$x\{(2x)^3 + (a)^3\}\{(2x)^3 - (a)^3\}$

$= x(2x + a)(4x^2 - 2ax + a^2)(2x - a)(4x^2 + 2ax + a^2)$

$= x(2x + a)(2x - a)(4x^2 - 2ax + a^2)(4x^2 + 2ax + a^2)$

Q.21 $\mathbf{x^3 - 27a^3}$

Sol: $= (x)^3 - (3a)^3$

$= (x - 3a)[(x)^2 + (x)(3a) + (3a)^2]$

$= (x - 3a)(x^2 + 3ax + 9a^2)$

Q.22 $\mathbf{x^3 + 27a^3}$

Sol: $= (x)^3 + (3a)^3$

$= (x + 3a)\{(x)^2 - (x)(3a) + (3a)^2\}$

$= (x + 3a)(x^2 - 3ax + 9a^2)$

[illegible]

I. ***Evaluate each of the polynomials for the value indicated.***

Q.1 $P(x) = 2x^3 - 5x^2 + 7x - 7;\ P(2)$

Sol: $P(x) = 2x^3 - 5x^2 + 7x - 7$

$P(2) = 2(2)^3 - 5(2)^2 + 7(2) - 7$

$= 2 \times 8 - 5 \times 4 + 7 \times 2 - 7$

$= 16 - 20 + 14 - 7$

$= 3$

Q.2 $\mathbf{P(x) = x^4 - 10x^2 + 25x - 2;\ P(-4)}$

Sol: $P(x) = x^4 - 10x^2 + 25x - 2$

$P(-4) = (-4)^4 - 10(-4)^2 + 25(-4) - 2$

$= 256 - 160 - 100 - 2$

$= -6$

Q.3 $\mathbf{P(x) = x^4 + 5x^3 - 13x^2 - 30;\ P(-1)}$

Sol: $P(x) = x^4 + 5x^3 - 13x^2 - 30$

$P(-1) = (-1)^4 + 5(-1)^3 - 13(-1)^2 - 30$

$= 1 - 5 - 13 - 30$

$= -47$

Q.4 $P(x) = x^5 - 10x^3 + 7x + 6$; $P(3)$

Sol: $P(x) = x^5 - 10x^3 + 7x + 6$

$P(3) = (3)^5 - 10(3)^3 + 7(3) + 6$

$= 243 - 270 + 21 + 6$

$= 0$

Q.5 $P(x) = x^4 + 4x^3 - 9x^2 + 19x + 6$; $P(-2)$

Sol: $P(x) = x^4 + 4x^3 - 9x^2 + 19x + 6$

$P(-2) = (-2)^4 + 4(-2)^3 - 9(-2)^2 + 19(-2) + 6$

$= 16 + 4(-8) - 9(4) - 38 + 6$

$= 16 - 32 - 36 - 38 + 6$

$= -106 + 22$

$= -84$

II. Determine whether the second polynomial is a factor of the first polynomial without dividing (Hint: evaluate directly and use the factor theorem).

Q.6 $x^{18} - 1$; $x + 1$

Sol: Let $P(x) = x^{18} - 1$

Here $x - a = x + 1$

Thus $a = -1$

Now $P(-1) = (-1)^{18} - 1$

$= 1 - 1 = 0$

Since $P(-1) = 0$

Then by factor theorem $x + 1$ is a factor of $x^{18} - 1$.

Q.7 $x^{18} - 1$; $x - 1$

Sol: Let $P(x) = x^{18} - 1$

Here $x - a = x - 1$

Thus $a = 1$

Now $P(-1) = (1)^{18} - 1$

$= 1 - 1 = 0$

Since $P(-1) = 0$

Then by factor theorem, $x - 1$ is a factor of $x^{18} - 1$.

Q.8 $x^9 - 2^9$; **x + 2**

Sol: Let $P(x) = x^9 - 2^9$

Here $x - a = x + 2$

Thus $a = -2$

Now $P(-2) = (-2)^9 - (2)^9$

$= -(2)^9 - (2)^9$

$= -ve \neq 0$

Since $P(-2) \neq 0$

Therefore, $x + 2$ is not a factor of $x^9 - 2^9$.

Q.9 $x^9 + 2^9 ; x - 2$

Sol: Let $P(x) = x^9 + 2^9$

Here $x - a = x - 2$

Thus $a = 2$

Now $P(2) = (2)^9 + (2)^9$

$\neq 0$

Since $P(2) \neq 0$

Therefore, $x - 2$ is not a factor of $x^9 + 2^9$.

Q.10 $3x^4 - 2x^3 + 5x - 6$; **x − 1**

Sol: Let $P(x) = 3x^4 - 2x^3 + 5x - 6$

Here $x - a = x - 1$

Thus $a = 1$

Now $P(1) = 3(1)^4 - 2(1)^3 + 5(1) - 6$

$= 3 - 2 + 5 - 6$

$= 8 - 8 = 0$

Since $P(1) = 0$

Then by factor theorem, $x - 1$ is a factor of $3x^4 - 2x^3 + 5x - 6$.

Q.11 $5x^6 - 7x^3 - 6x + x$; $x - 1$

Sol: Let $P(x) = 5x^6 - 7x^3 - 6x + x$

Here $x - a = x - 1$

Thus $a = 1$

Now $P(1) = 5(1)^6 - 7(1)^3 - 6(1) + (1)$

$= 5 - 7 - 6 + 1$

$= 6 - 13$

$= -7 \neq 0$

Since $P(1) \neq 0$

Then by factor theorem, $x - 1$ is not a factor of $5x^6 - 7x^3 - 6x + x$.

Q.12 $3x^3 - 7x^2 - 8x + 2$; $x + 1$

Sol: Let $P(x) = 3x^3 - 7x^2 - 8x + 2$

Here $x - a = x + 1$

therefore $a = -1$

Now $P(-1) = 3(-1)^3 - 7(-1)^2 - 8(-1) + 2$

$= -3 - 7 + 8 + 2$

$= 0$

Since $P(-1) = 0$

Then by factor theorem, $x+1$ is a factor of $3x^3 - 7x^2 - 8x + 2$.

Q.13 $5x^8 - 2x^5 + 3x^3 + 6x + 2$; $x + 1$

Sol: Let $P(x) = 5x^8 - 2x^5 + 3x^3 + 6x + 2$

Here $x - a = x + 1$

therefore $a = -1$

Now $P(-1) = 5(-1)^8 - 2(-1)^5 + 3(-1)^3 + 6(-1) + 2$

$= 5 + 2 - 3 - 6 + 2$

$= 9 - 9 = 0$

Since $P(-1) = 0$

Then by factor theorem, $x+1$ is a factor of $5x^8 - 2x^5 + 3x^3 + 6x + 2$.

Q.14 $6x^3 + 2x^2 - x + 9$; $x - 1$

Sol: Let $P(x) = 6x^3 + 2x^2 - x + 9$

Here $x - a = x - 1$

therefore $a = 1$

Now $P(1) = 6(1)^3 + 2(1)^2 - (1) + 9$

$= 6 + 2 - 1 + 9$

$= 17 - 1 = 16 \neq 0$

Since $P(1) \neq 0$

Then by factor theorem, $x - 1$ is not a factor of $6x^3 + 2x^2 - x + 9$.

Q.15 $4x^3 - 3x^2 - 8x + 4$; $x - 2$

Sol: Let $P(x) = 4x^3 - 3x^2 - 8x + 4$

Here $x - a = x - 2$

therefore $a = 2$

Now $P(2) = 4(2)^3 - 3(2)^2 - 8(2) + 4$

$= 4(8) - 3(4) - 8(2) + 4$

$= 32 - 12 - 16 + 4$

$= 36 - 28 = 8 \neq 0$

Since $P(2) \neq 0$

Then by factor theorem, $x - 2$ is not a factor of $4x^3 - 3x^2 - 8x + 4$.

Q.16 $5x^3 + 3x^2 - x + 1$; $x + 1$

Sol: Let $P(x) = 5x^3 + 3x^2 - x + 1$

Here $x - a = x + 1$

therefore $a = -1$

Now $P(-1) = 5(-1)^3 + 3(-1)^2 - (-1) + 1$

$= 5(-1) + 3(1) + 1 + 1$

$= -5 + 3 + 1 + 1$

$= -5 + 5$

$= 0$

Since $P(-1) = 0$

Then by factor theorem, $x+1$ is a factor of $5x^3 + 3x^2 - x + 1$.

Q.17 $2y^3 - 8y^2 + y - 4$; $y - 4$

Sol: Let $P(y) = 2y^3 - 8y^2 + y - 4$

Here $y - a = y - 4$

therefore $a = 4$

Now $P(4) = 2(4)^3 - 8(4)^2 + (4) - 4$

$= 2(64) - 8(16) + 4 - 4$

$= 128 - 128 + 4 - 4$

$= 132 - 132$

$= 0$

Since $P(4) = 0$

Then by factor theorem, $y - 4$ is not a factor of $2y^3 - 8y^2 + y - 4$.

Q.18 $z^3 - 5z^2 - 4z - 4$; $z + 2$

Sol: Let $P(z) = z^3 - 5z^2 - 4z - 4$

Here $z - a = z + 2$

therefore $a = -2$

Now $P(-2) = (-2)^3 - 5(-2)^2 - 4(-2) - 4$

$= -8 - 5(4) + 8 - 4$

$= - 8 - 20 + 8 - 4$

$= - 32 + 8$

$= -24 \neq 0$

Since $P(-2) \neq 0$

Then by factor theorem, $z+2$ is not a factor of $z^3 - 5z^2 - 4z - 4$.

III. Solve

Q.19. If $P(x) = x^3 - kx^2 + 3x + 5$ is divided by $x - 1$, find k, if remainder is 8.

Sol: Let $P(x) = x^3 - kx^2 + 3x + 5$

Here $x - a = x - 1$

therefore $a = 1$

Now $P(1) = (1)^3 - k(1)^2 + 3(1) + 5$

$= 1 - k + 3 + 5$

$= 9 - k =$ Remainder

But $9 - k = 8$

$-k = 8 - 9$

$-k = -1$

$k = 1$

Q.20. If $P(x) = 3x^3 + kx - 26$ is divided by $x - 2$, find k, if remainder is 0.

Sol: Lets $P(x) = 3x^3 + kx - 26$

Here $x - a = x - 2$

therefore $a = 2$

Now $P(2) = 3(2)^3 + k(2) - 26$

$= 3(8) + 2k - 26$

$= 24 + 2k - 26$

$= 2k - 2 =$ Remainder

But Remainder $= 0$

Thus, $2k - 2 = 0$

$2k = 2$

$k = \frac{2}{2}$

$k = 1$