

**Solve:**

1- If  $P(x) = x^4 + 3x^2 - 5x + 9$ , then find  $P(x)$ , for  $x = 0, x = 1$ .

Sol.  $P(x) = x^4 + 3x^2 - 5x + 9$

$$\begin{aligned} P(0) &= (0)^4 + 3(0)^2 - 5(0) + 9 \\ &= 0 + 0 - 0 + 9 \\ &= 9 \end{aligned}$$

and  $P(1) = (1)^4 + 3(1)^2 - 5(1) + 9$

$$\begin{aligned} &= 1 + 3 - 5 + 9 \\ &= 13 - 5 \\ &= 8 \end{aligned}$$

2- If  $P(x) = 2x^3 + 2x^2 + x - 1$ , then find  $P(-2)$

Sol.  $P(x) = 2x^3 + 2x^2 + x - 1$

Therefore,  $P(-2) = 2(-2)^3 + 2(-2)^2 + (-2) - 1$

$$\begin{aligned} &= 2(-8) + 2(4) - 2 - 1 \\ &= -16 + 8 - 2 - 1 \\ &= 8 - 16 - 2 - 1 \\ &= 8 - 19 \\ &= -11 \end{aligned}$$

3- If  $P(y) = 3y^2 + \frac{y}{4} + 9$ , then find  $P(0)$ ..

Sol.  $P(y) = 3y^2 + \frac{y}{4} + 9$

$$\begin{aligned} P(0) &= 3(0)^2 + \frac{0}{4} + 9 \\ &= 3(0) + 0 + 9 \\ &= 9 \end{aligned}$$

4- If  $P(x) = 9x^3 - 2x^2 + 3x + 1$ , then find  $P(1)$  and  $P(2)$ .

Sol.  $P(x) = 9x^3 - 2x^2 + 3x + 1$

$$P(1) = 9(1)^3 - 2(1)^2 + 3(1) + 1$$

$$\begin{aligned}
 &= 9(1) - 2(1) + 3 + 1 \\
 &= 9 - 2 + 3 + 1 \\
 &= 13 - 2 \\
 &= 11
 \end{aligned}$$

and  $P(2) = 9(2)^3 - 2(2)^2 + 3(2) + 1$

$$\begin{aligned}
 &\quad - 9(8) - 2(4) + 6 + 1 \\
 &= 72 - 8 + 6 + 1 \\
 &= 79 - 8 \\
 &= 71
 \end{aligned}$$

5- If  $P(x) = \frac{x^2 - 5x + 6}{x + 1}$ , then find  $P(1)$  and  $P(2)$ .

Sol.  $P(x) = \frac{x^2 - 5x + 6}{x + 1}$

$$\begin{aligned}
 P(1) &= \frac{(1)^2 - 5(1) + 6}{1+1} \\
 &= \frac{1 - 5 + 6}{1+1} \\
 &= \frac{2}{2} \\
 &= 1
 \end{aligned}$$

and  $P(2) = \frac{(2)^2 - 5(2) + 6}{2+1}$

$$\begin{aligned}
 &= \frac{4 - 10 + 6}{3} \\
 &= \frac{0}{3} \\
 &= 0
 \end{aligned}$$

6- If  $P(r) = 2\pi r$ , then find  $P(r)$ , for  $r = 3$  and  $\pi = \frac{22}{7}$ .

Sol.  $P(r) = 2\pi r$

By putting the values of  $r$  and  $\pi$

$$\begin{aligned}
 P(r) &= 2 \times \frac{22}{7} \times 3 \\
 &= \frac{132}{7} \\
 &= 18.9 \text{ (approximately)}
 \end{aligned}$$

7- If  $P(r) = 4\pi r^2$ , then find  $P(r)$ , for  $r = 8$  and  $\pi = \frac{22}{7}$ .

Sol.

$$P(r) = 4\pi r^2$$

By putting the values of  $r$  and  $\pi$

$$\begin{aligned}
 P(r) &= 4 \times \frac{22}{7} \times (8)^2 \\
 &= 4 \times \frac{22}{7} \times 64 \\
 &= \frac{5632}{7} \\
 &= 804.57
 \end{aligned}$$

8- If  $P(y) = y^4 + \frac{3y^3}{2} - y^2 + 1$ , then find  $P(y)$ , for  $y = 2$  and

$$y = -2.$$

Sol.  $P(y) = y^4 + \frac{3y^3}{2} - y^2 + 1$

for  $y = 2$

$$\begin{aligned}
 P(2) &= (2)^4 + \frac{3(2)^3}{2} - (2)^2 + 1 \\
 &= 16 + \frac{3 \times 8}{2} - 4 + 1 \\
 &= 16 + 12 - 4 + 1 \\
 &= 25
 \end{aligned}$$

$$P(-2) = (-2)^4 + \frac{3(-2)^3}{2} - (-2)^2 + 1 \quad \text{for } y = -2$$

$$\begin{aligned}
 &= 16 + \frac{3(-8)}{2} - 4 + 1 \\
 &= 16 - 12 - 4 + 1 \\
 &= 1
 \end{aligned}$$

Reduce the given rational expressions to lowest terms.

**Q.9**  $\frac{8x^2y^2}{12x^4y}$

$$\begin{aligned}
 \text{Sol. } &= \frac{2y^{2-1}}{3x^{4-2}} \\
 &= \frac{2y}{3x^2}
 \end{aligned}$$

**Q.10**  $\frac{25a^3b^2}{14a^2b^4}$

$$\begin{aligned}
 \text{Sol. } &= \frac{25a^{3-2}}{14b^{4-2}} \\
 &= \frac{25a}{14b^2}
 \end{aligned}$$

**Q.11**  $\frac{16a^6b^7}{12a^3b^5 + 20a^5b^4}$

$$\begin{aligned}
 \text{Sol. } &= \frac{16a^6b^7}{4a^3b^4(3b + 5a^2)} \\
 &= \frac{4a^{6-3}b^{7-4}}{(5a^2 + 3b)} \\
 &= \frac{4a^3b^3}{5a^2 + 3b}
 \end{aligned}$$

**Q.12**  $\frac{18m^5x^3}{27m^4x^8 - 36m^6x^6}$

$$\begin{aligned}
 \text{Sol. } &= \frac{18m^5x^3}{9m^4x^6(3x^2 - 4m^2)} \\
 &= \frac{2m^{5-4}}{x^{6-3}(3x^2 - 4m^2)}
 \end{aligned}$$

$$= \frac{2m}{x^3(3x^2 - 4m^2)}$$

$$= \frac{2m}{3x^5 - 4m^2x^3}$$

**Q.13**  $\frac{5c - 5d}{c^2 - d^2}$

Sol.  $= \frac{5(c - d)}{(c - d)(c + d)}$

$$= \frac{5}{(c + d)}$$

**Q.14**  $\frac{x^2 - y^2}{3y - 3x}$

Sol.  $= \frac{(x - y)(x + y)}{3(y - x)}$

$$= \frac{(x - y)(x + y)}{-3(x - y)}$$

$$= \frac{x + y}{-3}$$

Simplify:

**Q.15**  $\frac{x}{x - y} + \frac{x^2}{x^2 + y^2}$

Sol.  $= \frac{x(x^2 + y^2) + x^2(x - y)}{(x - y)(x^2 + y^2)}$

$$= \frac{x^3 + xy^2 + x^3 - x^2y}{x^3 + xy^2 - x^2y - y^3}$$

$$= \frac{2x^3 - x^2y + xy^2}{x^3 - x^2y + xy^2 - y^3}$$

**Q.16**  $\frac{x^2 + 2x}{x^2 + x - 2} + \frac{3x}{x + 1}$

Sol.  $= \frac{(x^2 + 2x)(x + 1) + 3x(x^2 + x - 2)}{(x^2 + x - 2)(x + 1)}$

$$= \frac{x^3 + x^2 + 2x^2 + 2x + 3x^3 + 3x^2 - 6x}{x^3 + x^2 - 2x + x^2 + x - 2}$$

$$= \frac{4x^3 + 6x^2 - 4x}{x^3 + 2x^2 - x - 2}$$

**Q.17**  $\frac{x+2}{x^2+3x+2} - \frac{x-5}{x^2-x-6}$

$$\begin{aligned} \text{Sol. } &= \frac{x+2}{x^2+x+2x+2} - \frac{x-5}{x^2-x-6} \\ &= \frac{x+2}{x(x+1)+2(x+1)} - \frac{x-5}{x^2-x-6} \\ &= \frac{x+2}{(x+2)(x+1)} - \frac{x-5}{x^2-x-6} \\ &= \frac{1}{x+1} - \frac{x-5}{x^2-x-6} \\ &= \frac{1(x^2-x-6) - (x+1)(x-5)}{(x+1)(x^2-x-6)} \\ &= \frac{x^2-x-6 - (x^2-5x+x-5)}{(x+1)(x^2-x-6)} \end{aligned}$$

$$= \frac{x^2-x-6 - x^2 + 5x - x + 5}{x^3 - x^2 - 6x + x^2 - x - 6}$$

$$= \frac{3x-1}{x^3 - 7x - 6}$$

**Q.18**  $\frac{8x^2+18y^2}{4x^2-9y^2} - \frac{2x+3y}{2x-3y}$

$$\begin{aligned} \text{Sol. } &= \frac{8x^2+18y^2}{(2x)^2-(3y)^2} - \frac{2x+3y}{2x-3y} \\ &= \frac{8x^2+18y^2}{(2x+3y)(2x-3y)} - \frac{2x+3y}{2x-3y} \\ &= \frac{8x^2+18y^2 - (2x+3y)(2x+3y)}{(2x+3y)(2x-3y)} \end{aligned}$$

$$\begin{aligned}
 &= \frac{8x^2 + 18y^2 - (4x^2 + 12xy + 9y^2)}{(2x + 3y)(2x - 3y)} \\
 &= \frac{8x^2 + 18y^2 - 4x^2 - 12xy - 9y^2}{(2x + 3y)(2x - 3y)} \\
 &= \frac{4x^2 - 12xy + 9y^2}{(2x + 3y)(2x - 3y)} \\
 &= \frac{(2x)^2 - 2(2x)(3y) + (3y)^2}{(2x + 3y)(2x - 3y)} \\
 &= \frac{(2x - 3y)^2}{(2x + 3y)(2x - 3y)} \\
 &= \frac{\cancel{(2x - 3y)}(2x - 3y)}{\cancel{(2x + 3y)}\cancel{(2x - 3y)}} \\
 &= \frac{2x - 3y}{2x + 3y}
 \end{aligned}$$

**Q.19**  $\frac{x}{x^2 + xy} - \frac{y}{x^2 - y^2}$

$$\begin{aligned}
 \text{Sol. } &= \frac{x}{x(x + y)} - \frac{y}{(x + y)(x - y)} \\
 &= \frac{1}{x + y} - \frac{y}{(x + y)(x - y)} \\
 &= \frac{1(x - y) - y}{(x + y)(x - y)} \\
 &= \frac{x - y - y}{(x + y)(x - y)} \\
 &= \frac{x - 2y}{x^2 - y^2}
 \end{aligned}$$

**Q.20**  $\frac{x + y}{xy + y^2} - \frac{x}{x^2 - xy}$

$$\text{Sol. } = \frac{x + y}{y(x + y)} - \frac{x}{x(x - y)}$$

$$\begin{aligned}
 &= \frac{1}{y} - \frac{1}{x-y} \\
 &= \frac{1(x-y) - 1(y)}{(y)(x-y)} \\
 &= \frac{x-y-y}{y(x-y)} \\
 &= \frac{x-2y}{xy-y^2}
 \end{aligned}$$

**Q.21**  $\frac{(x+1)^2}{x^2-1} - \frac{x^2+1}{x^2+1}$

$$\begin{aligned}
 \text{Sol. } &= \frac{(x+1)(x+1)}{(x+1)(x-1)} - \frac{x^2+1}{x^2+1} \\
 &= \frac{x+1}{x-1} - 1 \\
 &= \frac{x+1 - 1(x-1)}{x-1} \\
 &= \frac{x+1-x+1}{x-1} \\
 &= \frac{2}{x-1}
 \end{aligned}$$

**Q.22**  $\frac{5x}{x-9} + \frac{x^2-2x+1}{x^2-12x+27} - \frac{6x}{x-3}$

$$\begin{aligned}
 \text{Sol. } &= \frac{5x}{x-9} + \frac{x^2-2x+1}{x^2-9x-3x+27} - \frac{6x}{x-3} \\
 &= \frac{5x}{x-9} + \frac{x^2-2x+1}{x(x-9)-3(x-9)} - \frac{6x}{x-3} \\
 &= \frac{5x}{x-9} + \frac{x^2-2x+1}{(x-9)(x-3)} - \frac{6x}{x-3} \\
 &= \frac{5x(x-3)+x^2-2x+1-6x(x-9)}{(x-9)(x-3)}
 \end{aligned}$$

$$= \frac{5x^2 - 15x + x^2 - 2x + 1 - 6x^2 + 54x}{(x-9)(x-3)}$$

$$= \frac{37x + 1}{x^2 - 12x + 27}$$

**Q.23**  $\frac{x^2 - 4x + 4}{x^2 - 4} + \frac{x}{x-2}$

Sol.

$$= \frac{x^2 - 2x - 2x + 4}{(x)^2 - (2)^2} \times \frac{x-2}{x}$$

$$= \frac{x(x-2) - 2(x-2)}{(x+2)(x-2)} \times \frac{x-2}{x}$$

$$= \frac{(x-2)(x-2)}{x+2} \times \frac{1}{x}$$

$$= \frac{x^2 - 4x + 4}{x^2 + 2x}$$

**Q.24**  $\frac{x^2 - 36}{x^2 - 1} \div \frac{x-6}{1-x}$

Sol.

$$= \frac{(x)^2 - (6)^2}{(x)^2 - (1)^2} \times \frac{1-x}{x-6}$$

$$= \frac{(x-6)(x+6)}{(x-1)(x+1)} \times \frac{-(x-1)}{x-6}$$

$$= \frac{x+6}{x+1} \times (-1)$$

$$= -\frac{x+6}{x+1}$$

**Q.25**  $\frac{x^2 - 5x}{x-1} \div \frac{x^2 - 25}{x^2 + x + 20}$

Sol.

$$= \frac{x(x-5)}{x-1} \times \frac{x^2 + x + 20}{x^2 - 25}$$

$$= \frac{x(x-5)}{x-1} \times \frac{x^2 + x + 20}{(x)^2 - (5)^2}$$

$$= \frac{x(x-5)}{x-1} \times \frac{x^2 + x + 20}{(x-5)(x+5)}$$

$$= \frac{x(x^2 + x + 20)}{(x-1)(x+5)}$$

$$= \frac{x^3 + x^2 + 20x}{x^2 + 4x - 5}$$

Q.26  $\frac{2x^2 - 5x - 12}{4x^2 + 4x - 3} \div \frac{2x^2 - 7x - 4}{6x^2 + 5x - 4}$

$$\begin{aligned} \text{Sol. } &= \frac{2x^2 - 8x + 3x - 12}{4x^2 + 6x - 2x - 3} \times \frac{6x^2 + 5x - 4}{2x^2 - 7x - 4} \\ &= \frac{2x(x-4) + 3(x-4)}{2x(2x+3) - 1(2x+3)} \times \frac{6x^2 + 8x - 3x - 4}{2x^2 - 8x + x - 4} \\ &= \frac{(x-4)(2x+3)}{(2x+3)(2x-1)} \times \frac{2x(3x+4) - 1(3x+4)}{2x(x-4) + 1(x-4)} \\ &= \frac{x-4}{2x-1} \times \frac{(3x+4)(2x-1)}{(x-4)(2x+1)} \\ &= \frac{3x+4}{2x+1} \end{aligned}$$

Q.27  $\frac{x(2x-1)^2}{2x^2-1} + \frac{4x^2-1}{4x^2+4x+1}$

$$\begin{aligned} \text{Sol. } &= \frac{x(2x-1)(2x-1)}{2x^2-1} \times \frac{4x^2+4x+1}{4x^2-1} \\ &= \frac{x(2x-1)(2x-1)}{(2x^2-1)} \times \frac{4x^2+2x+2x+1}{(2x)^2-(1)^2} \\ &= \frac{x(2x-1)(2x-1)}{(2x^2-1)} \times \frac{2x(2x+1)+1(2x+1)}{(2x+1)(2x-1)} \\ &= \frac{x(2x-1)(2x-1)}{(2x^2-1)} \times \frac{(2x+1)(2x+1)}{(2x+1)(2x-1)} \\ &= \frac{x(2x-1)(2x+1)}{2x^2-1} \end{aligned}$$

$$= \frac{x(4x^2 - 1)}{2x^2 - 1}$$

$$= \frac{4x^3 - x}{2x^2 - 1}$$

**Q.28**  $\frac{x^2 + x}{x^2 - 1} \times \frac{x+1}{x^3 + 1}$

Sol.  $= \frac{x(x+1)}{(x+1)(x-1)} \times \frac{(x+1)}{(x+1)(x^2 - x + 1)}$

 $= \frac{x}{(x-1)(x^2 - x + 1)}$

$$= \frac{x}{x^3 - x^2 + x - x^2 + x - 1}$$

$$= \frac{x}{x^3 - 2x^2 + 2x - 1}$$

**Q.29**  $\frac{x^2 - 9}{x^2 - 6x + 9} \times \frac{x}{3x + 9}$

Sol.  $= \frac{(x)^2 - (3)^2}{x^2 - 3x - 3x + 9} \times \frac{x}{3(x+3)}$

 $= \frac{(x+3)(x-3)}{x(x-3) - 3(x-3)} \times \frac{x}{3(x+3)}$ 
 $= \frac{(x+3)(x-3)}{(x-3)(x-3)} \times \frac{x}{3(x+3)}$ 
 $= \frac{x}{3(x-3)}$

$$= \frac{x}{3x-9}$$

**Q.30**  $\frac{x+5}{x^2 + 6x} \times \frac{x^3 + 6x^2}{x+5}$

Sol.  $= \frac{(x+5)}{x(x+6)} \times \frac{x^2(x+6)}{(x+5)}$

$$\begin{aligned}
 &= \frac{x^2}{x} \\
 &= x^{2-1} \\
 &= x
 \end{aligned}$$

**Q.31**  $\frac{x^2 - 2x + 1}{x^2 - 1} \times \frac{x+1}{x-1}$

$$\begin{aligned}
 \text{Sol. } &= \frac{x^2 - x - x + 1}{(x)^2 - (1)^2} \times \frac{x+1}{x-1} \\
 &= \frac{x(x-1) - 1(x-1)}{(x+1)(x-1)} \times \frac{(x+1)}{(x-1)} \\
 &= \frac{(x-1)(x-1)}{(x+1)(x-1)} \times \frac{(x+1)}{(x-1)} \\
 &= 1
 \end{aligned}$$

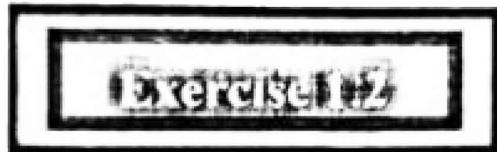
**Q.32**  $\frac{x^2 + 4x + 3}{x+3} \times \frac{x^2 - 2x + 1}{x^2 - 1}$

$$\begin{aligned}
 \text{Sol. } &= \frac{x^2 + x + 3x + 3}{x+3} \times \frac{x^2 - x - x + 1}{(x)^2 - (1)^2} \\
 &= \frac{x(x+1) + 3(x+1)}{(x+3)} \times \frac{x(x-1) - 1(x-1)}{(x-1)(x+1)} \\
 &= \frac{(x+1)(x+3)}{(x+3)} \times \frac{(x-1)(x-1)}{(x-1)(x+1)} \\
 &= x - 1
 \end{aligned}$$

### Formulae

- (i)  $(a+b)^2 = a^2 + 2ab + b^2$
- (ii)  $(a-b)^2 = a^2 - 2ab + b^2$
- (iii)  $(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$
- (iv)  $(a+b)^2 - (a-b)^2 = 4ab$
- (v)  $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$   
 $= a^2 + b^2 + c^2 + 2(ab + bc + ca)$
- (vi)  $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$$\begin{aligned}
 &= a^3 + b^3 + 3ab(a + b) \\
 (\text{vii}) \quad (a - b)^3 &= a^3 - 3a^2b + 3ab^2 - b^3 \\
 &= a^3 - b^3 - 3ab(a - b) \\
 (\text{viii}) \quad a^3 + b^3 &= (a + b)(a^2 - ab + b^2) \\
 (\text{ix}) \quad a^3 - b^3 &= (a - b)(a^2 + ab + b^2)
 \end{aligned}$$



**Solve the following questions using formulas.**

**Q.1**  $(x + 2y)^2 + (x - 2y)^2$

Sol.  $= 2[(x)^2 + (2y)^2]$  [Formula:  $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$ ]  
 $= 2[x^2 + 4y^2]$   
 $= 2x^2 + 8y^2$

**Q.2**  $(5x + 3y)^2 + (5x - 3y)^2$

Sol.  $= 2[(5x)^2 + (3y)^2]$  [Formula:  $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$ ]  
 $= 2[25x^2 + 9y^2]$   
 $= 50x^2 + 18y^2$

**Q.3**  $(3l + 2m)^2 - (3l - 2m)^2$

Sol.  $= 4(3l)(2m)$  [Formula:  $(a + b)^2 - (a - b)^2 = 4ab$ ]  
 $= 24lm$

**Q.4**  $(l + m)(l - m)(l^2 + m^2)(l^4 + m^4)$

Sol.  $= [(l + m)(l - m)](l^2 + m^2)(l^4 + m^4)$   
[Formula:  $(a + b)(a - b) = a^2 - b^2$ ]  
 $= (l^2 - m^2)(l^2 + m^2)(l^4 + m^4)$   
 $= [(l^2)^2 - (m^2)^2](l^4 + m^4)$   
 $= (l^4 - m^4)(l^4 + m^4)$   
 $= (l^4)^2 - (m^4)^2$   
 $= l^8 - m^8$