		$= a^3 + b^3 + 3ab(a + b)$
(vii)	$(a - b)^3$	$= a^3 - 3a^2b + 3ab^2 - b^3$
		$= a^3 - b^3 - 3ab(a - b)$
(viii)	$a^3 + b^3$	$= (a + b)(a^2 - ab + b^2)$
(ix)	$a^3 - b^3$	$= (a - b)(a^2 + ab + b^2)$

Exercise 1.2

Solve the following questions using formulas.

Q.1 $(x + 2y)^2 + (x - 2y)^2$

Sol. $= 2[(x)^2 + (2y)^2]$ [Formula: $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$]

$= 2[x^2 + 4y^2]$

$= 2x^2 + 8y^2$

Q.2 $(5x + 3y)^2 + (5x - 3y)^2$

Sol. $= 2[(5x)^2 + (3y)^2]$ [Formula: $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$]

$= 2[25x^2 + 9y^2]$

$= 50x^2 + 18y^2$

Q.3 $(3l + 2m)^2 - (3l - 2m)^2$

Sol. $= 4(3l)(2m)$ [Formula: $(a + b)^2 - (a - b)^2 = 4ab$]

$= 24lm$

Q.4 $(l + m)(l - m)(l^2 + m^2)(l^4 + m^4)$

Sol. $= [(l + m)(l - m)](l^2 + m^2)(l^4 + m^4)$

[Formula: $(a + b)(a - b) = a^2 - b^2$]

$= (l^2 - m^2)(l^2 + m^2)(l^4 + m^4)$

$= [(l^2)^2 - (m^2)^2](l^4 + m^4)$

$= (l^4 - m^4)(l^4 + m^4)$

$= (l^4)^2 - (m^4)^2$

$= l^8 - m^8$

Q.5 $\left(ab - \frac{1}{ab}\right)^3$

Sol. [Formula: $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$]

$= (ab)^3 - 3(ab)^2\left(\frac{1}{ab}\right) + 3ab\left(\frac{1}{ab}\right)^2 - \left(\frac{1}{ab}\right)^3$

$= a^3b^3 - 3ab + \frac{3}{ab} - \frac{1}{a^3b^3}$

Q.6 $(2x + 3y + 2)^2$

Sol. [$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$]

$= (2x)^2 + (3y)^2 + (2)^2 + 2(2x)(3y) + 2(3y)(2) + 2(2)(2x)$

$= 4x^2 + 9y^2 + 4 + 12xy + 12y + 8x$

Q.7 $(2p + q)^3$

Sol. [Formula: $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$]

$= (2p)^3 + 3(2p)^2(q) + 3(2p)(q)^2 + (q)^3$

$= 8p^3 + 3(4p^2)(q) + 6pq^2 + q^3$

$= 8p^3 + 12p^2q + 6pq^2 + q^3$

Q.8 $(3p + q + r)^2$

Sol. [Formula: $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$]

$= (3p)^2 + (q)^2 + (r)^2 + 2(3p)(q) + 2(q)(r) + 2(r)(3p)$

$= 9p^2 + q^2 + r^2 + 6pq + 2qr + 6rp$

Q.9 $(2x + 3y)^3$

Sol. [Formula: $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$]

$= (2x)^3 + 3(2x)^2(3y) + 3(2x)(3y)^2 + (3y)^3$

$= 8x^3 + 3(4x^2)(3y) + 3(2x)(9y^2) + 9y^3$

$= 8x^3 + 36x^2y + 54xy^2 + 9y^3$

Q.10 $(x + y)^3 - 1$

Sol. $= (x + y)^3 - (1)^3$

[Formula: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$]

$= (x + y - 1)[(x + y)^2 + (x + y)(1) + (1)^2]$

$= (x + y - 1)[x^2 + y^2 + 2xy + x + y + 1]$

Q.11 $(x - y)^3 + 64$

Sol. $= (x - y)^3 + (4)^3$

[Formula: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$]

$= (x - y + 4)[(x - y)^2 - (x - y)(4) + (4)^2]$

$= (x - y + 4)(x^2 + y^2 + 2xy - 4x + 4y + 16)$

Q.12 $8x^3 + 27y^3$

Sol. $= (2x)^3 + (3y)^3$

[Formula: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$]

$= (2x + 3y)[(2x)^2 - (2x)(3y) + (3y)^2]$

$= (2x + 3y)(4x^2 - 6xy + 9y^2)$

Q.13 $x^6 - 729y^6$

Sol. $= (x^3)^2 - (27y^3)^2$ [Formula: $a^2 - b^2 = (a + b)(a - b)$]

$= (x^3 + 27y^3)(x^3 - 27y^3)$

$= [(x)^3 + (3y)^3][(x)^3 - (3y)^3]$

(Formulas)

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$= (x + 3y)(x^2 - 3xy + 9y^2)(x - 3y)(x^2 + 3xy + 9y^2)$

$= (x + 3y)(x - 3y)(x^2 - 3xy + 9y^2)(x^2 + 3xy + 9y^2)$

Q.14 $64a^6 - b^6$

Sol. $= (8a^3)^2 - (b^3)^2$

$= (8a^3 - b^3)(8a^3 + b^3)$ [Formula: $a^2 - b^2 = (a - b)(a + b)$]

$= [(2a)^3 - (b)^3][(2a)^3 + (b)^3]$

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

(Formulas)

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$= (2a - b)(4a^2 + 2ab + b^2)(2a + b)(4a^2 - 2ab + b^2)$

$= (2a - b)(2a + b)(4a^2 + 2ab + b^2)(4a^2 - 2ab + b^2)$

Q.15 Find the value of $a^3 - b^3$ when $a - b = 4$ and $ab = -5$.

Sol. $a - b = 4$

$ab = -5$

$a^3 - b^3 = ?$

$a - b = 4$

$(a - b)^3 = (4)^3$ Taking cube

$a^3 - b^3 - 3(-5)(4) = 64$

$a^3 - b^3 + 60 = 64$

$a^3 - b^3 = 64 - 60$

$a^3 - b^3 = 4$

Q.16 Show that $\left(z + \frac{1}{z}\right)^2 - \left(z - \frac{1}{z}\right)^2 = 4$

$$L.H.S = \left(z + \frac{1}{z}\right)^2 - \left(z - \frac{1}{z}\right)^2$$

$$= \left(z^2 + 2(z)\left(\frac{1}{z}\right) + \frac{1}{z^2}\right) - \left(z^2 - 2(z)\left(\frac{1}{z}\right) + \frac{1}{z^2}\right)$$

$$= \left(z^2 + 2 + \frac{1}{z^2}\right) - \left(z^2 - 2 + \frac{1}{z^2}\right)$$

$$= z^2 + 2 + \frac{1}{z^2} - z^2 + 2 - \frac{1}{z^2}$$

$$= 4 = R.H.S$$

Q.17 Find the value of $a^2 + b^2$ and ab when $a + b = 5$ and $a - b = 3$.

Sol. $a + b = 5$

$a - b = 3$

$a^2 + b^2 = ?$

$ab = ?$

Formula:

$(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$

$(5)^2 + (3)^2 = 2(a^2 + b^2)$

$25 + 9 = 2(a^2 + b^2)$

$34 = 2(a^2 + b^2)$

Thus, $a^2 + b^2 = \frac{34}{2} = 17$

Formula: $(a + b)^2 - (a - b)^2 = 4ab$

$(5)^2 - (3)^2 = 4ab$

$25 - 9 = 4ab$

$4ab = 16$

$ab = \frac{16}{4}$

$ab = 4$

Q.18 Find the value of $a^2 + b^2 + c^2$ if $ab + bc + ca = 11$ and $a + b + c = 6$

Sol. $a + b + c = 6$

$ab + bc + ca = 11$

$a^2 + b^2 + c^2 = ?$

$a + b + c = 6$

$(a + b + c)^2 = (6)^2$ Taking square root

$a^2 + b^2 + c^2 + 2(ab + bc + ca) = 36$

By putting values $ab + bc + ca = 11$

$a^2 + b^2 + c^2 + 2(11) = 36$

$a^2 + b^2 + c^2 + 22 = 36$

$a^2 + b^2 + c^2 = 36 - 22$

$a^2 + b^2 + c^2 = 14$

Q.19 Find the value of $x^3 + y^3$ if $xy = 10$ and $x + y = 7$.

Sol. $x + y = 7$

$xy = 10$

$x^3 + y^3 = ?$

$x + y = 7$

$(x + y)^3 = (7)^3$ Taking cube

$x^3 + y^3 + 3xy(x + y) = 343$

$x^3 + y^3 + 3(10)(7) = 343$

$x^3 + y^3 + 210 = 343$

$x^3 + y^3 = 343 - 210$

$x^3 + y^3 = 133$

Q.20 Find the value of $(x - y)^2$ if $x^2 + y^2 = 86$ and $xy = -16$.

Sol. $xy = -16$ (i)

$x^2 + y^2 = 86$ (ii)

$(x - y)^2 = ?$

$(x - y)^2 = x^2 + y^2 - 2xy$

$= (x^2 + y^2) - 2(xy)$

$= 86 - 2(-16)$ From (i) and (ii)

$= 86 + 32$

$= 118$

Q.21 Find the value of ab + bc + ca when the values of

$a^2 + b^2 + c^2 = 81$, $a + b + c = 11$

Sol. $a + b + c = 11$............(i)

$a^2 + b^2 + c^2 = 81$............(ii)

$ab + bc + ca = ?$

$a + b + c = 11$

$(a + b + c)^2 = (11)^2$ Taking square root

$a^2 + b^2 + c^2 + 2(ab + bc + ca) = 121$

$81 + 2(ab + bc + ca) = 121$ from (ii)

$2(ab + bc + ca) = 121 - 81$

$2(ab + bc + ca) = 40$

$ab + bc + ca = \frac{40}{2}$

$ab + bc + ca = 20$

Q.22 Find the value of $(a + b + c)^2$ when the values of $a^2 + b^2 + c^2 = 32$ and $ab + bc + ca = 7$.

Sol. $ab + bc + ca = 7$ (i)

$a^2 + b^2 + c^2 = 32$ (ii)

$(a + b + c)^2 = ?$

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$= 32 + 2(7)$ From (i) and (ii)

$= 32 + 14$

$(a + b + c)^2 = 46$

Laws of Radicals

(i) $\left(\sqrt[n]{a}\right)^n = a$ (ii) $\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$

(iii) $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ (iv) $\left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$

Q.1 Remove the radical sign from the denominator:

(i) $\frac{1}{\sqrt{5}}$

Sol.

$= \frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$

$= \frac{\sqrt{5}}{\left(\sqrt{5}\right)^2}$

$= \frac{\sqrt{5}}{5}$