$$ab + bc + ca = \frac{40}{2}$$

$$ab + bc + ca = 20$$

Q.22 Find the value of $(a + b + c)^2$ when the values of $a^2 + b^2 + c^2 = 32$ and $ab + bc + ca = 7$.

Sol. $ab + bc + ca = 7$ (i)

$a^2 + b^2 + c^2 = 32$ (ii)

$(a + b + c)^2 = ?$

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$= 32 + 2(7)$ From (i) and (ii)

$= 32 + 14$

$(a + b + c)^2 = 46$

Laws of Radicals

(i) $\left(\sqrt[n]{a}\right)^n = a$ (ii) $\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$

(iii) $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ (iv) $\left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$

Q.1 Remove the radical sign from the denominator:

(i) $\frac{1}{\sqrt{5}}$

Sol.

$$= \frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$$

$$= \frac{\sqrt{5}}{\left(\sqrt{5}\right)^2}$$

$$= \frac{\sqrt{5}}{5}$$

(ii) $\frac{2}{\sqrt{2}} \cdot \frac{7}{\sqrt{3}}$

Sol.

$$=\frac{2\times7}{\sqrt{2\times3}}$$

$$=\frac{14}{\sqrt{6}}$$

$$=\frac{14}{\sqrt{6}}\times\frac{\sqrt{6}}{\sqrt{6}}$$

$$=\frac{14\times\sqrt{6}}{6}=\frac{7\sqrt{6}}{3}$$

(iii) $\frac{\sqrt{6}}{\sqrt{7}}$

Sol.

$$=\frac{\sqrt{6}}{\sqrt{7}}\times\frac{\sqrt{7}}{\sqrt{7}}$$

$$=\frac{\sqrt{6\times7}}{\sqrt{7\times7}}$$

$$=\frac{\sqrt{42}}{7}$$

Q.2 Simplify these expressions:

(i) $\sqrt{2}+\sqrt{8}$

Sol.

$$=\sqrt{2}+\sqrt{2\times2\times2}$$

$$=\sqrt{2}+2\sqrt{2}$$

$$=\sqrt{2}(1+2)$$

$$=3\sqrt{2}$$

(ii) $4\sqrt{50}+\sqrt{200}+\sqrt{50}$

Sol.

$$=4\sqrt{5\times5\times2}+\sqrt{5\times5\times2\times2\times2}+\sqrt{5\times5\times2}$$
$$=4\times5\sqrt{2}+5\times2\sqrt{2}+5\sqrt{2}$$
$$=20\sqrt{2}+10\sqrt{2}+5\sqrt{2}$$
$$=\sqrt{2}(20+10+5)$$
$$=\sqrt{2}(35)$$
$$=35\sqrt{2}$$

(iii) $(\sqrt{12}-\sqrt{2})(\sqrt{20}-3\sqrt{2})$

Sol.

$$=\sqrt{12}\times\sqrt{20}-\sqrt{12}\times3\sqrt{2}-\sqrt{2}\times\sqrt{20}+\sqrt{2}\times3\sqrt{2}$$
$$=\sqrt{2\times2\times3}\times\sqrt{2\times2\times5}-\sqrt{2\times2\times3}\times3\sqrt{2}$$
$$-\sqrt{2}\times\sqrt{2\times2\times5}+3\sqrt{2\times2}$$
$$=2\sqrt{3}\times2\sqrt{5}-2\sqrt{3}\times3\sqrt{2}-\sqrt{2}\times2\sqrt{5}+3\times2$$
$$=4\sqrt{3\times5}-6\sqrt{3\times2}-2\sqrt{2\times5}+6$$
$$=4\sqrt{15}-6\sqrt{6}-2\sqrt{10}+6$$

(iv) $(6+\sqrt{2})(5-\sqrt{5})$

Sol.

$$=6(5-\sqrt{5})+\sqrt{2}(5-\sqrt{5})$$
$$=30-6\sqrt{5}+5\sqrt{2}-\sqrt{2\times5}$$
$$=30-6\sqrt{5}+5\sqrt{2}-\sqrt{10}$$

(v) $(\sqrt{3}-2)(5-\sqrt{5})$

Sol.

$$=\sqrt{3}(5-\sqrt{5})-2(5-\sqrt{5})$$
$$=5\sqrt{3}-\sqrt{3\times5}-10+2\sqrt{5}$$
$$=5\sqrt{3}-\sqrt{15}-10+2\sqrt{5}$$

(vi) $(7+\sqrt{3})(5+\sqrt{2})$

Sol.

$$=7(5+\sqrt{2})+\sqrt{3}(5+\sqrt{2})$$
$$=35+7\sqrt{2}+5\sqrt{3}+\sqrt{3\times 2}$$
$$=35+7\sqrt{2}+5\sqrt{3}+\sqrt{6}$$

Q.3 Rationalize the denominators of the following:

(i) $\dfrac{1}{\sqrt{3}+2}$

Sol.

$$=\frac{1}{\sqrt{3}+2}\times\frac{\sqrt{3}-2}{\sqrt{3}-2}$$
$$=\frac{\sqrt{3}-2}{(\sqrt{3})^2-(2)^2}$$
$$=\frac{\sqrt{3}-2}{3-4}$$
$$=\frac{\sqrt{3}-2}{-1}$$
$$=2-\sqrt{3}$$

(ii) $\dfrac{1}{4-\sqrt{5}}$

Sol.

$$=\frac{1}{4-\sqrt{5}}\times\frac{4+\sqrt{5}}{4+\sqrt{5}}$$
$$=\frac{4+\sqrt{5}}{(4)^2-(\sqrt{5})^2}$$

$$= \frac{4+\sqrt{5}}{16-5}$$

$$= \frac{4+\sqrt{5}}{11}$$

(iii) $\frac{4\sqrt{3}}{\sqrt{7}+\sqrt{5}}$

Sol.

$$= \frac{4\sqrt{3}}{\sqrt{7}+\sqrt{5}} \times \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}}$$

$$= \frac{(4\sqrt{3})(\sqrt{7}-\sqrt{5})}{(\sqrt{7})^2-(\sqrt{5})^2}$$

$$= \frac{(4\sqrt{3})(\sqrt{7}-\sqrt{5})}{7-5}$$

$$= \frac{4\sqrt{3}(\sqrt{7}-\sqrt{5})}{2}$$

$$2\sqrt{3}(\sqrt{7}-\sqrt{5})$$

(iv) $\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$

Sol.

$$= \frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}} \times \frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}-\sqrt{y}}$$

$$= \frac{(\sqrt{x}-\sqrt{y})^2}{(\sqrt{x})^2-(\sqrt{y})^2}$$

$$= \frac{x+y-2\sqrt{xy}}{x-y}$$

(v) $\dfrac{5\sqrt{7}}{2+3\sqrt{7}}$

Sol.

$$=\frac{5\sqrt{7}}{2+3\sqrt{7}}\times\frac{2-3\sqrt{7}}{2-3\sqrt{7}}$$

$$=\frac{5\sqrt{7}(2-3\sqrt{7})}{(2)^2-(3\sqrt{7})^2}$$

$$=\frac{(5\sqrt{7})(2)-(5\sqrt{7})(3\sqrt{7})}{4-9\times 7}$$

$$=\frac{10\sqrt{7}-105}{4-63}$$

$$=\frac{10\sqrt{7}-105}{-59}$$

$$=\frac{105-10\sqrt{7}}{59}$$

(vi) $\dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

Sol.

$$=\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}\times\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

$$=\frac{(\sqrt{3}+\sqrt{2})^2}{(\sqrt{3})^2-(\sqrt{2})^2}$$

$$=\frac{3+2+2\sqrt{3\times 2}}{3-2}$$

$$=\frac{5+2\sqrt{6}}{1}$$

$$=5+2\sqrt{6}$$

(vii) $\dfrac{29}{11+3\sqrt{5}}$

Sol.

$$= \frac{29}{11+3\sqrt{5}} \times \frac{11-3\sqrt{5}}{11-3\sqrt{5}}$$

$$= \frac{29(11-3\sqrt{5})}{(11)^2-(3\sqrt{5})^2}$$

$$= \frac{29(11-3\sqrt{5})}{121-9\times5}$$

$$= \frac{29(11-3\sqrt{5})}{121-45}$$

$$= \frac{29(11-3\sqrt{5})}{76}$$

(viii) $\dfrac{17}{3\sqrt{7}+2\sqrt{3}}$

Sol.

$$= \frac{17}{3\sqrt{7}+2\sqrt{3}} \times \frac{3\sqrt{7}-2\sqrt{3}}{3\sqrt{7}-2\sqrt{3}}$$

$$= \frac{17(3\sqrt{7}-2\sqrt{3})}{(3\sqrt{7})^2-(2\sqrt{3})^2}$$

$$= \frac{17(3\sqrt{7}-2\sqrt{3})}{9\times7-4\times3}$$

$$= \frac{17(3\sqrt{7}-2\sqrt{3})}{63-12}$$

$$= \frac{17(3\sqrt{7}-2\sqrt{3})}{51}$$

$$= \frac{3\sqrt{7}-2\sqrt{3}}{3}$$

Q.4 If $x = \sqrt{5} + 2$, then find the values of (i) $x + \frac{1}{x}$ and (ii) $x^2 + \frac{1}{x^2}$.

Sol. (i) $x = \sqrt{5} + 2$ ....(1)

$$\frac{1}{x} = \frac{1}{\sqrt{5}+2}$$

$$= \frac{1}{\sqrt{5}+2} \times \frac{\sqrt{5}-2}{\sqrt{5}-2}$$

$$= \frac{\sqrt{5}-2}{(\sqrt{5})^2 - (2)^2}$$

$$= \frac{\sqrt{5}-2}{5-4}$$

$$\frac{1}{x} = \sqrt{5} - 2 \text{ ....(2)}$$

$$x + \frac{1}{x} = \sqrt{5} + 2 + \sqrt{5} - 2 \qquad \text{from (1) + (2)}$$

Thus $\boxed{x + \frac{1}{x} = 2\sqrt{5}}$

(ii) $\left(x + \frac{1}{x}\right)^2 = (2\sqrt{5})^2$ Taking square root

$$x^2 + \frac{1}{x^2} + 2 = 4 \times 5$$

$$x^2 + \frac{1}{x^2} + 2 = 20$$

$$x^2 + \frac{1}{x^2} = 20 - 2$$

Hence, $\boxed{x^2 + \frac{1}{x^2} = 18}$

Q.5 **If $x = 2 + \sqrt{3}$, then find the values of (i) $x - \frac{1}{x}$ and**

(ii) $x^2 + \frac{1}{x^2}$.

Sol. (i) $x = 2 + \sqrt{3}$ ....(1)

$$\frac{1}{x} = \frac{1}{2+\sqrt{3}}$$

$$= \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} \text{ (Rationalize the denominators)}$$

$$= \frac{2-\sqrt{3}}{(2)^2 - (\sqrt{3})^2}$$

$$= \frac{2-\sqrt{3}}{4-3} = \frac{2-\sqrt{3}}{1} = 2-\sqrt{3} \text{ ....(2)}$$

$$x - \frac{1}{x} = (2+\sqrt{3}) - (2-\sqrt{3}) \quad \text{from (1), (2)}$$

$$x - \frac{1}{x} = 2 + \sqrt{3} - 2 + \sqrt{3}$$

Hence, $\boxed{x - \frac{1}{x} = 2\sqrt{3}}$

(ii) $\left(x - \frac{1}{x}\right)^2 = (2\sqrt{3})^2$ Taking square root

$$x^2 + \frac{1}{x^2} - 2 = 4 \times 3$$

$$x^2 + \frac{1}{x^2} - 2 = 12$$

Hence, $\boxed{x^2 + \frac{1}{x^2} = 12 + 2 = 14}$

Q.6 **If $x = \sqrt{3} - \sqrt{2}$, then find the values of (i) $x - \frac{1}{x}$ and**

(ii) $x^2 + \frac{1}{x^2}$.

Sol. (i) $x = \sqrt{3} - \sqrt{2}$ ....(1)

$$\frac{1}{x} = \frac{1}{\sqrt{3} - \sqrt{2}}$$

$$= \frac{1}{(\sqrt{3} - \sqrt{2})} \times \frac{(\sqrt{3} + \sqrt{2})}{(\sqrt{3} + \sqrt{2})}$$

$$= \frac{(\sqrt{3} + \sqrt{2})}{(\sqrt{3})^2 - (\sqrt{2})^2}$$

$$= \frac{(\sqrt{3} + \sqrt{2})}{3 - 2}$$

$$\frac{1}{x} = \sqrt{3} + \sqrt{2} \text{ ....(2)}$$

$$x - \frac{1}{x} = (\sqrt{3} - \sqrt{2}) - (\sqrt{3} + \sqrt{2}) \quad \text{from (1), (2)}$$

$$= \sqrt{3} - \sqrt{2} - \sqrt{3} - \sqrt{2}$$

Hence, $\boxed{x - \frac{1}{x} = -2\sqrt{2}}$

(ii) $\left(x - \frac{1}{x}\right)^2 = (-2\sqrt{2})^2$ Taking square root

$$x^2 + \frac{1}{x^2} - 2 = 4 \times 2$$

$$x^2 + \frac{1}{x^2} - 2 = 8$$

$$x^2 + \frac{1}{x^2} = 8 + 2$$

Hence, $\boxed{x^2 + \frac{1}{x^2} = 10}$

Q.7 If $\frac{1}{x} = 3 - \sqrt{2}$, then evaluate (i) $x + \frac{1}{x}$ and (ii) $x - \frac{1}{x}$.

Sol. (i) $\frac{1}{x} = 3 - \sqrt{2}$ ....(1)

$$x = \frac{1}{3-\sqrt{2}}$$

$$= \frac{1}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}}$$

$$= \frac{3+\sqrt{2}}{(3)^2-(\sqrt{2})^2}$$

$$= \frac{3+\sqrt{2}}{9-2}$$

$$x = \frac{3+\sqrt{2}}{7} \quad \text{.................(2)}$$

$$x+\frac{1}{x} = \frac{3+\sqrt{2}}{7}+3-\sqrt{2} \qquad \text{Now from (1) + (2)}$$

$$= \frac{3+\sqrt{2}+7(3-\sqrt{2})}{7}$$

$$= \frac{3+\sqrt{2}+21-7\sqrt{2}}{7}$$

$$= \frac{24+\sqrt{2}-7\sqrt{2}}{7}$$

$$= \frac{24+\sqrt{2}(1-7)}{7}$$

$$= \frac{24+\sqrt{2}(-6)}{7}$$

Hence, $\boxed{x+\frac{1}{x} = \frac{24-6\sqrt{2}}{7}}$

(ii) $$x-\frac{1}{x} = \frac{3+\sqrt{2}}{7}-(3-\sqrt{2}) \qquad \text{from (1),(2)}$$

$$= \frac{3+\sqrt{2}}{7}-3+\sqrt{2}$$

$$= \frac{3+\sqrt{2}-21+7\sqrt{2}}{7}$$

$$= \frac{-18+\sqrt{2}+7\sqrt{2}}{7}$$

$$= \frac{-18+\sqrt{2}(1+7)}{7}$$

Hence, $\boxed{x - \frac{1}{x} = \frac{-18+8\sqrt{2}}{7}}$

Q.8 If $\frac{1}{p} = \sqrt{10}+3$, then evaluate (i) $\left(p+\frac{1}{p}\right)^2$ and

(ii) $\left(p-\frac{1}{p}\right)^2$

Sol.

(i) $\frac{1}{p} = \sqrt{10}+3$ ....(1)

$$p = \frac{1}{\sqrt{10}+3}$$

$$p = \frac{1}{\sqrt{10}+3} \times \frac{\sqrt{10}-3}{\sqrt{10}-3}$$ (Rationalize the denominators)

$$= \frac{\sqrt{10}-3}{(\sqrt{10})^2-(3)^2}$$

$$= \frac{\sqrt{10}-3}{10-9}$$

$$= \frac{\sqrt{10}-3}{1}$$

$= \sqrt{10}-3$ ....(2)

$p+\frac{1}{p} = (\sqrt{10}-3)+(\sqrt{10}+3)$ from (1)+(2)

$$= \sqrt{10}-3+\sqrt{10}+3$$

$$p+\frac{1}{p} = 2\sqrt{10}$$

$$\left(p+\frac{1}{p}\right)^2 = (2\sqrt{10})^2 \qquad \text{Taking square root}$$

$$= 4 \times 10$$

Hence, $\boxed{\left(p+\frac{1}{p}\right)^2 = 40}$

(ii) $$p-\frac{1}{p} = (\sqrt{10}-3)-(\sqrt{10}+3) \qquad \text{Now from (1) - (2)}$$

$$= \sqrt{10}-3-\sqrt{10}-3$$

$$p-\frac{1}{p} = -6$$

$$\left(p-\frac{1}{p}\right)^2 = (-6)^2 \qquad \text{Taking square root}$$

Hence, $\boxed{\left(p-\frac{1}{p}\right)^2 = 36}$

Q.9 Rationalize

(i) $$\frac{b+\sqrt{b^2-a^2}}{b-\sqrt{b^2-a^2}}$$

Sol. $$= \frac{b+\sqrt{b^2-a^2}}{b-\sqrt{b^2-a^2}} \times \frac{b+\sqrt{b^2-a^2}}{b+\sqrt{b^2-a^2}}$$

$$= \frac{(b+\sqrt{b^2-a^2})^2}{(b)^2-(\sqrt{b^2-a^2})^2}$$

$$= \frac{b^2+b^2-a^2+2b\sqrt{b^2-a^2}}{b^2-(b^2-a^2)}$$

$$= \frac{2b^2-a^2+2b\sqrt{b^2-a^2}}{b^2-b^2+a^2}$$

$$= \frac{2b^2-a^2+2b\sqrt{b^2-a^2}}{a^2}$$

(ii) $\frac{\sqrt{a+3}-\sqrt{a-3}}{\sqrt{a+3}+\sqrt{a-3}}$

Sol.

$$= \frac{\sqrt{a+3}-\sqrt{a-3}}{\sqrt{a+3}+\sqrt{a-3}} \times \frac{\sqrt{a+3}-\sqrt{a-3}}{\sqrt{a+3}-\sqrt{a-3}}$$

$$= \frac{(\sqrt{a+3}-\sqrt{a-3})^2}{(\sqrt{a+3})^2-(\sqrt{a-3})^2}$$

$$= \frac{(a+3)+(a-3)-2\sqrt{(a+3)(a-3)}}{(a+3)-(a-3)}$$

$$= \frac{a+3+a-3-2\sqrt{(a)^2-(3)^2}}{a+3-a+3}$$

$$= \frac{2a-2\sqrt{a^2-9}}{6}$$

$$= \frac{2(a-\sqrt{a^2-a})}{6}$$

$$= \frac{a-\sqrt{a^2-9}}{3}$$