + 100y^2)$

Q.6 $\mathbf{27x^3 - 64y^3}$

Sol: $= (3x)^3 - (4y)^3$

$= (3x - 4y)[(3x)^2 + (3x)(4y) + (4y)^2]$

$= (3x - 4y)(9x^2 + 12xy + 16y^2)$

Q.7 $\mathbf{x^3y^3 + z^3}$

Sol: $= (xy)^3 + (z)^3$

$= (xy + z)[(xy)^2 - (xy)(z) + (z)^2]$

$= (xy + z)(x^2y^2 - xyz + z^2)$

Q.8 $\mathbf{216p^3 - 343}$

Sol: $= (6p)^3 - (7)^3$

$= (6p - 7)[(6p)^2 + (6p)(7) + (7)^2]$

$= (6p - 7)(36p^2 + 42p + 49)$

Q.9 $8x^3 - \frac{1}{27}$

Sol: $= (2x)^3 - \left(\frac{1}{3}\right)^3$

$= \left(2x - \frac{1}{3}\right)\left[(2x)^2 + (2x)\left(\frac{1}{3}\right) + \left(\frac{1}{3}\right)^2\right]$

$= \left(2x - \frac{1}{3}\right)\left(4x^2 + \frac{2}{3}x + \frac{1}{9}\right)$

Q.10 $\mathbf{a^3 + b^3 + a + b}$

Sol: $= (a^3 + b^3) + (a + b)$

$= (a + b)(a^2 - ab + b^2) + (a + b)$

$= (a + b)[(a^2 - ab + b^2 + 1)]$

$= (a + b)(a^2 - ab + b^2 + 1)$

Q.11 $\mathbf{a - b - a^3 + b^3}$

Sol: $= (a - b) - (a^3 - b^3)$

$= (a - b) - [(a - b)(a^2 + ab + b^2)]$

$= (a - b)[1 - (a^2 + ab + b^2)]$

$= (a - b)(1 - a^2 - ab - b^2)$

Q.12 $\mathbf{x - 8xy^3}$

Sol: $= x(1 - 8y^3)$

$= x[(1)^3 - (2y)^3]$

$= x[(1 - 2y)\{(1)^2 + (1)(2y) + (2y)^2\}]$

$= x[(1 - 2y)(1 + 2y + 4y^2)]$

$= x(1 - 2y)(1 + 2y + 4y^2)$

Q.13 $\mathbf{x^{12} - y^{12}}$

Sol: $= (x^6)^2 - (y^6)^2$

$= (x^6 - y^6)(x^6 + y^6)$

$= \{(x^3)^2 - (y^3)^2\}\{(x^2)^3 + (y^2)^3\}$

$= (x^3 - y^3)(x^3 + y^3)(x^2 + y^2)(x^4 - x^2y^2 + y^4)$

$= (x - y)(x^2 + xy + y^2)(x + y)(x^2 - xy + y^2)(x^2 + y^2)$
$(x^4 - x^2y^2 + y^4)$

$= (x - y)(x + y)(x^2 + y^2)(x^2 + xy + y^2)(x^2 - xy + y^2)$
$(x^4 - x^2y^2 + y^4)$

Q.14 $1 - \dfrac{64p^3}{q^3}$

Sol: $= (1)^3 - \left(\dfrac{4p}{q}\right)^3$

$= \left(1 - \dfrac{4p}{q}\right)\left[(1)^2 + (1)\left(\dfrac{4p}{q}\right) + \left(\dfrac{4p}{q}\right)^2\right]$

$= \left(1 - \dfrac{4p}{q}\right)\left(1 + \dfrac{4p}{q} + \dfrac{16p^2}{q^2}\right)$

Q.15 $1 + 64u^3$

Sol: $= (1)^3 + (4u)^3$

$= (1 + 4u)[(1)^2 - (1)(4u) + (4u)^2]$

$= (1 + 4u)(1 - 4u + 16u^2)$

Q.16 $8x^3 - 6x - 9y + 27y^3$

Sol: $= 8x^3 + 27y^3 - 6x - 9y$

$= (8x^3 + 27y^3) - (6x + 9y)$

$= [(2x)^3 + (3y)^3] - 3(2x + 3y)$

$= (2x + 3y)[(2x)^2 - (2x)(3y) + (3y)^2] - 3(2x + 3y)$

$= (2x + 3y)(4x^2 - 6xy + 9y^2) - 3(2x + 3y)$

$= (2x + 3y)(4x^2 - 6xy + 9y^2 - 3)$

Q.17 $z^3 + 125$

Sol: $= (z)^3 + (5)^3$

$= (z + 5)\{(z)^2 - (z)(5) + (5)^2\}$

$= (z + 5)(z^2 - 5z + 25)$

Q.18 $x^9 + y^9$

Sol: $= (x^3)^3 + (y^3)^3$

$= (x^3 + y^3)[(x^3)^2 - (x^3)(y^3) + (y^3)^2]$

$= (x^3 + y^3)(x^6 - x^3y^3 + y^6)$

$= (x + y)(x^2 - xy + y^2)(x^6 - x^3y^3 + y^6)$

Q.19 $m^6 - n^6$

Sol: $= (m^3)^2 - (n^3)^2$

$= (m^3 + n^3)(m^3 - n^3)$

$= (m + n)(m^2 - mn + n^2)(m - n)(m^2 + mn + n^2)$

$= (m + n)(m - n)(m^2 - mn + n^2)(m^2 + mn + n^2)$

Q.20 $64x^7 - xa^6$

Sol: $= x(64x^6 - a^6)$

$= x\{(8x^3)^2 - (a^3)^2\}$

$= x(8x^3 + a^3)(8x^3 - a^3)$

$x\{(2x)^3 + (a)^3\}\{(2x)^3 - (a)^3\}$

$= x(2x + a)(4x^2 - 2ax + a^2)(2x - a)(4x^2 + 2ax + a^2)$

$= x(2x + a)(2x - a)(4x^2 - 2ax + a^2)(4x^2 + 2ax + a^2)$

Q.21 $\mathbf{x^3 - 27a^3}$

Sol: $= (x)^3 - (3a)^3$

$= (x - 3a)[(x)^2 + (x)(3a) + (3a)^2]$

$= (x - 3a)(x^2 + 3ax + 9a^2)$

Q.22 $\mathbf{x^3 + 27a^3}$

Sol: $= (x)^3 + (3a)^3$

$= (x + 3a)\{(x)^2 - (x)(3a) + (3a)^2\}$

$= (x + 3a)(x^2 - 3ax + 9a^2)$

EXERCISE

I. ***Evaluate each of the polynomials for the value indicated.***

Q.1 $P(x) = 2x^3 - 5x^2 + 7x - 7;\ P(2)$

Sol: $P(x) = 2x^3 - 5x^2 + 7x - 7$

$P(2) = 2(2)^3 - 5(2)^2 + 7(2) - 7$

$= 2 \times 8 - 5 \times 4 + 7 \times 2 - 7$

$= 16 - 20 + 14 - 7$

$= 3$

Q.2 $\mathbf{P(x) = x^4 - 10x^2 + 25x - 2;\ P(-4)}$

Sol: $P(x) = x^4 - 10x^2 + 25x - 2$

$P(-4) = (-4)^4 - 10(-4)^2 + 25(-4) - 2$

$= 256 - 160 - 100 - 2$

$= -6$

Q.3 $\mathbf{P(x) = x^4 + 5x^3 - 13x^2 - 30;\ P(-1)}$

Sol: $P(x) = x^4 + 5x^3 - 13x^2 - 30$