Exercise

Solve:

1- **If $P(x) = x^4 + 3x^2 - 5x + 9$, then find $P(x)$, for $x = 0$, $x = 1$.**

Sol. $P(x) = x^4 + 3x^2 - 5x + 9$

$P(0) = (0)^4 + 3(0)^2 - 5(0) + 9$

$= 0 + 0 - 0 + 9$

$= 9$

and $P(1) = (1)^4 + 3(1)^2 - 5(1) + 9$

$= 1 + 3 - 5 + 9$

$= 13 - 5$

$= 8$

2- **If $P(x) = 2x^3 + 2x^2 + x - 1$, then find $P(-2)$**

Sol. $P(x) = 2x^3 + 2x^2 + x - 1$

Therefore, $P(-2) = 2(-2)^3 + 2(-2)^2 + (-2) - 1$

$= 2(-8) + 2(4) - 2 - 1$

$= -16 + 8 - 2 - 1$

$= 8 - 16 - 2 - 1$

$= 8 - 19$

$= -11$

3- **If $P(y) = 3y^2 + \frac{y}{4} + 9$, then find $P(0)$..**

Sol. $P(y) = 3y^2 + \frac{y}{4} + 9$

$P(0) = 3(0)^2 + \frac{0}{4} + 9$

$= 3(0) + 0 + 9$

$= 9$

4- **If $P(x) = 9x^3 - 2x^2 + 3x + 1$, then find $P(1)$ and $P(2)$.**

Sol. $P(x) = 9x^3 - 2x^2 + 3x + 1$

$P(1) = 9(1)^3 - 2(1)^2 + 3(1) + 1$

$= 9(1) - 2(1) + 3 + 1$

$= 9 - 2 + 3 + 1$

$= 13 - 2$

$= 11$

and $P(2) = 9(2)^3 - 2(2)^2 + 3(2) + 1$

$= 9(8) - 2(4) + 6 + 1$

$= 72 - 8 + 6 + 1$

$= 79 - 8$

$= 71$

5- If $P(x) = \dfrac{x^2 - 5x + 6}{x + 1}$, then find P(1) and P(2).

Sol. $P(x) = \dfrac{x^2 - 5x + 6}{x + 1}$

$P(1) = \dfrac{(1)^2 - 5(1) + 6}{1 + 1}$

$= \dfrac{1 - 5 + 6}{1 + 1}$

$= \dfrac{2}{2}$

$= 1$

and $P(2) = \dfrac{(2)^2 - 5(2) + 6}{2 + 1}$

$= \dfrac{4 - 10 + 6}{3}$

$= \dfrac{0}{3}$

$= 0$

6- If P(r) = 2πr, then find P(*r*), for r = 3 and $\pi = \dfrac{22}{7}$.

Sol. $P(r) = 2\pi r$

By putting the values of *r* and π

$$P(r) = 2\times\frac{22}{7}\times 3$$

$$= \frac{132}{7}$$

$$= 18.9 \text{ (approximately)}$$

7- **If $P(r) = 4\pi r^2$, then find $P(r)$, for $r = 8$ and $\pi = \frac{22}{7}$.**

Sol. $P(r) = 4\pi r^2$

By putting the values of r and π

$$P(r) = 4\times\frac{22}{7}\times(8)^2$$

$$= 4\times\frac{22}{7}\times 64$$

$$= \frac{5632}{7}$$

$$= 804.57$$

8- **If $P(y) = y^4 + \frac{3y^3}{2} - y^2 + 1$, then find $P(y)$, for $y = 2$ and $y = -2$.**

Sol. $P(y) = y^4 + \frac{3y^3}{2} - y^2 + 1$

for $y = 2$

$$P(2) = (2)^4 + \frac{3(2)^3}{2} - (2)^2 + 1$$

$$= 16 + \frac{3\times 8}{2} - 4 + 1$$

$$= 16 + 12 - 4 + 1$$

$$= 25$$

$$P(-2) = (-2)^4 + \frac{3(-2)^3}{2} - (-2)^2 + 1$$ for $y = -2$

$$= 16 + \frac{3(-8)}{2} - 4 + 1$$

$$= 16 - 12 - 4 + 1$$

$$= 1$$

Reduce the given rational expressions to lowest terms.

Q.9 $\frac{8x^2y^2}{12x^4y}$

Sol. $= \frac{2y^{2-1}}{3x^{4-2}}$

$= \frac{2y}{3x^2}$

Q.10 $\frac{25a^3b^2}{14a^2b^4}$

Sol. $= \frac{25a^{3-2}}{14b^{4-2}}$

$= \frac{25a}{14b^2}$

Q.11 $\frac{16a^6b^7}{12a^3b^5 + 20a^5b^4}$

Sol. $= \frac{16a^6b^7}{4a^3b^4(3b + 5a^2)}$

$= \frac{4a^{6-3}b^{7-4}}{(5a^2 + 3b)}$

$= \frac{4a^3b^3}{5a^2 + 3b}$

Q.12 $\frac{18m^5x^3}{27m^4x^8 - 36m^6x^6}$

Sol. $= \frac{18m^5x^3}{9m^4x^6(3x^2 - 4m^2)}$

$= \frac{2m^{5-4}}{x^{6-3}(3x^2 - 4m^2)}$

$$= \frac{2m}{x^3(3x^2 - 4m^2)}$$

$$= \frac{2m}{3x^5 - 4m^2x^3}$$

Q.13 $\frac{5c - 5d}{c^2 - d^2}$

Sol. $= \frac{5(c-d)}{(c-d)(c+d)}$

$$= \frac{5}{(c+d)}$$

Q.14 $\frac{x^2 - y^2}{3y - 3x}$

Sol. $= \frac{(x-y)(x+y)}{3(y-x)}$

$$= \frac{(x-y)(x+y)}{-3(x-y)}$$

$$= \frac{x+y}{-3}$$

Simplify:

Q.15 $\frac{x}{x-y} + \frac{x^2}{x^2+y^2}$

Sol. $= \frac{x(x^2+y^2) + x^2(x-y)}{(x-y)(x^2+y^2)}$

$$= \frac{x^3 + xy^2 + x^3 - x^2y}{x^3 + xy^2 - x^2y - y^3}$$

$$= \frac{2x^3 - x^2y + xy^2}{x^3 - x^2y + xy^2 - y^3}$$

Q.16 $\frac{x^2 + 2x}{x^2 + x - 2} + \frac{3x}{x+1}$

Sol. $= \frac{(x^2+2x)(x+1) + 3x(x^2+x-2)}{(x^2+x-2)(x+1)}$

$$= \frac{x^3 + x^2 + 2x^2 + 2x + 3x^3 + 3x^2 - 6x}{x^3 + x^2 - 2x + x^2 + x - 2}$$

$$= \frac{4x^3 + 6x^2 - 4x}{x^3 + 2x^2 - x - 2}$$

Q.17 $\frac{x+2}{x^2+3x+2} - \frac{x-5}{x^2-x-6}$

Sol. $= \frac{x+2}{x^2+x+2x+2} - \frac{x-5}{x^2-x-6}$

$$= \frac{x+2}{x(x+1)+2(x+1)} - \frac{x-5}{x^2-x-6}$$

$$= \frac{x+2}{(x+2)(x+1)} - \frac{x-5}{x^2-x-6}$$

$$= \frac{1}{x+1} - \frac{x-5}{x^2-x-6}$$

$$= \frac{1(x^2-x-6)-(x+1)(x-5)}{(x+1)(x^2-x-6)}$$

$$= \frac{x^2-x-6-(x^2-5x+x-5)}{(x+1)(x^2-x-6)}$$

$$= \frac{x^2-x-6-x^2+5x-x+5}{x^3-x^2-6x+x^2-x-6}$$

$$= \frac{3x-1}{x^3-7x-6}$$

Q.18 $\frac{8x^2+18y^2}{4x^2-9y^2} - \frac{2x+3y}{2x-3y}$

Sol. $= \frac{8x^2+18y^2}{(2x)^2-(3y)^2} - \frac{2x+3y}{2x-3y}$

$$= \frac{8x^2+18y^2}{(2x+3y)(2x-3y)} - \frac{2x+3y}{2x-3y}$$

$$= \frac{8x^2+18y^2-(2x+3y)(2x+3y)}{(2x+3y)(2x-3y)}$$

$$= \frac{8x^2 + 18y^2 - (4x^2 + 12xy + 9y^2)}{(2x+3y)(2x-3y)}$$

$$= \frac{8x^2 + 18y^2 - 4x^2 - 12xy - 9y^2}{(2x+3y)(2x-3y)}$$

$$= \frac{4x^2 - 12xy + 9y^2}{(2x+3y)(2x-3y)}$$

$$= \frac{(2x)^2 - 2(2x)(3y) + (3y)^2}{(2x+3y)(2x-3y)}$$

$$= \frac{(2x-3y)^2}{(2x+3y)(2x-3y)}$$

$$= \frac{(2x-3y)\cancel{(2x-3y)}}{(2x+3y)\cancel{(2x-3y)}}$$

$$= \frac{2x-3y}{2x+3y}$$

Q.19 $\dfrac{x}{x^2+xy} - \dfrac{y}{x^2-y^2}$

Sol. $= \dfrac{x}{x(x+y)} - \dfrac{y}{(x+y)(x-y)}$

$$= \frac{1}{x+y} - \frac{y}{(x+y)(x-y)}$$

$$= \frac{1(x-y) - y}{(x+y)(x-y)}$$

$$= \frac{x-y-y}{(x+y)(x-y)}$$

$$= \frac{x-2y}{x^2-y^2}$$

Q.20 $\dfrac{x+y}{xy+y^2} - \dfrac{x}{x^2-xy}$

Sol. $= \dfrac{x+y}{y(x+y)} - \dfrac{x}{x(x-y)}$

$$= \frac{1}{y} - \frac{1}{x-y}$$

$$= \frac{1(x-y)-1(y)}{(y)(x-y)}$$

$$= \frac{x-y-y}{y(x-y)}$$

$$= \frac{x-2y}{xy-y^2}$$

Q.21 $\dfrac{(x+1)^2}{x^2-1} - \dfrac{x^2+1}{x^2+1}$

Sol. $= \dfrac{(x+1)(x+1)}{(x+1)(x-1)} - \dfrac{x^2+1}{x^2+1}$

$$= \frac{x+1}{x-1} - 1$$

$$= \frac{x+1-1(x-1)}{x-1}$$

$$= \frac{x+1-x+1}{x-1}$$

$$= \frac{2}{x-1}$$

Q.22 $\dfrac{5x}{x-9} + \dfrac{x^2-2x+1}{x^2-12x+27} - \dfrac{6x}{x-3}$

Sol. $= \dfrac{5x}{x-9} + \dfrac{x^2-2x+1}{x^2-9x-3x+27} - \dfrac{6x}{x-3}$

$$= \frac{5x}{x-9} + \frac{x^2-2x+1}{x(x-9)-3(x-9)} - \frac{6x}{x-3}$$

$$= \frac{5x}{x-9} + \frac{x^2-2x+1}{(x-9)(x-3)} - \frac{6x}{x-3}$$

$$= \frac{5x(x-3)+x^2-2x+1-6x(x-9)}{(x-9)(x-3)}$$

$$= \frac{5x^2 - 15x + x^2 - 2x + 1 - 6x^2 + 54x}{(x-9)(x-3)}$$

$$= \frac{37x + 1}{x^2 - 12x + 27}$$

Q.23 $\frac{x^2 - 4x + 4}{x^2 - 4} \div \frac{x}{x-2}$

Sol. $= \frac{x^2 - 2x - 2x + 4}{(x)^2 - (2)^2} \times \frac{x-2}{x}$

$$= \frac{x(x-2) - 2(x-2)}{(x+2)(x-2)} \times \frac{x-2}{x}$$

$$= \frac{(x-2)(x-2)}{x+2} \times \frac{1}{x}$$

$$= \frac{x^2 - 4x + 4}{x^2 + 2x}$$

Q.24 $\frac{x^2 - 36}{x^2 - 1} \div \frac{x-6}{1-x}$

Sol. $= \frac{(x)^2 - (6)^2}{(x)^2 - (1)^2} \times \frac{1-x}{x-6}$

$$= \frac{(x-6)(x+6)}{(x-1)(x+1)} \times \frac{-(x-1)}{x-6}$$

$$= \frac{x+6}{x+1} \times (-1)$$

$$= -\frac{x+6}{x+1}$$

Q.25 $\frac{x^2 - 5x}{x-1} \div \frac{x^2 - 25}{x^2 + x + 20}$

Sol. $= = \frac{x(x-5)}{x-1} \times \frac{x^2 + x + 20}{x^2 - 25}$

$$= \frac{x(x-5)}{x-1} \times \frac{x^2 + x + 20}{(x)^2 - (5)^2}$$

$$= \frac{x(x-5)}{x-1} \times \frac{x^2+x+20}{(x-5)(x+5)}$$

$$= \frac{x(x^2+x+20)}{(x-1)(x+5)}$$

$$= \frac{x^3+x^2+20x}{x^2+4x-5}$$

Q.26 $\frac{2x^2-5x-12}{4x^2+4x-3} \div \frac{2x^2-7x-4}{6x^2+5x-4}$

Sol. $= \frac{2x^2-8x+3x-12}{4x^2+6x-2x-3} \times \frac{6x^2+5x-4}{2x^2-7x-4}$

$$= \frac{2x(x-4)+3(x-4)}{2x(2x+3)-1(2x+3)} \times \frac{6x^2+8x-3x-4}{2x^2-8x+x-4}$$

$$= \frac{(x-4)(2x+3)}{(2x+3)(2x-1)} \times \frac{2x(3x+4)-1(3x+4)}{2x(x-4)+1(x-4)}$$

$$= \frac{x-4}{2x-1} \times \frac{(3x+4)(2x-1)}{(x-4)(2x+1)}$$

$$= \frac{3x+4}{2x+1}$$

Q.27 $\frac{x(2x-1)^2}{2x^2-1} \div \frac{4x^2-1}{4x^2+4x+1}$

Sol. $= \frac{x(2x-1)(2x-1)}{2x^2-1} \times \frac{4x^2+4x+1}{4x^2-1}$

$$= \frac{x(2x-1)(2x-1)}{(2x^2-1)} \times \frac{4x^2+2x+2x+1}{(2x)^2-(1)^2}$$

$$= \frac{x(2x-1)(2x-1)}{(2x^2-1)} \times \frac{2x(2x+1)+1(2x+1)}{(2x+1)(2x-1)}$$

$$= \frac{x(2x-1)(2x-1)}{(2x^2-1)} \times \frac{(2x+1)(2x+1)}{(2x+1)(2x-1)}$$

$$= \frac{x(2x-1)(2x+1)}{2x^2-1}$$

$$= \frac{x(4x^2-1)}{2x^2-1}$$

$$= \frac{4x^3-x}{2x^2-1}$$

Q.28 $\frac{x^2+x}{x^2-1} \times \frac{x+1}{x^3+1}$

Sol. $= \frac{x(x+1)}{(x+1)(x-1)} \times \frac{(x+1)}{(x+1)(x^2-x+1)}$

$$= \frac{x}{(x-1)(x^2-x+1)}$$

$$= \frac{x}{x^3-x^2+x-x^2+x-1}$$

$$= \frac{x}{x^3-2x^2+2x-1}$$

Q.29 $\frac{x^2-9}{x^2-6x+9} \times \frac{x}{3x+9}$

Sol. $= \frac{(x)^2-(3)^2}{x^2-3x-3x+9} \times \frac{x}{3(x+3)}$

$$= \frac{(x+3)(x-3)}{x(x-3)-3(x-3)} \times \frac{x}{3(x+3)}$$

$$= \frac{(x+3)(x-3)}{(x-3)(x-3)} \times \frac{x}{3(x+3)}$$

$$= \frac{x}{3(x-3)}$$

$$= \frac{x}{3x-9}$$

Q.30 $\frac{x+5}{x^2+6x} \times \frac{x^3+6x^2}{x+5}$

Sol. $= \frac{(x+5)}{x(x+6)} \times \frac{x^2(x+6)}{(x+5)}$

$$= \frac{x^2}{x}$$

$$= x^{2-1}$$

$$= x$$

Q.31 $\frac{x^2-2x+1}{x^2-1} \times \frac{x+1}{x-1}$

Sol. $= \frac{x^2-x-x+1}{(x)^2-(1)^2} \times \frac{x+1}{x-1}$

$$= \frac{x(x-1)-1(x-1)}{(x+1)(x-1)} \times \frac{(x+1)}{(x-1)}$$

$$= \frac{(x-1)(x-1)}{(x+1)(x-1)} \times \frac{(x+1)}{(x-1)}$$

$$= 1$$

Q.32 $\frac{x^2+4x+3}{x+3} \times \frac{x^2-2x+1}{x^2-1}$

Sol. $= \frac{x^2+x+3x+3}{x+3} \times \frac{x^2-x-x+1}{(x)^2-(1)^2}$

$$= \frac{x(x+1)+3(x+1)}{(x+3)} \times \frac{x(x-1)-1(x-1)}{(x-1)(x+1)}$$

$$= \frac{(x+1)(x+3)}{(x+3)} \times \frac{(x-1)(x-1)}{(x-1)(x+1)}$$

$$= x - 1$$

Formulae

(i) $(a+b)^2 = a^2 + 2ab + b^2$

(ii) $(a-b)^2 = a^2 - 2ab + b^2$

(iii) $(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$

(iv) $(a+b)^2 - (a-b)^2 = 4ab$

(v) $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$= a^2 + b^2 + c^2 + 2(ab + bc + ca)$

(vi) $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$= a^3 + b^3 + 3ab(a + b)$

(vii) $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$= a^3 - b^3 - 3ab(a - b)$

(viii) $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

(ix) $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Exercise 1.2

Solve the following questions using formulas.

Q.1 $(x + 2y)^2 + (x - 2y)^2$

Sol. $= 2[(x)^2 + (2y)^2]$ [Formula: $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$]

$= 2[x^2 + 4y^2]$

$= 2x^2 + 8y^2$

Q.2 $(5x + 3y)^2 + (5x - 3y)^2$

Sol. $= 2[(5x)^2 + (3y)^2]$ [Formula: $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$]

$= 2[25x^2 + 9y^2]$

$= 50x^2 + 18y^2$

Q.3 $(3l + 2m)^2 - (3l - 2m)^2$

Sol. $= 4(3l)(2m)$ [Formula: $(a + b)^2 - (a - b)^2 = 4ab$]

$= 24lm$

Q.4 $(l + m)(l - m)(l^2 + m^2)(l^4 + m^4)$

Sol. $= [(l + m)(l - m)](l^2 + m^2)(l^4 + m^4)$

[Formula: $(a + b)(a - b) = a^2 - b^2$]

$= (l^2 - m^2)(l^2 + m^2)(l^4 + m^4)$

$= [(l^2)^2 - (m^2)^2](l^4 + m^4)$

$= (l^4 - m^4)(l^4 + m^4)$

$= (l^4)^2 - (m^4)^2$

$= l^8 - m^8$

Q.5 $\left(ab - \frac{1}{ab}\right)^3$

Sol. [Formula: $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$]

$= (ab)^3 - 3(ab)^2\left(\frac{1}{ab}\right) + 3ab\left(\frac{1}{ab}\right)^2 - \left(\frac{1}{ab}\right)^3$

$= a^3b^3 - 3ab + \frac{3}{ab} - \frac{1}{a^3b^3}$

Q.6 $(2x + 3y + 2)^2$

Sol. [$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$]

$= (2x)^2 + (3y)^2 + (2)^2 + 2(2x)(3y) + 2(3y)(2) + 2(2)(2x)$

$= 4x^2 + 9y^2 + 4 + 12xy + 12y + 8x$

Q.7 $(2p + q)^3$

Sol. [Formula: $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$]

$= (2p)^3 + 3(2p)^2(q) + 3(2p)(q)^2 + (q)^3$

$= 8p^3 + 3(4p^2)(q) + 6pq^2 + q^3$

$= 8p^3 + 12p^2q + 6pq^2 + q^3$

Q.8 $(3p + q + r)^2$

Sol. [Formula: $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$]

$= (3p)^2 + (q)^2 + (r)^2 + 2(3p)(q) + 2(q)(r) + 2(r)(3p)$

$= 9p^2 + q^2 + r^2 + 6pq + 2qr + 6rp$

Q.9 $(2x + 3y)^3$

Sol. [Formula: $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$]

$= (2x)^3 + 3(2x)^2(3y) + 3(2x)(3y)^2 + (3y)^3$

$= 8x^3 + 3(4x^2)(3y) + 3(2x)(9y^2) + 9y^3$

$= 8x^3 + 36x^2y + 54xy^2 + 9y^3$

Q.10 $(x + y)^3 - 1$

Sol. $= (x + y)^3 - (1)^3$

[Formula: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$]

$= (x + y - 1)[(x + y)^2 + (x + y)(1) + (1)^2]$

$= (x + y - 1)[x^2 + y^2 + 2xy + x + y + 1]$

Q.11 $(x-y)^3+64$

Sol. $=(x-y)^3+(4)^3$

[Formula: $a^3+b^3=(a+b)(a^2-ab+b^2)$]

$=(x-y+4)[(x-y)^2-(x-y)(4)+(4)^2]$

$=(x-y+4)(x^2+y^2+2xy-4x+4y+16)$

Q.12 $8x^3+27y^3$

Sol. $=(2x)^3+(3y)^3$

[Formula: $a^3+b^3=(a+b)(a^2-ab+b^2)$]

$=(2x+3y)[(2x)^2-(2x)(3y)+(3y)^2]$

$=(2x+3y)(4x^2-6xy+9y^2)$

Q.13 x^6-729y^6

Sol. $=(x^3)^2-(27y^3)^2$ [Formula: $a^2-b^2=(a+b)(a-b)$]

$=(x^3+27y^3)(x^3-27y^3)$

$=[(x)^3+(3y)^3][(x)^3-(3y)^3]$

(Formulas)

$a^3+b^3=(a+b)(a^2-ab+b^2)$

$a^3-b^3=(a-b)(a^2+ab+b^2)$

$=(x+3y)(x^2-3xy+9y^2)(x-3y)(x^2+3xy+9y^2)$

$=(x+3y)(x-3y)(x^2-3xy+9y^2)(x^2+3xy+9y^2)$

Q.14 $64a^6-b^6$

Sol. $=(8a^3)^2-(b^3)^2$

$=(8a^3-b^3)(8a^3+b^3)$ [Formula: $a^2-b^2=(a-b)(a+b)$]

$=[(2a)^3-(b)^3][(2a)^3+(b)^3]$

$a^3+b^3=(a+b)(a^2-ab+b^2)$

(Formulas)

$a^3-b^3=(a-b)(a^2+ab+b^2)$

$=(2a-b)(4a^2+2ab+b^2)(2a+b)(4a^2-2ab+b^2)$

$=(2a-b)(2a+b)(4a^2+2ab+b^2)(4a^2-2ab+b^2)$

Q.15 Find the value of $a^3 - b^3$ when $a - b = 4$ and $ab = -5$.

Sol. $a - b = 4$

$ab = -5$

$a^3 - b^3 = ?$

$a - b = 4$

$(a - b)^3 = (4)^3$ Taking cube

$a^3 - b^3 - 3(-5)(4) = 64$

$a^3 - b^3 + 60 = 64$

$a^3 - b^3 = 64 - 60$

$a^3 - b^3 = 4$

Q.16 Show that $\left(z + \frac{1}{z}\right)^2 - \left(z - \frac{1}{z}\right)^2 = 4$

$$L.H.S = \left(z + \frac{1}{z}\right)^2 - \left(z - \frac{1}{z}\right)^2$$

$$= \left(z^2 + 2(z)\left(\frac{1}{z}\right) + \frac{1}{z^2}\right) - \left(z^2 - 2(z)\left(\frac{1}{z}\right) + \frac{1}{z^2}\right)$$

$$= \left(z^2 + 2 + \frac{1}{z^2}\right) - \left(z^2 - 2 + \frac{1}{z^2}\right)$$

$$= z^2 + 2 + \frac{1}{z^2} - z^2 + 2 - \frac{1}{z^2}$$

$= 4 = R.H.S$

Q.17 Find the value of $a^2 + b^2$ and ab when $a + b = 5$ and $a - b = 3$.

Sol. $a + b = 5$

$a - b = 3$

$a^2 + b^2 = ?$

$ab = ?$

Formula:

$(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$

$(5)^2 + (3)^2 = 2(a^2 + b^2)$

$25 + 9 = 2(a^2 + b^2)$

$34 = 2(a^2 + b^2)$

Thus, $a^2 + b^2 = \frac{34}{2} = 17$

Formula: $(a + b)^2 - (a - b)^2 = 4ab$

$(5)^2 - (3)^2 = 4ab$

$25 - 9 = 4ab$

$4ab = 16$

$ab = \frac{16}{4}$

$ab = 4$

Q.18 Find the value of $a^2 + b^2 + c^2$ if $ab + bc + ca = 11$ and $a + b + c = 6$

Sol. $a + b + c = 6$

$ab + bc + ca = 11$

$a^2 + b^2 + c^2 = ?$

$a + b + c = 6$

$(a + b + c)^2 = (6)^2$ Taking square root

$a^2 + b^2 + c^2 + 2(ab + bc + ca) = 36$

By putting values $ab + bc + ca = 11$

$a^2 + b^2 + c^2 + 2(11) = 36$

$a^2 + b^2 + c^2 + 22 = 36$

$a^2 + b^2 + c^2 = 36 - 22$

$a^2 + b^2 + c^2 = 14$

Q.19 Find the value of $x^3 + y^3$ if $xy = 10$ and $x + y = 7$.

Sol. $x + y = 7$

$xy = 10$

$x^3 + y^3 = ?$

$x + y = 7$

$(x + y)^3 = (7)^3$ Taking cube

$x^3 + y^3 + 3xy(x + y) = 343$

$x^3 + y^3 + 3(10)(7) = 343$

$x^3 + y^3 + 210 = 343$

$x^3 + y^3 = 343 - 210$

$x^3 + y^3 = 133$

Q.20 Find the value of $(x - y)^2$ if $x^2 + y^2 = 86$ and $xy = -16$.

Sol. $xy = -16$ (i)

$x^2 + y^2 = 86$ (ii)

$(x - y)^2 = ?$

$(x - y)^2 = x^2 + y^2 - 2xy$

$= (x^2 + y^2) - 2(xy)$

$= 86 - 2(-16)$ From (i) and (ii)

$= 86 + 32$

$= 118$

Q.21 Find the value of ab + bc + ca when the values of

$a^2 + b^2 + c^2 = 81$, $a + b + c = 11$

Sol. $a + b + c = 11$............(i)

$a^2 + b^2 + c^2 = 81$............(ii)

$ab + bc + ca = ?$

$a + b + c = 11$

$(a + b + c)^2 = (11)^2$ Taking square root

$a^2 + b^2 + c^2 + 2(ab + bc + ca) = 121$

$81 + 2(ab + bc + ca) = 121$ from (ii)

$2(ab + bc + ca) = 121 - 81$

$2(ab + bc + ca) = 40$

$$ab + bc + ca = \frac{40}{2}$$

$$ab + bc + ca = 20$$

Q.22 Find the value of $(a + b + c)^2$ when the values of $a^2 + b^2 + c^2 = 32$ and $ab + bc + ca = 7$.

Sol.

$ab + bc + ca = 7$ (i)

$a^2 + b^2 + c^2 = 32$ (ii)

$(a + b + c)^2 = ?$

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$= 32 + 2(7)$ From (i) and (ii)

$= 32 + 14$

$(a + b + c)^2 = 46$

Laws of Radicals

(i) $\left(\sqrt[n]{a}\right)^n = a$ (ii) $\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$

(iii) $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ (iv) $\left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$

Q.1 Remove the radical sign from the denominator:

(i) $\frac{1}{\sqrt{5}}$

Sol.

$$= \frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$$

$$= \frac{\sqrt{5}}{\left(\sqrt{5}\right)^2}$$

$$= \frac{\sqrt{5}}{5}$$

(ii) $\frac{2}{\sqrt{2}} \cdot \frac{7}{\sqrt{3}}$

Sol.

$$= \frac{2 \times 7}{\sqrt{2 \times 3}}$$

$$= \frac{14}{\sqrt{6}}$$

$$= \frac{14}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$$

$$= \frac{14 \times \sqrt{6}}{6} = \frac{7\sqrt{6}}{3}$$

(iii) $\frac{\sqrt{6}}{\sqrt{7}}$

Sol.

$$= \frac{\sqrt{6}}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$$

$$= \frac{\sqrt{6 \times 7}}{\sqrt{7 \times 7}}$$

$$= \frac{\sqrt{42}}{7}$$

Q.2 Simplify these expressions:

(i) $\sqrt{2} + \sqrt{8}$

Sol.

$$= \sqrt{2} + \sqrt{2 \times 2 \times 2}$$

$$= \sqrt{2} + 2\sqrt{2}$$

$$= \sqrt{2}(1 + 2)$$

$$= 3\sqrt{2}$$

(ii) $4\sqrt{50}+\sqrt{200}+\sqrt{50}$

Sol.

$$= 4\sqrt{5\times5\times2}+\sqrt{5\times5\times2\times2\times2}+\sqrt{5\times5\times2}$$

$$= 4\times5\sqrt{2}+5\times2\sqrt{2}+5\sqrt{2}$$

$$= 20\sqrt{2}+10\sqrt{2}+5\sqrt{2}$$

$$= \sqrt{2}(20+10+5)$$

$$= \sqrt{2}(35)$$

$$= 35\sqrt{2}$$

(iii) $(\sqrt{12}-\sqrt{2})(\sqrt{20}-3\sqrt{2})$

Sol.

$$= \sqrt{12}\times\sqrt{20}-\sqrt{12}\times3\sqrt{2}-\sqrt{2}\times\sqrt{20}+\sqrt{2}\times3\sqrt{2}$$

$$= \sqrt{2\times2\times3}\times\sqrt{2\times2\times5}-\sqrt{2\times2\times3}\times3\sqrt{2}$$
$$-\sqrt{2}\times\sqrt{2\times2\times5}+3\sqrt{2\times2}$$

$$= 2\sqrt{3}\times2\sqrt{5}-2\sqrt{3}\times3\sqrt{2}-\sqrt{2}\times2\sqrt{5}+3\times2$$

$$= 4\sqrt{3\times5}-6\sqrt{3\times2}-2\sqrt{2\times5}+6$$

$$= 4\sqrt{15}-6\sqrt{6}-2\sqrt{10}+6$$

(iv) $(6+\sqrt{2})(5-\sqrt{5})$

Sol.

$$= 6(5-\sqrt{5})+\sqrt{2}(5-\sqrt{5})$$

$$= 30-6\sqrt{5}+5\sqrt{2}-\sqrt{2\times5}$$

$$= 30-6\sqrt{5}+5\sqrt{2}-\sqrt{10}$$

(v) $(\sqrt{3}-2)(5-\sqrt{5})$

Sol.

$$= \sqrt{3}(5-\sqrt{5})-2(5-\sqrt{5})$$

$$= 5\sqrt{3}-\sqrt{3\times5}-10+2\sqrt{5}$$

$$= 5\sqrt{3}-\sqrt{15}-10+2\sqrt{5}$$

(vi) $(7+\sqrt{3})(5+\sqrt{2})$

Sol.

$$= 7(5+\sqrt{2})+\sqrt{3}(5+\sqrt{2})$$
$$= 35+7\sqrt{2}+5\sqrt{3}+\sqrt{3\times 2}$$
$$= 35+7\sqrt{2}+5\sqrt{3}+\sqrt{6}$$

Q.3 Rationalize the denominators of the following:

(i) $\dfrac{1}{\sqrt{3}+2}$

Sol.

$$= \frac{1}{\sqrt{3}+2}\times\frac{\sqrt{3}-2}{\sqrt{3}-2}$$
$$= \frac{\sqrt{3}-2}{(\sqrt{3})^2-(2)^2}$$
$$= \frac{\sqrt{3}-2}{3-4}$$
$$= \frac{\sqrt{3}-2}{-1}$$
$$= 2-\sqrt{3}$$

(ii) $\dfrac{1}{4-\sqrt{5}}$

Sol.

$$= \frac{1}{4-\sqrt{5}}\times\frac{4+\sqrt{5}}{4+\sqrt{5}}$$
$$= \frac{4+\sqrt{5}}{(4)^2-(\sqrt{5})^2}$$

$$=\frac{4+\sqrt{5}}{16-5}$$

$$=\frac{4+\sqrt{5}}{11}$$

(iii) $\frac{4\sqrt{3}}{\sqrt{7}+\sqrt{5}}$

Sol.

$$=\frac{4\sqrt{3}}{\sqrt{7}+\sqrt{5}}\times\frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}}$$

$$=\frac{(4\sqrt{3})(\sqrt{7}-\sqrt{5})}{(\sqrt{7})^2-(\sqrt{5})^2}$$

$$=\frac{(4\sqrt{3})(\sqrt{7}-\sqrt{5})}{7-5}$$

$$=\frac{4\sqrt{3}(\sqrt{7}-\sqrt{5})}{2}$$

$$2\sqrt{3}(\sqrt{7}-\sqrt{5})$$

(iv) $\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$

Sol.

$$=\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}\times\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}-\sqrt{y}}$$

$$=\frac{(\sqrt{x}-\sqrt{y})^2}{(\sqrt{x})^2-(\sqrt{y})^2}$$

$$=\frac{x+y-2\sqrt{xy}}{x-y}$$

(v) $\frac{5\sqrt{7}}{2+3\sqrt{7}}$

Sol.

$$=\frac{5\sqrt{7}}{2+3\sqrt{7}}\times\frac{2-3\sqrt{7}}{2-3\sqrt{7}}$$

$$=\frac{5\sqrt{7}(2-3\sqrt{7})}{(2)^2-(3\sqrt{7})^2}$$

$$=\frac{(5\sqrt{7})(2)-(5\sqrt{7})(3\sqrt{7})}{4-9\times 7}$$

$$=\frac{10\sqrt{7}-105}{4-63}$$

$$=\frac{10\sqrt{7}-105}{-59}$$

$$=\frac{105-10\sqrt{7}}{59}$$

(vi) $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

Sol.

$$=\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}\times\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

$$=\frac{(\sqrt{3}+\sqrt{2})^2}{(\sqrt{3})^2-(\sqrt{2})^2}$$

$$=\frac{3+2+2\sqrt{3\times 2}}{3-2}$$

$$=\frac{5+2\sqrt{6}}{1}$$

$$=5+2\sqrt{6}$$

(vii) $\frac{29}{11+3\sqrt{5}}$

Sol.

$$=\frac{29}{11+3\sqrt{5}}\times\frac{11-3\sqrt{5}}{11-3\sqrt{5}}$$

$$=\frac{29(11-3\sqrt{5})}{(11)^2-(3\sqrt{5})^2}$$

$$=\frac{29(11-3\sqrt{5})}{121-9\times5}$$

$$=\frac{29(11-3\sqrt{5})}{121-45}$$

$$=\frac{29(11-3\sqrt{5})}{76}$$

(viii) $\frac{17}{3\sqrt{7}+2\sqrt{3}}$

Sol.

$$=\frac{17}{3\sqrt{7}+2\sqrt{3}}\times\frac{3\sqrt{7}-2\sqrt{3}}{3\sqrt{7}-2\sqrt{3}}$$

$$=\frac{17(3\sqrt{7}-2\sqrt{3})}{(3\sqrt{7})^2-(2\sqrt{3})^2}$$

$$=\frac{17(3\sqrt{7}-2\sqrt{3})}{9\times7-4\times3}$$

$$=\frac{17(3\sqrt{7}-2\sqrt{3})}{63-12}$$

$$=\frac{17(3\sqrt{7}-2\sqrt{3})}{51}$$

$$=\frac{3\sqrt{7}-2\sqrt{3}}{3}$$

Q.4 If $x = \sqrt{5} + 2$, then find the values of (i) $x + \frac{1}{x}$ and (ii) $x^2 + \frac{1}{x^2}$.

Sol. (i) $x = \sqrt{5} + 2$ ....(1)

$$\frac{1}{x} = \frac{1}{\sqrt{5}+2}$$

$$= \frac{1}{\sqrt{5}+2} \times \frac{\sqrt{5}-2}{\sqrt{5}-2}$$

$$= \frac{\sqrt{5}-2}{(\sqrt{5})^2-(2)^2}$$

$$= \frac{\sqrt{5}-2}{5-4}$$

$$\frac{1}{x} = \sqrt{5}-2 \text{ ....(2)}$$

$$x + \frac{1}{x} = \sqrt{5}+2+\sqrt{5}-2$$ from (1) + (2)

Thus $$\boxed{x + \frac{1}{x} = 2\sqrt{5}}$$

(ii) $$\left(x + \frac{1}{x}\right)^2 = (2\sqrt{5})^2$$ Taking square root

$$x^2 + \frac{1}{x^2} + 2 = 4 \times 5$$

$$x^2 + \frac{1}{x^2} + 2 = 20$$

$$x^2 + \frac{1}{x^2} = 20 - 2$$

Hence, $$\boxed{x^2 + \frac{1}{x^2} = 18}$$

Q.5 **If** $x = 2+\sqrt{3}$**, then find the values of (i)** $x-\frac{1}{x}$ **and**

(ii) $x^2+\frac{1}{x^2}$.

Sol. (i) $x = 2+\sqrt{3}$ ....(1)

$$\frac{1}{x} = \frac{1}{2+\sqrt{3}}$$

$$= \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$$ (Rationalize the denominators)

$$= \frac{2-\sqrt{3}}{(2)^2-(\sqrt{3})^2}$$

$$= \frac{2-\sqrt{3}}{4-3} = \frac{2-\sqrt{3}}{1} = 2-\sqrt{3}$$ ....(2)

$$x-\frac{1}{x} = (2+\sqrt{3})-(2-\sqrt{3})$$ from (1), (2)

$$x-\frac{1}{x} = 2+\sqrt{3}-2+\sqrt{3}$$

Hence, $$\boxed{x-\frac{1}{x} = 2\sqrt{3}}$$

(ii) $$\left(x-\frac{1}{x}\right)^2 = (2\sqrt{3})^2$$ Taking square root

$$x^2+\frac{1}{x^2}-2 = 4\times 3$$

$$x^2+\frac{1}{x^2}-2 = 12$$

Hence, $$\boxed{x^2+\frac{1}{x^2} = 12+2 = 14}$$

Q.6 **If** $x = \sqrt{3}-\sqrt{2}$**, then find the values of (i)** $x-\frac{1}{x}$ **and**

(ii) $x^2+\frac{1}{x^2}$.

Sol. (i) $x = \sqrt{3} - \sqrt{2}$ ....(1)

$$\frac{1}{x} = \frac{1}{\sqrt{3}-\sqrt{2}}$$

$$= \frac{1}{(\sqrt{3}-\sqrt{2})} \times \frac{(\sqrt{3}+\sqrt{2})}{(\sqrt{3}+\sqrt{2})}$$

$$= \frac{(\sqrt{3}+\sqrt{2})}{(\sqrt{3})^2-(\sqrt{2})^2}$$

$$= \frac{(\sqrt{3}+\sqrt{2})}{3-2}$$

$$\frac{1}{x} = \sqrt{3}+\sqrt{2} \text{ ....(2)}$$

$$x - \frac{1}{x} = (\sqrt{3}-\sqrt{2}) - (\sqrt{3}+\sqrt{2}) \qquad \text{from (1), (2)}$$

$$= \sqrt{3}-\sqrt{2}-\sqrt{3}-\sqrt{2}$$

Hence, $\boxed{x - \frac{1}{x} = -2\sqrt{2}}$

(ii) $\left(x - \frac{1}{x}\right)^2 = (-2\sqrt{2})^2$ Taking square root

$$x^2 + \frac{1}{x^2} - 2 = 4 \times 2$$

$$x^2 + \frac{1}{x^2} - 2 = 8$$

$$x^2 + \frac{1}{x^2} = 8 + 2$$

Hence, $\boxed{x^2 + \frac{1}{x^2} = 10}$

Q.7 If $\frac{1}{x} = 3 - \sqrt{2}$, then evaluate (i) $x + \frac{1}{x}$ and (ii) $x - \frac{1}{x}$.

Sol. (i) $\frac{1}{x} = 3 - \sqrt{2}$ ....(1)

$$x = \frac{1}{3-\sqrt{2}}$$

$$= \frac{1}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}}$$

$$= \frac{3+\sqrt{2}}{(3)^2-(\sqrt{2})^2}$$

$$= \frac{3+\sqrt{2}}{9-2}$$

$$x = \frac{3+\sqrt{2}}{7} \quad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

$$x+\frac{1}{x} = \frac{3+\sqrt{2}}{7}+3-\sqrt{2}$$ Now from (1) + (2)

$$= \frac{3+\sqrt{2}+7(3-\sqrt{2})}{7}$$

$$= \frac{3+\sqrt{2}+21-7\sqrt{2}}{7}$$

$$= \frac{24+\sqrt{2}-7\sqrt{2}}{7}$$

$$= \frac{24+\sqrt{2}(1-7)}{7}$$

$$= \frac{24+\sqrt{2}(-6)}{7}$$

Hence, $\boxed{x+\frac{1}{x} = \frac{24-6\sqrt{2}}{7}}$

(ii) $$x-\frac{1}{x} = \frac{3+\sqrt{2}}{7}-(3-\sqrt{2})$$ from (1),(2)

$$= \frac{3+\sqrt{2}}{7}-3+\sqrt{2}$$

$$= \frac{3+\sqrt{2}-21+7\sqrt{2}}{7}$$

$$= \frac{-18+\sqrt{2}+7\sqrt{2}}{7}$$

$$= \frac{-18+\sqrt{2}(1+7)}{7}$$

Hence, $\boxed{x-\frac{1}{x}=\frac{-18+8\sqrt{2}}{7}}$

Q.8 **If $\frac{1}{p}=\sqrt{10}+3$, then evaluate (i) $\left(p+\frac{1}{p}\right)^2$ and**

(ii) $\left(p-\frac{1}{p}\right)^2$

Sol.

(i) $\frac{1}{p} = \sqrt{10}+3$ ....(1)

$$p = \frac{1}{\sqrt{10}+3}$$

$$p = \frac{1}{\sqrt{10}+3} \times \frac{\sqrt{10}-3}{\sqrt{10}-3}$$ (Rationalize the denominators)

$$= \frac{\sqrt{10}-3}{(\sqrt{10})^2-(3)^2}$$

$$= \frac{\sqrt{10}-3}{10-9}$$

$$= \frac{\sqrt{10}-3}{1}$$

$= \sqrt{10}-3$ ....(2)

$p+\frac{1}{p} = (\sqrt{10}-3)+(\sqrt{10}+3)$ from (1)+(2)

$= \sqrt{10}-3+\sqrt{10}+3$

$$p+\frac{1}{p} = 2\sqrt{10}$$

$$\left(p+\frac{1}{p}\right)^2 = (2\sqrt{10})^2 \qquad \text{Taking square root}$$

$$= 4 \times 10$$

Hence, $$\boxed{\left(p+\frac{1}{p}\right)^2 = 40}$$

(ii) $$p-\frac{1}{p} = (\sqrt{10}-3)-(\sqrt{10}+3) \qquad \text{Now from (1) - (2)}$$

$$= \sqrt{10}-3-\sqrt{10}-3$$

$$p-\frac{1}{p} = -6$$

$$\left(p-\frac{1}{p}\right)^2 = (-6)^2 \qquad \text{Taking square root}$$

Hence, $$\boxed{\left(p-\frac{1}{p}\right)^2 = 36}$$

Q.9 Rationalize

(i) $$\frac{b+\sqrt{b^2-a^2}}{b-\sqrt{b^2-a^2}}$$

Sol. $$=\frac{b+\sqrt{b^2-a^2}}{b-\sqrt{b^2-a^2}} \times \frac{b+\sqrt{b^2-a^2}}{b+\sqrt{b^2-a^2}}$$

$$=\frac{(b+\sqrt{b^2-a^2})^2}{(b)^2-(\sqrt{b^2-a^2})^2}$$

$$=\frac{b^2+b^2-a^2+2b\sqrt{b^2-a^2}}{b^2-(b^2-a^2)}$$

$$=\frac{2b^2-a^2+2b\sqrt{b^2-a^2}}{b^2-b^2+a^2}$$

$$=\frac{2b^2-a^2+2b\sqrt{b^2-a^2}}{a^2}$$

(ii) $\dfrac{\sqrt{a+3}-\sqrt{a-3}}{\sqrt{a+3}+\sqrt{a-3}}$

Sol.

$$=\frac{\sqrt{a+3}-\sqrt{a-3}}{\sqrt{a+3}+\sqrt{a-3}}\times\frac{\sqrt{a+3}-\sqrt{a-3}}{\sqrt{a+3}-\sqrt{a-3}}$$

$$=\frac{(\sqrt{a+3}-\sqrt{a-3})^2}{(\sqrt{a+3})^2-(\sqrt{a-3})^2}$$

$$=\frac{(a+3)+(a-3)-2\sqrt{(a+3)(a-3)}}{(a+3)-(a-3)}$$

$$=\frac{a+3+a-3-2\sqrt{(a)^2-(3)^2}}{a+3-a+3}$$

$$=\frac{2a-2\sqrt{a^2-9}}{6}$$

$$=\frac{2(a-\sqrt{a^2-a})}{6}$$

$$=\frac{a-\sqrt{a^2-9}}{3}$$

Exercise 2.1

Factorize:

Q.1 $3a(x + y) - 7b(x + y)$

Sol: $= (x + y)(3a - 7b)$

Q.2 $ax + ay - x^2 - xy$

Sol: $= (ax + ay) - (x^2 + xy)$

$= a(x + y) - x(x + y)$

$= (x + y)(a - x)$

Q.3 $a^3 + a - 3a^2 - 3$

Sol: $= (a^3 + a) - (3a^2 + 3)$

$= a(a^2 + 1) - 3(a^2 + 1)$

$= (a^2 + 1)(a - 3)$

Q.4 $x^3 + y - xy - x$

Sol: $= x^3 - x - xy + y$ (writing in order)

$= x(x^2 - 1) - y(x - 1)$

$= x(x - 1)(x + 1) - y(x - 1)$

$= (x - 1)[x(x + 1) - y]$

$= (x - 1)(x^2 + x - y)$

Q.5 $3ax + 6ay - 8by - 4bx$

Sol: $= 3ax - 4bx + 6ay - 8by$ (writing in order)

$= x(3a - 4b) + 2y(3a - 4b)$

$= (3a - 4b)(x + 2y)$

Q.6 $2a^2 - bc - 2ab + ac$

Sol: $= 2a^2 + ac - 2ab - bc$ (writing in order)

$= a(2a + c) - b(2a + c)$

$= (2a + c)(a - b)$

$= (a - b)(2a + c)$

Q.7 $a(a - b + c) - bc$

Sol: $= a^2 - ab + ca - bc$

$= a(a - b) + c(a - b)$

$= (a - b)(a + c)$

Q.8 $8 - 4a - 2a^3 + a^4$

Sol: $= 4(2 - a) - a^3(2 - a)$

$= (2 - a)(4 - a^3)$

$= (4 - a^3)(2 - a)$

Q.9 $16x^2 - 24xa + 9a^2$

Sol: $= 16x^2 - 12xa - 12xa + 9a^2$

$= 4x(4x - 3a) - 3a(4x - 3a)$

$= (4x - 3a)(4x - 3a)$

$= (4x - 3a)^2$

Q.10 $1 - 14x + 49x^2$

Sol: $= (1)^2 - 2(1)(7x) + (7x)^2$

$= (1 - 7x)^2$

Q.11 $20x^2 + 5 - 20x$

Sol: $= 20x^2 - 20x + 5$

$= 5[4x^2 - 4x + 1]$

$= 5[(2x)^2 - 2(2x)(1) + (1)^2]$

$= 5(2x - 1)^2$

Q.12 $2a^3b + 2ab^3 - 4a^2b^2$

Sol: $= 2ab[a^2 + b^2 - 2ab]$

$= 2ab[a^2 - 2ab + b^2]$

$= 2ab[(a)^2 - 2(a)(b) + (b)^2]$

$= 2ab(a - b)^2$

Q.13 $x^2 + x + \frac{1}{4}$

Sol: $= (x)^2 + 2(x)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2$

$= \left(x + \frac{1}{2}\right)^2$

Q.14 $x^2 + \frac{1}{x^2} - 2$

Sol: $= x^2 - 2 + \frac{1}{x^2}$

$= (x)^2 - 2(x)\left(\frac{1}{x}\right) + \left(\frac{1}{x}\right)^2$

$= \left(x - \frac{1}{x}\right)^2$

Q.15 $5x^3 - 30x^2 + 45x$

Sol: $= 5x[x^2 - 6x + 9]$

$= 5x[(x)^2 - 2(x)(3) + (3)^2]$

$= 5x(x - 3)^2$

Q.16 $a^2 + b^2 + 2ab + 2bc + 2ac$

Sol: $= (a^2 + 2ab + b^2) + 2ac + 2bc$

$= [(a)^2 + 2(a)(b) + (b)^2] + 2c(a + b)$

$= (a + b)^2 + 2c(a + b)$

$= (a + b)[(a + b) + 2c]$

$= (a + b)(a + b + 2c)$

Exercise 2.2

Resolve into factors.

Q.1 $x^2 + 2xy + y^2 - a^2$

Sol: $= (x^2 + 2xy + y^2) - a^2$

$= [(x)^2 + 2(x)(y) + (y)^2] - (a)^2$

$= (x + y)^2 - (a)^2$

$= (x + y + a)(x + y - a)$

Q.2 $4a^2 - 4ab + b^2 - 9c^2$

Sol: $= (4a^2 - 4ab + b^2) - 9c^2$

$= [(2a)^2 - 2(2a)(b) + (b)^2] - (3c)^2$

$= (2a - b)^2 - (3c)^2$

$= (2a - b + 3c)(2a - b - 3c)$

Q.3 $x^2 + 6ax + 9a^2 - 16b^2$

Sol: $= (x^2 + 6ax + 9a)^2 - 16b^2$

$= [(x)^2 + 2(x)(3a) + (3a)^2] - (4b)^2$

$= (x + 3a)^2 - (4b)^2$

$= (x + 3a + 4b)(x + 3a - 4b)$

Q.4 $y^2 - c^2 + 2cx - x^2$

Sol: $= y^2 - (c^2 - 2cx + x^2)$

$= (y)^2 - [(c)^2 - 2(c)(x) + (x)^2]$

$= (y)^2 - (c - x)^2$ Formula: $[a^2 - b^2 = (a + b)(a - b)]$

$= [y + (c - x)][y - (c - x)]$

$= (y + c - x)(y - c + x)$

$= (y - x + c)(y + x - c)$

Q.5 $x^2 + y^2 + 2xy - 4x^2y^2$

Sol: $= (x^2 + 2xy + y^2) - 4x^2y^2$

$= [(x)^2 + 2(x)(y) + (y)^2] - (2xy)^2$

$= (x + y)^2 - (2xy)^2$

$= (x + y + 2xy)(x + y - 2xy)$

Q.6 $a^2 - 4ab + 4b^2 - 9a^2c^2$

Sol: $= (a^2 - 4ab + 4b^2) - (9a^2c^2)$

$= [(a)^2 - 2(a)(2b) + (2b)^2] - [3ac]^2$

$= (a - 2b)^2 - (3ac)^2$

$= (a - 2b + 3ac)(a - 2b - 3ac)$

Q.7 $x^2 - 2xy + y^2 - a^2 + 2ab - b^2$

Sol: $= (x^2 - 2xy + y^2) - (a^2 - 2ab + b^2)$

$= [(x)^2 - 2(x)(y) + (y)^2]\,[(a)^2 - 2(a)(b) + (b)^2]$

$= (x - y)^2 - (a - b)^2$

$= [(x - y) + (a - b)][(x - y) - (a - b)]$

$= (x - y + a - b)(x - y - a + b)$

Q.8 $y^4 + 4$

Sol: $= (y^2)^2 + (2)^2 + (2)(y^2)(2) - 2(y^2)(2)$(completing square root)

$= [(y^2)^2 + (2)^2 + 4y^2] - 4y^2$

$= (y^2 + 2)^2 - (2y)^2$

$= (y^2 + 2 + 2y)(y^2 + 2 - 2y)$

$= (y^2 + 2y + 2)(y^2 - 2y + 2)$

Q.9 $z^4 + 64y^4$

Sol: $= (z^2)^2 + (8y^2)^2 + 2(z^2)(8y^2) - 2(z^2)(8y^2)$ (completing square root)

$= (z^2 + 8y^2)^2 - 16z^2y^2$

$= (z^2 + 8y^2)^2 - (4zy)^2$

$= (z^2 + 8y^2 - 4zy)(z^2 + 8y^2 + 4zy)$

Q.10 $x^4 + 324$

Sol: $= (x^2)^2 + (18)^2 + 2(x^2)(18) - 2(x^2)(18)$ (completing square root)

$= (x^2 + 18)^2 - 36x^2$

$= (x^2 + 18)^2 - (6x)^2$

$= (x^2 + 18 + 6x)(x^2 + 18 - 6x)$

$= (x^2 + 6x + 18)(x^2 - 6x + 18)$

Q.11 $z^4 - z^2 + 16$

Sol: $= (z^2)^2 + (4)^2 + 2(z^2)(4) - 9z^2$ (completing square root)

$= (z^2 + 4)^2 - (3z)^2$

$= (z^2 + 4 - 3z)(z^2 + 4 + 3z)$

$= (z^2 - 3z + 4)(z^2 + 3z + 4)$

Q.12 $4x^4 - 5x^2y^2 + y^4$

Sol: $= (2x^2)^2 - 5x^2y^2 + (y^2)^2$ (completing square root)

$= (2x^2)^2 + 2(2x^2)(y^2) + (y^2)^2 - 9x^2y^2$ (completing square root)

$= (2x^2 + y^2)^2 - (3xy)^2$

$= (2x^2 + y^2 - 3xy)(2x^2 + y^2 + 3xy)$

Exercise 2.3

Factorize:

Q.1 $x^2 + 9x + 20$

Sol: $= x^2 + 4x + 5x + 20$

$= (x^2 + 4x) + (5x + 20)$

$= x(x + 4) + 5(x + 4)$

$= (x + 4)(x + 5)$

Q.2 $x^2 + 5x - 14$

Sol: $= x^2 + 7x - 2x - 14$

$= (x^2 + 7x) - (2x + 14)$

$= x(x + 7) - 2(x + 7)$

$= (x + 7)(x - 2)$

Q.3 $x^2 + 5x - 6$

Sol: $= x^2 + 6x - x - 6$

$= (x^2 + 6x) - (x + 6)$

$= x(x + 6) - 1(x + 6)$

$= (x + 6)(x - 1)$

Q.4 $x^2 - 7x + 12$

Sol: $= x^2 - 3x - 4x + 12$

$= (x^2 - 3x) - (4x - 12)$

$= x(x - 3) -4(x - 3)$

$= (x - 3)(x - 4)$

Q.5 $x^2 - x - 156$

Sol: $= x^2 - 13x + 12x - 156$

$= (x^2 - 13x) + (12x - 156)$

$= x(x - 13) + 12(x - 13)$

$= (x - 13)(x + 12)$

Q.6 $x^2 - x - 2$

Sol: $= x^2 - 2x + x - 2$

$= (x^2 - 2x) + (x - 2)$

$= x(x - 2) +1 (x - 2)$

$= (x - 2)(x + 1)$

Q.7 $x^2 - 9x - 90$

Sol: $= x^2 - 15x + 6x - 90$

$= (x^2 - 15x) + (6x - 90)$

$= x(x - 15) + 6(x - 15)$

$= (x - 15)(x + 6)$

Q.8 $\mathbf{a^2 - 12a - 85}$

Sol: $= a^2 - 17a + 5a - 85$

$= (a^2 - 17a) + (5a - 85)$

$= a(a - 17) + 5(a - 17)$

$= (a - 17)(a + 5)$

Q.9 $\mathbf{98 - 7x - x^2}$

Sol: $= 98 - 14x + 7x - x^2$

$= (98 - 14x) + (7x - x^2)$

$= 14(7 - x) + x(7 - x)$

$= (7 - x)(14 + x)$

Q.10 $\mathbf{y^2 - 11y - 152}$

Sol: $= y^2 - 19y + 8y - 152$

$= (y^2 - 19y) + (8y - 152)$

$= y(y - 19) + 8(y - 19)$

$= (y - 19)(y + 8)$

Q.11 $\mathbf{2x^2 + 3x + 1}$

Sol: $= 2x^2 + 2x + x + 1$

$= (2x^2 + 2x) + (x + 1)$

$= 2x(x + 1) + 1(x + 1)$

$= (x + 1)(2x + 1)$

Q.12 $\mathbf{3x^2 + 5x + 2}$

Sol: $= 3x^2 + 3x + 2x + 2$

$= (3x^2 + 3x) + (2x + 2)$

$= 3x(x + 1) + 2(x + 1)$

$= (x + 1)(3x + 2)$

Q.13 $\mathbf{2x^2 - x - 1}$

Sol: $= 2x^2 - 2x + x - 1$

$= (2x^2 - 2x) + (x - 1)$

$= 2x(x - 1) + 1(x - 1)$

$= (x - 1)(2x + 1)$

Q.14 $6x^2 + 7x - 3$

Sol: $= 6x^2 + 9x - 2x - 3$

$= (6x^2 + 9x) - (2x + 3)$

$= 3x(2x + 3) - 1(2x + 3)$

$= (2x + 3)(3x - 1)$

Q.15 $2 - 3x - 2x^2$

Sol: $= 2 - 4x + x - 2x^2$

$= (2 - 4x) + (x - 2x^2)$

$= 2(1 - 2x) + x(1 - 2x)$

$= (1 - 2x)(2 + x)$

Q.16 $8 + 6x - 5x^2$

Sol: $= 8 + 10x - 4x - 5x^2$

$= (8 + 10x) - (4x + 5x^2)$

$= 2(4 + 5x) - x(4 + 5x)$

$= (4 + 5x)(2 - x)$

Q.17 $3u^2 - 10u + 8$

Sol: $= 3u^2 - 6u - 4u + 8$

$= (3u^2 - 6u) - (4u - 8)$

$= 3u(u - 2) - 4(u - 2)$

$= (u - 2)(3u - 4)$

Q.18 $10x^2 - 7x - 12$

Sol: $= 10x^2 - 15x + 8x - 12$

$= (10x^2 - 15x) + (8x - 12)$

$= 5x(2x - 3) + 4(2x - 3)$

$= (2x - 3)(5x + 4)$

Q.19 $5x^2 - 32x + 12$

Sol: $= 5x^2 - 30x - 2x + 12$

$= (5x^2 - 30x) - (2x - 12)$

$= 5x(x - 6) - 2(x - 6)$

$= (x - 6)(5x - 2)$

Q.20 $4\sqrt{3}x^2 + 5x - 2\sqrt{3}$

Sol: $4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3}$

$(4\sqrt{3}x^2 + 8x) - (3x + 2\sqrt{3})$

$4x(\sqrt{3}x + 2) - \sqrt{3}(\sqrt{3}x + 2)$

$= (\sqrt{3}x + 2)(4x - \sqrt{3})$

Formulae

(i) $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

(ii) $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

(iii) $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

(iv) $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Exercise 2.4

Factorize:

Q.1 $8x^3 - y^3$

Sol: $(2x)^3 - (y)^3$

$(2x - y)[(2x)^2 + (2x)(y) + (y)^2]$

$(2x - y)(4x^2 + 2xy + y^2)$

Q.2 $27x^3 + 1$

Sol: $= (3x)^3 + (1)^3$

$= (3x + 1)[(3x)^2 - (3x)(1) + (1)^2]$

$= (3x + 1)(9x^2 - 3x + 1)$

Q.3 $1 - 343x^3$

Sol: $= (1)^3 - (7x)^3$

$= (1 - 7x)[(1)^2 + 1(7x) + (7x)^2]$

$= (1 - 7x)(1 + 7x + 49x^2)$

Q.4 $a^3b^3 + 512$

Sol: $= (ab)^3 + (8)^3$

$(ab + 8)[(ab)^2 - (ab)(8) + (8)^2]$

$= (ab + 8)(a^2b^2 - 8ab + 64)$

Q.5 **$27 - 1000y^3$**

Sol: $= (3)^3 - (10y)^3$

$= (3 - 10y)[(3)^2 + (3)(10y) + (10y)^2]$

$= (3 - 10y)(9 + 30y + 100y^2)$

Q.6 **$27x^3 - 64y^3$**

Sol: $= (3x)^3 - (4y)^3$

$= (3x - 4y)[(3x)^2 + (3x)(4y) + (4y)^2]$

$= (3x - 4y)(9x^2 + 12xy + 16y^2)$

Q.7 **$x^3y^3 + z^3$**

Sol: $= (xy)^3 + (z)^3$

$= (xy + z)[(xy)^2 - (xy)(z) + (z)^2]$

$= (xy + z)(x^2y^2 - xyz + z^2)$

Q.8 **$216p^3 - 343$**

Sol: $= (6p)^3 - (7)^3$

$= (6p - 7)[(6p)^2 + (6p)(7) + (7)^2]$

$= (6p - 7)(36p^2 + 42p + 49)$

Q.9 $8x^3 - \frac{1}{27}$

Sol: $= (2x)^3 - \left(\frac{1}{3}\right)^3$

$$= \left(2x - \frac{1}{3}\right)\left[(2x)^2 + (2x)\left(\frac{1}{3}\right) + \left(\frac{1}{3}\right)^2\right]$$

$$= \left(2x - \frac{1}{3}\right)\left(4x^2 + \frac{2}{3}x + \frac{1}{9}\right)$$

Q.10 **$a^3 + b^3 + a + b$**

Sol: $= (a^3 + b^3) + (a + b)$

$= (a + b)(a^2 - ab + b^2) + (a + b)$

$= (a + b)[(a^2 - ab + b^2 + 1)]$

$= (a + b)(a^2 - ab + b^2 + 1)$

Q.11 $\mathbf{a - b - a^3 + b^3}$

Sol: $= (a - b) - (a^3 - b^3)$

$= (a - b) - [(a - b)(a^2 + ab + b^2)]$

$= (a - b) [1 - (a^2 + ab + b^2)]$

$= (a - b)(1 - a^2 - ab - b^2)$

Q.12 $\mathbf{x - 8xy^3}$

Sol: $= x(1 - 8y^3)$

$= x[(1)^3 - (2y)^3]$

$= x[(1 - 2y)\{(1)^2 + (1)(2y) + (2y)^2\}]$

$= x[(1 - 2y)(1 + 2y + 4y^2)]$

$= x(1 - 2y)(1 + 2y + 4y^2)$

Q.13 $\mathbf{x^{12} - y^{12}}$

Sol: $= (x^6)^2 - (y^6)^2$

$= (x^6 - y^6)(x^6 + y^6)$

$= \{(x^3)^2 - (y^3)^2\}\{(x^2)^3 + (y^2)^3\}$

$= (x^3 - y^3)(x^3 + y^3)(x^2 + y^2)(x^4 - x^2y^2 + y^4)$

$= (x - y)(x^2 + xy + y^2)(x + y)(x^2 - xy + y^2)(x^2 + y^2)$
$(x^4 - x^2y^2 + y^4)$

$= (x - y)(x + y)(x^2 + y^2)(x^2 + xy + y^2)(x^2 - xy + y^2)$
$(x^4 - x^2y^2 + y^4)$

Q.14 $1 - \dfrac{64p^3}{q^3}$

Sol: $= (1)^3 - \left(\dfrac{4p}{q}\right)^3$

$= \left(1 - \dfrac{4p}{q}\right)\left[(1)^2 + (1)\left(\dfrac{4p}{q}\right) + \left(\dfrac{4p}{q}\right)^2\right]$

$= \left(1 - \dfrac{4p}{q}\right)\left(1 + \dfrac{4p}{q} + \dfrac{16p^2}{q^2}\right)$

Q.15 $1 + 64u^3$

Sol: $= (1)^3 + (4u)^3$

$= (1 + 4u)[(1)^2 - (1)(4u) + (4u)^2]$

$= (1 + 4u)(1 - 4u + 16u^2)$

Q.16 $8x^3 - 6x - 9y + 27y^3$

Sol: $= 8x^3 + 27y^3 - 6x - 9y$

$= (8x^3 + 27y^3) - (6x + 9y)$

$= [(2x)^3 + (3y)^3] - 3(2x + 3y)$

$= (2x + 3y)[(2x)^2 - (2x)(3y) + (3y)^2] - 3(2x + 3y)$

$= (2x + 3y)(4x^2 - 6xy + 9y^2) - 3(2x + 3y)$

$= (2x + 3y)(4x^2 - 6xy + 9y^2 - 3)$

Q.17 $z^3 + 125$

Sol: $= (z)^3 + (5)^3$

$= (z + 5)\{(z)^2 - (z)(5) + (5)^2\}$

$= (z + 5)(z^2 - 5z + 25)$

Q.18 $x^9 + y^9$

Sol: $= (x^3)^3 + (y^3)^3$

$= (x^3 + y^3)[(x^3)^2 - (x^3)(y^3) + (y^3)^2]$

$= (x^3 + y^3)(x^6 - x^3y^3 + y^6)$

$= (x + y)(x^2 - xy + y^2)(x^6 - x^3y^3 + y^6)$

Q.19 $m^6 - n^6$

Sol: $= (m^3)^2 - (n^3)^2$

$= (m^3 + n^3)(m^3 - n^3)$

$= (m + n)(m^2 - mn + n^2)(m - n)(m^2 + mn + n^2)$

$= (m + n)(m - n)(m^2 - mn + n^2)(m^2 + mn + n^2)$

Q.20 $64x^7 - xa^6$

Sol: $= x(64x^6 - a^6)$

$= x\{(8x^3)^2 - (a^3)^2\}$

$= x(8x^3 + a^3)(8x^3 - a^3)$

$x\{(2x)^3 + (a)^3\}\{(2x)^3 - (a)^3\}$

$= x(2x + a)(4x^2 - 2ax + a^2)(2x - a)(4x^2 + 2ax + a^2)$

$= x(2x + a)(2x - a)(4x^2 - 2ax + a^2)(4x^2 + 2ax + a^2)$

Q.21 $\mathbf{x^3 - 27a^3}$

Sol: $= (x)^3 - (3a)^3$

$= (x - 3a)[(x)^2 + (x)(3a) + (3a)^2]$

$= (x - 3a)(x^2 + 3ax + 9a^2)$

Q.22 $\mathbf{x^3 + 27a^3}$

Sol: $= (x)^3 + (3a)^3$

$= (x + 3a)\{(x)^2 - (x)(3a) + (3a)^2\}$

$= (x + 3a)(x^2 - 3ax + 9a^2)$

EXERCISE [illegible]

I. Evaluate each of the polynomials for the value indicated.

Q.1 $P(x) = 2x^3 - 5x^2 + 7x - 7;\ P(2)$

Sol: $P(x) = 2x^3 - 5x^2 + 7x - 7$

$P(2) = 2(2)^3 - 5(2)^2 + 7(2) - 7$

$= 2 \times 8 - 5 \times 4 + 7 \times 2 - 7$

$= 16 - 20 + 14 - 7$

$= 3$

Q.2 $\mathbf{P(x) = x^4 - 10x^2 + 25x - 2;\ P(-4)}$

Sol: $P(x) = x^4 - 10x^2 + 25x - 2$

$P(-4) = (-4)^4 - 10(-4)^2 + 25(-4) - 2$

$= 256 - 160 - 100 - 2$

$= -6$

Q.3 $\mathbf{P(x) = x^4 + 5x^3 - 13x^2 - 30;\ P(-1)}$

Sol: $P(x) = x^4 + 5x^3 - 13x^2 - 30$

$P(-1) = (-1)^4 + 5(-1)^3 - 13(-1)^2 - 30$

$= 1 - 5 - 13 - 30$

$= -47$

Q.4 $P(x) = x^5 - 10x^3 + 7x + 6$; $P(3)$

Sol: $P(x) = x^5 - 10x^3 + 7x + 6$

$P(3) = (3)^5 - 10(3)^3 + 7(3) + 6$

$= 243 - 270 + 21 + 6$

$= 0$

Q.5 $P(x) = x^4 + 4x^3 - 9x^2 + 19x + 6$; $P(-2)$

Sol: $P(x) = x^4 + 4x^3 - 9x^2 + 19x + 6$

$P(-2) = (-2)^4 + 4(-2)^3 - 9(-2)^2 + 19(-2) + 6$

$= 16 + 4(-8) - 9(4) - 38 + 6$

$= 16 - 32 - 36 - 38 + 6$

$= -106 + 22$

$= -84$

II. ***Determine whether the second polynomial is a factor of the first polynomial without dividing (Hint: evaluate directly and use the factor theorem).***

Q.6 $x^{18} - 1$; $x + 1$

Sol: Let $P(x) = x^{18} - 1$

Here $x - a = x + 1$

Thus $a = -1$

Now $P(-1) = (-1)^{18} - 1$

$= 1 - 1 = 0$

Since $P(-1) = 0$

Then by factor theorem $x + 1$ is a factor of $x^{18} - 1$.

Q.7 $x^{18} - 1$; $x - 1$

Sol: Let $P(x) = x^{18} - 1$

Here $x - a = x - 1$

Thus $a = 1$

Now $P(-1) = (1)^{18} - 1$

$= 1 - 1 = 0$

Since $P(-1) = 0$

Then by factor theorem, $x - 1$ is a factor of $x^{18} - 1$.

Q.8 $x^9 - 2^9$; $x + 2$

Sol: Let $P(x) = x^9 - 2^9$

Here $x - a = x + 2$

Thus $a = -2$

Now $P(-2) = (-2)^9 - (2)^9$

$= -(2)^9 - (2)^9$

$= -ve \neq 0$

Since $P(-2) \neq 0$

Therefore, $x + 2$ is not a factor of $x^9 - 2^9$.

Q.9 $x^9 + 2^9$; $x - 2$

Sol: Let $P(x) = x^9 + 2^9$

Here $x - a = x - 2$

Thus $a = 2$

Now $P(2) = (2)^9 + (2)^9$

$\neq 0$

Since $P(2) \neq 0$

Therefore, $x - 2$ is not a factor of $x^9 + 2^9$.

Q.10 $3x^4 - 2x^3 + 5x - 6$; $x - 1$

Sol: Let $P(x) = 3x^4 - 2x^3 + 5x - 6$

Here $x - a = x - 1$

Thus $a = 1$

Now $P(1) = 3(1)^4 - 2(1)^3 + 5(1) - 6$

$= 3 - 2 + 5 - 6$

$= 8 - 8 = 0$

Since $P(1) = 0$

Then by factor theorem, $x-1$ is a factor of $3x^4-2x^3+5x-6$.

Q.11 $5x^6-7x^3-6x+x$; $x-1$

Sol: Let $P(x)=5x^6-7x^3-6x+x$

Here $x-a=x-1$

Thus $a=1$

Now $P(1)=5(1)^6-7(1)^3-6(1)+(1)$

$=5-7-6+1$

$=6-13$

$=-7\neq 0$

Since $P(1)\neq 0$

Then by factor theorem, $x-1$ is not a factor of $5x^6-7x^3-6x+x$.

Q.12 $3x^3-7x^2-8x+2$; $x+1$

Sol: Let $P(x)=3x^3-7x^2-8x+2$

Here $x-a=x+1$

therefore $a=-1$

Now $P(-1)=3(-1)^3-7(-1)^2-8(-1)+2$

$=-3-7+8+2$

$=0$

Since $P(-1)=0$

Then by factor theorem, $x+1$ is a factor of $3x^3-7x^2-8x+2$.

Q.13 $5x^8-2x^5+3x^3+6x+2$; $x+1$

Sol: Let $P(x)=5x^8-2x^5+3x^3+6x+2$

Here $x-a=x+1$

therefore $a=-1$

Now $P(-1)=5(-1)^8-2(-1)^5+3(-1)^3+6(-1)+2$

$=5+2-3-6+2$

$=9-9=0$

Since $P(-1)=0$

Then by factor theorem, $x+1$ is a factor of $5x^8-2x^5+3x^3+6x+2$.

Q.14 $6x^3 + 2x^2 - x + 9$; $x - 1$

Sol: Let $P(x) = 6x^3 + 2x^2 - x + 9$

Here $x - a = x - 1$

therefore $a = 1$

Now $P(1) = 6(1)^3 + 2(1)^2 - (1) + 9$

$= 6 + 2 - 1 + 9$

$= 17 - 1 = 16 \neq 0$

Since $P(1) \neq 0$

Then by factor theorem, $x - 1$ is not a factor of $6x^3 + 2x^2 - x + 9$.

Q.15 $4x^3 - 3x^2 - 8x + 4$; $x - 2$

Sol: Let $P(x) = 4x^3 - 3x^2 - 8x + 4$

Here $x - a = x - 2$

therefore $a = 2$

Now $P(2) = 4(2)^3 - 3(2)^2 - 8(2) + 4$

$= 4(8) - 3(4) - 8(2) + 4$

$= 32 - 12 - 16 + 4$

$= 36 - 28 = 8 \neq 0$

Since $P(2) \neq 0$

Then by factor theorem, $x - 2$ is not a factor of $4x^3 - 3x^2 - 8x + 4$.

Q.16 $5x^3 + 3x^2 - x + 1$; $x + 1$

Sol: Let $P(x) = 5x^3 + 3x^2 - x + 1$

Here $x - a = x + 1$

therefore $a = -1$

Now $P(-1) = 5(-1)^3 + 3(-1)^2 - (-1) + 1$

$= 5(-1) + 3(1) + 1 + 1$

$= -5 + 3 + 1 + 1$

$= -5 + 5$

$= 0$

Since $P(-1) = 0$

Then by factor theorem, $x+1$ is a factor of $5x^3 + 3x^2 - x + 1$.

Q.17 $2y^3 - 8y^2 + y - 4$; $y - 4$

Sol: Let $P(y) = 2y^3 - 8y^2 + y - 4$

Here $y - a = y - 4$

therefore $a = 4$

Now $P(4) = 2(4)^3 - 8(4)^2 + (4) - 4$

$= 2(64) - 8(16) + 4 - 4$

$= 128 - 128 + 4 - 4$

$= 132 - 132$

$= 0$

Since $P(4) = 0$

Then by factor theorem, $y - 4$ is not a factor of $2y^3 - 8y^2 + y - 4$.

Q.18 $z^3 - 5z^2 - 4z - 4$; $z + 2$

Sol: Let $P(z) = z^3 - 5z^2 - 4z - 4$

Here $z - a = z + 2$

therefore $a = -2$

Now $P(-2) = (-2)^3 - 5(-2)^2 - 4(-2) - 4$

$= -8 - 5(4) + 8 - 4$

$= -8 - 20 + 8 - 4$

$= -32 + 8$

$= -24 \neq 0$

Since $P(-2) \neq 0$

Then by factor theorem, $z+2$ is not a factor of $z^3 - 5z^2 - 4z - 4$.

III. Solve

Q.19. If $P(x) = x^3 - kx^2 + 3x + 5$ is divided by $x - 1$, find k, if remainder is 8.

Sol: Let $P(x) = x^3 - kx^2 + 3x + 5$

Here $x - a = x - 1$

therefore $a = 1$

Now $P(1) = (1)^3 - k(1)^2 + 3(1) + 5$

$= 1 - k + 3 + 5$

$= 9 - k =$ Remainder

But $9 - k = 8$

$-k = 8 - 9$

$-k = -1$

$k = 1$

Q.20. If $P(x) = 3x^3 + kx - 26$ is divided by $x - 2$, find k, if remainder is 0.

Sol: Lets $P(x) = 3x^3 + kx - 26$

Here $x - a = x - 2$

therefore $a = 2$

Now $P(2) = 3(2)^3 + k(2) - 26$

$= 3(8) + 2k - 26$

$= 24 + 2k - 26$

$= 2k - 2 =$ Remainder

But Remainder $= 0$

Thus, $2k - 2 = 0$

$2k = 2$

$k = \frac{2}{2}$

$k = 1$

UNIT 3 Algebraic Manipulation

- H.C.F and L.C.M
- Basic Operations on Algebraic Fractions
- Square Roots of Algebraic Fractions

After complation of this unit, the students will be able to:

- find highest common factor (HCF) and least common multiple (LCM) of algebraic expressions.
- use factor or division method to determine HCF and LCM.
- know the relationship between HCF and LCM.
- use HCF and LCM to reduce fractional expressions involving +, − , ×, ÷.
- find square root of an algebraic expression by factorization and division.

Exercise 3.1

Find H.C.F by factorization.

Q.1 $abxy, a^2bc$.

Sol.

Factorization of $abxy$ $= \boxed{a} \times \boxed{b} \times x \times y$

Factorization of a^2bc $= \boxed{a} \times \boxed{b} \times a \times c$

common factors $= a, b$

Thus, H.C.F $= a \times b$

$= ab$

Q.2 $6pqr, 15qrs$

Sol.

Factorization of $6pqr = 2 \times \boxed{3} \times p \times \boxed{q} \times \boxed{r}$

Factorization of $15qrs = 5 \times \boxed{3} \times \boxed{q} \times \boxed{r} \times s$

Common factors $= 3, q, r$

Thus, H.C.F $= 3 \times q \times r$

$= 3qr$

Q.3 $8xy^2z^3, 12x^2y^2z^2$

Sol.

Factorization of $8xy^2z^3 = \boxed{2} \times \boxed{2} \times 2 \times \boxed{x} \times \boxed{y} \times \boxed{y} \times z \times \boxed{z} \times \boxed{z}$

Factorization of $12x^2y^2z^2 = \boxed{2} \times \boxed{2} \times 3 \times \boxed{x} \times x \times \boxed{y} \times \boxed{y} \times \boxed{z} \times \boxed{z}$

Common factors $= 2, 2, x, y, y, z, z$

Thus, H.C.F $= 2 \times 2 \times x \times y \times y \times z \times z$

$= 4xy^2z^2$

Q.4 $14a^2bc, 21ab^2$

Sol.

Factorization of $14a^2bc = 2 \times \boxed{7} \times \boxed{a} \times a \times \boxed{b} \times c$

Factorization of $21ab^2 = 3 \times \boxed{7} \times \boxed{a} \times \boxed{b} \times b$

Common factors $= 7, a, b$

Thus, H.C.F $= 7 \times a \times b$

$= 7ab$

Q.5 $3x^5y^2, 12x^2y^4, 15x^3y^2$

Sol.

Factorization of $3x^5y^2 = \boxed{3} \times x \times x \times \boxed{x} \times \boxed{x} \times x \times \boxed{y} \times \boxed{y}$

Factorization of $12x^2y^4 = \boxed{3} \times 2 \times 2 \times \boxed{x} \times \boxed{x} \times \boxed{y} \times \boxed{y} \times y \times y$

Factorization of $15x^3y^2 = \boxed{3} \times 5 \times \boxed{x} \times \boxed{x} \times x \times \boxed{y} \times \boxed{y}$

Common factors $= 3, x, x, y, y$

Thus, H.C.F $= 3 \times x \times x \times y \times y$

$= 3x^2y^2$

Q.6 $4abc^3, 8a^3bc, 6ab^3c$

Sol.

$$\begin{aligned}
\text{Factorization of } 4abc^3 &= \boxed{2} \times 2 \times \boxed{a} \times \boxed{b} \times c \times \boxed{c} \times c \\
\text{Factorization of } 8a^3bc &= 2 \times \boxed{2} \times 2 \times \boxed{a} \times a \times a \times \boxed{b} \times \boxed{c} \\
\text{Factorization of } 6ab^3c &= \boxed{2} \times 3 \times \boxed{a} \times b \times \boxed{b} \times b \times \boxed{c}
\end{aligned}$$

Common factors $= 2, a, b, c$

Thus, H.C.F $= 2 \times a \times b \times c$

$= 2abc$

Q.7 $x^3 + 64, x^2 - 16$

Sol.

Factorization of $x^3 + 64 = (x)^3 + (4)^3$

$= (x + 4)[(x)^2 - (x)(4) + (4)^2]$

$= (x + 4)(x^2 - 4x + 16)$ .... (i)

Factorization of $x^2 - 16 = (x)^2 - (4)^2$

$= (x + 4)(x - 4)$.................................(ii)

Common factors are: $(x + 4)$

Thus, H.C.F $= x + 4$

Q.8 $x^2 - y^2, x^4 - y^4, x^6 - y^6$

Factorization of $x^2 - y^2 = (x + y)(x - y)$...................(i)

Factorization of $x^4 - y^4 = (x^2)^2 - (y^2)^2$

$= (x^2 + y^2)(x^2 - y^2)$

$= (x^2 + y^2)(x + y)(x - y)$.............(ii)

Factorization of $x^6 - y^6 = (x^3)^2 - (y^3)^2$

$= (x^3 - y^3)(x^3 + y^3)$

$= (x-y)(x^2+xy+y^2)(x+y)(x^2-xy+y^2)$.....(iii)

Common factors are : $(x + y), (x - y)$

Thus, H.C.F $= (x + y)(x - y)$

$= x^2 - y^2$

Q.9 $t^2 - 9, (t + 3)^2, t^2 + t - 6$

Sol.

Factorization of $t^2 - 9 = (t)^2 - (3)^2$

$= (t + 3)(t - 3)$........................(i)

Factorization of $(t + 3)^2 = (t + 3)(t + 3)$........................(ii)

Factorization of $t^2 + t - 6 = t^2 + 3t - 2t - 6$

$= t(t + 3) - 2(t + 3)$

$= (t + 3)(t - 2)$........................(iii)

Common factor is : $(t + 3)$

Thus, H.C.F $= (t + 3)$

Q.10 $x^2 - x - 2, x^2 + x - 6, x^2 - 3x + 2$

Sol.

Factorization of $x^2 - x - 2 = x^2 - 2x + x - 2$

$= (x^2 - 2x) + (x - 2)$

$= x(x - 2) + 1(x - 2)$

$= (x - 2)(x + 1)$..............................(i)

Factorization of $x^2 + x - 6 = x^2 + 3x - 2x - 6$

$= (x^2 + 3x) - (2x + 6)$

$= x(x + 3) - 2(x + 3)$

$= (x + 3)(x - 2)$..............................(ii)

Factorization of $x^2 - 3x + 2 = x^2 - x - 2x + 2$

$= (x^2 - x) - (2x - 2)$

$= x(x - 1) - 2(x - 1)$

$= (x - 1)(x - 2)$..............................(iii)

Common factor is : $(x - 2)$

Thus, H.C.F $= (x - 2)$

Q.11 $1 - x^2, x^3 + 1, 1 - x - 2x^2$

Sol.

Factorization of $1 - x^2 = (1)^2 - (x)^2$

$= (1 + x)(1 - x)$..............................(i)

Factorization of $x^3 + 1 = (x)^3 + (1)^3$

$= (x + 1)(x^2 - x + 1)$......................(ii)

Factorization of $1 - x - 2x^2 = 1 - 2x + x - 2x^2$

$= (1 - 2x) + (x - 2x^2)$

$= 1(1 - 2x) + x(1 - 2x)$

$= (1 - 2x)(1 + x)$

Common factor is : $1 + x$

Thus, H.C.F $= 1 + x$

Q.12 $x^3 - 8, x^2 - 7x + 10$

Sol:

Factorization of $x^3 - 8 = (x)^3 - (2)^3$

$= (x - 2)[(x)^2 + (x)(2) + (2)^2]$

$= (x - 2)(x^2 + 2x + 4)$.........................(i)

Factorization of $x^2 - 7x + 10 = x^2 - 2x - 5x + 10$

$= (x^2 - 2x) - (5x - 10)$

$= x(x - 2) - 5(x - 2)$

$= (x - 2)(x - 5)$.............................(ii)

Common factors are: $x - 2$

Thus, H.C.F $= x - 2$

Q.13 $x^2 + 3x + 2, x^2 + 4x + 3, x^2 + 5x + 4$

Sol:

Factorization of $x^2 + 3x + 2 = x^2 + x + 2x + 2$

$= (x^2 + x) + (2x + 2)$

$= x(x + 1) + 2(x + 1)$

$= (x + 1)(x + 2)$..........................(i)

Factorization of $x^2 + 4x + 3 = x^2 + x + 3x + 3$

$= (x^2 + x) + (3x + 3)$

$= x(x + 1) + 3(x + 1)$

$= (x + 1)(x + 3)$............................(ii)

Factorization of $x^2 + 5x + 4 = x^2 + x + 4x + 4$

$= (x^2 + x) + (4x + 4)$

$= x(x + 1) + 4(x + 1)$

$= (x + 1)(x + 4)$ ....(iii)

Common factors are: $(x + 1)$

Thus, H.C.F $= x + 1$

Q.14: $x^4 + x^3 - 6x^2,\ x^4 - 9x^2,\ x^3 + x^2 - 6x$

Sol:

Factorization of $x^4 + x^3 - 6x^2 = x^2(x^2 + x - 6)$

$= x^2(x^2 + 3x - 2x - 6)$

$= x^2\left[(x^2 + 3x) - (2x + 6)\right]$

$= x^2\left[x(x + 3) - 2(x + 3)\right]$

$= x^2(x + 3)(x - 2)$

$= x \times x(x + 3)(x - 2)$ ....(i)

Factorization of $x^4 - 9x^2 = x^2(x^2 - 9)$

$= x^2\left[(x)^2 - (3)^2\right]$

$= x^2(x - 3)(x + 3)$

$= x \times x(x - 3)(x + 3)$ ....(ii)

Factorization of $x^3 + x^2 - 6x = x(x^2 + x - 6)$

$= x(x^2 + 3x - 2x - 6)$

$= x\left[(x^2 + 3x) - (2x + 6)\right]$

$= x\left[x(x + 3) - 2(x + 3)\right]$

$= x(x + 3)(x - 2)$ ....(iii)

Common factors are: x, $x + 3$

Thus, H.C.F $= x(x + 3)$

$= x^2 + 3x$

Q.15 $35a^2c^3b, 45a^3cb^2, 30ac^2b^3$

Sol:

Factorization of $35a^2c^3b$ =	5	×7×	a	×a×c×	c	×c×	b	
Factorization of $45a^3cb^2$ =	5	×3×3×	a	×a×a×	c	×	b	×b
Factorization of $30ac^2b^3$ =	5	×2×3×	a	×	c	×c×	b	×b×b

Common factors = 5, a, b, c

Thus, H.C.F $= 5 \times a \times b \times c$

$= 5abc$

Exercise 3.2

Find the H.C.F by Division Method.

Q.1 $x^4 + x^2 + 1, x^4 + x^3 + x + 1$

Sol:

Divisor $x^4 + x^2 + 1$; dividend $x^4 + x^3 + x + 1$; quotient 1

$x^4 + x^3 + x + 1$
$\underline{\pm x^4 \pm x^2 \pm 1}$

Take common x | $x^3 - x^2 + x$

Divisor $x^2 - x + 1$; dividend $x^4 + x^2 + 1$; quotient $x^2 + x + 1$

$x^4 + x^2 + 1$
$\underline{\pm x^4 \mp x^3 \pm x^2}$
$x^3 + 1$
$\underline{\pm x^3 \mp x^2 \pm x}$
$x^2 - x + 1$
$\underline{\pm x^2 \mp x \pm 1}$
0

H.C.F $= x^2 - x + 1$

Q.2. $6x^3 + 7x^2 - 9x + 2,\ 8x^4 + 6x^3 - 15x^2 + 9x - 2$

Sol:

$$\begin{array}{r|l} & 4x + 5 \\ 6x^3 + 7x^2 - 9x + 2 & 8x^4 + 6x^3 - 15x^2 + 9x - 2 \\ & \times 3 \qquad \text{multiply by 3} \\ \hline & 24x^4 + 18x^3 - 45x^2 + 27x - 6 \\ & \underset{-}{\pm} 24x^4 \underset{-}{\pm} 28x^3 \underset{+}{\mp} 36x^2 \underset{-}{\pm} 8x \\ \hline & -10x^3 - 9x^2 + 19x - 6 \\ & \times(-3) \\ \hline & 30x^3 + 27x^2 - 57x + 18 \\ & \underset{-}{\pm} 30x^3 \underset{-}{\pm} 35x^2 \underset{+}{\mp} 45x \underset{-}{\pm} 10 \\ \hline & -4 \,|\, -8x^2 - 12x + 8 \end{array}$$

$$\begin{array}{r|l} & 3x - 1 \\ 2x^2 + 3x - 2 & 6x^3 + 7x^2 - 9x + 2 \\ & \underset{-}{\pm} 6x^3 \underset{-}{\pm} 9x^2 \underset{+}{\mp} 6x \\ \hline & -2x^2 - 3x + 2 \\ & \underset{+}{\mp} 2x^2 \underset{+}{\mp} 3x \underset{-}{\pm} 2 \\ \hline & 0 \end{array}$$

H.C.F $= 2x^2 + 3x - 2$

Q.3 $4x^3 + 2x^2 - 6x,\ 4x^3 - 8x + 4$

Sol:

$$2 \,\underline{|\, 4x^3 + 2x^2 - 6x} \qquad 4 \,\underline{|\, 4x^3 - 8x + 4}$$
$$\quad 2x^3 + x^2 - 3x \qquad\qquad x^3 - 2x + 1$$

Now we find H.C.F of $x^3 - 2x + 1$ and $2x^3 + x^2 - 3x$ a n d

H.C.F of 2, 4 is 2.

$$
\begin{array}{l}
\qquad\qquad\qquad\quad 2 \\
x^2 - 2x + 1 \overline{\big)\; 2x^3 + x^2 - 3x} \\
\qquad\qquad\quad\; \underline{\pm 2x^3 \quad \mp 4x \pm 2} \\
\qquad\qquad\qquad x^2 + x - 2 \big)\; x^3 - 2x + 1 \big(x \\
\qquad\qquad\qquad\qquad\qquad\quad \underline{\pm x^3 \mp 2x \quad \pm x^2} \\
\qquad\qquad\qquad\qquad\qquad\qquad -x^2 + 1
\end{array}
$$

$$
\begin{array}{l}
\qquad\qquad\qquad -1 \\
-x^2 + 1 \overline{\big)\; x^2 + x - 2} \\
\qquad\quad\;\; \underline{\pm x^2 \quad \mp 1} \\
\qquad\qquad x - 1 \big)\; -x^2 + 1 \big(-x - 1 \\
\qquad\qquad\qquad\quad \underline{\mp x^2 \quad \pm x} \\
\qquad\qquad\qquad\quad -x + 1 \\
\qquad\qquad\qquad\quad \underline{\mp x \pm 1} \\
\qquad\qquad\qquad\qquad 0
\end{array}
$$

Required H.C.F = $2(x - 1)$

Q.4 $\quad x^3 + 7x^2 + 12x\,, \quad x^3 - 2x^2 - 15x$

Sol:

$$
\begin{array}{l|l}
x & x^3 + 7x^2 + 12x \\
\hline
 & x^2 + 7x + 12
\end{array}
\qquad
\begin{array}{l|l}
x & x^3 - 2x^2 - 15x \\
\hline
 & x^2 - 2x - 15
\end{array}
$$

H.C.F of x and x is x.

Now we find H.C.F of $x^2 - 2x - 15$ and $x^2 + 7x + 12$.

$$
\begin{array}{r|l}
 & \quad 1 \\
x^2 + 7x + 12 & x^2 - 2x - 15 \\
 & \pm x^2 \pm 7x \pm 12 \\
\hline
-9 & -9x - 27 \\
\end{array}
$$

$$
\begin{array}{r|l}
x + 3 & +x^2 + 7x + 12 \; (x + 4 \\
 & \pm x^2 \pm 3x \\
\hline
 & \quad + 4x + 12 \\
 & \quad \pm 4x \pm 12 \\
\hline
 & \qquad 0
\end{array}
$$

Required H.C.F $= x(x + 3)$

Q.5 $x^3 - x^2 - x + 1$, $x^4 - 2x^3 + 2x - 1$

Sol:

$$
\begin{array}{r|l}
 & \qquad x - 1 \\
x^3 - x^2 - x + 1 & x^4 - 2x^3 \quad + 2x - 1 \\
 & \pm x^4 \mp x^3 \mp x^2 \pm x \\
\hline
 & \quad -x^3 + x^2 + x - 1 \\
 & \quad \mp x^3 \pm x^2 \pm x \mp 1 \\
\hline
 & \qquad 0
\end{array}
$$

Required H.C.F $= x^3 - x^2 - x + 1$

$= x^2 (x - 1) - 1(x - 1)$

$= (x^2 - 1)(x - 1)$

$= \left[(x)^2 - (1)^2\right](x - 1)$

$= (x - 1)(x + 1)(x - 1)$

$= (x + 1)(x - 1)^2$

Q.6 $x^3 - x^2 - x - 2$, $x^3 + 3x^2 - 6x - 8$

Sol:

$$\begin{array}{r|l} & \quad 1 \\ x^3 - x^2 - x - 2 & x^3 + 3x^2 - 6x - 8 \\ & \pm x^3 \mp x^2 \mp x \mp 2 \\ \hline \end{array}$$

$$\begin{array}{r|l} 4x^2 - 5x - 6 & x^3 - x^2 - x - 2 \\ & \times 4 \\ \hline \end{array}$$

$$\begin{array}{r|l} 4x^2 - 5x - 6 & 4x^3 - 4x^2 - 4x - 8 \ (x \\ & \pm 4x^3 \mp 5x^2 \mp 6x \\ \hline x^2 + 2x - 8 & 4x^2 - 5x - 6 \ (4 \\ & \pm 4x^2 \pm 8x \mp 32 \\ \hline -13 & -13x + 26 \\ \hline & x - 2 \end{array}$$

$$\begin{array}{r|l} & \quad x + 4 \\ x - 2 & x^2 + 2x - 8 \\ & \pm x^2 \mp 2x \\ \hline & \quad 4x - 8 \\ & \quad \pm 4x \mp 8 \\ \hline & \quad 0 \end{array}$$

Required H.C.F $= x - 2$

Q.7 $x^2 + 3x - 4$, $x^3 - 2x^2 - 2x + 3$

Sol:

$$\begin{array}{r|l} & \quad x - 5 \\ x^2 + 3x - 4 & x^3 - 2x^2 - 2x + 3 \\ & \pm x^3 \pm 3x^2 \mp 4x \\ \hline & -5x^2 + 2x + 3 \\ & \mp 5x^2 \mp 15x \pm 20 \\ \hline 17 & 17x - 17 \\ \hline x - 1 & x^2 + 3x - 4 \ (x + 4 \\ & \pm x^2 \mp x \\ \hline & \quad 4x - 4 \\ & \quad \pm 4x \mp 4 \\ \hline & \quad 0 \end{array}$$

Required H.C.F $= x - 1$

Q.8 $3x^3 - 14x^2 + 9x + 10,\ 15x^3 - 34x^2 + 21x - 10$

Sol:

$$5$$

$$3x^3 - 14x^2 + 9x + 10 \;\big|\; 15x^3 - 34x^2 + 21x - 10$$

$$\pm 15x^3 \mp 70x^2 \pm 45x \pm 50$$

$$12 \;\big|\; 36x^2 - 24x - 60$$

$$3x^2 - 2x - 5 \;\big|\; 3x^3 - 14x^2 + 9x + 10 \;\big|\; x - 4$$

$$\pm 3x^3 \mp 2x^2 \mp 5x$$

$$-12x^2 + 14x + 10$$

$$\mp 12x^2 \pm 8x \pm 20$$

$$2 \;\big|\; 6x - 10$$

$$3x - 5$$

$$x + 1$$

$$3x - 5 \;\big|\; 3x^2 - 2x - 5$$

$$\pm 3x^2 \mp 5x$$

$$3x - 5$$

$$\pm 3x \mp 5$$

$$0$$

Required H.C.F $= 3x - 5$

Q.9 $2x^4 + x^3 + 4x + 2,\ 6x^3 + 5x^2 + x,\ 2x^4 + 3x^3 + x^2 + 2x + 1$

Sol: First we will find the H.C.F of $2x^4 + x^3 + 4x + 2$ and $2x^4 + 3x^3 + x^2 + 2x + 1$

$$1$$

$$2x^4 + x^3 + 4x + 2 \;\big|\; +2x^4 + 3x^3 + x^2 + 2x + 1$$

$$\pm 2x^4 \pm x^3 \qquad \pm 4x \pm 2 \qquad\qquad x$$

$$2x^3 + x^2 - 2x - 1 \;\big|\; +2x^4 + x^3 \qquad + 4x + 2x$$

$$\pm 2x^4 \pm x^3 \mp 2x^2 \mp x$$

$$2x^2 + 5x + 2$$

$$x - 2$$

$$2x^2 + 5x + 2 \;\big|\; 2x^3 + x^2 - 2x - 1$$

$$\pm 2x^3 \pm 5x^2 \pm 2x$$

$$-4x^2 - 4x - 1$$

$$\mp 4x^2 \mp 10x \mp 4$$

$$3 \;\big|\; 6x + 3$$

$$\text{H.C.F} \quad 2x + 1 \;\big|\; 2x^2 + 5x + 2$$

$$\pm 2x^2 \pm x$$

$$4x + 2$$

$$\pm 4x \pm 2$$

$$0$$

Now we will find H.C.F of $2x + 1$ and $6x^3 + 5x^2 + x$

$$3x^2 + x$$

$$2x + 1 \;\big|\; 6x^3 + 5x^2 + x$$

$$\pm 6x^3 \pm 3x^2$$

$$2x^2 + x$$

$$\pm 2x^2 \pm x$$

$$0$$

Required H.C.F $= 2x + 1$

Q.10 $x^3 + x^2 - 5x + 3,\ x^3 - 7x + 6,\ x^3 + 2x^2 - 2x + 3$

Sol: First we will find the H.C.F of $x^3 - 7x + 6$ and $x^3 + x^2 - 5x + 3$.

1

$x^3 - 7x + 6 \;\big|\; x^3 + x^2 - 5x + 3$

$\pm x^3 \quad \mp 7x \pm 6 \qquad x - 2$

$x^2 + 2x - 3 \;\big|\; x^3 \quad - 7x + 6$

$\pm x^3 \pm 2x^2 \mp 3x$

$- 2x^2 - 4x + 6$

$\mp 2x^2 \mp 4x \pm 6$

0

Now we will find H.C.F of $x^2 + 2x - 3$ and $x^3 + 2x^2 + 3$.

x

$x^2 + 2x - 3 \;\big|\; x^3 + 2x^2 - 2x + 3$

$\pm x^3 \pm 2x^2 \mp 3x \qquad x - 1$

$x + 3 \;\big|\; x^2 + 2x - 3$

$\pm x^2 \pm 3x$

$- x - 3$

$\mp x \mp 3$

0

H.C.F $\quad x + 3$

Least Common Multiple (LCM)

The least common multiple of two or more algebraic expressions is the expression of lowest degree which is divisible by each of them without remainder.

The abbreviation of the words ***least common multiple*** is ***L.C.M***.

Exercise 4.[illegible]

Find L.C.M by Factorization.

Q.1 $21a^4x^3y,\ 35a^2x^4y,\ 28a^3xy^4$

Sol:

Factorization of $21a^4x^3y = 3 \times 7 \times a \times a \times a \times a \times x \times x \times x \times y$

Factorization of $35a^2x^4y = 5 \times 7 \times a \times a \times x \times x \times x \times x \times y$

Factorization of $28a^3xy^4 = 2 \times 2 \times 7 \times a \times a \times a \times x \times y \times y \times y \times y$

Product of common factors $= 7 \times a \times a \times a \times x \times y \times x \times x$

$= 7a^3x^3y$ ....(i)

Product of uncommon factors $= 3 \times 5 \times 2 \times 2 \times a \times x \times y \times y \times y$

$= 60axy^3$..........(ii)

L.C.M $= \text{(i)} \times \text{(ii)}$

$= 7a^3x^3y \times 60axy^3$

$= 420a^4x^4y^4$

Q.2 $3a^4b^2c^3,\ 5a^2b^3c^5$

Sol:

Fac of $3a^4b^2c^3 = 3 \times a \times a \times a \times a \times b \times b \times c \times c \times c$

Fac of $5a^2b^3c^5 = 5 \times a \times a \times b \times b \times b \times c \times c \times c \times c \times c$

Product of common factors $= a \times a \times b \times b \times c \times c \times c$

$= a^2b^2c^3$ .... (i)

Product of uncommon factors $= 3 \times 5 \times a \times a \times b \times c \times c$

$= 15a^2bc^2$ .... (ii)

L.C.M $= \text{(i)} \times \text{(ii)}$

$= (a^2b^2c^3)(15a^2bc^2)$

$= 15a^4b^3c^5$

Q.3 $2ab, 3ab, 4ca$

Sol:

Factorization of $2ab$ =		2	×	a	×	b		
Factorization of $3ab$ =	$3\times$		×	a	×	b		
Factorization of $4ca$ =	$2\times$	2	×	a	×			c

Product of common factors= $2\times a\times b = 2ab$.........(i)

Product of uncommon factors= $3\times 2\times c = 6c$.........(ii)

$$\begin{aligned} \text{L.C.M} &= \text{(i)}\times\text{(ii)} \\ &= (2ab)(6c) \\ &= 12abc \end{aligned}$$

Q.4 x^2yz, xy^2z, xyz^2

Sol:

Factorization of x^2yz =	x	$\times x$		×	y	×		z
Factorization of xy^2z =	x	×	y	×	y	×		z
Factorization of xyz^2 =	x	×	y	×			$z\times$	z

Product of common factors= $x\times y\times y\times z = xy^2z$.........(i)

Product of uncommon factors= $x\times z = xz$..................(ii)

$$\begin{aligned} \text{L.C.M} &= \text{(i)}\times\text{(ii)} \\ &= xy^2z\times xz \\ &= x^2y^2z^2 \end{aligned}$$

Q.5 $p^3q - pq^3, p^5q^2 - p^2q^5$

Sol:

Factorization of $p^3q - pq^3 = pq(p^2 - q^2)$

$= pq(p-q)(p+q)$.........(i)

Factorization of $p^5q^2 - p^2q^5 = p^2q^2(p^3 - q^3)$

$= ppqq(p - q)(p^2 + pq + q^2)$..........(ii)

In (i) and (ii)

Product of common factors $= pq(p - q)$..........(iii)

Product of uncommon factors $= pq(p + q)(p^2 + pq + q^2)$........(iv)

$$\text{L.C.M} = \text{(iii)} \times \text{(iv)}$$

$$= \left[pq(p - q)\right]\left[pq(p + q)(p^2 + pq + q^2)\right]$$

$$= p^2q^2(p - q)(p + q)(p^2 + pq + q^2)$$

Q. $x^3 + 64,\ x^2 - 16$

Sol:

Factorization of $x^3 + 64 = (x)^3 + (4)^3$

$= (x + 4)\left[(x)^2 - (x)(4) + (4)^2\right]$

$= (x + 4)(x^2 - 4x + 16)$..........(i)

Factorization of $x^2 - 16 = (x)^2 - (4)^2$

$= (x + 4)(x - 4)$....................(ii)

In (i) and (ii)

Product of common factors $= (x + 4)$........... (iii)

Product of uncommon factors $= (x - 4)(x^2 - 4x + 16)$......(iv)

$$\text{L.C.M} = \text{(iii)} \times \text{(iv)}$$

$$= (x + 4)(x - 4)(x^2 - 4x + 16)$$

Q. $x^2 - x - 2,\ x^2 + x - 6,\ x^2 - 3x + 2$

Sol:

Factorization of $x^2 - x - 2 = x^2 - 2x + x - 2$

$= (x^2 - 2x) + (x - 2)$

$= x(x - 2) + 1(x - 2)$

$= (x - 2)(x + 1)$........(i)

Factorization of $x^2 + x - 6 = x^2 + 3x - 2x - 6$

$= (x^2 + 3x) - (2x + 6)$

$= x(x+3) - 2(x+3)$

$= (x+3)(x-2)$.........(ii)

Factorization of $x^2 - 3x + 2 = x^2 - x - 2x + 2$

$= (x^2 - x) - (2x + 2)$

$= x(x-1) - 2(x-1)$

$= (x-1)(x-2)$..........(iii)

In (i), (ii) and (iii)

Product of common factors = $(x-2)$......(iv)

Product of uncommon factors = $(x+1)(x+3)(x-1)$........(v)

L.C.M $= \text{(iv)} \times \text{(v)}$

$= (x-2)(x+1)(x+3)(x-1)$

Q.8 $\mathbf{y^2 - 9,\ (y+3)^2,\ y^2 + y - 6}$

Sol:

Factorization of $y^2 - 9 = (y)^2 - (3)^2$

$= (y+3)(y-3)$..........(i)

Factorization of $(y+3)^2 = (y+3)(y+3)$.........(ii)

Factorization of $y^2 + y - 6 = y^2 + 3y - 2y - 6$

$= (y^2 + 3y) - (2y + 6)$

$= y(y+3) - 2(y+3)$

$= (y+3)(y-2)$........(iii)

In (i), (ii) and (iii)

Product of common factors = $(y+3)$...........(iv)

Product of uncommon factors = $(y-3)(y+3)(y-2)$........(v)

L.C.M $= \text{(iv)} \times \text{(v)}$

$= (y+3)(y-3)(y+3)(y-2)$

Q.9 $\mathbf{1 - y^2,\ y^3 + 1,\ 1 - y - 2y^2}$

Sol:

Factorization of $1 - y^2 = (1)^2 - (y)^2$

$= (1 + y)(1 - y)$..........(i)

Factorization of $y^3 + 1 = (y)^3 + (1)^3$

$= (y + 1)(y^2 - y + 1)$......(ii)

Factorization of $1 - y - 2y^2 = 1 - 2y + y - 2y^2$

$= (1 - 2y) + (y - 2y^2)$

$= 1(1 - 2y) + y(1 - 2y)$

$= (1 - 2y)(1 + y)$..................(iii)

In (i), (ii) and (iii)

Product of common factors $= (1 + y)$..................................(iv)

Product of uncommon factors $= (1 - y)(1 - 2y)(y^2 - y + 1)$ ...(v)

L.C.M $= \text{(iv)} \times \text{(v)}$

$= (1 + y)(1 - y)(1 - 2y)(y^2 - y + 1)$

Q.10 $x^2 - y^2$, $x^4 - y^4$, $x^6 - y^6$

Sol:

Factorization of $x^2 - y^2 = (x + y)(x - y)$.........(i)

$= (x)^2 - (y)^2$

Factorization of $x^4 - y^4 = (x^2)^2 - (y^2)^2$

$= (x^2 + y^2)(x^2 - y^2)$

$= (x^2 + y^2)\left[(x)^2 - (y)^2\right]$

$= (x^2 + y^2)(x + y)(x - y)$.......(ii)

Factorization of $x^6 - y^6 = (x^3)^2 - (y^3)^2$

$= (x^3 + y^3)(x^3 - y^3)$

$= (x + y)(x^2 - xy + y^2)(x - y)(x^2 + xy + y^2)$(iii)

In (i), (ii) and (iii)

Product of common factors $= (x + y)(x - y)$............(iv)

Product of uncommon factors $= (x^2+y^2)(x^2-xy+y^2)(x^2+xy+y^2)$ (v)

L.C.M $= \text{(iv)} \times \text{(v)}$

$= (x + y)(x - y)(x^2 + y^2)$

$$(x^2 - xy + y^2)(x^2 + xy + y^2)$$
$$= (x + y)(x - y)(x^2 + y^2)$$
$$(x^4 + x^2y^2 + y^4)$$

Q.11 $x^3 + 1$, $x^4 + x^2 + 1$, $(x^2 + x + 1)^2$

Sol:

Factorization of $x^3 + 1 = (x)^3 + (1)^3$

$= (x + 1)(x^2 - x + 1)$..............(i)

Factorization of $x^4 + x^2 + 1 = x^4 + 2x^2 + 1 - x^2$ (completing square)

$= (x^2 + 1)^2 - (x)^2$

$= (x^2 + 1 + x)(x^2 + 1 - x)$

$= (x^2 + x + 1)(x^2 - x + 1)$......(ii)

Factorization of $(x^2 + x + 1)^2 = (x^2 + x + 1)(x^2 + x + 1)$......(iii)

In (i), (ii) and (iii)

Product of common factors $= (x^2 + x + 1)(x^2 - x + 1)$....(iv)

Product of uncommon factors $= (x + 1)(x^2 + x + 1)$.....(v)

L.C.M $= \text{(iv)} \times \text{(v)}$

$= (x^2 + x + 1)(x^2 - x + 1)(x + 1)(x^2 + x + 1)$

$= (x + 1)(x^2 - x + 1)(x^2 + x + 1)^2$

Q.12 $x^3 + y^3$, $x^4 - y^4$, $x^6 + y^6$

Sol:

Factorization of $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$........(i)

Factorization of $x^4 - y^4 = (x^2)^2 - (y^2)^2$

$= (x^2 + y^2)(x^2 - y^2)$

$= (x^2 + y^2)\left[(x^2) - (y)^2\right]$

$= (x^2 + y^2)(x + y)(x - y)$........(ii)

Factorization of $x^6 + y^6 = (x^2)^3 + (y^2)^3$

$= (x^2 + y^2)[(x^2)^2 - (x^2)(y^2) + (y^2)^2]$

$= (x^2 + y^2)(x^4 - x^2y^2 + y^4)$........(iii)

In (i), (ii) and (iii)

Product of common factors = $(x+y)(x^2+y^2)$.....(iv)

Product of uncommon factors = $(x^2-xy+y^2)(x-y)(x^4-x^2y^2+y^4)$..(v)

$$\text{L.C.M} = \text{(iv)} \times \text{(v)}$$

$$= (x+y)(x^2+y^2)(x^2-x+y^2)(x-y)$$
$$(x^4-x^2y^2+y^4)$$

$$= (x+y)(x-y)(x^2+y^2)$$
$$(x^2-xy+y^2)(x^4-x^2y^2+y^4)$$

Q.13 $2x^2+5x+3,\ x^2+2x+1,\ 2x^2+9x+9$

Sol:

Factorization of $2x^2+5x+3 = 2x^2+2x+3x+3$

$= (2x^2+2x)+(3x+3)$

$= 2x(x+1)+3(x+1)$

$= (x+1)(2x+3)$..........(i)

Factorization of $x^2+2x+1 = x^2+x+x+1$

$= (x^2+x)+(x+1)$

$= x(x+1)+1(x+1)$

$= (x+1)(x+1)$..........(ii)

Factorization of $2x^2+9x+9 = 2x^2+3x+6x+9$

$= (2x^2+3x)+(6x+9)$

$= x(2x+3)+3(2x+3)$

$= (2x+3)(x+3)$...........(iii)

In (i), (ii) and (iii)

Product of common factors = $(x+1)(2x+3)$......(iv)

Product of uncommon factors = $(x+1)(x+3)$.........(v)

$$\text{L.C.M} = \text{(iv)} \times \text{(v)}$$

$$= (x+1)(2x+3)(x+1)(x+3)$$

$$= (x+1)^2(2x+3)(x+3)$$

Q.14 $x^4 + x^3 - 6x^2$, $x^4 - 9x^2$, $x^3 + x^2 - 6x$

Sol:

Factorization of $x^4 + x^3 - 6x^2 = x^2(x^2 + x - 6)$

$= x^2(x^2 + 3x - 2x - 6)$

$= x^2[(x^2 + 3x) - (2x + 6)]$

$= x^2[x(x + 3) - 2(x + 3)]$

$= x^2(x + 3)(x - 2)$...............(i)

Factorization of $x^4 - 9x^2 = x^2(x^2 - 9)$

$= x^2[(x^2 - 3^2)]$

$= x^2(x + 3)(x - 3)$......................(ii)

Factorization of $x^3 + x^2 - 6x = x(x^2 + x - 6)$

$= x(x^2 + 3x - 2x - 6)$

$= x[x(x + 3) - 2(x + 3)]$

$= x(x + 3)(x - 2)$.................(iii)

In (i), (ii) and (iii)

Product of common factors $= x^2(x + 3)(x - 2)$.......(iv)

Product of uncommon factors $= (x - 3)$..................... (v)

L.C.M $=$ (iv) $\times$ (v)

$= x^2(x + 3)(x - 2)(x - 3)$

Q.15 $x^2 + 4xy + 4y^2$, $x^2 + 3xy + 2y^2$, $x^2 + 2xy + y^2$

Sol:

Factorization of $x^2 + 4xy + 4y^2 = x^2 + 2xy + 2xy + 4y^2$

$= (x^2 + 2xy) + (2xy + 4y^2)$

$= x(x + 2y) + 2y(x + 2y)$

$= (x + 2y)(x + 2y)$......................(i)

Factorization of $x^2 + 3xy + 2y^2 = x^2 + xy + 2xy + 2y^2$

$= (x^2 + xy) + (2xy + 2y^2)$

$= x(x + y) + 2y(x + y)$

$= (x + y)(x + 2y)$........................(ii)

Factorization of $x^2 + 2xy + y^2 = x^2 + xy + xy + y^2$

$= (x^2 + xy) + (xy + y^2)$

$= x(x + y) + y(x + y)$

$= (x + y)(x + y)$..........................(iii)

In (i), (ii) and (iii)

Product of common factors $= (x + 2y)(x + y)$.........(iv)

Product of uncommon factors $= (x + 2y)(x + y)$.........(v)

L.C.M $= \text{(iv)} \times \text{(v)}$

$= (x + 2y)(x + y)(x + 2y)(x + y)$

$= (x + y)(x + y)(x + 2y)(x + 2y)$

$= (x + y)^2(x + 2y)^2$

Relationship between HCF and LCM

If A and B are two algebraic expressions and ***H.C.F*** and ***L.C.M*** of these is represented by H and L respectively, then the relation among them can be expressed as:

$$\boxed{A \times B = H \times L}$$

It is called a formula between ***L.C.M*** and ***H.C.F.***

PROOF: Suppose that

$$\frac{A}{H} = x \qquad \textit{and} \qquad \frac{B}{H} = y$$

$A = Hx$ ....(*i*)

$B = Hy$ ....(*ii*)

Since there is no common factor between x and y.

Therefore $L = H.\,x.\,y$

$HL = H\,(H.x.y)$ (multiplying both the sides by H)

$= (Hx).\,(Hy)$

$HL = A.B.$

(*i*) $L = \dfrac{A \times B}{H}$

(*ii*) $H = \dfrac{A \times B}{L}$

(*iii*) $A = \dfrac{H \times L}{B}$

Exercise 3.4

Find the H.C.F and L.C.M of the following.

Q.1 $x^3 + x^2 + x + 1, x^3 - x^2 + x - 1$

Sol:

First we find H.C.F

$$
\begin{array}{r|l}
x^3 + x^2 + x + 1 & \overset{1}{\overline{x^3 - x^2 + x - 1}} \\
 & \pm x^3 \pm x^2 \pm x \pm 1 \\
\hline
-2 & -2x^2 - 2 \\
\hline
\end{array}
$$

$$
\begin{array}{r|l}
x^2 + 1 & \overset{x + 1}{\overline{x^3 + x^2 + x + 1}} \\
 & \pm x^3 \quad\quad \pm x \\
\hline
 & x^2 \quad\quad + 1 \\
 & \pm x^2 \quad\quad \pm 1 \\
\hline
 & 0
\end{array}
$$

H.C.F $= x^2 + 1$

Now we will find L.C.M

$$\text{L.C.M} = \frac{(x^3 + x^2 + x + 1)\overset{(x-1)}{\cancel{(x^3 - x^2 + x - 1)}}}{\cancel{(x^2 + 1)}}$$

$$= (x^3 + x^2 + x + 1)(x - 1)$$

$$= x^4 + x^3 + x^2 + x - x^3 - x^2 - x - 1$$

L.C.M $= x^4 - 1$

Working

$$
\begin{array}{r|l}
x^2 + 1 & \overset{x - 1}{\overline{x^3 - x^2 + x - 1}} \\
 & \pm x^3 \quad\quad \pm x \\
\hline
 & -x^2 \quad\quad -1 \\
 & +x^2 \quad\quad +1 \\
\hline
 & 0
\end{array}
$$

Q.2 $x^3 - 3x^2 - 4x + 12,\ x^3 - x^2 - 4x + 4$

Sol: First we find H.C.F

$$1$$

$$x^3 - 3x^2 - 4x + 12 \;\big)\; x^3 - x^2 - 4x + 4$$

$$\pm x^3 \mp 3x^2 \mp 4x \pm 12$$

$$2 \;\big|\; 2x^2 - 8 \qquad x - 3$$

$$x^2 - 4 \;\big)\; x^3 - 3x^2 - 4x + 12$$

$$\pm x^3 \qquad \mp 4x$$

$$-3x^2 \qquad + 12$$

$$\mp 3x^2 \qquad \pm 12$$

$$0$$

H.C.F $= x^2 - 4$

Now we will find L.C.M

$$\text{L.C.M} = \frac{\overset{x-3}{\cancel{(x^3 - 3x^2 - 4x + 12)}}(x^3 - x^2 - 4x + 4)}{\cancel{(x^2 - 4)}}$$

$= (x - 3)(x^3 - x^2 - 4x + 4)$

$= x^4 - x^3 - 4x^2 + 4x - 3x^3 + 3x^2 + 12x - 12$

$= x^4 - 4x^3 - x^2 + 16x - 12$

Q.3 $2x^3 + 2x^2 + x + 1,\ 2x^3 - 2x^2 + x - 1$

Sol: First we find H.C.F

$$1$$

$$2x^3 + 2x^2 + x + 1 \;\big)\; 2x^3 - 2x^2 + x - 1$$

$$\pm 2x^3 \pm 2x^2 \pm x \pm 1$$

$$-2 \;\big|\; -4x^2 - 2 \qquad x + 1$$

$$\begin{array}{r|l} 2x^2+1 & 2x^3+2x^2+x+1 \\ & \pm 2x^3 \qquad \pm x \\ \hline & \quad 2x^2 \qquad +1 \\ & \quad \pm 2x^2 \qquad \pm 1 \\ \hline & \quad 0 \end{array}$$

H.C.F $= 2x^2 + 1$

Now we will find L.C.M

$$\text{L.C.M} = \frac{\overset{x+1}{\cancel{(2x^3+2x^2+x+1)}}(2x^3-2x^2+x-1)}{\cancel{(2x^2+1)}}$$

$= (x+1)(2x^3 - 2x^2 + x - 1)$

$= 2x^4 - 2x^3 + x^2 - x + 2x^3 - 2x^2 + x - 1$

$= 2x^4 - x^2 - 1$

Q.4 $6x^3 + 7x^2 - 9x + 2,\ 8x^4 + 6x^3 - 15x^2 + 9x - 2$

Sol:

First we find H.C.F

$$\begin{array}{r|l} & \quad 4x+5 \\ \hline 6x^3+7x^2-9x+2 & 8x^4+6x^3-15x^2+9x-2 \\ & \times 3 \\ \hline & 24x^4+18x^3-45x^2+27x-6 \\ & \pm 24x^4 \pm 28x^3 \mp 36x^2 \pm 8x \\ \hline -1 & -10x^3-9x^2+19x-6 \\ \hline & 10x^3+9x^2-19x+6 \\ & \times 3 \\ \hline & 30x^3+27x^2-57x+18 \\ & \pm 30x^3 \pm 35x^2 \mp 45x \pm 10 \end{array}$$

$$
\begin{array}{r|l}
-1 & -8x^2 - 12x + 8 \\
\hline
4 & 8x^2 + 12x - 8 \qquad 3x - 1
\end{array}
$$

$$
\begin{array}{r|l}
2x^2 + 3x - 2 & 6x^3 + 7x^2 - 9x + 2 \\
& \pm 6x^3 \pm 9x^2 \mp 6x \\
\hline
& -2x^2 - 3x + 2 \\
& \mp 2x^2 \mp 3x \pm 2 \\
\hline
& 0
\end{array}
$$

H.C.F $= 2x^2 + 3x - 2$

Now we will find L.C.M

$$\text{L.C.M} = \frac{\overset{3x-1}{\cancel{(6x^3 + 7x^2 - 9x + 2)}}(8x^4 + 6x^3 - 15x^2 + 9x - 2)}{\cancel{(2x^2 + 3x - 2)}}$$

$$= (3x - 1)(8x^4 + 6x^3 - 15x^2 + 9x - 2)$$

Q.5 $\mathbf{3x^4 + 17x^3 + 27x^2 + 7x - 6,\ 6x^4 + 7x^3 - 27x^2 + 17x - 3}$

Sol: First we find H.C.F

$$
\begin{array}{r|l}
 & \quad 2 \\
3x^4+17x^3+27x^2+7x-6 & 6x^4 + 7x^3 - 27x^2 + 17x - 3 \\
& \pm 6x^4 \pm 34x^3 \pm 54x^2 \pm 14x \mp 12 \\
\hline
-3 & -27x^3 - 81x^2 + 3x + 9 \\
\hline
& 9x^3 + 27x^2 - x - 3
\end{array}
$$

$$
\begin{array}{r|l}
 & x + 2 + 2 \\
9x^3 + 27x^2 - x - 3 & 3x^4 + 17x^3 + 27x^2 + 7x - 6 \\
& \times 3 \\
\hline
& 9x^4 + 51x^3 + 81x^2 + 21x - 18 \\
& \pm 9x^4 \pm 27x^3 \mp x^2 \mp 3x
\end{array}
$$

$$24x^3 + 82x^2 + 24x - 18$$
$$\pm 18x^3 \pm 54x^2 \mp 2x \mp 6$$
$$6x^3 + 28x^2 + 26x - 12$$
$$\times 3$$
$$18x^3 + 84x^2 + 78x - 36$$
$$\pm 18x^3 \pm 54x^2 \mp 2x \mp 6$$
$$2 \,|\, 30x^2 + 80x - 30$$
$$5 \,|\, 15x^2 + 40x - 15 \qquad 3x + 1$$
$$3x^2 + 8x - 3 \,|\, 9x^3 + 27x^2 - x - 3$$
$$\pm 9x^3 \pm 24x^2 \mp 9x$$
$$3x^2 + 8x - 3$$
$$\pm 3x^2 \pm 8x \mp 3$$
$$0$$

H.C.F $= 3x^2 + 8x - 3$

Now we will find L.C.M

$$\text{L.C.M} = \frac{(3x^4 + 17x^3 + 27x^2 + 7x - 6)(6x^4 + 7x^3 - 27x^2 + 17x - 3)}{(3x^2 + 8x - 3)}$$

(the factor $6x^4 + 7x^3 - 27x^2 + 17x - 3$ and the denominator $3x^2 + 8x - 3$ are struck through, with $2x^2 - 3x + 1$ written above)

$$= (2x^2 - 3x + 1)(3x^4 + 17x^3 + 27x^2 + 7x - 6)$$

Working

$$2x^2 - 3x + 1$$
$$3x^2 + 8x - 3 \,\Big|\, 6x^4 + 7x^3 - 27x^2 + 17x - 3$$
$$\pm 6x^4 \pm 16x^3 \mp 6x^2$$
$$-9x^3 - 21x^2 + 17x - 3$$
$$\mp 9x^3 \mp 24x^2 \pm 9x$$
$$3x^2 + 8x - 3$$
$$\pm 3x^2 \pm 8x \mp 3$$
$$0$$

Q.6 $2x^4 + 3x^3 - 13x^2 - 7x + 15,\ 2x^4 + x^3 - 20x^2 - 7x + 24$

Sol: First we find H.C.F

$$
\begin{array}{r|l}
 & \quad 1 \\
2x^4+3x^3-13x^2-7x+15 & 2x^4 + x^3 - 20x^2 - 7x + 24 \\
 & \pm 2x^4 \pm 3x^3 \mp 13x^2 \mp 7x \pm 15 \\
\hline
-1 & -2x^3 - 7x^2 + 9
\end{array}
$$

$$
\begin{array}{r|l}
 & \quad x - 2 \\
2x^3 + 7x^2 - 9 & 2x^4+3x^3-13x^2-7x+15 \\
 & \pm 2x^4 \pm 7x^3 \qquad \mp 9x \\
\hline
 & -4x^3 - 13x^2 + 2x + 15 \\
 & \mp 4x^3 \mp 14x^2 \quad \pm 18 \\
\hline
 & x^2 + 2x - 3
\end{array}
$$

$$
\begin{array}{r|l}
 & \quad 2x^2 - x - 5 \\
x^2 + 2x - 3 & 2x^4 + 3x^3 - 13x^2 - 7x + 15 \\
 & \pm 2x^4 \pm 4x^3 \mp 6x^2 \\
\hline
 & -x^3 - 7x^2 - 7x + 15 \\
 & \mp x^3 \mp 2x^2 \pm 3x \\
\hline
 & -5x^2 - 10x + 15 \\
 & \mp 5x^2 \mp 10x \pm 15 \\
\hline
 & 0
\end{array}
$$

H.C.F $= x^2 + 2x - 3$

Now we will find L.C.M

$$\text{L.C.M} = \frac{\overset{2x^2 - x - 5}{\cancel{(2x^4 + 3x^3 - 13x^2 - 7x + 15)}}(2x^4 + x^3 - 20x^2 - 7x + 24)}{\cancel{x^2 + 2x - 3}}$$

$$= (2x^2 - x - 5)(2x^4 + x^3 - 20x^2 - 7x + 24)$$

Q.7 $x^4 - x^3 - x + 1,\ x^4 + x^3 - x - 1$

Sol: First we find H.C.F

$$
\begin{array}{r|l}
 & 1 \\
x^4 - x^3 - x + 1 & x^4 + x^3 - x - 1 \\
 & \pm x^4 \mp x^3 \mp x \pm 1 \\
\hline
2 & 2x^3 - 2 \\
\end{array}
$$

$$
\begin{array}{r|l}
 & x - 1 \\
x^3 - 1 & x^4 - x^3 - x + 1 \\
 & \pm x^4 \qquad \mp x \\
\hline
 & -x^3 + 1 \\
 & -x^3 + 1 \\
\hline
 & 0
\end{array}
$$

H.C.F $= x^3 - 1$

Now we will find L.C.M

$$\text{L.C.M} = \frac{\overset{(x-1)}{\cancel{(x^4 - x^3 - x + 1)}}(x^4 + x^3 - x - 1)}{\cancel{(x^3 - 1)}}$$

$$= (x - 1)(x^4 + x^3 - x - 1)$$

Q.8 $x^4 + x^3 + x + 1, x^4 + x^3 - x - 1$

Sol: First we find H.C.F

$$
\begin{array}{r|l}
 & 1 \\
x^4 + x^3 + x + 1 & x^4 + x^3 - x - 1 \\
 & \pm x^4 \pm x^3 \pm x \pm 1 \\
\hline
-2 & -2x - 2 \\
\end{array}
$$

$$
\begin{array}{r|l}
 & x^3 + 1 \\
x + 1 & x^4 + x^3 + x + 1 \\
 & \pm x^4 \pm x^3 \\
\hline
 & x + 1 \\
 & \pm x \pm 1 \\
\hline
 & 0
\end{array}
$$

H.C.F $= x + 1$

Now we will find L.C.M.

$$\text{L.C.M} = \frac{\overset{(x^3+1)}{\cancel{(x^4+x^3+x+1)}}(x^4+x^3-x-1)}{\cancel{(x+1)}}$$

$$= (x^3+1)(x^4+x^3-x+1)$$

Find the Required Polynomial.

Q.9. **$A = x^2 - 5x - 14$, $H = x-7$, $L = x^3 - 10x^2 + 11x + 70$, $B = ?$**

Sol: Formula: $B = \dfrac{H \times L}{A}$

$$= \frac{(x-7)\overset{(x-5)}{\cancel{(x^3-10x^2+11x+70)}}}{\cancel{(x^2-5x-14)}}$$

$= (x-7)(x-5)$

$= x^2 - 12x + 35$

Working

$$\begin{array}{r|l} & \quad x-5 \\ \hline x^2-5x-14 & x^3-10x^2+11x+70 \\ & \pm x^3 \mp 5x^2 \mp 14x \\ \hline & \quad -5x^2+25x+70 \\ & \quad \mp 5x^2 \pm 25x \pm 70 \\ \hline & \qquad 0 \end{array}$$

Q.10. $B = 3x^2 + 14x + 8$, $H = 3x + 2$, $L = 6x^3 + 25x^2 + 2x - 8$, $A = ?$

Sol: Formula: $A = \dfrac{H \times L}{B}$

$$= \frac{(3x+2)\overset{(2x-1)}{\cancel{(6x^3+25x^2+2x-8)}}}{\cancel{(3x^2+14x+8)}}$$

$= (3x + 2)(2x - 1)$

$= 6x^2 + x - 2$

Working

$$
\begin{array}{r|l}
 & \quad 2x - 1 \\
3x^2 + 14x + 18 & \ 6x^3 + 25x^2 + 2x - 8 \\
 & \pm 6x^3 \pm 28x^2 \pm 16x \\
\hline
 & \qquad -3x^2 - 14x - 8 \\
 & \qquad +3x^2 + 14x + 8 \\
\hline
 & \qquad\qquad 0
\end{array}
$$

Q.11. The product of two polynomials and their L.C.M. are $x^4 + 6x^3 - 3x^2 - 56x - 48$ and $x^3 + 2x^2 - 11x - 12$ respectively. Find their H.C.F.

Sol:

$A \times B = x^4 + 6x^3 - 3x^2 - 56x - 48$

$H = x^3 + 2x^2 - 11x - 12$

$L = ?$

Formula: $L = \dfrac{A \times B}{H}$

$$= \frac{x^4 + 6x^3 - 3x^2 - 56x - 48}{x^3 + 2x^2 - 11x - 12}$$

$= x + 4$

Working

$$
\begin{array}{r|l}
 & \quad x + 4 \\
x^3 + 2x^2 - 11x - 12 & \ x^4 + 6x^3 - 3x^2 - 56x - 48 \\
 & \pm x^4 \pm 2x^3 \mp 11x^2 \mp 12x \\
\hline
 & \qquad + 4x^3 + 8x^2 - 44x - 48 \\
 & \qquad \pm 4x^3 \pm 8x^2 \mp 44x \mp 48 \\
\hline
 & \qquad\qquad 0
\end{array}
$$

Q.12. The product of two polynomials and their L.C.M. are $x^4 + 5x^3 - x^2 - 17x + 12$ and $x^3 + 6x^2 + 5x - 12$ respectively.

Find their H.C.F.

Sol: A × B $= x^4 + 5x^3 - x^2 - 17x + 12$

L $= x^3 + 6x^2 + 5x - 12$

H = ?

Formula: $H = \frac{A \times B}{L}$

$$= \frac{x^4 + 5x^3 - x^2 - 17x + 12}{x^3 + 6x^2 + 5x - 12}$$

H $= x - 1$

Working

$$\begin{array}{r|l} & x - 1 \\ \hline x^3 + 6x^2 + 5x - 12 & x^4 + 5x^3 - x^2 - 17x + 12 \\ & \pm x^4 \pm 6x^3 \pm 5x^2 \mp 12x \\ \hline & -x^3 - 6x^2 - 5x + 12 \\ & \underline{+} x^3 \underline{+} 6x^2 \underline{+} 5x \pm 12 \\ \hline & 0 \end{array}$$

Q.13. The product of two polynomials and their H.C.F. are $x^4 - 12x^3 + 53x^2 - 102x + 72$ and $x - 3$ respectively. Find L.C.M.

Sol:

A × B $= x^4 - 12x^3 + 53x^2 - 102x + 72$

L $= x - 3$

H = ?

Formula: $H = \frac{A \times B}{L}$

$$= \frac{x^4 - 12x^3 + 53x^2 - 102x + 72}{(x - 3)}$$

H $= x^3 - 9x^2 + 26x - 24$

Working

$$
\begin{array}{r|l}
 & x^3 - 9x^2 + 26x - 24 \\
\hline
x - 3 & x^4 - 12x^3 + 53x^2 - 102x + 72 \\
 & \pm x^4 \mp 3x^3 \\
\hline
 & -9x^3 + 53x^2 - 102x + 72 \\
 & \mp 9x^3 \pm 27x^2 \\
\hline
 & 26x^2 - 102x + 72 \\
 & \pm 26x^2 \mp 78x \\
\hline
 & -24x + 72 \\
 & \mp 24x \pm 72 \\
\hline
 & 0
\end{array}
$$

Q.14. The product of two polynomials and their H.C.F. is $x^4 - 5x^3 + 2x^2 + 20x - 24$ and $x + 2$ respectively. Find their L.C.M.

Sol:

A × B $= x^4 - 5x^3 + 2x^2 + 20x - 24$

H $= x + 2$

L $= ?$

Formula $L = \frac{A \times B}{H}$

$$= \frac{x^4 - 5x^3 + 2x^2 + 20x - 24}{(x + 2)}$$

H $= x^3 - 7x^2 + 16x - 12$

Working

$$
\begin{array}{r|l}
 & x^3 - 7x^2 + 16x - 12 \\
\hline
x + 2 & x^4 - 5x^3 + 2x^2 + 20x - 24 \\
 & \pm x^4 \pm 2x^3 \\
\hline
 & -7x^3 + 2x^2 + 20x - 24
\end{array}
$$

$$\mp 7x^3 \mp 14x^2$$

$$+16x^2 + 20x - 24$$
$$\pm 16x^3 \pm 32x$$

$$-12x - 24$$
$$\mp 12x \mp 24$$

$$0$$

Q.15. One algebraic expression is $x^3 + 3x^2 - 4x - 12$ and $x^3 + 5x^2 - 4x - 20$ other one is . Their H.C.F is $x^2 - 4$. Find their L.C.M.

Sol:

A $= x^3 + 3x^2 - 4x - 12$

B $= x^3 + 5x^2 - 4x - 20$

H $= x^2 - 4$

L $= ?$

Formula: $L = \dfrac{A \times B}{H}$

$$= \frac{\overset{x+3}{(x^3 + 3x^2 - 4x - 12)}(x^3 + 5x^2 - 4x - 20)}{(x^2 - 4)}$$

$$= (x + 3)(x^3 + 5x^2 - 4x - 20)$$

$$= x^4 + 8x^3 + 11x^2 - 32x - 60$$

Working

$$x + 3$$

$$x^2 - 4 \,\big|\, x^3 + 3x^2 - 4x - 12$$
$$\pm x^3 \quad \mp 4x$$

$$3x^2 \quad - 12$$
$$\pm 3x^3 \quad \mp 12$$

$$0$$

Q.16. One algebraic expression is $x^3 - x^2 + 2x - 2$ and other one is $x^3 - x^2 - 2x + 2$. Their H.C.F is $x - 1$. Find their L.C.M.

Sol:

A $= x^3 - x^2 + 2x - 2$

B $= x^3 - x^2 - 2x + 2$

H $= x - 1$

L $= ?$

Formula: $L = \dfrac{A \times B}{H}$

$$(x^2 + 2)$$

$$= \frac{\cancel{(x^3 - x^2 + 2x - 2)}(x^3 - x^2 - 2x + 2)}{\cancel{(x - 1)}}$$

$$= (x^2 + 2)(x^3 - x^2 - 2x + 2)$$

$$= x^5 - x^4 - 2x + 4$$

Working

$$x^2 + 2$$

$$x - 1 \,\overline{)\, x^3 - x^2 + 2x - 2}$$

$$\pm x^3 \mp x^2$$

$$2x^2 - 2$$

$$\pm 2x^2 \mp 2$$

$$0$$

Q.17. Prove that $H^3 + L^3 = A^3 + B^3$ where $H + L = A + B$ 'H' and 'L' stand for H.C.F and L.C.M respectively and 'A, B' represent two polynomials.

Sol:

Proof: We know th:

$H + L = A + B$

$(H + L)^3 = (A + B)^3$ Taking cube

$H^3 + L^3 + 3HL(H + L) = A^3 + B^3 + 3AB(A + B)$

$H^3 + L^3 = A^3 + B^3 + 3AB(A + B) - 3HL(H + L)$ (i)

Now $H + L = A + B$

and $H \times L = A \times B$

Putting values in (i) H + L and HL

$H^3 + L^3 = A^3 + B^3 + 3AB(A + B) - 3AB(A + B)$

$H^3 + L^3 = A^3 + B^3$ Proved

Exercise 3.5

Simplify

Q.1 $\frac{1}{a} + \frac{2}{a+1} - \frac{3}{a+2}$

Sol: $= \frac{1(a+1)(a+2) + 2(a)(a+2) - 3(a)(a+1)}{(a)(a+1)(a+2)}$

$= \frac{a^2 + 3a + 2 + 2a^2 + 4a - 3a^2 - 3a}{(a)(a+1)(a+2)}$

$= \frac{4a + 2}{(a)(a+1)(a+2)}$

$= \frac{2(2a+1)}{(a)(a+1)(a+2)}$

Q.2 $\frac{2a}{(x-2a)} - \frac{x-a}{x^2 - 5ax + 6a^2} + \frac{2}{x-3a}$

Sol: $= \frac{2a}{(x-2a)} - \frac{x-a}{x^2 - 2ax - 3ax + 6a^2} + \frac{2}{x-3a}$

$$= \frac{2a}{(x-2a)} - \frac{x-a}{x(x-2a)-3a(ax-2a)} + \frac{2}{x-3a}$$

$$= \frac{2a}{(x-2a)} - \frac{x-a}{(x-2a)(x-3a)} + \frac{2}{x-3a}$$

$$= \frac{2a(x-3a)-(x-a)+2(x-2a)}{(x-2a)(x-3a)}$$

$$= \frac{2ax-6a^2-x+a+2x-4a}{(x-2a)(x-3a)}$$

$$= \frac{2ax+x-3a-6a^2}{(x-2a)(x-3a)}$$

Q.3 $\frac{1}{a^2+1} - \frac{a^4}{a^2+1} + \frac{a^6}{a^2-1} - \frac{1}{a^2-1}$

Sol: $= \frac{1(a^2-1)-a^4(a^2-1)+a^6(a^2+1)-1(a^2+1)}{(a^2+1)(a^2-1)}$

$$= \frac{a^2-1-a^6+a^4+a^8+a^6-a^2-1}{(a^2+1)(a^2-1)}$$

$$= \frac{a^8+a^4-2}{(a^2+1)(a^2-1)}$$

$$= \frac{a^8+2a^4-a^4-2}{(a^2+1)(a^2-1)}$$

$$= a^4(a^4+2)-1(a^4+2)$$

$$= \frac{(a^4+2)(a^4-1)}{(a^4-1)}$$

$$= a^4+2$$

Q.4 $\dfrac{1}{x^2+x+1}-\dfrac{1}{x^2-x+1}+\dfrac{2x+1}{x^4+x^2+1}$

Sol: $=\dfrac{1}{x^2+x+1}-\dfrac{1}{x^2-x+1}+\dfrac{2x+1}{(x^2+x+1)(x^2-x+1)}$

$$=\frac{1(x^2-x+1)-1(x^2+x+1)+2x+1}{(x^2+x+1)(x^2-x+1)}$$

$$=\frac{x^2-x+1-x^2-x-1+2x+1}{(x^2+x+1)(x^2-x+1)}$$

$$=\frac{1}{(x^2+x+1)(x^2-x+1)}$$

$$=\frac{1}{(x^4+x^2+1)}$$

Q.5 $\dfrac{a^2(b-c)}{(a+b)(a+c)}-\dfrac{b^2(c-a)}{(b+c)(b+a)}+\dfrac{c^2(a-b)}{(c+a)(c+b)}$

Sol: $=\dfrac{a^2(b-c)(b+c)-b^2(c-a)(c+a)+c^2(a-b)(a+b)}{(a+b)(a+c)(b+c)}$

$$=\frac{a^2(b^2-c^2)-b^2(c^2-a^2)+c^2(a^2-b^2)}{(a+b)(a+c)(b+c)}$$

$$=\frac{a^2b^2-a^2c^2-b^2c^2+a^2b^2+a^2c^2-b^2c^2}{(a+b)(a+c)(b+c)}$$

$$=\frac{2a^2b^2-2b^2c^2}{(a+b)(a+c)(b+c)}$$

$$=\frac{2b^2(a^2-c^2)}{(a+b)(a+c)(b+c)}$$

$$=\frac{2b^2(a+c)(a-c)}{(a+b)(a+c)(b+c)}$$

$$=\frac{2b^2(a-c)}{(a+b)(b+c)}$$

Q.6 $\dfrac{1}{x-1}+\dfrac{1}{x+1}-\dfrac{x+2}{x^2+x+1}-\dfrac{x-2}{x^2-x+1}$

Sol: Changing order

$$=\left(\frac{1}{x-1}-\frac{x+2}{x^2+x+1}\right)+\left(\frac{1}{x+1}-\frac{x-2}{x^2-x+1}\right)$$

$$=\frac{(x^2+x+1)-(x-1)(x+2)}{(x-1)(x^2+x+1)}+\frac{(x^2-x+1)-(x-2)(x+1)}{(x+1)(x^2-x+1)}$$

$$=\frac{(x^2+x+1)-(x^2+x-2)}{x^3-1}+\frac{(x^2-x+1)-(x^2-x-2)}{x^3+1}$$

$$=\frac{x^2+x+1-x^2-x+2}{x^3-1}+\frac{x^2-x+1-x^2+x+2}{x^3+1}$$

$$=\frac{3}{x^3-1}+\frac{3}{x^3+1}$$

$$=\frac{3(x^3+1)+3(x^3-1)}{(x^3-1)(x^3+1)}$$

$$=\frac{3x^3+3+3x^3-3}{x^6-1}$$

$$=\frac{6x^3}{x^6-1}$$

Q.7 $\dfrac{a^2+ab+b^2}{a+b}+\dfrac{a^2-ab+b^2}{a-b}$

Sol:

$$=\frac{(a-b)(a^2+ab+b^2)+(a+b)(a^2-ab+b^2)}{(a+b)(a-b)}$$

$$=\frac{(a^3-b^3)+(a^3+b^5)}{a^2-b^2}$$

$$=\frac{a^3-b^3+a^3+b^3}{a^2-b^2}$$

$$=\frac{2a^3}{a^2-b^2}.$$

Q.8 $\dfrac{x^4-y^4}{x^2-2xy+y^2}\times\dfrac{x-y}{x(x+y)}\div\dfrac{x^2+y^2}{x}$

Sol: $=\dfrac{(x^2+y^2)(x^2-y^2)}{(x-y)^2}\times\dfrac{x-y}{x(x+y)}\times\dfrac{x}{(x^2+y^2)}$

$=\dfrac{(x+y)(x-y)}{(x-y)(x-y)}\times\dfrac{(x-y)}{x(x+y)}\times x$

$=1$

Q.9 $\dfrac{x^2-1}{x^2+x-2}\times\dfrac{x^3+8}{x^4+4x^2+16}\div\dfrac{x^2+x}{x^3+2x^2+4x}$

Sol: $=\dfrac{(x+1)(x-1)}{x^2+2x-x-2}\times\dfrac{(x)^3+(2)^3}{x^4+4x^2+16}\times\dfrac{x^3+2x^2+4x}{x^2+x}$

$=\dfrac{(x+1)(x-1)}{x(x+2)-1(x+2)}\times\dfrac{(x+2)(x^2-2x+4)}{x^4+8x^2+16-4x^2}\times\dfrac{x(x^2+2x+4)}{x(x+1)}$

(Completing square)

$=\dfrac{(x+1)(x-1)}{(x+2)(x-1)}\times\dfrac{(x+2)(x^2-2x+4)}{(x^2+4)^2-(2x)^2}\times\dfrac{x^2+2x+4}{x+1}$

$=\dfrac{(x^2-2x+4)(x^2+2x+4)}{(x^2+4-2x)(x^2+4+2x)}$

$=\dfrac{(x^2-2x+4)(x^2+2x+4)}{(x^2-2x+4)(x^2+2x+4)}$

$=1$

Q.10 $\dfrac{a^3+64b^3}{a^2+20ab+64b^2}\div\dfrac{a^2-4ab+16b^2}{a^2+4ab+16b^2}\times\dfrac{a^2+12ab-64b^2}{a^3-64b^3}$

Sol:

$=\dfrac{(a)^3+(4b)^3}{a^2+4ab+16ab+64b^2}\times\dfrac{a^2+4ab+16b^2}{a^2-4ab+16b^2}\times\dfrac{a^2+16ab-4ab-64b^2}{(a)^3-(4b)^3}$

$=\dfrac{(a+4b)(a^2-4ab+16b^2)}{a(a+4b)+16b(a+4b)}\times\dfrac{a^2+4ab+16b^2}{a^2-4ab+16b^2}\times\dfrac{a(a+16b)-4b(a+16b)}{(a-4b)(a^2+4ab+16b^2)}$

$=\dfrac{(a+4b)(a^2-4ab+16b^2)}{(a+4b)(a+16b)}\times\dfrac{a^2+4ab+16b^2}{a^2-4ab+16b^2}\times\dfrac{(a+16b)(a-4b)}{(a-4b)(a^2+4ab+16b^2)}$

$=1$

Q.11 $\frac{a}{(a+b)^2-2ab}\times\frac{a^4-b^4}{(a+b)^3-3ab(a+b)}\div\frac{(a+b)^2-4ab}{(a+b)^2-3ab}$

Sol: $=\frac{a}{a^2+b^2+2ab-2ab}\times\frac{(a^2)^2-(b^2)^2}{a^3+b^3+3ab(a+b)-3ab(a+b)}\times\frac{(a+b)^2-3ab}{(a+b)^2-4ab}$

$$=\frac{a}{(a^2+b^2)}\times\frac{(a^2+b^2)(a^2-b^2)}{a^3+b^3}\times\frac{a^2+b^2+2ab-3ab}{a^2+b^2+2ab-4ab}$$

$$=a\times\frac{(a+b)(a-b)}{(a+b)(a^2-ab+b^2)}\times\frac{(a^2+b^2-ab)}{a^2+b^2-2ab}$$

$$=\frac{a\times(a-b)}{a^2-2ab+b^2}=\frac{a(a-b)}{(a-b)^2}$$

$$=\frac{a(a-b)}{(a-b)(a-b)}$$

$$=\frac{a}{a-b}$$

Q.12 $\frac{a^2-1}{a^2-a-2}\div\frac{a^2+5a+6}{a^2-5a+6}\div\frac{a^2-4a+3}{a^2+4a+3}$

Sol: $=\frac{(a)^2-(1)^2}{a^2-2a+a-2}\times\frac{a^2-5a+6}{a^2+5a+6}\times\frac{a^2+4a+3}{a^2-4a+3}$

$$=\frac{(a-1)(a+1)}{a(a-2)+1(a-2)}\times\frac{a^2-2a-3a+6}{a^2+2a+3a+6}\times\frac{a^2+a+3a+3}{a^2-a-3a+3}$$

$$=\frac{(a-1)(a+1)}{(a-2)(a+1)}\times\frac{a(a-2)-3(a-2)}{a(a+2)+3(a+2)}\times\frac{a(a+1)+3(a+1)}{a(a-1)-3(a-1)}$$

$$=\frac{(a-1)(a+1)}{(a-2)(a+1)}\times\frac{(a-2)(a-3)}{(a+2)(a+3)}\times\frac{(a+1)(a+3)}{(a-1)(a-3)}$$

$$=\frac{a+1}{a+2}$$

Exercise 3.6

Find the Square Root of the Following.

Q.1 $16x^2 + 24xy + 9y^2$

Sol: $= 16x^2 + 12xy + 12xy + 9y^2$

$= (16x^2 + 12xy) + (12xy + 9y^2)$

$= 4x(4x + 3y) + 3y(4x + 3y)$

$= (4x + 3y)(4x + 3y)$

$= (4x + 3y)^2$

$\sqrt{16x^2 + 24xy + 9y^2} = \sqrt{(4x + 3y)^2}$

$= \pm(4x + 3y)$

Q.2 $(x^2 - 7x + 12)(x^2 - 9x + 20)(x^2 - 8x + 15)$

Sol: $= [x^2 - 3x - 4x + 12][x^2 - 4x - 5x + 20][x^2 - 3x - 5x + 15]$

$= [(x^2 - 3x) - (4x - 12)][(x^2 - 4x) - (5x - 20)][(x^2 - 3x) - (5x - 15)]$

$= [x(x - 3) - 4(x - 3)][x(x - 4) - 5(x - 4)][x(x - 3) - 5(x - 5)]$

$= (x - 3)(x - 4)(x - 4)(x - 5)(x - 3)(x - 5)$

$= (x - 3)^2(x - 4)^2(x - 5)^2$

Now we will take square root.

$= \sqrt{(x - 3)^2(x - 4)^2(x - 5)^2}$

$= \pm(x - 3)(x - 4)(x - 5)$

Q.3 $(x^2 + 8x + 7)(2x^2 - x - 3)(2x^2 + 11x - 21)$

Sol: $= (x^2 + x + 7x + 7)(2x^2 - 3x + 2x - 3)(2x^2 + 14x - 3x - 21)$

$= [(x^2 + x) + (7x + 7)][(2x^2 - 3x) + (2x - 3)][(2x^2 + 14x) - (3x + 21)]$

$= [x(x + 1) + 7(x + 1)][x(2x - 3) + 1(2x - 3)][2x(x + 7) - 3(x + 7)]$

$= [(x + 1)(x + 7)][(2x - 3)(x + 1)][(x + 7)(2x - 3)]$

$= (x + 1)^2(x + 7)^2(2x - 3)^2$

$= \sqrt{(x + 1)^2(x + 7)^2(2x - 3)^2}$ (Taking square root)

$= \pm(x + 1)(x + 7)(2x - 3)$

Q.4 $x(x+2)(x+4)(x+6)+16$

Sol: $= x(x+6)(x+2)(x+4)+16$ (Rearranging)

$= [x(x+6)][(x+2)(x+4)]+16$

$= (x^2+6x)(x^2+6x-8)+16$......................(i)

Let $x^2+6x = y$

$= (y)(y+8)+16$ from (i)

$= y^2+8y+16$

$= y^2+4y+4y+16$

$= (y^2+4y)+(4y+16)$

$= y(y+4)+4(y+4)$

$= (y+4)(y+4)$

$= (y+4)^2$

Putting values of y in x^2+6x

$= (x^2+6x+4)^2$

$= \sqrt{(x^2+6x+4)^2}$ (Taking square root)

$= \pm(x^2+6x+4)$

Q.5 $(2x+1)(2x+3)(2x+5)(2x+7)+16$

Sol: (Rearranging)

$= (2x+1)(2x+7)(2x+3)(2x+5)+16$

$= [(2x+1)(2x+7)][(2x+3)(2x+5)]+16$

$= [4x^2+16x+7][(4x^2+16x+15]+16$

Let $4x^2+16x = y$

$= (y+7)(y+15)+16$

$= y^2+22y+105+16$

$= y^2+22y+121$

$= y^2+11y+11y+121$

$= (y^2+11y)+(11y+121)$

$= y(y + 11) + 11(y + 11)$

$= (y + 11)(y + 11)$

$= (y + 11)^2$

Putting value of y

$= (4x^2 + 16x + 11)^2$

$= \sqrt{(4x^2 + 16x + 11)^2}$ (Taking square root)

$= \pm(4x^2 + 16x + 11)$

Q.6 $\left(x^2 + \frac{1}{x^2}\right) - 10\left(x + \frac{1}{x}\right) + 27$ **; $x \neq 0$**

Sol: $\left(x^2 + \frac{1}{x^2}\right) - 10\left(x + \frac{1}{x}\right) + 27$ (i)

Let $x + \frac{1}{x} = y$

$x^2 + \frac{1}{x^2} + 2 = y^2$ (Squaring both sides.)

$x^2 + \frac{1}{x^2} = y^2 - 2$

Putting values $x + \frac{1}{x} = y$ and $x^2 + \frac{1}{x^2} = y^2 - 2$ in (i).

$= y^2 - 2 - 10y + 27$

$= y^2 - 10y + 25$

$= y^2 - 5y - 5y + 25$

$= y(y - 5) - 5(y - 5)$

$= (y - 5)(y - 5)$

$= (y - 5)^2$

$= \left(x + \frac{1}{x} - 5\right)^2$

$= \sqrt{\left(x + \frac{1}{x} - 5\right)^2}$ (Taking square root)

$$= \pm\left(x+\frac{1}{x}-5\right)$$

Q.7 $\left(t-\frac{1}{t}\right)^2-4\left(t+\frac{1}{t}\right)+8, (t \neq 0)$

Sol: Let $t+\frac{1}{t}=y$

We know that

$$\left(t-\frac{1}{t}\right)^2 = \left(t+\frac{1}{t}\right)^2-4t\times\frac{1}{t}$$

$$= y^2-4$$

Now $t+\frac{1}{t}=y$ and $\left(t+\frac{1}{t}\right)^2=y^2-4$ putting in given values

$= y^2-4-4y+8$

$= y^2-4y+4$

$= y^2-2y-2y+4$

$= y(y-2)-2(y-2)$

$= (y-2)(y-2)$

$= (y-2)^2$

Putting value of y.

$$= \left(t+\frac{1}{t}-2\right)^2$$

$$= \sqrt{\left(t+\frac{1}{t}-2\right)^2} \quad \text{(Taking square root)}$$

$$= \pm\left(t+\frac{1}{t}-2\right)$$

Q.8 $\left(x^2+\frac{1}{x^2}\right)^2-4\left(x+\frac{1}{x}\right)^2+12$ **; $x \neq 0$**

Sol: Let $x+\frac{1}{x}=y$

$$x+\frac{1}{x} \quad = y$$

Squaring both sides

$$\left(x+\frac{1}{x}\right)^2 \quad = y^2$$

$$x^2+\frac{1}{x^2}+2 = y^2 \text{.................................(i)}$$

$$x^2+\frac{1}{x^2} \quad = y^2-2$$

Putting $x+\frac{1}{x}=y$ and $x^2+\frac{1}{x^2}=y^2-2$ in the given statement.

$= (y^2 - 2)^2 - 4y^2 + 12$
$= y^4 - 4y^2 + 4 - 4y^2 + 12$
$= y^4 - 8y^2 + 16$
$= y^4 - 4y^2 - 4y^2 + 16$
$= y^2(y^2 - 4) - 4(y^2 - 4)$
$= (y^2 - 4)(y^2 - 4)$
$= (y^2 - 4)^2$

From (i), putting values $y^2 = x^2+\frac{1}{x^2}+2$.

$$= \left(x^2+\frac{1}{x^2}+2-4\right)^2$$

$$= \left(x^2+\frac{1}{x^2}-2\right)^2$$

$$= \sqrt{\left(x^2+\frac{1}{x^2}-2\right)^2} \quad \text{(Taking square root)}$$

$$= \pm\left(x^2+\frac{1}{x^2}-2\right)$$

Q.9 $4x^4 + 12x^3 + 25x^2 + 24x + 16$

Sol:

$$\begin{array}{r|l}
 & \quad 2x^2 + 3x + 4 \\
\hline
2x^2 & 4x^4 + 12x^3 + 25x^2 + 24x + 16 \\
 & \pm 4x^4 \\
\hline
4x^2 + 3x & 12x^3 + 25x^2 + 24x + 16 \\
 & \pm 12x^3 \pm 9x^2 \\
\hline
4x^2 + 6x + 4 & +16x^2 + 24x + 16 \\
 & \pm 16x^2 \pm 24x \pm 16 \\
\hline
 & 0
\end{array}$$

Square root $= \pm(2x^2 + 3x + 4)$

Q.10 $\frac{9x^2}{4y^2} - \frac{3x}{2y} - \frac{7}{4} + \frac{2y}{3x} + \frac{4y^2}{9x^2} (x \neq 0,\ y \neq 0)$

Sol:

$$\text{Square root} = \pm\left(\frac{3x}{2y} - \frac{1}{2} - \frac{2y}{3x}\right)$$

Q.11. For what value of x, $x^4 + 4x^2 + x + \frac{8}{x^2} + \frac{4}{x^4}$ is a complete square, where $x \neq 0$

Sol: First we will find square root

$$x^2 + 2 + \frac{2}{x^2}$$

x^2	$x^4 + 4x^2 + x + \frac{8}{x^2} + \frac{4}{x^4}$
	$\pm x^4$
$2x^2 + 2$	$4x^2 + x + \frac{8}{x^2} + \frac{4}{x^4}$
	$\pm 4x^2 \pm 4$
$2x^2 + 4 + \frac{2}{x^2}$	$-4 + x + \frac{8}{x^2} + \frac{4}{x^4}$
	$\pm 4 \pm \frac{8}{x^2} \pm \frac{4}{x^4}$
	$-8 + x$

For completing square remainder must be zero.

Therefore $-8 + x = 0$

$\Rightarrow \quad x = 8$

Q.12. If $x^4 + lx^3 + mx^2 + 12x + 9$ is a complete square then find the values of l and m.

Sol:

$$x^2 + 2x + 3$$

x^2	$x^4 + lx^3 + mx^2 + 12x + 9$
	$\pm x^4$
$2x^2 + 2x$	$lx^3 + mx^2 + 12x + 9$
	$\pm 4x^3 \pm 4x^2$
$2x^2 + 4x + 3$	$(l-4)x^3 + mx^2 - 4x^2 + 12x + 9$
	$\pm 6x^2 \pm 12x \pm 9$
	$(l-4)x^3 + mx^2 - 10x^2$

For completing square remainder must be zero

$(l - 4)x^3 + (m - 10)x^2 = 0$

Therefore, $l - 4 = 0$ and $m - 10 = 0$

$l = 4$, $m = 10$

Exercise 4.1

Solve:

Q.1. (i) $3x + 20 = 44$

Sol.

$$3x = 44 - 20$$

$$3x = 24$$

$$\frac{3x}{3} = \frac{24}{3} \quad \text{(Dividing by 3)}$$

$$x = 8$$

(ii) $\frac{4x}{5} - \frac{3x}{4} = 4$

Sol. Mulitiplied by 4, 5 L.C.M. 20.

$$\frac{4x}{5}(20) - \frac{3x}{4}(20) = 4(20)$$

$$16x - 15x = 80$$

$$x = 80$$

(iii) $3x + 3(x + 1) = 69$

Soi.

$$3x + 3x + 3 = 69$$

$$6x + 3 = 69$$

$$6x = 69 - 3$$

$$6x = 66$$

$$\frac{6x}{6} = \frac{66}{6} \quad \text{(Dividing by 6)}$$

$$x = 11$$

(iv) $(90 - 9x) + 27 = 90 + 9$

Sol.
$$90 - 9x + 27 = 99$$
$$-9x = 99 - 90 - 27$$
$$-9x = -18$$
$$\frac{-9x}{-9} = \frac{-18}{-9} \text{ (Dividing by } -9)$$
$$x = 2$$

Q.2. $3(x + 3) = 14 + x$

Sol.
$$3x + 9 = 14 + x$$
$$3x - x = 14 - 9$$
$$2x = 5$$
$$\frac{2x}{2} = \frac{5}{2} \text{ (Dividing by 2)}$$
$$x = \frac{5}{2}$$

Q.3. $3(2x + 5) = 25 + x$

Sol.
$$6x + 15 = 25 + x$$
$$6x - x = 25 - 15$$
$$5x = 10$$
$$\frac{5x}{5} = \frac{10}{5} \text{ (Dividing by 5)}$$
$$x = 2$$

Q.4. $9x - 3 = 3(2x - 8)$

Sol.
$$9x - 3 = 6x - 24$$
$$9x - 6x = -24 + 3$$

$$3x = -21$$

$$\frac{3x}{3} = \frac{-21}{3} \text{ (Dividing by 3)}$$

$$x = -7$$

Q.5. $\mathbf{3(2x - 1) = 5(x - 1)}$

Sol.

$$6x - 3 = 5x - 5$$

$$6x - 5x = -5 + 3 \text{ (By Transposing)}$$

$$x = -2$$

Q.6. $\mathbf{2(7x - 6) = 3(1 + 3x)}$

Sol.

$$14x - 12 = 3 + 9x$$

$$14x - 9x = 3 + 12$$

$$5x = 15$$

$$\frac{x}{5} = \frac{15}{5} \text{ (Dividing by 5)}$$

$$x = 3$$

Q.7. $$\frac{10x - 1}{2x + 5} = 3$$

Sol.

$$10x - 1 = 3(2x + 5)$$

$$10x - 1 = 6x + 15$$

$$10x - 6x = 15 + 1$$

$$4x = 16$$

$$\frac{x}{4} = \frac{16}{4} \text{ (Dividing by 4)}$$

$$x = 4$$

Q.8. $\frac{2x+1}{x+5} = 1$

Sol.
$$2x + 1 = 1(x + 5)$$
$$2x + 1 = x + 5$$
$$2x - x = 5 - 1$$
$$x = 4$$

Q.9. $\frac{5x+3}{x+6} = 2$

Sol.
$$5x + 3 = 2(x + 6)$$
$$5x + 3 = 2x + 12$$
$$5x - 2x = 12 - 3$$
$$3x = 9$$
$$\frac{x}{3} = \frac{9}{3} \quad \text{(Dividing by 3)}$$
$$x = 3$$

Q.10. $y - 6 + \sqrt{y} = 0$

Sol.
$$\sqrt{y} = 6 - y$$
$$\left(\sqrt{y}\right)^2 = (6 - y)^2 \quad \text{(Squaring on both sides)}$$
$$y = 36 + y^2 - 12y$$
$$0 = 36 + y^2 - 12y - y$$
$$0 = 36 + y^2 - 13y$$
$$\Rightarrow y^2 - 13y + 36 = 0$$
$$y^2 - 4y - 9y + 36 = 0$$
$$y(y - 4) - 9(y - 4) = 0$$
$$(y - 4)(y - 9) = 0$$

If $y - 4 = 0$

Then, $y = 4$

and if $y-9=0$

Then, $y=9$

Check to eliminate extraneous root $y=9$.

Q.11 $x=15-2\sqrt{x}$

Sol. $2\sqrt{x}=15-x$

$\left(2\sqrt{x}\right)^2=(15-x)^2$ (Squaring on both sides)

$4x=225+x^2-30x$

$0=225+x^2-30x-4x$

$0=x^2+225-34x$

$\Rightarrow x^2-34x+225=0$

$x^2-9x-25x+225=0$

$x(x-9)-25(x-9)=0$

$(x-9)(x-25)=0$

If $x-9=0$

then $x=9$

And if $x-25=0$

then $x=25$

Check to eliminate extraneous roots $x=25$

Q.12. $m-13=\sqrt{m+7}$

Sol. $(m-13)^2=\left(\sqrt{m+7}\right)^2$ Squaring both sides

$m^2-26m+169=m+7$

$m^2-26m+169-m-7=0$

$$m^2 - 27m + 162 = 0$$

$$m^2 - 18m - 9m + 162 = 0$$

$$m(m - 18) - 9(m - 18) = 0$$

$$(m - 18)(m - 9) = 0$$

If $m - 18 = 0$

then $m = 18$

If any $m - 9 = 0$

Then $m = 9$

Check to eliminate extraneous roots $m = 9$

Q.13 $\sqrt{5n+9} = n - 1$

Sol. $\left(\sqrt{5n+9}\right)^2 = (n-1)^2$ (Squaring both sides)

$$5n + 9 = n^2 - 2n + 1$$

$$0 = n^2 - 2n + 1 - 5n - 9$$

$$0 = n^2 - 7n - 8$$

$$\Rightarrow n^2 - 7n - 8 = 0$$

$$n^2 - 8n + n - 8 = 0$$

$$n(n - 8) + 1(n - 8) = 0$$

$$(n - 8)(n + 1) = 0$$

If $n - 8 = 0$

Then $n = 8$

If any $n + 1 = 0$

then $n = -1$

Check to eliminate extraneous roots $n = -1$

Q.14. $3 + \sqrt{2x-1} = 0$

Sol. $\sqrt{2x-1} = 0 - 3$

$$\sqrt{2x-1} = -3$$

Square root is always positive.

Therefore = { } S.S.

Let solve

$$\sqrt{2x-1} = -3$$

$$\left(\sqrt{2x-1}\right)^2 = (-3)^2 \text{ (Squaring both sides)}$$

$$2x-1 = 9$$

$$2x = 9+1$$

$$2x = 10$$

$$\frac{x}{2} = \frac{10}{2} \text{ (Dividing by 2)}$$

$$x = 5$$

$$\text{L. H. S.} = \sqrt{2x-1} \quad \text{To check}$$

$$= \sqrt{2\times 5-1} \quad \text{(Putting } x = 5\text{)}$$

$$= \sqrt{10-1}$$

$$= \sqrt{9}$$

$$= 3$$

Square root is always positive.

Hence, S.S. = { }

Q.15 $\sqrt{x+5}+7 = 0$

Sol. $\sqrt{x+5} = 0-7$

$$\sqrt{x+5} = -7$$

Square root is always taken positive.

Hence, S.S. = { }

Q.16. $\sqrt{2x-1}-\sqrt{x-4}=2$

Sol. $\sqrt{2x-1}=2+\sqrt{x-4}$

Squaring root on both sides

$$\left(\sqrt{2x-1}\right)^2=\left(2+\sqrt{x-4}\right)^2$$

$$2x-1=4+x-4+4\sqrt{x-4}$$

$$2x-1-x=4\sqrt{x-4}$$

$$x-1=4\sqrt{x-4}$$

(Again squaring) $(x-1)^2=\left(4\sqrt{x-4}\right)^2$

$$x^2-2x+1=16(x-4)$$

$$x^2-2x+1=16x-64$$

$$x^2-2x-16x+1+64=0$$

$$x^2-18x+65=0$$

$$x^2-13x-5x+65=0$$

$$x(x-13)-5(x-13)=0$$

$$(x-13)(x-5)=0$$

If $x-13=0$

Then $x=13$

If any $x-5=0$

then $x=5$

S.S. = {13, 5}

Q.17. $\sqrt{x+1}=3$

Sol. $\left(\sqrt{x+1}\right)^2=(3)^2$ (Squaring both sides)

$$x+1=9$$

$$x = 9 - 1$$

$$x = 8$$

Q.18. $\sqrt{2x-1} = 5$

Sol. $\left(\sqrt{2x-1}\right)^2 = (5)^2$ (Squaring both sides)

$$2x - 1 = 25$$

$$2x = 25 + 1$$

$$2x = 26$$

$$x = \frac{26}{2}$$

$$x = 13$$

Q.19. $\sqrt{x-1} = 10$

Sol. $\left(\sqrt{x-1}\right)^2 = (10)^2$ (Squaring both sides)

$$x - 1 = 100$$

$$x = 100 + 1$$

$$x = 101$$

Q.20. $\sqrt{3x+4} = 7$

Sol. $\left(\sqrt{3x+4}\right)^2 = (7)^2$ (Squaring both sides)

$$3x + 4 = 49$$

$$3x = 49 - 4$$

$$3x = 45$$

$$x = \frac{45}{3}$$

$$x = 15$$

Exercise 4.2

Solve and Check:

Q.1. $|x| = 9$

Sol. $x = 9$ And $-x = 9$

$x = 9$

Hence, $x = \pm 9$

Check, $|\pm 9| = 9$ Check

Q.2. $|x - 3| = 4$

Sol.

$x - 3 = 4$	$-(x - 3) = 4$
$x = 4 + 3$	$-x + 3 = 4$
$x = 7$	$-x = 4 - 3$
	$-x = 1$
	$x = -1$

Hence: $x = -1, 7$

Check: $|-1 - 3| = |-4| = 4$ اور $|7 - 3| = |4| = 4$

Q.3. $|x + 1| = 5$

Sol. $x + 1 = 5$	$-(x + 1) = 5$
$x = 5 - 1$	$-x - 1 = 5$
$x = 4$	$-x = 5 + 1$
	$-x = 6$
	$x = -6$

Hence, $x = 4, -6$

Check: $|4 + 1| = |5| = 5$ اور $|-6 + 1| = |-5| = 5$

Q.4. $|2x-3|=5$

Sol.

$$2x-3=5 \qquad\qquad -(2x-3)=5$$
$$2x=5+3 \qquad\qquad -2x+3=5$$
$$2x=8 \qquad\qquad -2x=5-3$$
$$\frac{x}{2}=\frac{8}{2} \qquad\qquad \frac{-2x}{2}=\frac{2}{-2}$$
$$x=4 \qquad\qquad x=-1$$

S.S. = {4, − 1}

Check: $|2(4)-3|=|8-3|=|5|=5$ اور $|2(-1)-3|$

$=|-2-3|=|-5|=5$

Q.5. $|3x+4|=9$

Sol.

$$3x+4=9 \qquad\qquad -(3x+4)=9$$
$$3x=9-4 \qquad\qquad -3x-4=9$$
$$3x=5 \qquad\qquad -3x=9+4$$
$$x=\frac{5}{3} \qquad\qquad -3x=13$$
$$x=-\frac{13}{3}$$

$$\text{S. S.}=\left\{\frac{5}{3},-\frac{13}{3}\right\}$$

Check: Putting $\frac{5}{3}$

$$\left|3\left(\frac{5}{3}\right)+4\right|=|5+4|=|9|=9$$

Putting $-\frac{13}{3}$

$$\left|3\left(-\frac{13}{3}\right)+4\right|=|-13+4|=|-9|=9$$

Q.6. $$3(x-2) < 2x+1$$

Sol. $$3x-6 < 2x+1$$

$$3x-2x < 1+6$$

$$x < 7$$

Check: Let $x = 6$

These putting values

$$3(6-2) < 2(6)+1$$

$$3(4) < 12+1$$

$$12 < 13$$

Which is true

Therefore, $x < 7$

Q.7. $$3(x+5) > 2(x+2)+8$$

Sol. $$3x+15 > 2x+4+8$$

$$3x-2x > 4+8-15$$

$$x > -3$$

Check: Suppose that $x = -2$

$$3(-2+5) > 2(-2+2)+8$$

$$3(3) > 2(0)+8$$

$$9 > 8$$

Which is true

Hence, $x > -3$

Q.8. $$\frac{1}{2}(2-x) > \frac{1}{4}(3-x)+\frac{1}{2}$$

Sol.
$$\frac{2}{2}-\frac{x}{2} > \frac{3}{4}-\frac{x}{4}+\frac{1}{2}$$
$$1-\frac{x}{2} \quad \frac{3}{4}-\frac{x}{4}+\frac{1}{2}$$
$$-\frac{x}{2}+\frac{x}{4} > \frac{3}{4}+\frac{1}{2}-1$$
$$\frac{-2x+x}{4} > \frac{3+2-4}{4}$$
$$-\frac{x}{4} > \frac{1}{4}$$
$$-x > 1 \qquad \text{(Multiplying by 4)}$$

Therefore, $x < -1$

Now, Suppose that $= -2$

- x

Putting values $x = -2$
$$\frac{1}{2}(2+2) > \frac{1}{4}(3+2)+\frac{1}{2}$$
$$\frac{4}{2} > \frac{5}{4}+\frac{1}{2}$$
$$2 > \frac{5+2}{4}$$
$$2 > \frac{7}{4} \qquad \text{(Which is true)}$$

Hence, $x < -1$

Q.9. $$\frac{x-2}{4}+\frac{2}{3} < \frac{x-4}{6}$$

Sol. Mulitiply by L.C.M. of 4, 3, 6 by 12.

$$12\frac{(x-2)}{4}+12\left(\frac{2}{3}\right) < 12\frac{(x-4)}{6}$$

$$3(x-2)+4(2) < 2(x-4)$$

$$3x-6+8 < 2x-8$$

$$3x-2x < -8-8+6$$

$$x < -10$$

Q.10. $$\frac{3x+4}{5}-\frac{x+1}{3} > 1-\frac{x+5}{3}$$

Sol. Multiply by L.C.M of 5, 3 by 15.

$$15\left(\frac{3x+4}{5}\right)-15\left(\frac{x+1}{3}\right) > 15\times1-15\left(\frac{x+5}{3}\right)$$

$$3(3x+4)-5(x+1) > 15-5(x+5)$$

$$9x+12-5x-5 > 15-5x-25$$

$$9x-5x+5x > 15-25-12+5$$

$$9x > -17$$

$$x > -\frac{17}{9}$$

Q.11. $$\frac{x+1}{2}-\frac{x+3}{3} > \frac{x+1}{4}+1$$

Sol. Multiply by 12, L.C.M of 4, 3, 2.

$$12\left(\frac{x+1}{2}\right)-12\left(\frac{x+3}{3}\right) > 12\left(\frac{x+1}{4}\right)+12\times1$$

$$6(x+1)-4(x+3) > 3(x+1)+12$$

$$6x+6-4x-12 > 3x+3+12$$

$$6x-4x-3x > 3+12-6+12$$

$$-x > 21$$

Therefore, $x < -21$ N.T.S

Q.12. $$\frac{x+3}{4}-\frac{x+2}{5} < 1+\frac{x+5}{6}$$

Sol. Multiply by 60, L.C.M. of 4, 5, 6.

$$60\left(\frac{x+3}{4}\right)-60\left(\frac{x+2}{5}\right) < 60\times 1+60\left(\frac{x+5}{6}\right)$$

$$15(x+3)-12(x+2) < 60+10(x+5)$$

$$15x+45-12x-24 < 60+10x+50$$

$$15x-12x-10x < 60+50-45+24$$

$$-7x < 89$$

$$7x < -89 \text{ (Changing symbol)}$$

$$x > -\frac{89}{7}$$

$$x > -12\frac{5}{7}$$

Q.13. $$\frac{1}{2}x \geq 1+\frac{1}{3}x$$

Sol. Multiply by 6, L.C.M. of 2, 3.

$$6\left(\frac{1}{2}x\right) \geq 6\times 1+6\times\frac{1}{3}x$$

$$3x \geq 6+2x$$

$$3x-2x \geq 6$$

$$x \geq 6$$

Q.14. $$\frac{1}{4}(2x+3) \leq (7-4x)$$

Sol. $$4\times\frac{1}{4}(2x+3) \leq 4(7-4x) \text{ (Multiply by 4)}$$

$$2x+3 \leq 28-16x$$

$$+16x \leq 28-3$$

$$18x \leq 25$$

$$\frac{18x}{18} \leq \frac{25}{18} \quad \text{(Dividing by 18)}$$

$$x \leq 1\frac{7}{18}$$

Q.15. $$\frac{4}{3}(2x+3) \geq 10 - \frac{4x}{3}$$

Sol.

$$3 \times \frac{4}{3}(2x+3) \geq 3 \times 10 - 3 \times \frac{4x}{3} \quad \text{(Mulitply by 3)}$$

$$4(2x+3) \geq 30 - 4x$$

$$8x + 12 \geq 30 - 4x$$

$$8x + 4x \geq 30 - 12$$

$$12x \geq 18$$

$$x \geq \frac{18}{12}$$

$$x \geq \frac{3}{2}$$

$$x \geq 1\frac{1}{2}$$

Q.16. $$\frac{x-2}{4} - \frac{x-5}{6} \geq \frac{1}{3}$$

Sol. Multiply by 12, L.C.M. of 4, 6, 3.

$$12\left(\frac{x-2}{4}\right) - 12\left(\frac{x-5}{6}\right) \geq 12 \times \frac{1}{3}$$

$$3(x-2) - 2(x-5) \geq 4$$

$$3x - 6 - 2x + 10 \geq 4$$

$$3x - 2x \geq 4 + 6 - 10$$

$$x \geq 0$$

[illegible]

-I Solve by using factrization method.

Q.1. $x^2 - 4x - 12 = 0$

Sol. $x^2 - 6x + 2x - 12 = 0$

$(x^2 - 6x) + (2x - 12) = 0$

$x(x - 6) + 2(x - 6) = 0$

$(x - 6)(x + 2) = 0$

If $x - 6 = 0$

then $\boxed{x = 6}$

and if $x + 2 = 0$

then $\boxed{x = -2}$

S. S. $= \{6, -2\}$

Q.2. $x^2 - 6x + 5 = 0$

Sol. $x^2 - x - 5x + 5 = 0$

$(x^2 - x) - (5x - 5) = 0$

$x(x - 1) - 5(x - 1) = 0$

$(x - 1)(x - 5) = 0$

If $x - 1 = 0$

then $\boxed{x = 1}$

and if $x - 5 = 0$

then $\boxed{x = 5}$

$$S.S. = \{1, 5\}$$

Q.3. $$x^2 = 8 - 7x$$

Sol.

$$x^2 + 7x - 8 = 0$$

$$x^2 + 8x - x - 8 = 0$$

$$(x^2 + 8x) - (x + 8) = 0$$

$$x(x + 8) - 1(x + 8) = 0$$

$$(x + 8)(x - 1) = 0$$

If $x + 8 = 0$

then $\boxed{x = -8}$

and if $x - 1 = 0$

then $\boxed{x = 1}$

$$\text{Solution set} = \{-8, 1\}$$

Q.4. $$5x = x^2 + 6$$

Sol.

$$0 = x^2 + 6 - 5x$$

$$x^2 + 6 - 5x = 0$$

$$x^2 - 5x + 6 = 0$$

$$x^2 - 2x - 3x + 6 = 0$$

$$(x^2 - 2x) - (3x - 6) = 0$$

$$x(x - 2) - 3(x - 2) = 0$$

$$(x - 2)(x - 3) = 0$$

If $x - 2 = 0$

then $\boxed{x = 2}$

and if $x - 3 = 0$

then $\boxed{x = 3}$

Solution set $= \{2, 3\}$

Q.5. $3x^2 - 10x + 8 = 0$

Sol. $3x^2 - 6x - 4x + 8 = 0$

$(3x^2 - 6x) - (4x - 8) = 0$

$3x(x - 2) - 4(x - 2) = 0$

$(x - 2)(3x - 4) = 0$

If $x - 2 = 0$

then $\boxed{x = 2}$

and if $3x - 4 = 0$

$3x = 4$

then $\boxed{x = \frac{4}{3}}$

Solution set $= \left\{2, \frac{4}{3}\right\}$

Q.6. $2x^2 + 15x - 8 = 0$

Sol. $2x^2 - x + 16x - 8 = 0$

$(2x^2 - x) + (16x - 8) = 0$

$x(2x - 1) + 8(2x - 1) = 0$

$(2x - 1)(x + 8) = 0$

If $2x - 1 = 0$

$2x = 1$

then $\boxed{x = \frac{1}{2}}$

and if $x + 8 = 0$

then $\boxed{x = -8}$

Solution set $= \left\{\frac{1}{2}, -8\right\}$

Q.7. $\frac{x}{4}(x+1) = 3$

Sol. $4\left(\frac{x}{4}\right)(x+1) = 3 \times 4$ (Muliply by 4)

$x(x + 1) = 12$

$x^2 + x = 12$

$x^2 + x - 12 = 0$

$x^2 + 4x - 3x - 12 = 0$

$(x^2 + 4x) - (3x + 12) = 0$

$x(x + 4) - 3(x + 4) = 0$

$(x + 4)(x - 3) = 0$

If $x + 4 = 0$

then $\boxed{x = -4}$

and if $x - 3 = 0$

then $\boxed{x = 3}$

Solution set $= \{-4, 3\}$

Q.8. $$3x^2 - 8x - 3 = 0$$

Sol.

$$3x^2 - 9x + x - 3 = 0$$

$$(3x^2 - 9x) + (x - 3) = 0$$

$$3x(x - 3) + 1(x - 3) = 0$$

$$(x - 3)(3x + 1) = 0$$

If $x - 3 = 0$

then $\boxed{x = 3}$

and if $3x + 1 = 0$

$$3x = -1$$

then $\boxed{x = -\frac{1}{3}}$

$$\text{Solution set} = \left\{3, -\frac{1}{3}\right\}$$

Q.9. $$2x = \frac{2}{x} + 3$$

Sol. (Multiply by x)

$$x(2x) = x\left(\frac{2}{x}\right) + 3(x)$$

$$2x^2 = 2 + 3x$$

$$2x^2 - 3x - 2 = 0$$

$$2x^2 - 4x + x - 2 = 0$$

$$(2x^2 - 4x) + (x - 2) = 0$$

$$2x(x - 2) + 1(x - 2) = 0$$

$$(x - 2)(2x + 1) = 0$$

If $x - 2 = 0$

then $\boxed{x = 2}$

and if $2x + 1 = 0$

$2x = -1$

then $\boxed{x = -\frac{1}{2}}$

Solution set $= \left\{2, -\frac{1}{2}\right\}$

Q.10. $5x^2 - 6x - 8 = 0$

Sol. $5x^2 - 10x + 4x - 8 = 0$

$(5x^2 - 10x) + (4x - 8) = 0$

$5x(x - 2) + 4(x - 2) = 0$

$(x - 2)(5x + 4) = 0$

If $x - 2 = 0$

then $\boxed{x = 2}$

and if $5x + 4 = 0$

$5x = -4$

then $\boxed{x = -\frac{4}{5}}$

Solution set $= \left\{2, -\frac{4}{5}\right\}$

Q.11. $(2x + 3)(x - 2) = 0$

Sol. If $x - 2 = 0$

then $\boxed{x = 2}$

and if $2x + 3 = 0$

$$2x = -3$$

then $\boxed{x = -\frac{3}{2}}$

Solution set $= \left\{2, -\frac{3}{2}\right\}$.

Q.12. $\mathbf{(2x + 1)(5x - 4) = 0}$

Sol. If $2x + 1 = 0$

$$2x = -1$$

then $\boxed{x = -\frac{1}{2}}$

and if $5x - 4 = 0$

$$5x = 4$$

then $\boxed{x = \frac{4}{5}}$

Solution set $= \left\{-\frac{1}{2}, \frac{4}{5}\right\}$

Q.13. $\mathbf{4x(3x - 1) - 2 = (2x - 1)(5x + 1)}$

Sol. $12x^2 - 4x - 2 = 10x^2 - 3x - 1$

$$12x^2 - 10x^2 - 4x + 3x - 2 + 1 = 0$$

$$2x^2 - x - 1 = 0$$

$$2x^2 - 2x + x - 1 = 0$$

$$(2x^2 - 2x) + (x - 1) = 0$$

$$2x(x - 1) + 1(x - 1) = 0$$

$$(x-1)(2x+1) = 0$$

If $x - 1 = 0$

then $\boxed{x = 1}$

and if $2x + 1 = 0$

$$2x = -1$$

then $\boxed{x = -\frac{1}{2}}$

Solution set $= \left\{1, -\frac{1}{2}\right\}$

II- Solve by completing the square method.

Q.14. $x^2 - 10x - 3 = 0$

Sol. $x^2 - 10x = 3$

$\frac{10}{2} = 5$ then add $(5)^2$ on both sides

$$x^2 - 10x + (5)^2 = 3 + (5)^2$$

$$(x-5)^2 = 3 + 25$$

$$(x-5)^2 = 28$$

(By taking square root on both sides)

$$x - 5 = \sqrt{28}$$

$$x - 5 = \pm 2\sqrt{7}$$

$$x = 5 \pm 2\sqrt{7}$$

Q.15. $x^2 - 6x - 3 = 0$

Sol. $x^2 - 6x = 3$

$\frac{6}{2} = 3$ then add $(3)^2$ on both sides

$$x^2 - 6x + (3)^2 = 3 + (3)^2$$

$$(x-3)^2 = 3 + 9$$

$$(x-3)^2 = 12$$

(By taking square root on both sides)

$$x - 3 = \pm\sqrt{12}$$

$$x = \pm 2\sqrt{3}$$

$$x = 3 \pm 2\sqrt{3}$$

Q.16. $x^2 + x - 1 = 0$

Sol. $x^2 + x = 1$

Adding $\left(\frac{1}{2}\right)^2$ on both sides

$$x^2 + x + \left(\frac{1}{2}\right)^2 = 1 + \left(\frac{1}{2}\right)^2$$

$$\left(x + \frac{1}{2}\right)^2 = 1 + \frac{1}{4}$$

$$\left(x + \frac{1}{2}\right)^2 = \frac{4+1}{4}$$

$$\left(x + \frac{1}{2}\right)^2 = \frac{5}{4}$$

$$x + \frac{1}{2} = \pm\sqrt{\frac{5}{4}} \text{ (Taking square root)}$$

$$x = -\frac{1}{2} \pm \sqrt{\frac{5}{4}}$$

$$= -\frac{1}{2} \pm \frac{\sqrt{5}}{2}$$

$$= \frac{-1 \pm \sqrt{5}}{2}$$

Q.17. $x^2 + 6x - 3 = 0$

Sol. $x^2 + 6x = 3$

$\frac{6}{2} = 3$, then $(3)^2$ adding on both sides

$$x^2 + 6x + (3)^2 = 3 + (3)^2$$

$$(x+3)^2 = 3 + 9$$

$$(x+3)^2 = 12$$

$$x + 3 = \pm\sqrt{12} \text{ (Taking square root)}$$

$$x + 3 = \pm 2\sqrt{3}$$

$$x = -3 \pm 2\sqrt{3}$$

Q.18. $2x^2 - 4x + 1 = 0$

Sol. $2x^2 - 4x = -1$

Divided by co-efficient 2 of x.

$$x^2 - 2x = -\frac{1}{2}$$

$\frac{2}{2} = 1$, add $(1)^2$ on both sides

$$x^2 - 2x + (1)^2 = -\frac{1}{2} + (1)^2$$

$$(x-1)^2 = -\frac{1}{2} + 1$$

$$(x-1)^2 = \frac{-1+2}{2}$$

$$(x-1)^2 = \frac{1}{2}$$

$$x - 1 = \pm\sqrt{\frac{1}{2}} \text{ (Taking square root)}$$

thus, $x = 1 \pm \sqrt{\frac{1}{2}}$

Q.19. $$2x^2 - 6x + 3 = 0$$

Sol. $$2x^2 - 6x = -3$$

Divided by co-efficient 2 of x.

$$x^2 - 3x = -\frac{3}{2}$$

Adding $\left(\frac{3}{2}\right)^2$ on both sides

$$x^2 - 3x + \left(\frac{3}{2}\right)^2 = -\frac{3}{2} + \left(\frac{3}{2}\right)^2$$

$$\left(x - \frac{3}{2}\right)^2 = -\frac{3}{2} + \frac{9}{2}$$

$$\left(x - \frac{3}{2}\right)^2 = \frac{-3+9}{2}$$

$$\left(x - \frac{3}{2}\right)^2 = \frac{6}{2}$$

$$\left(x - \frac{3}{2}\right)^2 = 3$$

$$x - \frac{3}{2} = \pm\sqrt{3} \text{ (Taking square root)}$$

$$x = \frac{3}{2} \pm \sqrt{3}$$

$$x = \frac{3 \pm 2\sqrt{3}}{2}$$

Q.20. $$3x^2 + 5x - 4 = 0$$

Sol. $$3x^2 + 5x = 4$$

Divided by 3 co-efficient of x.

$$x^2 + \frac{5}{3}x = \frac{4}{3}$$

$\frac{1}{2}\left(\frac{5}{3}\right)=\frac{5}{6}$, Adding $\left(\frac{5}{6}\right)^2$ on both sides

$$x^2+\frac{5}{3}x+\left(\frac{5}{6}\right)^2 = \left(\frac{5}{6}\right)^2+\frac{4}{3}$$

$$\left(x+\frac{5}{6}\right)^2 = \frac{25}{36}+\frac{4}{3}$$

$$\left(x+\frac{5}{6}\right)^2 = \frac{25+48}{36}$$

$$\left(x+\frac{5}{6}\right)^2 = \frac{73}{36}$$

$$x+\frac{5}{6} = \pm\frac{\sqrt{73}}{6} \text{ (Taking square root)}$$

$$x = -\frac{5}{6}\pm\frac{\sqrt{73}}{6}$$

$$x = \frac{-5\pm\sqrt{73}}{6}$$

Q21. $x^2+mx+n = 0$

Sol. $x^2+mx = -n$

Adding $\left(\frac{m}{2}\right)^2$ on both sides.

$$x^2+mx+\left(\frac{m}{2}\right)^2 = \left(\frac{m}{2}\right)^2-n$$

$$\left(x+\frac{m}{2}\right)^2 = \frac{m^2}{4}-n$$

$$\left(x+\frac{m}{2}\right)^2 = \frac{m^2-4n}{4}$$

$$x+\frac{m}{2} = \pm\sqrt{\frac{m^2-4n}{4}} \text{ (Taking square root)}$$

$$x = -\frac{m}{2} \pm \sqrt{\frac{m^2-4n}{4}}$$

$$x = -\frac{m}{2} \pm \frac{\sqrt{m^2-4n}}{2}$$

$$x = \frac{-m \pm \sqrt{m^2-4n}}{2}$$

Q.22 $11x^2 = 6x+21$

Sol. $11x^2 - 6x = 21$

$$x^2 - \frac{6}{11}x = \frac{21}{11} \text{ (Divided by 11)}$$

$$\left(\frac{6}{11} \times \frac{1}{2}\right)^2 = \frac{3}{11} \text{ adding } \left(\frac{3}{11}\right)^2 \text{ on both sides}$$

$$x^2 - \frac{6}{11}x + \left(\frac{3}{11}\right)^2 = \left(\frac{3}{11}\right)^2 + \frac{21}{11}$$

$$\left(x - \frac{3}{11}\right)^2 = \frac{9}{121} + \frac{21}{11}$$

$$\left(x - \frac{3}{11}\right)^2 = \frac{9+231}{121}$$

$$\left(x - \frac{3}{11}\right)^2 = \frac{240}{121}$$

$$x - \frac{3}{11} = \pm\sqrt{\frac{240}{121}} \text{ (Taking square root)}$$

$$x - \frac{3}{11} = \pm\sqrt{\frac{16 \times 15}{11 \times 11}}$$

$$x - \frac{3}{11} = \pm\frac{4\sqrt{15}}{11}$$

$$x = \frac{3}{11} \pm \frac{4\sqrt{15}}{11}$$

$$x = \frac{3 \pm 4\sqrt{15}}{11}$$

Q23. $2x^2 + 8x - 26 = 0$

Sol. $2x^2 + 8x = 26$

$x^2 + 4x = 13$ (Dividing by 2)

$\frac{4}{2} = 2$ adding $(2)^2$ on both sides.

$$x^2 + 4x + (2)^2 = (2)^2 + 13$$

$$(x+2)^2 = 4 + 13$$

$$(x+2)^2 = 17$$

$$x + 2 = \pm\sqrt{17} \text{ (Taking square root)}$$

$$x = -2 \pm \sqrt{17}$$

Q24. $5x^2 - 20x - 28 = 0$

Sol. $5x^2 - 20x = 28$

$x^2 - 4x = \frac{28}{5}$ (Divided by 5)

$\frac{4}{2} = 2$, Adding $(2)^2$ on both sides.

$$x^2 - 4x + (2)^2 = (2)^2 + \frac{28}{5}$$

$$(x-2)^2 = 4 + \frac{28}{5}$$

$$(x-2)^2 = \frac{20+28}{5}$$

$$(x-2)^2 = \frac{48}{5}$$

$$x - 2 = \pm\sqrt{\frac{48}{5}} \quad \text{(Taking square root)}$$

$$x - 2 = \pm\frac{4\sqrt{3}}{\sqrt{5}}$$

$$x = 2 \pm \frac{4\sqrt{3}}{\sqrt{5}} \quad \text{(Rationalize)}$$

$$x = 2 \pm 4\frac{\sqrt{3}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$$

$$x = 2 \pm \frac{4\sqrt{15}}{5}$$

$$x = \frac{10 \pm 4\sqrt{15}}{5} \quad \text{Ans.}$$

Q25. $x^2 - 11x - 26 = 0$

Sol. $x^2 - 11x = 26$

Adding $\left(\frac{11}{2}\right)^2$ on both sides.

$$x^2 - 11x + \left(\frac{11}{2}\right)^2 = \left(\frac{11}{2}\right)^2 + 26$$

$$\left(x - \frac{11}{2}\right)^2 = \frac{121}{4} + 26$$

$$\left(x - \frac{11}{2}\right)^2 = \frac{121 + 104}{4}$$

$$\left(x - \frac{11}{2}\right)^2 = \frac{225}{4}$$

$$x - \frac{11}{2} = \pm\sqrt{\frac{225}{4}} \quad \text{(Taking square root)}$$

$$x - \frac{11}{2} = \pm\frac{15}{2}$$

$$x = \frac{11}{2} \pm \frac{15}{2}$$

$$x = \frac{11 \pm 15}{2}$$

$$x = \frac{11+15}{2}, \frac{11-15}{2}$$

$$x = \frac{26}{2}, \frac{-4}{2}$$

$$x = 13, -2$$

$$\text{S.S.} = \{13, -2\}$$

5.3 The Quadratic Formula

If $\quad ax^2 + bx + c = 0 \quad a \neq 0$

then $\quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Exercise 5.3

Solve using quadratic formula:

Q.1. $\quad x^2 - 5x + 6 = 0$

Sol. Here $\quad a = 1$

$b = -5$

$c = 6$

(Formula) $\quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Putting values of a, b, c

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(6)}}{2(1)}$$

$$x = \frac{5 \pm \sqrt{25 - 24}}{2}$$

$$x = \frac{5 \pm \sqrt{1}}{2}$$

$$x = \frac{5 \pm 1}{2}$$

$$x = \frac{5+1}{2}, \frac{5-1}{2}$$

$$x = \frac{6}{2}, \frac{4}{2}$$

$$x = 3, 2$$

Solutin set $= \{3, 2\}$

Q.2. $(3 - 4x) = (4x - 3)^2$

Sol. $3 - 4x = 16x^2 - 24x + 9$

$$0 = 16x^2 - 24x + 9 - 3 + 4x$$

$$0 = 16x^2 - 20x + 6$$

or $16x^2 - 20x + 6 = 0$

$8x^2 - 10x + 3 = 0$ (Divided by 2)

Here $a = 8$

$b = -10$

$c = 3$

(Formula) $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Putting the values of a, b, c

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(8)(3)}}{2(8)}$$

$$x = \frac{10 \pm \sqrt{100 - 96}}{16}$$

$$x = \frac{10 \pm \sqrt{4}}{16}$$

$$x = \frac{10 \pm 2}{16}$$

$$x = \frac{10 + 2}{16}, \frac{10 - 2}{16}$$

$$x = \frac{12}{16}, \frac{8}{16}$$

$$x = \frac{3}{4}, \frac{1}{2}$$

Solution set $= \left\{\frac{3}{4}, \frac{1}{2}\right\}$

Q.3. $3x^2 + x - 2 = 0$

Sol. Here $a = 3$

$b = 1$

$c = -2$

(Formula) $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

By putting the values of a, b, c

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(3)(-2)}}{2(3)}$$

$$x = \frac{-1 \pm \sqrt{1 + 24}}{6}$$

$$x = \frac{-1 \pm \sqrt{25}}{6}$$

$$x = \frac{-1 \pm 5}{6}$$

$$x = \frac{-1+5}{6}, \frac{-1-5}{6}$$

$$x = \frac{4}{6}, \frac{-6}{6}$$

$$x = \frac{2}{3}, -1$$

$$\text{Solution set} = \left\{\frac{2}{3}, -1\right\}$$

Q.4. $10x^2 - 5x = 15$

Sol. $10x^2 - 5x - 15 = 0$

$2x^2 - x - 3 = 0$ (Divided by 5)

Here $a = 2$

$b = -1$

$c = -3$

Formula: $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

By putting the values of a, b, c

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-3)}}{2(2)}$$

$$x = \frac{1 \pm \sqrt{1+24}}{4}$$

$$x = \frac{1 \pm \sqrt{25}}{4}$$

$$x = \frac{1 \pm 5}{4}$$

$$x = \frac{1+5}{4}, \frac{1-5}{4}$$

$$x = \frac{6}{4}, \frac{-4}{4}$$

$$x = \frac{3}{2}, -1$$

Solution set $= \left\{\frac{3}{2}, -1\right\}$

Q.5. $\mathbf{(x-1)(x+3)-12=0}$

Sol. $x^2+2x-3-12=0$

$$x^2+2x-15=0$$

Here $a = 1$

$b = 2$

$c = -15$

(Formula) $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

By putting the values of a, b, c

$$x = \frac{-2 \pm \sqrt{(2)^2-4(1)(-15)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{4+60}}{2}$$

$$x = \frac{-2 \pm \sqrt{64}}{2}$$

$$x = \frac{-2 \pm 8}{2}$$

$$x = \frac{-2+8}{2}, \frac{-2-8}{2}$$

$$x = \frac{6}{2}, \frac{-10}{2}$$

$$x = 3, -5$$

Solution set $= \{3, -5\}$

Q.6. $\mathbf{x(2x + 7) - 3(2x + 7) = 0}$

Sol. $2x^2 + 7x - 6x - 21 = 0$

$$2x^2 + x - 21 = 0$$

Here $a = 2$

$b = 1$

$c = -21$

(Formula) $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

By putting the values of a, b, c

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(2)(-21)}}{2(2)}$$

$$x = \frac{-1 \pm \sqrt{1 + 168}}{4}$$

$$x = \frac{-1 \pm \sqrt{169}}{4}$$

$$x = \frac{-1 \pm 13}{4}$$

$$x = \frac{-1 + 13}{4}, \frac{-1 - 13}{4}$$

$$x = \frac{12}{4}, \frac{-14}{4}$$

$$x = 3, -\frac{7}{2}$$

Solution set $= \left\{3, -\frac{7}{2}\right\}$

Q.7. $\dfrac{x+1}{x+4} = \dfrac{2x-1}{x+6}$, **where** $x \neq -4, -6$

Sol. $$(x+1)(x+6) = (2x-1)(x+4)$$

$$x^2 + 7x + 6 = 2x^2 + 7x - 4$$

$$x^2 - 2x^2 + 7x - 7x + 6 + 4 = 0$$

$$-x^2 + 10 = 0$$

$$x^2 - 10 = 0$$

Here $a = 1$

$b = 0$

$c = -10$

(Formula) $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

By putting the values of a, b, c

$$x = \frac{-(0) \pm \sqrt{(0)^2 - 4(1)(-10)}}{2(1)}$$

$$x = \frac{0 \pm \sqrt{0 + 40}}{2}$$

$$x = \frac{0 \pm \sqrt{40}}{2}$$

$$x = \frac{0 \pm 2\sqrt{10}}{2}$$

$$x = \pm \frac{2\sqrt{10}}{2}$$

$$x = \pm\sqrt{10}$$

Solution set $= \left\{\pm\sqrt{10}\right\}$

Q.8. $$\frac{x}{6}+\frac{6}{x}=\frac{4}{x}+\frac{x}{4}, \text{ where } x \neq 0$$

Sol. Multiplying by L.C.M. $12x$

$$12x\left(\frac{x}{6}\right)+12x\left(\frac{6}{x}\right)=12x\left(\frac{4}{x}\right)+12x\left(\frac{x}{4}\right)$$

$$2x^2+72=48+3x^2$$

$$2x^2-3x^2+72-48=0$$

$$-x^2+24=0$$

$$x^2-24=0$$

Here $a=1$

$b=0$

$c=-24$

(Formula) $$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

By putting the values of a, b, c

$$x=\frac{-(0)\pm\sqrt{(0)^2-4(1)(-24)}}{2(1)}$$

$$x=\frac{\pm\sqrt{0+96}}{2}$$

$$x=\frac{\pm\sqrt{96}}{2}$$

$$x=\frac{\pm\sqrt{16\times 6}}{2}$$

$$x=\frac{\pm 4\sqrt{6}}{2}$$

$$x=\pm 2\sqrt{6}$$

Solution set $=\left\{\pm 2\sqrt{6}\right\}$

Q.9. $\frac{x+4}{x-4}+\frac{x-4}{x+4}=\frac{10}{3}$ where $x \neq -4$

Sol. Multiplying by $3(x-4)(x+4)$

$$3(x+4)(x+4)+3(x-4)(x-4) = 10(x-4)(x+4)$$

$$3(x^2+8x+16)+3(x^2-8x+16) = 10(x^2-16)$$

$$3x^2+24x+48+3x^2-24x+48 = 10x^2-160$$

$$3x^2+3x^2-10x^2+24x-24x+48+48+160=0$$

$$-4x^2+256 = 0$$

$$4x^2-256 = 0$$

$$x^2-64 = 0 \quad \text{(Divided by 4)}$$

Here $a = 1$

$b = 0$

$c = -64$

(Formula) $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

By putting the values of a, b, c

$$x = \frac{-0 \pm \sqrt{(0)^2-4(1)(-64)}}{2(1)}$$

$$x = \frac{\pm\sqrt{0+256}}{2}$$

$$x = \frac{\pm\sqrt{256}}{2}$$

$$x = \frac{\pm 16}{2}$$

$$x = \pm 8$$

Solution set $= \{8, -8\}$

Q.10. $$\frac{1}{x-1}+\frac{1}{x-2}=\frac{2}{x-3} \quad \text{where} \quad x \neq 1,2,3$$

Sol. Multiplying by $(x-1)(x-2)(x-3)$

$$(x-1)(x-2)(x-3)\times\frac{1}{(x-1)}+(x-1)(x-2)(x-3)\times\frac{1}{(x-2)}=(x-1)(x-2)(x-3)\times\frac{2}{(x-3)}$$

$$(x-2)(x-3)+(x-1)(x-3)=(x-1)(x-2)(2)$$

$$x^2-5x+6+x^2-4x+3=2(x^2-3x+2)$$

$$\cancel{2x^2}-9x+9=\cancel{2x^2}-6x+4$$

$$-9x+6x=4-9$$

$$-3x=-5$$

$$x=\frac{-5}{-3}$$

$$x=\frac{5}{3}$$

Q.11. $(x+4)(x-1)+(x+5)(x+2)=6$

Sol. $x^2+3x-4+x^2+7x+10=6$

$$2x^2+10x-4+10-6=0$$

$$2x^2+10x=0$$

Here $a=2$

$b=10$

$c=0$

(Formula) $$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

By putting the values of a, b, c

$$x=\frac{-10\pm\sqrt{(10)^2-4(2)(0)}}{2(2)}$$

$$x = \frac{-10 \pm \sqrt{100}}{4}$$

$$x = \frac{-10 \pm 10}{4}$$

$$x = \frac{-10+10}{4}, \frac{-10-10}{4}$$

$$x = \frac{0}{4}, \frac{-20}{4}$$

$$x = 0, -5$$

Solution set $= \{0, -5\}$

Q.12 $(2x+4)^2 - (4x-6)^2 = 0$

Sol.

$$(4x^2 + 16x + 16) - (16x^2 - 48x + 36) = 0$$

$$-12x^2 + 64x - 20 = 0$$

(Dividing by −4)

$$3x^2 - 16x + 5 = 0$$

Here $a = 3$

$b = -16$

$c = 5$

(Formula) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

By putting the values of a, b, c

$$x = \frac{-(-16) \pm \sqrt{(-16)^2 - 4(3)(5)}}{2(3)}$$

$$x = \frac{16 \pm \sqrt{256 - 60}}{6}$$

$$x = \frac{16 \pm \sqrt{196}}{6}$$

$$x = \frac{16 \pm 14}{6}$$

$$x = \frac{16+14}{6}, \frac{16-14}{6}$$

$$x = \frac{30}{6}, \frac{2}{6}$$

$$x = 5, \frac{1}{3}$$

$$\text{Solution set} = \left\{5, \frac{1}{3}\right\}$$

Exercise 5.3

Q.1. Find two consecutive positive odd numbers such that the sum of their squares is 74.

Sol. Let 1^{st} odd number $= 2x + 1$

2^{nd} odd number $= 2x + 3$

According to statement

$$(2x+1)^2 + (2x+3)^2 = 74$$

$$4x^2 + 4x + 1 + 4x^2 + 12x + 9 = 74$$

$$8x^2 + 16x + 10 - 74 = 0$$

$$8x^2 + 16x - 64 = 0$$

$$x^2 + 2x - 8 = 0 \text{ (Divided by 8)}$$

$$x^2 + 4x - 2x - 8 = 0$$

$$(x^2 + 4x) - (2x + 8) = 0$$

$$x(x+4)-2(x+4)=0$$

$$(x+4)(x-2)=0$$

If $x-2=0$

then $\boxed{x=2}$

and if $x+4=0$

then $\boxed{x=-4}$

when $x=2$ then 1st number $=2x+1$

$=2(2)+1$

$=4+1$

$=5$

and 2nd number $=2x+3$

$=2(2)+3$

$=4+3$

$=7$

Required odd number $=5,7$

Q.2. Find two consecutive positive even numbers such that the sum of their squares is 164.

Sol Let 1st number $=2x$

2nd number $=2x+2$

According to the statement

$$(2x)^2+(2x+2)^2=164$$

$$4x^2+4x^2+8x+4=164$$

$$8x^2 + 8x + 4 - 164 = 0$$

$$8x^2 + 8x - 160 = 0$$

$$x^2 + x - 20 = 0 \quad \text{(Divided by 8)}$$

$$x^2 + 5x - 4x - 20 = 0$$

$$(x^2 + 5x) - (4x + 20) = 0$$

$$x(x + 5) - 4(x + 5) = 0$$

$$(x + 5)(x - 4) = 0$$

If $x - 4 = 0$

then $\boxed{x = 4}$

and if $x + 5 = 0$

then $\boxed{x = -5}$

when $x = 4$, then 1st number $= 2x$

$= 2(4) = 8$

And 2nd number $= 2x + 2$

$= 2(4) + 2 = 8 + 2 = 10$

Required even number = 8, 10

Q.3. The difference of two numbers is 9 and the product of the numbers is 162. Find the two numbers.

Sol. Let 1st number $= x$

2nd number $= x + 9$

According to the statement

$$x^2 + 9x = 162$$

$$x^2 + 9x = 162$$

$$x^2 + 9x - 162 = 0$$

$$x^2 + 18x - 9x - 162 = 0$$

$$(x^2 + 18x) - (9x + 162) = 0$$

$$x(x + 18) - 9(x + 18) = 0$$

$$(x - 9)(x + 18) = 0$$

If $x - 9 = 0$

then $\boxed{x = 9}$

and if $x + 18 = 0$

$\boxed{x = -18}$

If $x = 9$, then 1st number $= x = 9$

2nd number $= x + 9$

$= 9 + 9 = 18$

Required numbers $= \boxed{9, 18}$

If $x = -18$

1st number $= x = -18$

2nd number $= x + 9$

$= -18 + 9 = -9$

Required numbers $= \boxed{-18, -9}$

Q.4. **The base and height of a triangle are *(x + 3)cm* and *(2x − 5)cm* respectively. If the area of the triangle is *20cm²*, find *x*.**

Sol.

$$\text{Length of base} = x + 3\text{cm}$$

$$\text{Length of altitude} = 2x - 5\text{cm}$$

$$\text{Area of } \Delta = 20\text{cm}^2$$

According to the statement

$$\frac{(x+3)(2x-5)}{2} = 20$$

$$\text{Area of } \Delta = \frac{B \times A}{2}$$

$$\frac{2x^2 + x - 15}{2} = 20$$

$$2x^2 + x - 15 = 2 \times 20$$

$$2x^2 + x - 15 = 40$$

$$2x^2 + x - 15 - 40 = 0$$

$$2x^2 + x - 55 = 0$$

$$2x^2 + 11x - 10x - 55 = 0$$

$$(2x^2 + 11x) - (10x + 55) = 0$$

$$x(2x + 11) - 5(2x + 11) = 0$$

$$(2x + 11)(x - 5) = 0$$

If $x - 5 = 0$

then $\boxed{x = 5}$

Therefore, Length of base $= x + 3$

$= 5 + 3$

$= 8cm$

Length of altitude $= 2x - 5$

$= 2(5) - 5$

$= 10 - 5$

$= 5cm$

and if $2x + 11 = 0$

$2x = -11$

then $\boxed{x = -\frac{11}{2}}$

Cancelling due to negativity.

Q.5. The perimeter and area of a rectangle are 22cm and $30cm^2$ respectively. Find the length and breadth of the rectangle.

Sol. Rectangle's perimeter $= 22cm$

Rectangle's area $= 30cm^2$

Let, Length of rectangle $= x$ cm

Breadth of rectangle $= \frac{30}{x}$ cm

$\therefore$ Perimeter $= (\text{length} + \text{breadth}) \times 2$

$$22 = \left(\frac{30}{x} + x\right) \times 2$$

$$11 = x + \frac{30}{x} \text{ (Divided by 2)}$$

$$11x = 30 + x^2 \text{ (Multiplying by } x\text{)}$$

$$0 = 30 + x^2 - 11x$$

$$x^2 - 11x + 30 = 0$$

$$x^2 - 5x - 6x + 30 = 0$$

$$(x^2 - 5x) - (6x - 30) = 0$$

$$x(x - 5) - 6(x - 5) = 0$$

$$(x - 5)(x - 6) = 0$$

If $x - 5 = 0$

then $\boxed{x = 5}$

Length of rectangle $= x$

$= 5\text{cm}$

and Breadth of rectangle $= \dfrac{30}{x}$

$= \dfrac{30}{5}$

$= 6 \text{ cm}$

Length of rectangle $= 5\text{cm}$

Breadth of rectangle $= 6\text{cm}$

and if $x - 6 = 0$

then $\boxed{x = 6}$

Length of rectangle $= x$

$= 6\text{cm}$

and Breadth of rectangle $= \dfrac{30}{x}$

$= \dfrac{30}{6}$

$= 5\text{cm}.$

Length of rectangle = 6cm

So, Breadth of rectangle = 5cm

Q.6. The product of two consecutive positive numbers is 156. Find the numbers.

Sol. Let, 1^{st} number $= x$

2^{nd} number $= x + 1$

According to the statement

$$(x)(x+1) = 156$$

$$x^2 + x = 156$$

$$x^2 + x - 156 = 0$$

$$x^2 + 13x - 12x - 156 = 0$$

$$(x^2 + 13x) - (12x + 156) = 0$$

$$x(x+13) - 12(x+13) = 0$$

$$(x-12)(x+13) = 0$$

If $x - 12 = 0$

then $\boxed{x = 12}$

Required numbers $= x, x + 1$

$= 12, 12 + 1$

$= \boxed{12, 13}$

and if $x + 13 = 0$

then $\boxed{x = -13}$

Required numbers $= x, x + 1$

$$= -13, -13+1$$

$$= \boxed{-13, -12}$$

Q.7. Find two consecutive positive odd numbers given that the difference between their reciprocals is $\frac{2}{63}$.

Sol. Suppose that 1^{st} number $= 2x+1$

2^{nd} number $= 2x+3$

According to given condition

$$\frac{1}{2x+1} - \frac{1}{2x+3} = \frac{2}{63}$$

Multiplying by $(2x+1)(2x+3)(63)$

$$63(2x+1)(2x+3)\times\frac{1}{(2x+1)} - 63(2x+1)(2x+3)\times\frac{1}{(2x+3)} = \frac{2}{63}\times 63(2x+1)(2x+3)$$

$$63(2x+3) - 63(2x+1) = 2(2x+1)(2x+3)$$

$$126x + 189 - 126x - 63 = 2(4x^2+8x+3)$$

$$126 = 8x^2+16x+6$$

$$8x^2+16x+6-126 = 0$$

$$8x^2+16x-120 = 0$$

$$x^2+2x-15 = 0 \quad \text{(Dividing by 8)}$$

$$x^2+5x-3x-15 = 0$$

$$(x^2+5x)-(3x+15) = 0$$

$$x(x+5)-3(x+5) = 0$$

$$(x+5)(x-3) = 0$$

If $\quad x-3 = 0$

then $\boxed{x=3}$

Required numbers: 1st number $= 2x+1$ required numbers

$$= 2(3)+1$$

$$= 6+1$$

$$\boxed{1st\ number = 7}$$

2nd number $= 2x+3$

$$= 2(3)+3$$

$$= 6+3$$

$$\boxed{2nd\ number = 9}$$

and if $\quad x+5 = 0$

then $\boxed{x=-5}$

Cancelling due to negativity.

Q.8. The sum of the two positive number is 12 and the sum of whose squares is 80. Find the numbers.

Sol. Suppose that 1st number $= x$

2nd number $= 12-x$

According to given condition

$$(x)^2+(12-x)^2 = 80$$

$$x^2+144+x^2-24x = 80$$

$$2x^2-24x+144-80 = 0$$

$$2x^2-24x+64 = 0$$

$$x^2-12x+32 = 0 \quad \text{(Dividing by 2)}$$

$$x^2 - 4x - 8x + 32 = 0$$

$$x(x-4) - 8(x-4) = 0$$

$$(x-4)(x-8) = 0$$

If $\quad x - 4 = 0$

then $\quad \boxed{x = 4}$

Required numbers: 1st number $= \boxed{x = 4}$

2nd number $= 12 - x$

$$\boxed{2nd\ number = 12 - 4 = 8}$$

and if $\quad x - 8 = 0$

$$x = 8$$

Hence $\quad \boxed{1st\ number = 8}$

2nd number $= 12 - x$

$= 12 - 8$

$$\boxed{2nd\ number = 4}$$

Required numbers $= 8, 4$

or $\quad$ Required numbers $= 4, 8$

Exercise 6.1

With the help of the given matrices answer the questions from 1 to 3.

$$A=\begin{bmatrix}2 & -2\\ -5 & 0\end{bmatrix},\ B=\begin{bmatrix}-3 & -2\\ 0 & 4\end{bmatrix},\ C=\begin{bmatrix}3\\ -1\\ 0\end{bmatrix},$$

$$D=\begin{bmatrix}-3 & 2 & 0\\ 0 & 1 & 5\\ 4 & -2 & 2\end{bmatrix},\ E=\begin{bmatrix}-3 & 2 & 0\end{bmatrix},\ F=\begin{bmatrix}-3 & 4\\ 0 & 5\\ 3 & -1\end{bmatrix}$$

1- **What are the orders of matrices A, C and F?**

2- **What are the orders of matrices B, D and E?**

3- **What element is in the second row and third column of matrix D?**

Answers:

1.(i) (R) Number of rows in matrix A = 2

(C) Number of columns in matrix B = 2

Order of matrix A (R × C) = 2-by-2

(ii) (R) Number of rows in matrix C = 3

(C) Number of columns in matrix C = 1

Order of matrix C(R × C) = 3-by-1

(iii) (R) Number of rows in matrix F = 3

(C) Number of columns in matrix F = 2

Order of matrix F(R × C) = 3-by-2

2.(i) (R) Number of rows in matrix B = 2

(C) Number of columns in matrix B = 2

Order of matrix B(R × C) = 2-by-2

(ii) (R) Number of rows in matrix D = 3

(C) Number of columns in matrix D = 3

Order of matrix D(R × C) = 3-by-3

(iii) (R) Number of rows in matrix E = 1

(C) Number of columns in matrix E = 3

Order of matrix E(R × C) = 1-by-3

3. The element is in the second row and third column of matrix D is 5.

Q.4. Which of the following matrices are equal and which of them are not?

$$A=\begin{bmatrix}4\end{bmatrix},\ B=\begin{bmatrix}1 & 2\end{bmatrix},\ C=\begin{bmatrix}6\\9\end{bmatrix},\ D=\begin{bmatrix}2+2\end{bmatrix},$$

$$E=\begin{bmatrix}3+3\\8+1\end{bmatrix},\ F=\begin{bmatrix}\frac{5}{5} & \frac{4}{2}\end{bmatrix},\ G=\begin{bmatrix}1 & 3\\6 & 8\end{bmatrix},$$

$$H=\begin{bmatrix}1 & 2 & 5\\0 & 3 & 4\\2 & 6 & 3\end{bmatrix},\ I=\begin{bmatrix}1 & 3\\6 & 7\end{bmatrix},\ J=\begin{bmatrix}1 & 3\\6 & \frac{16}{2}\end{bmatrix}$$

$$K = \begin{bmatrix} 1 & 2 & 3+2 \\ 0 & 3 & 4 \\ 2 & 4+2 & 3 \end{bmatrix}, \; L = \begin{bmatrix} 1 & 3 & 5 \\ 0 & 3 & 4 \\ 2 & 6 & 3 \end{bmatrix}$$

Answers:

(i) B = F, G = J, H = K, C = E, A = D

(ii) I and L are not equal to any matrix.

TYPES OF MATRICES

(i) ***Row Matrix:***

A matrix with only one row is called a row matrix.

For example: B = [2 3 4] is of order 1 – by – 3.

(ii) ***Column Matrix:***

A matrix with only one column is called a column matrix.

For example: $D = \begin{bmatrix} 3 \\ 2 \\ 7 \end{bmatrix}$ is of order 3 – by – 1.

(iii) ***Rectangular Matrix:***

If in a matrix, the number of rows and the number of columns are not equal, then the matrix is called a rectangular matrix.

For example: $C = \begin{bmatrix} 1 & 2 & 7 \\ 3 & 4 & 5 \end{bmatrix}$

(iv) ***Square Matrix:***

If a matrix has equal number of rows and columns, it is called a square matrix.

For example: $Q = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \\ 3 & 5 & 7 \end{bmatrix}$

(v) ***Zero or Null Matrix:***

If all the elements in a matrix are zeros, it is called a zero matrix or null matrix. A null matrix is denoted by the letter O.

For example: $O = [0]$ is of order 1 - by - 1.

$O = [0 \quad 0]$ is of order 1 - by - 2.

(vi) ***Diagonal Matrix:***

A square matrix in which all the elements except at least the one element in the diagonal are zeros is called a diagonal matrix.

For example: $A = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ 0 & 3 \end{bmatrix}$

(vii) ***Scalar Matrix:***

A diagonal matrix having equal elements is called a scalar matrix.

For example: $B = \begin{bmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{bmatrix}$

(viii) ***Unit Matrix or Identity Matrix:***

A scalar matrix having each element equal to 1 is called a unit or identity matrix. Identity or unit matrix is generally denoted by I.

For example: $I = [1]$ $\quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(ix) ***Transpose of a Matrix:***

If A is a matrix of order (m - by - n), then a matrix (n - by - m) obtained by interchanging the rows and columns of A is called the transpose of A. It is denoted by A^t.

For example: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, *then* $A^t = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$

(x) ***Symmetric Matrix:***

A square matrix A is called symmetric if $A' = A$

For example: $A = \begin{bmatrix} p & q \\ q & r \end{bmatrix}$, *and* $A' = \begin{bmatrix} p & q \\ q & r \end{bmatrix}$

(xi) ***Skew-Symmetric Matrix:***

A square matrix A is called skew symmetric (or anti-symmetric) if $A' = -A$

For example: $A = \begin{bmatrix} 0 & -2 & 3 \\ 2 & 0 & 4 \\ -3 & -4 & 0 \end{bmatrix}$

$$A' = \begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & -4 \\ 3 & 4 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -2 & 3 \\ 2 & 0 & 4 \\ -3 & -4 & 0 \end{bmatrix} = -A$$

$A' = -A$ Hence A is skew symmetric.

Exercise 6.2

1- Identify row matrices, column matrices, square matrices, and rectangular matrices in the following matrices.

$$A = [3 \ \ 1 \ \ 1 \ \ 1], B = \begin{bmatrix} 5+2 & 4 \\ 2 & 6 \end{bmatrix}, C = \begin{bmatrix} a+x \\ b+y \end{bmatrix},$$

$$D = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}, E = \begin{bmatrix} x & -2 \\ b & 5 \end{bmatrix}, F = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 4 & 5 \\ 1 & -5 & 0 \end{bmatrix},$$

$$G = \begin{bmatrix} 1 & 2 & 4 \\ 5 & 7 & 8 \end{bmatrix}, H = [\,0\,]$$

Solution:

$$A = \begin{bmatrix} 3 & 1 & 1 & 1 \end{bmatrix}$$ Row Matrix

$$C = \begin{bmatrix} (a+x) \\ (b+y) \end{bmatrix}$$ Column Matrix

$$B = \begin{bmatrix} (5+2) & 4 \\ 2 & 6 \end{bmatrix}, D = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$$ Square Matrix

$$E = \begin{bmatrix} x & -2 \\ b & 5 \end{bmatrix}, F = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 4 & 5 \\ 1 & -5 & 0 \end{bmatrix}, H = [0]$$

$$C = \begin{bmatrix} (a+x) \\ (b+y) \end{bmatrix}, G = \begin{bmatrix} 1 & 2 & 4 \\ 5 & 7 & 8 \end{bmatrix}$$ Rectangle Matrix

2. **Identify, diagonal matrices, scalar matrices, identity matrices.**

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix}, C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 4 \end{bmatrix},$$

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, E = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}, F = \begin{bmatrix} 8 & 0 \\ 0 & 0 \end{bmatrix},$$

$$G = \begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix}$$

Solution:

$$G = \begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix}$$ Diagonal Matrix

$$A=\begin{bmatrix}3&0&0\\0&1&0\\0&0&0\end{bmatrix}, C=\begin{bmatrix}0&0&0\\0&0&0\\0&0&4\end{bmatrix}, F=\begin{bmatrix}8&0\\0&0\end{bmatrix}$$

Scalar Matrix

$$B=\begin{bmatrix}7&0&0\\0&7&0\\0&0&7\end{bmatrix}, E=\begin{bmatrix}5&0\\0&5\end{bmatrix}, G=\begin{bmatrix}k&0&0\\0&k&0\\0&0&k\end{bmatrix}$$

$$D=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$ Identity Matrix

3. **Find transpose of the following matrices.**

$$A=\begin{bmatrix}3&4\\-1&4\end{bmatrix}, B=\begin{bmatrix}-3&-2\\-1&4\end{bmatrix},$$

$$C=\begin{bmatrix}a&-b\\c&d\end{bmatrix}, D=\begin{bmatrix}l&m&n\\p&q&r\\a&b&c\end{bmatrix}$$

Solution:

$$A=\begin{bmatrix}3&4\\-1&4\end{bmatrix}, A'=\begin{bmatrix}3&-1\\4&4\end{bmatrix}$$

$$B=\begin{bmatrix}-3&-2\\-1&4\end{bmatrix}, B'=\begin{bmatrix}-3&-1\\-2&4\end{bmatrix}$$

$$C=\begin{bmatrix}a&-b\\c&d\end{bmatrix}, C'=\begin{bmatrix}a&c\\-b&d\end{bmatrix}$$

$$D = \begin{bmatrix} l & m & n \\ p & q & r \\ a & b & c \end{bmatrix}, \; D' = \begin{bmatrix} l & p & a \\ m & q & b \\ n & r & c \end{bmatrix}$$

4. Identify all row matrices, if:

$$A = [3 \;\; 4 \;\; 5], \; B = \begin{bmatrix} -1 & 3 \\ 4 & 6 \end{bmatrix}, C = [e \;\; f \;\; g],$$

$$D = \begin{bmatrix} 3 & 7 & 5 \\ 4 & 6 & 2 \\ 1 & 9 & 8 \end{bmatrix}, \; F = \begin{bmatrix} 1 & 4 & 6 \\ 3 & 7 & 3 \end{bmatrix}$$

Solution: Row Matrix

$$A = [3 \;\; 4 \;\; 5], \; C = [e \;\; f \;\; g]$$

5. Identify all column matrices, if:

$$A = \begin{bmatrix} 3 \\ 6 \\ 10 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 \\ 6 & 5 \\ 4 & 7 \end{bmatrix}, C = \begin{bmatrix} a \\ b \\ c \end{bmatrix},$$

$$D = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 6 & 5 \\ -2 & 3 & 4 \end{bmatrix}, E = \begin{bmatrix} 5 \\ 7 \\ -4 \end{bmatrix}, \; F = [9 \;\; 7 \;\; 1]$$

Solution: Column Matrix

$$A = \begin{bmatrix} 3 \\ 6 \\ 10 \end{bmatrix}, \; C = \begin{bmatrix} a \\ b \\ c \end{bmatrix}, \; E = \begin{bmatrix} 5 \\ 7 \\ -4 \end{bmatrix}$$

6. **Identify all column matrices, if:**

$$A=\begin{bmatrix}1 & 3\\ 2 & 2\end{bmatrix}, B=\begin{bmatrix}3 & -2 & 4\\ 1 & 6 & 5\\ 7 & 3 & 4\end{bmatrix}, C=\begin{bmatrix}1\\ 2\\ 9\end{bmatrix},$$

$$D=\begin{bmatrix}7 & 8\\ 6 & 5\end{bmatrix}, E=\begin{bmatrix}3 & 7 & 5\end{bmatrix}, F=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$$

Solution: Column Matrix

$$C=\begin{bmatrix}1\\ 2\\ 9\end{bmatrix}$$

7. **Identify all 3-by-3 square matrices, if:**

$$A=\begin{bmatrix}2 & -3 & 6\\ 1 & 5 & 4\\ 3 & 6 & -3\end{bmatrix}, B=\begin{bmatrix}2\\ 4\\ 7\end{bmatrix}, C=[7 \quad 3 \quad 4]$$

Solution:

$$A=\begin{bmatrix}2 & -3 & 6\\ 1 & 5 & 4\\ 3 & 6 & -3\end{bmatrix}$$

A Scalar Multiplication

Any element from the set of real numbers is also called a scalar. We define the product of a matrix A and a scalar k, denoted by kA, to be the matrix formed by multiplying each element of A by k.

For example: $A=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$, then $kA=\begin{bmatrix}ka & kb\\ kc & kd\end{bmatrix}$

Laws of Addition of Matrices

Commutative Law:

For any two matrices A and B of the same order

A + B = B + A

This law is called commutative law of matrices with respect to addition.

Associative Law:

For three matrices A, B and C of same order.

(A + B) + C = A + (B + C)

This law is called associative law of matrices with respect to addition.

Additive Identity of Matrices

$A + O = O + A = A$

Additive Inverse of a Matrix

If two matrices A and B are such that their sum (A + B) is a zero matrix, then A and B are called additive inverse of each other.

For example: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, and $B = \begin{bmatrix} -a & -b \\ -c & -d \end{bmatrix}$

Exercise 6.3

1- If $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 5 & 5 \\ 4 & 9 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 1 & 5 \\ 2 & 3 & 6 \\ 1 & 4 & -2 \end{bmatrix}$

Find

(i) A + B (ii) A - B (iii) B - A

(iv) 2A + 3B (v) 3A - 4B (vi) A - 2B

Solutions:

(i)

$$A + B = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 5 & 5 \\ 4 & 9 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 5 \\ 2 & 3 & 6 \\ 1 & 4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2+0 & 3+1 & 4+5 \\ 1+2 & 5+3 & 5+6 \\ 4+1 & 9+4 & 3-2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 & 9 \\ 3 & 8 & 11 \\ 5 & 13 & 1 \end{bmatrix}$$

(ii)

$$A-B = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 5 & 5 \\ 4 & 9 & 3 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 5 \\ 2 & 3 & 6 \\ 1 & 4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2-0 & 3-1 & 4-5 \\ 1-2 & 5-3 & 5-6 \\ 4-1 & 9-4 & 3+2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 & -1 \\ -1 & 2 & -1 \\ 3 & 5 & 5 \end{bmatrix}$$

(iii)

$$B-A = \begin{bmatrix} 0 & 1 & 5 \\ 2 & 3 & 6 \\ 1 & 4 & -2 \end{bmatrix} - \begin{bmatrix} 2 & 3 & 4 \\ 1 & 5 & 5 \\ 4 & 9 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0-2 & 1-3 & 5-4 \\ 2-1 & 3-5 & 6-5 \\ 1-4 & 4-9 & -2-3 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -2 & 1 \\ 1 & -2 & 1 \\ -3 & -5 & -5 \end{bmatrix}$$

(iv)

$$2A + 3B = 2\begin{bmatrix} 2 & 3 & 4 \\ 1 & 5 & 5 \\ 4 & 9 & 3 \end{bmatrix} + 3\begin{bmatrix} 0 & 1 & 5 \\ 2 & 3 & 6 \\ 1 & 4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 6 & 8 \\ 2 & 10 & 10 \\ 8 & 18 & 6 \end{bmatrix} + \begin{bmatrix} 0 & 3 & 15 \\ 6 & 9 & 18 \\ 3 & 12 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} 4+0 & 6+3 & 8+15 \\ 2+6 & 10+9 & 10+18 \\ 8+3 & 18+12 & 6-6 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 9 & 23 \\ 8 & 19 & 28 \\ 11 & 30 & 0 \end{bmatrix}$$

(v)

$$3A-4B = 3\begin{bmatrix} 2 & 3 & 4 \\ 1 & 5 & 5 \\ 4 & 9 & 3 \end{bmatrix} - 4\begin{bmatrix} 0 & 1 & 5 \\ 2 & 3 & 6 \\ 1 & 4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 9 & 12 \\ 3 & 15 & 15 \\ 12 & 27 & 9 \end{bmatrix} - \begin{bmatrix} 0 & 4 & 20 \\ 8 & 12 & 24 \\ 4 & 16 & -8 \end{bmatrix}$$

$$= \begin{bmatrix} 6-0 & 9-4 & 12-20 \\ 3-8 & 15-12 & 15-24 \\ 12-4 & 27-16 & 9+8 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 5 & -8 \\ -5 & 3 & -9 \\ 8 & 11 & 17 \end{bmatrix}$$

(vi)

$$A-2B = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 5 & 5 \\ 4 & 9 & 3 \end{bmatrix} - 2\begin{bmatrix} 0 & 1 & 5 \\ 2 & 3 & 6 \\ 1 & 4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 & 4 \\ 1 & 5 & 5 \\ 4 & 9 & 3 \end{bmatrix} - \begin{bmatrix} 0 & 2 & 10 \\ 4 & 6 & 12 \\ 2 & 8 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 2-0 & 3-2 & 4-10 \\ 1-4 & 5-6 & 5-12 \\ 4-2 & 9-8 & 3+4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & -6 \\ -3 & -1 & -7 \\ 2 & 1 & 7 \end{bmatrix}$$

2- **Find the additive inverse of the following matrices.**

$$A = \begin{bmatrix} 4 & 3 \\ 2 & 6 \end{bmatrix}, B = \begin{bmatrix} \sqrt{2} & 3 \\ 4 & \sqrt{3} \end{bmatrix}, C = \begin{bmatrix} 1 \\ -7 \\ 4 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 3 & 4 \\ 2 & -1 & -3 \end{bmatrix}, E = \begin{bmatrix} 2 & 5 & -3 \end{bmatrix}$$

Sol:

Matrix	**Additive Inverse**
$A = \begin{bmatrix} 4 & 3 \\ 2 & 6 \end{bmatrix}$,	$-A = \begin{bmatrix} -4 & -3 \\ -2 & -6 \end{bmatrix}$,

$$B=\begin{bmatrix}\sqrt{2} & 3\\ 4 & \sqrt{3}\end{bmatrix}, \qquad -B=\begin{bmatrix}-\sqrt{2} & -3\\ -4 & -\sqrt{3}\end{bmatrix},$$

$$C=\begin{bmatrix}1\\ -7\\ 4\end{bmatrix}, \qquad -C=\begin{bmatrix}-1\\ 7\\ -4\end{bmatrix},$$

$$D=\begin{bmatrix}1 & 0 & -2\\ 0 & 3 & 4\\ 2 & -1 & -3\end{bmatrix}, \qquad -D=\begin{bmatrix}-1 & 0 & 2\\ 0 & -3 & -4\\ -2 & 1 & 3\end{bmatrix},$$

$$E=[2 \quad 5 \quad -3] \qquad -E=[-2 \quad -5 \quad 3]$$

3- **If** $A=\begin{bmatrix}2 & 3\\ 1 & 5\end{bmatrix}$ **and** $B=\begin{bmatrix}1 & 7\\ 4 & 6\end{bmatrix}$ **then show that**

(i) $4A-3A=A$ **(ii)** $3B-3A=3(B-A)$

Sol:

(i)

$$\begin{aligned}4A-3A &= 4\begin{bmatrix}2 & 3\\ 1 & 5\end{bmatrix}-3\begin{bmatrix}2 & 3\\ 1 & 5\end{bmatrix}\\ &= \begin{bmatrix}8 & 12\\ 4 & 20\end{bmatrix}-\begin{bmatrix}6 & 9\\ 3 & 15\end{bmatrix}\\ &= \begin{bmatrix}8-6 & 12-9\\ 4-3 & 20-15\end{bmatrix}\\ 4A-3A &= \begin{bmatrix}2 & 3\\ 1 & 5\end{bmatrix}=A\end{aligned}$$

(Proved)

(ii)

$$3B-3A = 3(B-A)$$

L.H.S. $3B - 3A = 3\begin{bmatrix} 1 & 7 \\ 4 & 6 \end{bmatrix} - 3\begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$

$$= \begin{bmatrix} 3 & 21 \\ 12 & 18 \end{bmatrix} - \begin{bmatrix} 6 & 9 \\ 3 & 15 \end{bmatrix}$$

$$= \begin{bmatrix} 3-6 & 21-9 \\ 12-3 & 18-15 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 12 \\ 9 & 3 \end{bmatrix} \ldots\ldots\ldots\ldots (i)$$

R.H.S. $= 3(B - A) = 3\left(\begin{bmatrix} 1 & 7 \\ 4 & 6 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}\right)$

$$= 3\left(\begin{bmatrix} 1-2 & 7-3 \\ 4-1 & 6-5 \end{bmatrix}\right)$$

$$= 3\begin{bmatrix} -1 & 4 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 12 \\ 9 & 3 \end{bmatrix} \quad \textbf{(ii)}$$

$3B - 3A = 3(B - A)$ From (i) and (ii)

4- **Find x and y if** $\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$

Sol:

$$x + 3 = 2$$

$$x = 2 - 3$$

$$\boxed{x = -1}$$

and $3y - 4 = 2$

$$3y = 2 + 4$$

$$3y = 6$$

$$y = \frac{6}{3}$$

$$\boxed{y = 2}$$

5- If $A = \begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 7 \\ 6 & 5 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 6 \\ 3 & -2 \end{bmatrix}$ then

prove that,

(i) $A + B = B + A$ (ii) $A + (B + C) = (A + B) + C$

(i) **A + B = B + A**

Sol:

$$A + B = \begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} 4 & 7 \\ 6 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4 & 3+7 \\ 4+6 & 5+5 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 10 \\ 10 & 10 \end{bmatrix} \text{........ } (i)$$

and $B + A = \begin{bmatrix} 4 & 7 \\ 6 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix}$

$$= \begin{bmatrix} 4+1 & 7+3 \\ 6+4 & 5+5 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 10 \\ 10 & 10 \end{bmatrix} \text{........ } (ii)$$

From (i) and (ii) $A + B = B + A$

(ii) **A + (B + C) = (A + B) + C**

$$A + (B + C) = \begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix} + \left(\begin{bmatrix} 4 & 7 \\ 6 & 5 \end{bmatrix} + \begin{bmatrix} 2 & 6 \\ 3 & -2 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} 4+2 & 7+6 \\ 6+3 & 5-2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} 6 & 13 \\ 9 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+6 & 3+13 \\ 4+9 & 5+3 \end{bmatrix} = \begin{bmatrix} 7 & 16 \\ 13 & 8 \end{bmatrix} \ldots\ldots (i)$$

and

$$(A + B) + C = \left(\begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} 4 & 7 \\ 6 & 5 \end{bmatrix} \right) + \begin{bmatrix} 2 & 6 \\ 3 & -2 \end{bmatrix}$$

$$= \left(\begin{bmatrix} 1+4 & 3+7 \\ 4+6 & 5+5 \end{bmatrix} \right) + \begin{bmatrix} 2 & 6 \\ 3 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 10 \\ 10 & 10 \end{bmatrix} + \begin{bmatrix} 2 & 6 \\ 3 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 5+2 & 10+6 \\ 10+3 & 10-2 \end{bmatrix} = \begin{bmatrix} 7 & 16 \\ 13 & 8 \end{bmatrix} \ldots\ldots (ii)$$

From (i) and (ii)

A + (B + C) = (A + B) + C

6- Solve the matrix equation for X.

$3X - 2A = B$ if $A = \begin{bmatrix} 2 & 3 \\ -4 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -3 \\ 4 & 4 \end{bmatrix}$

Solution:

$$3X - 2A = B$$

$$3X = B + 2A$$

$$3X = \begin{bmatrix} 2 & -3 \\ 4 & 4 \end{bmatrix} + 2\begin{bmatrix} 2 & 3 \\ -4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3 \\ 4 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 6 \\ -8 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2+4 & -3+6 \\ 4-8 & 4+2 \end{bmatrix}$$

$$3X = \begin{bmatrix} 6 & 3 \\ -4 & 6 \end{bmatrix}$$

$$X = \frac{1}{3}\begin{bmatrix} 6 & 3 \\ -4 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 6 \times \frac{1}{3} & 3 \times \frac{1}{3} \\ -4 \times \frac{1}{3} & 6 \times \frac{1}{3} \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \\ -\frac{4}{3} & 2 \end{bmatrix}$$

7- Find a, b, c, d, e and f such that

$$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} - \begin{bmatrix} 3 & -2 & 1 \\ 5 & 0 & -4 \end{bmatrix} = \begin{bmatrix} -1 & -2 & 3 \\ -2 & 4 & 6 \end{bmatrix}$$

Sol: $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} - \begin{bmatrix} 3 & -2 & 1 \\ 5 & 0 & -4 \end{bmatrix} = \begin{bmatrix} -1 & -2 & 3 \\ -2 & 4 & 6 \end{bmatrix}$

$$\begin{bmatrix} a-3 & b+2 & c-1 \\ d-5 & e-0 & f+4 \end{bmatrix} = \begin{bmatrix} -1 & -2 & 3 \\ -2 & 4 & 6 \end{bmatrix}$$

Hence

$a-3=-1,\qquad b+2=-2,\qquad c-1=3$

$a=-1+3\qquad b=-2-2\qquad c=3+1$

$\boxed{a=2}\qquad \boxed{b=-4}\qquad \boxed{c=4}$

and

$d-5=-2,\qquad e-0=4,\qquad f+4=6$

$d=-2+5\qquad \boxed{e=4}\qquad f=6-4$

$\boxed{d=3}\qquad \boxed{f=2}$

8- Find w, x, y, z such that

$$\begin{bmatrix} w & x \\ y & z \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ -1 & 5 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 6 & -3 \end{bmatrix}$$

Sol: $\begin{bmatrix} w+3 & x+0 \\ y-1 & z+5 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 6 & -3 \end{bmatrix}$

Hence $w+3=2,\qquad x+0=1$

$w=2-3\qquad \boxed{x=1}$,

$\boxed{w=-1}$

$y-1=6\qquad z+5=-3$

$y=6+1\qquad z=-3-5$

$\boxed{y = 7}$ $\boxed{z = -8}$

9- If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then what is the additive inverse of A?

Sol: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$-A = -\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$ **(Additive Inverse of A)**

$$= \begin{bmatrix} -a & -b \\ -c & -d \end{bmatrix}$$

10- **Given that** $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ verify that $A^2 - 4A + 5I = 0$.

Sol: $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

L.H.S $= A^2 - 4A + 5I$

$$= \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} - 4\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} + 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2 & -1-3 \\ 2+6 & -2+9 \end{bmatrix} - \begin{bmatrix} 4 & -4 \\ 8 & 12 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -4 \\ 8 & 7 \end{bmatrix} - \begin{bmatrix} 4 & -4 \\ 8 & 12 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -1-4 & -4+4 \\ 8-8 & 7-12 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & 0 \\ 0 & -5 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -5+5 & 0+0 \\ 0+0 & -5+5 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

Hence proved

$$A^2 - 4A + 5I = 0$$

11- If $A = \begin{bmatrix} 2 & 4 \\ 1 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -2 \\ 4 & 6 \end{bmatrix}$, then verify that

$$(A+B)^t = A^t + B^t$$

Sol:

$$A + B = \begin{bmatrix} 2 & 4 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} 3 & -2 \\ 4 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 2+3 & 4-2 \\ 1+4 & 5+6 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 2 \\ 5 & 11 \end{bmatrix}$$

$$(A+B)^t = \begin{bmatrix} 5 & 5 \\ 2 & 11 \end{bmatrix} \text{.......} (i)$$

Now

$$A = \begin{bmatrix} 2 & 4 \\ 1 & 5 \end{bmatrix}$$

$$A^t = \begin{bmatrix} 2 & 1 \\ 4 & 5 \end{bmatrix} \text{.......} (ii)$$

and $$B = \begin{bmatrix} 3 & -2 \\ 4 & 6 \end{bmatrix}$$

$$B' = \begin{bmatrix} 3 & 4 \\ -2 & 6 \end{bmatrix} \ldots\ldots (iii)$$

$$A' + B' = \begin{bmatrix} 2 & 1 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ -2 & 6 \end{bmatrix}$$ From (i) and (ii)

$$= \begin{bmatrix} 2+3 & 1+4 \\ 4-2 & 5+6 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 5 \\ 2 & 11 \end{bmatrix} \ldots\ldots (iv)$$

$$(A+B)' = A' + B'$$ From (i) and (iv)

12- If $A = \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -7 \\ 5 & 8 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 5 \\ 0 & 2 \end{bmatrix}$ **then**

show that $A + B - C = \begin{bmatrix} 2 & -10 \\ 8 & 2 \end{bmatrix}$

Solution:

$$A + B - C = \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix} + \begin{bmatrix} 2 & -7 \\ 5 & 8 \end{bmatrix} - \begin{bmatrix} 1 & 5 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2 & 2-7 \\ 3+5 & -4+8 \end{bmatrix} - \begin{bmatrix} 1 & 5 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -5 \\ 8 & 4 \end{bmatrix} - \begin{bmatrix} 1 & 5 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3-1 & -5-5 \\ 8-0 & 4-2 \end{bmatrix} = \begin{bmatrix} 2 & -10 \\ 8 & 2 \end{bmatrix}$$

Hence proved

$$A + B - C = \begin{bmatrix} 2 & -10 \\ 8 & 2 \end{bmatrix}$$

MULTIPLICATION OF MATRICES

Two matrices A and B are said to be conformable for the product AB, if the number of columns in A is equal to the number of rows in B.

> ***Remember that:***
>
> For multiplication AB of two matrices A and B the following points should be kept in mind.
>
> (i) The number of columns in A = number of rows in B.
>
> (ii) The product of matrices A and B is denoted by $A \times B$ or AB.
>
> (iii) If A is a m-by-p matrix and B is a p-by-n matrix then AB is m-by-n matrix.

Associative Law of Matrices with respect to Multiplication

If three matrices A, B and C are conformable for multiplication, then

$$A(BC) = (AB)C$$

is called associative law with respect to multiplication.

Distributive Laws:

If the matrices A, B and C are conformable for addition and multiplication, then

(i) $A(B + C) = AB + AC$ (left distributive law for matrices)

(ii) $(A + B)C = AC + BC$ (right distributive law for matrices)

(i) and (ii) are called distributive laws.

Exercise 6.4

In Problems 1 to 8 Verify Each Statement, Using

$$A=\begin{bmatrix}4 & 2\\ 0 & 0\end{bmatrix},\ B=\begin{bmatrix}2 & 1\\ -2 & 4\end{bmatrix},\ C=\begin{bmatrix}-1 & 2\\ 4 & 2\end{bmatrix}$$

1. $(AB)C = A(BC)$

L.H.S = (AB)C

$$(AB)C=\left(\begin{bmatrix}4 & 2\\ 0 & 0\end{bmatrix}\begin{bmatrix}2 & 1\\ -2 & 4\end{bmatrix}\right)\times\begin{bmatrix}-1 & 2\\ 4 & 2\end{bmatrix}$$

$$=\left(\begin{bmatrix}8-4 & 4+8\\ 0+0 & 0+0\end{bmatrix}\right)\times\begin{bmatrix}-1 & 2\\ 4 & 2\end{bmatrix}$$

$$=\begin{bmatrix}4 & 12\\ 0 & 0\end{bmatrix}\times\begin{bmatrix}-1 & 2\\ 4 & 2\end{bmatrix}$$

$$=\begin{bmatrix}-4+48 & 8+24\\ 0+0 & 0+0\end{bmatrix}=\begin{bmatrix}44 & 32\\ 0 & 0\end{bmatrix}\text{.......}(i)$$

R.H.S = A(BC)

$$A(BC)=\begin{bmatrix}4 & 2\\ 0 & 0\end{bmatrix}\times\left(\begin{bmatrix}2 & 1\\ -2 & 4\end{bmatrix}\times\begin{bmatrix}-1 & 2\\ 4 & 2\end{bmatrix}\right)$$

$$=\begin{bmatrix}4 & 2\\ 0 & 0\end{bmatrix}\times\left(\begin{bmatrix}-2+4 & 4+2\\ 2+16 & -4+8\end{bmatrix}\right)$$

$$=\begin{bmatrix}4 & 2\\ 0 & 0\end{bmatrix}\times\begin{bmatrix}2 & 6\\ 18 & 4\end{bmatrix}$$

$$= \begin{bmatrix} 8+36 & 24+8 \\ 0+0 & 0+0 \end{bmatrix} = \begin{bmatrix} 44 & 32 \\ 0 & 0 \end{bmatrix} \ldots\ldots (ii)$$

$(AB)C = A(BC)$ from (i) and (ii)

2. $AB \neq BA$

Sol: $$AB = \begin{bmatrix} 4 & 2 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 8-4 & 4+8 \\ 0+0 & 0+0 \end{bmatrix} = \begin{bmatrix} 4 & 12 \\ 0 & 0 \end{bmatrix} \ldots\ldots (i)$$

$$BA = \begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix} \times \begin{bmatrix} 4 & 2 \\ 0 & 0 \end{bmatrix}$$ ب

$$= \begin{bmatrix} 8+0 & 4+0 \\ -8+0 & -4+0 \end{bmatrix} = \begin{bmatrix} 8 & 4 \\ -8 & -4 \end{bmatrix} \ldots\ldots (ii)$$

$AB \neq BA$ from (i) and (ii)

3. $A(B+C) = AB + AC$

Sol: L.H.S = A(B + C)

Putting values of A, B, C

$$A(B+C) = \begin{bmatrix} 4 & 2 \\ 0 & 0 \end{bmatrix} \left(\begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 4 & 2 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 4 & 2 \\ 0 & 0 \end{bmatrix} \left(\begin{bmatrix} 2-1 & 1+2 \\ -2+4 & 4+2 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 4 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 4+4 & 12+12 \\ 0+0 & 0+0 \end{bmatrix} = \begin{bmatrix} 8 & 24 \\ 0 & 0 \end{bmatrix} \text{......} (i)$$

R.H.S = AB + AC

Putting values of A, B, C

$$AB + AC = \begin{bmatrix} 4 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 8-4 & 4+8 \\ 0+0 & 0+0 \end{bmatrix} + \begin{bmatrix} -4+8 & 8+4 \\ 0+0 & 0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 12 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 12 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 4+4 & 12+12 \\ 0+0 & 0+0 \end{bmatrix} = \begin{bmatrix} 8 & 24 \\ 0 & 0 \end{bmatrix} \text{........} (ii)$$

from (i) and (ii)

$$A(B+C) = AB + AC$$

4. $(B+C)A = BA + CA$

Sol: L.H.S = (B + C)A

(B+C)A $(B+C)A = \left(\begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 4 & 2 \end{bmatrix} \right) \begin{bmatrix} 4 & 2 \\ 0 & 0 \end{bmatrix}$

$$= \left(\begin{bmatrix} 2-1 & 1+2 \\ -2+4 & 4+2 \end{bmatrix} \right) \begin{bmatrix} 4 & 2 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix} \begin{bmatrix} 4 & 2 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 4+0 & 2+0 \\ 8+0 & 4+0 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 8 & 4 \end{bmatrix} \text{.......} (i)$$

R.H.S = BA + CA

$BA + CA$

$$BA + CA = \begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} 4 & 2 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} 4 & 2 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 8+0 & 4+0 \\ -8+0 & -4+0 \end{bmatrix} + \begin{bmatrix} -4+0 & -2+0 \\ 16+0 & 8+0 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 4 \\ -8 & -4 \end{bmatrix} + \begin{bmatrix} -4 & -2 \\ 16 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 8-4 & 4-2 \\ -8+16 & -4+8 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 8 & 4 \end{bmatrix} \text{.......} (ii)$$

from (i) and (ii)

$$(B + C)A = BA + CA$$

5. $(B + C)(B - C) \neq B^2 - C^2$

Sol: L.H.S = $(B + C)(B - C)$

Putting values of B, C

$$(B + C)(B - C) = \left(\begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 4 & 2 \end{bmatrix}\right)\left(\begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix} - \begin{bmatrix} -1 & 2 \\ 4 & 2 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 2-1 & 1+2 \\ -2+4 & 4+2 \end{bmatrix}\begin{bmatrix} 2+1 & 1-2 \\ -2-4 & 4-2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ -6 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3-18 & -1+6 \\ 6-36 & -2+12 \end{bmatrix}$$

$$= \begin{bmatrix} -15 & 5 \\ -30 & 10 \end{bmatrix} \text{........} (i)$$

R.H.S $B^2 - C^2 = B \times B - C \times C$

$$= \begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix} - \begin{bmatrix} -1 & 2 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4-2 & 2+4 \\ -4-8 & -2+16 \end{bmatrix} - \begin{bmatrix} 1+8 & -2+4 \\ -4+8 & 8+4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 6 \\ -12 & 14 \end{bmatrix} - \begin{bmatrix} 9 & 2 \\ 4 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 2-9 & 6-2 \\ -12-4 & 14-12 \end{bmatrix}$$

$$= \begin{bmatrix} -7 & 4 \\ -16 & 2 \end{bmatrix} \text{........} (ii)$$

$(B + C)(B - C) \neq B^2 - C^2$ from (i) and (ii)

6. $(BC)' = C'B'$

Sol: L.H.S = $(BC)'$

$$BC = \begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -2+4 & 4+2 \\ 2+16 & -4+8 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 6 \\ 18 & 4 \end{bmatrix}$$

$$(BC)' = \begin{bmatrix} 2 & 18 \\ 6 & 4 \end{bmatrix} \text{........} (i)$$ and

Now R.H.S = $C'B'$

$$C = \begin{bmatrix} -1 & 2 \\ 4 & 2 \end{bmatrix}$$

$$C' = \begin{bmatrix} -1 & 4 \\ 2 & 2 \end{bmatrix}$$

and $$B = \begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix}$$

$$B' = \begin{bmatrix} 2 & -2 \\ 1 & 4 \end{bmatrix}$$

Now $$C'B' = \begin{bmatrix} -1 & 4 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 2 & -2 \\ 1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -2+4 & 2+16 \\ 4+2 & -4+8 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 18 \\ 6 & 4 \end{bmatrix} \ldots\ldots (ii)$$

$(BC)' = C'B'$ from (i) and (ii)

7. $\boldsymbol{BI = B}$

Sol: L.H.S $= BI$

$$BI = \begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$BI = \begin{bmatrix} 2+0 & 0+1 \\ -2+0 & 0+4 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix} = B$$

8. $BC \neq CB$

Sol: $BC = \begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 4 & 2 \end{bmatrix}$

$= \begin{bmatrix} -2+4 & 4+2 \\ 2+16 & -4+8 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 18 & 4 \end{bmatrix}$ .... (i)

$CB = \begin{bmatrix} -1 & 2 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix}$

$= \begin{bmatrix} -2-4 & -1+8 \\ 8-4 & 4+8 \end{bmatrix} = \begin{bmatrix} -6 & 7 \\ 4 & 12 \end{bmatrix}$ .... (ii)

from (i) and (ii)

$BC \neq CB$

Find the Matrix Products.

9. $\begin{bmatrix} 2 & 5 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

Sol: $= \begin{bmatrix} 2(1)+5(2) & 2(-1)+(5)(3) \end{bmatrix}$

$= \begin{bmatrix} 2+10 & -2+15 \end{bmatrix}$

$= \begin{bmatrix} 12 & 13 \end{bmatrix}$

10. $\begin{bmatrix} 3 & 4 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

Sol: $= \begin{bmatrix} 3(-1)+4(2) \\ -1(-1)+(-2)(2) \end{bmatrix}$

$= \begin{bmatrix} -3+8 \\ +1-4 \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$

11. $\begin{bmatrix} 2 & -3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & -2 \end{bmatrix}$

Sol: $= \begin{bmatrix} 2(1)+(-3)(0) & 2(-1)+(-3)(-2) \\ 1(1)+2(0) & 1(-1)+(2)(-2) \end{bmatrix}$

$= \begin{bmatrix} 2+0 & -2+6 \\ 1+0 & -1-4 \end{bmatrix}$

$= \begin{bmatrix} 2 & 4 \\ 1 & -5 \end{bmatrix}$

12. $\begin{bmatrix} -3 & 2 \\ 4 & -1 \end{bmatrix} \begin{bmatrix} -1 & 5 \\ -1 & 3 \end{bmatrix}$

Sol: $= \begin{bmatrix} -3(-1)+(2)(-1) & -3(5)+(2)(3) \\ 4(-1)+(-1)(-1) & 4(5)+(-1)(3) \end{bmatrix}$

$= \begin{bmatrix} 3-2 & -15+6 \\ -4+1 & 20-3 \end{bmatrix}$

$= \begin{bmatrix} 1 & -9 \\ -3 & 17 \end{bmatrix}$

13. $= \begin{bmatrix} -5 & -2 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ 0 & -3 \end{bmatrix}$

Sol: $= \begin{bmatrix} (-5)(-2)+(-2)(0) & (-5)(1)+(-2)(-3) \\ 1(-2)+(-3)(0) & (1)(1)+(-3)(-3) \end{bmatrix}$

$= \begin{bmatrix} 10+0 & -5+6 \\ -2+0 & 1+9 \end{bmatrix}$

$= \begin{bmatrix} 10 & 1 \\ -2 & 10 \end{bmatrix}$

14. $\begin{bmatrix} -2 & 4 \\ 0 & -3 \end{bmatrix}\begin{bmatrix} -5 & -5 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & -0 \end{bmatrix}$

Sol: $= \left(\begin{bmatrix} -2 & 4 \\ 0 & -3 \end{bmatrix}\begin{bmatrix} -5 & -5 \\ 1 & -3 \end{bmatrix}\right)\begin{bmatrix} 1 & -1 \\ 2 & -0 \end{bmatrix}$

$$= \begin{bmatrix} (-2)(-5)+(4)(1) & (-2)(-5)+(4)(-3) \\ (0)(-5)+(-3)(1) & (0)(-5)+(-3)(-3) \end{bmatrix}$$

$$= \begin{bmatrix} 10+4 & 10-12 \\ 0-3 & 0+9 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & -2 \\ -3 & 9 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 14(1)+(-2)(2) & 14(-1)+(-2)(0) \\ -3(1)+9(2) & -3(-1)+9(0) \end{bmatrix}$$

$$= \begin{bmatrix} 14-4 & -14+0 \\ -3+18 & 3+0 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & -14 \\ 15 & 3 \end{bmatrix}$$

15. If $\begin{bmatrix} 1 & 5 \\ 3 & a \end{bmatrix}\begin{bmatrix} b \\ 7 \end{bmatrix} = \begin{bmatrix} 35 \\ 10 \end{bmatrix}$, then find the values of a and b.

Sol: $\begin{bmatrix} 1 & 5 \\ 3 & a \end{bmatrix}\begin{bmatrix} b \\ 7 \end{bmatrix} = \begin{bmatrix} 35 \\ 10 \end{bmatrix}$

$$\begin{bmatrix} 1(b)+5(7) \\ 3(b)+(a)(7) \end{bmatrix} = \begin{bmatrix} 35 \\ 10 \end{bmatrix}$$

$$\begin{bmatrix} b+35 \\ 3b+7a \end{bmatrix} = \begin{bmatrix} 35 \\ 10 \end{bmatrix}$$

Now $b + 35 = 35$

Hence $\boxed{b = 0}$

and $3b + 7a = 10$

$3(0) + 7a = 10$ (Putting values of b)

$7a = 10$

$$\boxed{a = \frac{10}{7}}$$

16. If $A = \begin{bmatrix} 2 & 6 \\ 7 & 8 \end{bmatrix}, B = \begin{bmatrix} -1 & -3 \\ 2 & 0 \end{bmatrix}$, then verify $(AB)' = B'A'$.

Sol:

L.H.S = $(AB)'$

$$AB = \begin{bmatrix} 2 & 6 \\ 7 & 8 \end{bmatrix}\begin{bmatrix} -1 & -3 \\ 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2(-1)+6(2) & 2(-3)+6(0) \\ 7(-1)+8(2) & 7(-3)+8(0) \end{bmatrix}$$

$$= \begin{bmatrix} -2+12 & -6+0 \\ -7+16 & -21+0 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & -6 \\ 9 & -21 \end{bmatrix}$$

$$(AB)' = \begin{bmatrix} 10 & 9 \\ -6 & -21 \end{bmatrix} \ldots\ldots (i)$$

R.H.S = $B'A'$

$$B = \begin{bmatrix} -1 & -3 \\ 2 & 0 \end{bmatrix}$$

$$B' = \begin{bmatrix} -1 & 2 \\ -3 & 0 \end{bmatrix}$$

and $$A = \begin{bmatrix} 2 & 6 \\ 7 & 8 \end{bmatrix}$$

$$A' = \begin{bmatrix} 2 & 7 \\ 6 & 8 \end{bmatrix}$$

Now $$B'A' = \begin{bmatrix} -1 & 2 \\ -3 & 0 \end{bmatrix} \begin{bmatrix} 2 & 7 \\ 6 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} -1(2) + 2(6) & -1(7) + 2(8) \\ -3(2) + (0)(6) & -3(7) + 0(8) \end{bmatrix}$$

$$= \begin{bmatrix} -2 + 12 & -7 + 16 \\ -6 + 0 & -21 + 0 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 9 \\ -6 & -21 \end{bmatrix} \ldots\ldots (ii)$$

$(AB)' = B'A'$ from (i) and (ii)

MULTIPLICATIVE INVERSE OF A MATRIX

Determinant Function

If A is a square matrix, then det A or $|A|$ read "The determinant of A" is used to denote the unique real number.

Singular Matrix:

A square matrix A is called a singular matrix. If *det* A = 0

$$Example : A = \begin{bmatrix} 12 & 6 \\ 6 & 3 \end{bmatrix}$$

$$\det \quad A = \begin{bmatrix} 12 & 6 \\ 6 & 3 \end{bmatrix} = 36 - 36$$

$$\det \quad A = 0. Hence\ matrix\ A\ is\ singular.$$

Non-Singular Matrix:

A square matrix A is called non-singular matrix, if *det* $A \neq 0$.

Example

$$If \quad A = \begin{bmatrix} 2 & 5 \\ 6 & 8 \end{bmatrix}$$

$$\det \quad A = \begin{bmatrix} 2 & 5 \\ 6 & 8 \end{bmatrix} = 16 - 30$$

$$\det \quad A = -14. Hence\ matrix\ A\ is\ non-singular.$$

Adjoint of a Matrix:

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a square matrix of order 2-by-2.

Then the matrix obtained by interchanging the elements of the diagonal (i.e a and d) and by changing the signs of the other elements b and c is called the adjoint of the matrix A.

Multiplicative Inverse

In the set of real numbers, we know that for each real number a (except zero) there exists a real number a^{-1} such that $aa^{-1} = 1$.

The number a^{-1} is called the multiplicative inverse of a.

$$\boxed{A^{-1} = \frac{adj\ A}{|A|}},\ |A| \neq 0$$

> If A is a singular matrix then the multiplicative inverse of A does not exist.

Remember that:

(i) Inverse of square matrix A is denoted by A^{-1}.
(ii) Only non-singular matrices have inverses.
(iii) Inverse of square matrix A is always unique.
(iv) Non-square matrices cannot possess inverses.
(v) $A^{-1} = \frac{adj\ A}{|A|}$

Exercise 6.5

1- Find the determinants of the following matrices.

(i) $\begin{bmatrix} u & v \\ x & y \end{bmatrix}$ (ii) $\begin{bmatrix} -2 & 5 \\ 1 & 4 \end{bmatrix}$

(iii) $\begin{bmatrix} -8 & -4 \\ -4 & -2 \end{bmatrix}$ (iv) $\begin{bmatrix} \frac{1}{1} & \frac{3}{8} \\ \frac{1}{8} & \frac{1}{4} \end{bmatrix}$

(i). $A = \begin{bmatrix} u & v \\ x & y \end{bmatrix}$

Sol: Let $|A| = \begin{vmatrix} u & v \\ x & y \end{vmatrix} = uy - vx$

(ii) $P = \begin{bmatrix} -2 & 5 \\ 1 & 4 \end{bmatrix}$

Sol: Let $|P| = \begin{vmatrix} -2 & 5 \\ 1 & 4 \end{vmatrix}$

$= (-2)(4) - (5)(1)$

$= -8 - 5 = -13$

(iii) $B = \begin{bmatrix} -8 & -4 \\ -4 & -2 \end{bmatrix}$

Sol: Let $| B | = \begin{vmatrix} -8 & -4 \\ -4 & -2 \end{vmatrix}$

$= (-8)(-2) - (-4)(-4)$

$= 16 - 16 = 0$

(iv) $A = \begin{bmatrix} \frac{1}{1} & \frac{3}{8} \\ \frac{1}{8} & \frac{1}{4} \end{bmatrix}$

Sol: Let $|A| = \begin{vmatrix} \frac{1}{1} & \frac{3}{8} \\ \frac{1}{8} & \frac{1}{4} \end{vmatrix}$

$$= (1)\left(\frac{1}{4}\right) - \left(\frac{3}{8}\right)\left(\frac{1}{8}\right)$$

$$= \frac{1}{4} - \frac{3}{64}$$

$$= \frac{16-3}{64} = \frac{13}{64}$$

2. Identify the singular and non-singular matrices.

(i) $\begin{bmatrix} -1 & 3 \\ 1 & -3 \end{bmatrix}$

Sol: Let $A = \begin{bmatrix} -1 & 3 \\ 1 & -3 \end{bmatrix}$

$$|A| = \begin{vmatrix} -1 & 3 \\ 1 & -3 \end{vmatrix}$$

$$= (-1)(-3) - (3)(1)$$

$$= 3 - 3 = 0$$

Therefore, $\begin{bmatrix} -1 & 3 \\ 1 & -3 \end{bmatrix}$ is singular matrix

(ii) $\begin{bmatrix} 3 & 8 \\ 4 & 9 \end{bmatrix}$

Sol: Let $B = \begin{bmatrix} 3 & 8 \\ 4 & 9 \end{bmatrix}$

$$|B| = \begin{vmatrix} 3 & 8 \\ 4 & 9 \end{vmatrix}$$

$$= (3)(9) - (8)(4)$$

$= 27 - 32 = -5 \neq 0$

Therefore, $\begin{bmatrix} 3 & 8 \\ 4 & 9 \end{bmatrix}$ is non-singular matrix.

(iii) $\begin{bmatrix} -a & b \\ a & b \end{bmatrix}$

Sol Let $P = \begin{bmatrix} -a & b \\ a & b \end{bmatrix}$

$|P| = \begin{vmatrix} -a & b \\ a & b \end{vmatrix}$

$= (-a)(b) - (a)(b)$

$= -ab - ab$

$= -2ab \neq 0$

Therefore, $\begin{bmatrix} -a & b \\ a & b \end{bmatrix}$ is non-singular matrix.

3. Find the inverse of each matrix A and show that $A^{-1}A = I$. If the inverse does not exist, give reason.

(i) $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$

Sol: $|A| = \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix}$

$= (1)(3) - (2)(1)$

$= 3 - 2 = 1$

$$A^{-1} = \frac{adj\ A}{|A|}$$

$$= \frac{\begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}}{1}$$

$$= \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$$

Now $A^{-1}A = ?$

$$A^{-1}A = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3(1) + (-2)(1) & 3(2) + (-2)(3) \\ -1(1) + (1)(1) & -1(2) + (1)(3) \end{bmatrix}$$

$$= \begin{bmatrix} 3 - 2 & 6 - 6 \\ -1 + 1 & -2 + 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$A^{-1}A = I$ (proved)

(ii) $A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$

Sol: $|A| = \begin{vmatrix} 2 & 1 \\ 5 & 3 \end{vmatrix} = (2)(3) - (1)(5) = 6 - 5 = 1$

$$A^{-1} = \frac{adj\ A}{|A|}$$

$$= \frac{\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}}{1}$$

$$= \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$$

and $A^{-1}A = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 3(2)+(-1)(5) & 3(1)+(-1)(3) \\ -5(2)+(2)(5) & -5(1)+(2)(3) \end{bmatrix}$$

$$= \begin{bmatrix} 6-5 & 3-3 \\ -10+10 & -5+6 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$A^{-1}A = I$ (proved)

(iii) $A = \begin{bmatrix} 2 & 0 \\ -1 & 3 \end{bmatrix}$

Sol: $|A| = (2)(3) - (-1)(0)$

$$|A| = \begin{bmatrix} 2 & 0 \\ -1 & 3 \end{bmatrix} = (2)(3) - (-1)(0)$$

$$= 6 - 0 = 6$$

$$A^{-1} = \frac{adj\ A}{|A|}$$

$$A^{-1} = \frac{\begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix}}{6}$$

$$= \frac{1}{6}\begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix}$$

Now $A^{-1}A = \frac{1}{6}\begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ -1 & 3 \end{bmatrix}$

$$= \frac{1}{6}\begin{bmatrix} (3)(2)+(0)(-1) & (3)(0)+(0)(3) \\ (1)(2)+(2)(-1) & (1)(0)+(2)(3) \end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix} 6+0 & 0+0 \\ 2-2 & 0+6 \end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{6}{6} & \frac{0}{6} \\ \frac{0}{6} & \frac{6}{6} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$A^{-1}A = I$ (proved)

(iv) $A = \begin{bmatrix} -6 & 4 \\ 3 & -2 \end{bmatrix}$

Sol: $|A| = \begin{vmatrix} -6 & 4 \\ 3 & -2 \end{vmatrix}$

$$= (-6)(-2) - (4)(3)$$

$$= 12 - 12 = 0$$

It is a singular matrix, its inverse is not possible.

(v) $A=\begin{bmatrix}1 & 3\\ 2 & 8\end{bmatrix}$

Sol: $|A|=\begin{vmatrix}1 & 3\\ 2 & 8\end{vmatrix}$

$=(1)(8)-(3)(2)$

$=8-6=2$

$A^{-1}=\dfrac{adj\ A}{|A|}$

$=\dfrac{\begin{bmatrix}8 & -3\\ -2 & 1\end{bmatrix}}{2}$

$=\dfrac{1}{2}\begin{bmatrix}8 & -3\\ -2 & 1\end{bmatrix}$

Now $A^{-1}A=\dfrac{1}{2}\begin{bmatrix}8 & -3\\ -2 & 1\end{bmatrix}\begin{bmatrix}1 & 3\\ 2 & 8\end{bmatrix}$

$=\dfrac{1}{2}\begin{bmatrix}8(1)+(-3)(2) & 8(3)+(-3)(8)\\ -2(1)+1(2) & -2(3)+(1)(8)\end{bmatrix}$

$=\dfrac{1}{2}\begin{bmatrix}8-6 & 24-24\\ -2+2 & -6+8\end{bmatrix}$

$=\dfrac{1}{2}\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}$

$=\begin{bmatrix}\frac{2}{2} & \frac{0}{2}\\ \frac{0}{2} & \frac{2}{2}\end{bmatrix}$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$A^{-1}A = I$ (proved)

(vi) $A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

Sol: $|A| = \begin{vmatrix} -1 & 0 \\ 0 & -1 \end{vmatrix}$

$$= (-1)(-1) - (0)(0)$$

$$= 1 - 0 = 1$$

$$A^{-1} = \frac{adj\ A}{|A|}$$

$$= \frac{\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}}{1}$$

$$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

Now $A^{-1}A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

$$A^{-1}A = \begin{bmatrix} (-1)(-1) + (0)(0) & (-1)(0) + (0)(-1) \\ 0(-1) + (-1)(0) & (0)(0) + (-1)(-1) \end{bmatrix}$$

$$= \begin{bmatrix} 1 + 0 & 0 + 0 \\ 0 + 0 & 0 + 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$A^{-1}A = I$ (proved)

(vii) $$A=\begin{bmatrix}\frac{3}{5} & \frac{-4}{5}\\ \frac{4}{5} & \frac{3}{5}\end{bmatrix}$$

Sol: $$|A|=\begin{vmatrix}\frac{3}{5} & \frac{-4}{5}\\ \frac{4}{5} & \frac{3}{5}\end{vmatrix}$$

$$=\left(\frac{3}{5}\right)\left(\frac{3}{5}\right)-\left(-\frac{4}{5}\right)\left(\frac{4}{5}\right)$$

$$=\frac{9}{25}+\frac{16}{25}$$

$$=\frac{9+16}{25}$$

$$=\frac{25}{25}=1$$

$$A^{-1}=\frac{adj\ A}{|A|}$$

$$=\frac{\begin{bmatrix}\frac{3}{5} & \frac{4}{5}\\ -\frac{4}{5} & \frac{3}{5}\end{bmatrix}}{1}$$

$$=\begin{bmatrix}\frac{3}{5} & \frac{4}{5}\\ -\frac{4}{5} & \frac{3}{5}\end{bmatrix}$$

Now $A^{-1}A = \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ -\frac{4}{5} & \frac{3}{5} \end{bmatrix} \begin{bmatrix} \frac{3}{5} & -\frac{4}{5} \\ \frac{4}{5} & \frac{3}{5} \end{bmatrix}$

$$= \begin{bmatrix} \left(\frac{3}{5}\right)\left(\frac{3}{5}\right)+\left(\frac{4}{5}\right)\left(\frac{4}{5}\right) & \left(\frac{3}{5}\right)\left(-\frac{4}{5}\right)+\left(\frac{4}{5}\right)\left(\frac{3}{5}\right) \\ \left(-\frac{4}{5}\right)\left(\frac{3}{5}\right)+\left(\frac{3}{5}\right)\left(\frac{4}{5}\right) & \left(-\frac{4}{5}\right)\left(-\frac{4}{5}\right)+\left(\frac{3}{5}\right)\left(\frac{3}{5}\right) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{9}{25}+\frac{16}{25} & -\frac{12}{25}+\frac{12}{25} \\ -\frac{12}{25}+\frac{12}{25} & \frac{16}{25}+\frac{9}{25} \end{bmatrix}$$

$$A^{-1}A = \begin{bmatrix} \frac{25}{25} & \frac{0}{25} \\ \frac{0}{25} & \frac{25}{25} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$A^{-1}A = I$ Hence proved

4. Let $M = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

(a) Find M^{-1}

(b) Verify that $M^{-1}M = MM^{-1}$

Sol: $M = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$$|M| = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$$

$$= (1)(4) - (2)(3)$$

$$= 4 - 6 = -2$$

$$M^{-1} = \frac{adj\ M}{|M|}$$

$$= \frac{\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}}{-2}$$

$$= \frac{1}{-2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 4\left(-\frac{1}{2}\right) & (-2)\left(-\frac{1}{2}\right) \\ -3\left(-\frac{1}{2}\right) & (1)\left(-\frac{1}{2}\right) \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$$

$$M^{-1}M = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} (-2)(1) + (1)(3) & (-2)(2) + (1)(4) \\ \left(\frac{3}{2}\right)(1) + \left(-\frac{1}{2}\right)(3) & \left(\frac{3}{2}\right)(2) + \left(-\frac{1}{2}\right)(4) \end{bmatrix}$$

$$= \begin{bmatrix} -2 + 3 & -4 + 4 \\ \frac{3}{2} - \frac{3}{2} & 3 - 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{..........}(i)$$

and now $MM^{-1} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$

$$= \begin{bmatrix} (1)(-2) + (2)\left(\frac{3}{2}\right) & (1)(1) + (2)\left(-\frac{1}{2}\right) \\ (3)(-2) + (4)\left(\frac{3}{2}\right) & (3)(1) + (4)\left(-\frac{1}{2}\right) \end{bmatrix}$$

$$= \begin{bmatrix} -2+3 & 1-1 \\ -6+6 & 3-2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{.......} (ii)$$

$M^{-1}M = MM^{-1}$ from (i) and (ii)

5. If $A = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$.

Sol: L.H.S = $(AB)^{-1}$

$$AB = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} (5)(4) + (2)(3) & (5)(2) + (2)(-1) \\ (2)(4) + (1)(3) & (2)(2) + (1)(-1) \end{bmatrix}$$

$$= \begin{bmatrix} 20+6 & 10-2 \\ 8+3 & 4-1 \end{bmatrix}$$

$$= \begin{bmatrix} 26 & 8 \\ 11 & 3 \end{bmatrix}$$

Now $|AB| = \begin{vmatrix} 26 & 8 \\ 11 & 3 \end{vmatrix}$

$= (26)(3) - (11)(8)$

$= 78 - 88$

$= -10$

$$(AB)^{-1} = \frac{adj\,(AB)}{|AB|}$$

$$= \frac{\begin{bmatrix} 3 & -8 \\ -11 & 26 \end{bmatrix}}{-10} = -\frac{1}{10}\begin{bmatrix} 3 & -8 \\ -11 & 26 \end{bmatrix}$$

$$= \begin{bmatrix} (3)\left(-\frac{1}{10}\right) & (-8)\left(-\frac{1}{10}\right) \\ (-11)\left(-\frac{1}{10}\right) & (26)\left(-\frac{1}{10}\right) \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{3}{10} & \frac{4}{5} \\ \frac{11}{10} & -\frac{13}{5} \end{bmatrix} \text{.............}(i)$$

R.H.S $= B^{-1}A^{-1}$

$B^{-1}A^{-1} = ?$

$B = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}$

$|B| = \begin{vmatrix} 4 & 2 \\ 3 & -1 \end{vmatrix}$

$= (4)(-1) - (2)(3)$

$= -4 - 6$

$= -10$

$$B^{-1} = \frac{adj\ B}{|B|}$$

$$B^{-1} = \frac{\begin{bmatrix} -1 & -2 \\ -3 & 4 \end{bmatrix}}{-10}$$

$$= -\frac{1}{10}\begin{bmatrix} -1 & -2 \\ -3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} (-1)\left(-\frac{1}{10}\right) & (-2)\left(-\frac{1}{10}\right) \\ (-3)\left(-\frac{1}{10}\right) & (4)\left(-\frac{1}{10}\right) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{10} & \frac{1}{5} \\ \frac{3}{10} & -\frac{2}{5} \end{bmatrix}$$

and $A = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$

$$|A| = \begin{vmatrix} 5 & 2 \\ 2 & 1 \end{vmatrix}$$

$= (5)(1) - (2)(2)$

$= 5 - 4$

$= 1$

$$A^{-1} = \frac{adj\ A}{|A|}$$

$$= \frac{\begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix}}{1} = \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix}$$

Now $B^{-1}A^{-1} = \begin{bmatrix} \frac{1}{10} & \frac{1}{5} \\ \frac{3}{10} & -\frac{2}{5} \end{bmatrix} \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix}$

$$= \begin{bmatrix} \left(\frac{1}{10}\right)(1) + \left(\frac{1}{5}\right)(-2) & \left(\frac{1}{10}\right)(-2) + \left(\frac{1}{5}\right)(5) \\ \left(\frac{3}{10}\right)(1) + \left(-\frac{2}{5}\right)(-2) & \left(\frac{3}{10}\right)(-2) + \left(-\frac{2}{5}\right)(5) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{10} - \frac{2}{5} & -\frac{1}{5} + 1 \\ \frac{3}{10} + \frac{4}{5} & -\frac{3}{5} - 2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1-4}{10} & \frac{-1+5}{5} \\ \frac{3+8}{10} & \frac{-3-10}{5} \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{3}{10} & \frac{4}{5} \\ \frac{11}{10} & \frac{13}{5} \end{bmatrix} \quad \text{.......}(ii)$$

from (i) and (ii)

$(AB)^{-1} = B^{-1}A^{-1}$

SOLUTION OF SIMULTANEOUS LINEAR EQUATIONS

To determine the value of two variables, we need a pair of equations. Such a pair of equations is called a system of simultaneous linear equations.

1- Write the equation $2x + ky = 7$ and $4x - 9y = 4$ in matrix form. Also find the value of k if the matrix of the coefficients is singular.

Sol:

$$2x + ky = 7$$

$$4x - 9y = 4$$

In matrix form

$$\begin{bmatrix} 2 & k \\ 4 & -9 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \end{bmatrix}$$

$$A \;\; X = B$$

$$A = \begin{bmatrix} 2 & k \\ 4 & -9 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} 7 \\ 4 \end{bmatrix}$$

$$|A| = 0$$

Now, if A is singular matrix.
Therefore,

$$\begin{bmatrix} 2 & k \\ 4 & -9 \end{bmatrix} = 0$$

$$(2)(-9) - (k)(4) = 0$$

$$-18 - 4k = 0$$

$$-4k = 18$$

$$k = \frac{18}{-4}$$

$$= -\frac{9}{2}$$

2. Solve the simultaneous equations by the matrix inversion method where possible. Where there is no solution, explain why this is so

(i)
$$2x - 5y = 1$$
$$3x - 7y = 2$$

Sol. In matrix form

$$\begin{bmatrix} 2 & -5 \\ 3 & -7 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$A \quad X = B$$

$$A = \begin{bmatrix} 2 & -5 \\ 3 & -7 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

and $$A = \begin{bmatrix} 2 & -5 \\ 3 & -7 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & -5 \\ 3 & -7 \end{vmatrix}$$

$$= (2)(-7) - (-5)(3)$$

$$= -14 + 15 = 1 \neq 0$$

A is non-singular matrix. Therefore, equations can be solve

$$A^{-1} = \frac{adj\ A}{|A|}$$

$$= \frac{\begin{bmatrix} -7 & 5 \\ -3 & 2 \end{bmatrix}}{1}$$

$$= \begin{bmatrix} -7 & 5 \\ -3 & 2 \end{bmatrix}$$

because $X = A^{-1}B$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -7 & 5 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -7(1) + (5)(2) \\ -3(1) + (2)(2) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -7 + 10 \\ -3 + 4 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

Hence, $x = 3$

$y = 1$

$$S.S = \{(3,1)\}$$

(ii) $3x + 2y = 10$

$2y - 3x = -4$

Sol: Write equations again

$3x + 2y = 10$

$-3x + 2y = -4$

In matrix form

$$\begin{bmatrix} 3 & 2 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ -4 \end{bmatrix}$$

$AX = B$

$$A = \begin{bmatrix} 3 & 2 \\ -3 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 10 \\ -4 \end{bmatrix}$$

$$X = A^{-1}B$$

Now $A = \begin{bmatrix} 3 & 2 \\ -3 & 2 \end{bmatrix}$

$$|A| = \begin{vmatrix} 3 & 2 \\ -3 & 2 \end{vmatrix}$$

$$= (3)(2) - (2)(-3)$$

$$= 6 + 6 = 12 \neq 0$$

A is non-singular matrix, therefore equations can be solve

$$A^{-1} = \frac{adj\ A}{|A|}$$

$$= \frac{\begin{bmatrix} 2 & -2 \\ 3 & 3 \end{bmatrix}}{12}$$

$$= \frac{1}{12}\begin{bmatrix} 2 & -2 \\ 3 & 3 \end{bmatrix}$$

Now $\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{12}\begin{bmatrix} 2 & -2 \\ 3 & 3 \end{bmatrix}\begin{bmatrix} 10 \\ -4 \end{bmatrix}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{12}\begin{bmatrix} 2(10) + (-2)(-4) \\ 3(10) + (3)(-4) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{12}\begin{bmatrix} 20 + 8 \\ 30 - 12 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{12}\begin{bmatrix} 28 \\ 18 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{28}{12} \\ \frac{18}{12} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{7}{3} \\ \frac{3}{2} \end{bmatrix}$$

Hence, $x = \frac{7}{3}$

$y = \frac{3}{2}$

$$S.S = \left\{ \left(\frac{7}{3}, \frac{3}{2} \right) \right\}$$

(iii) $4x + 5y = 0$

$2x + 5y = 1$

Sol: $\begin{bmatrix} 4 & 5 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ In matrix form

Let $A \; X = B$

Now $A^{-1} A \; X = A^{-1} B$

$X = A^{-1} B$

Now $A = \begin{bmatrix} 4 & 5 \\ 2 & 5 \end{bmatrix}$

$$|A| = \begin{vmatrix} 4 & 5 \\ 2 & 5 \end{vmatrix}$$

$$|A| = (4)(5) - (5)(2)$$

$$= 20 - 10 = 10 \neq 0$$

A is non-singular matrix, therefore equations can be solve.

$$|A| = 10$$

$$A^{-1} = \frac{adj\ A}{|A|}$$

$$= \frac{\begin{bmatrix} 5 & -5 \\ -2 & 4 \end{bmatrix}}{10} = \frac{1}{10}\begin{bmatrix} 5 & -5 \\ -2 & 4 \end{bmatrix}$$

Now $A^{-1}B = \frac{1}{10}\begin{bmatrix} 5 & -5 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$= \frac{1}{10}\begin{bmatrix} (5)(0) + (-5)(1) \\ (-2)(0) + (4)(1) \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 0 - 5 \\ 0 + 4 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} -5 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-5}{10} \\ \frac{4}{10} \end{bmatrix}$$

$$A^{-1}B = \begin{bmatrix} -\frac{1}{2} \\ \frac{2}{5} \end{bmatrix}$$

But $X = A^{-1}B$

Thus

$$X = \begin{bmatrix} -\frac{1}{2} \\ \frac{2}{5} \end{bmatrix}$$

or

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} \\ \frac{2}{5} \end{bmatrix}$$

Hence, $x = -\frac{1}{2}$

$$y = \frac{2}{5}$$

$$S.S = \left\{\left(-\frac{1}{2}, \frac{2}{5}\right)\right\}$$

(iv)

$$5x + 6y = 25$$

$$3x + 4y = 17$$

Sol: $\begin{bmatrix} 5 & 6 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 25 \\ 17 \end{bmatrix}$ In matrix form

Let $A \quad X = B$

$$A^{-1}AX = A^{-1}B$$

$$X = A^{-1}B$$

Now $A = \begin{bmatrix} 5 & 6 \\ 3 & 4 \end{bmatrix}$

$$|A| = \begin{vmatrix} 5 & 6 \\ 3 & 4 \end{vmatrix}$$

$$= (5)(4) - (6)(3)$$

$$= 20 - 18$$

$$|A| = 2 \neq 0$$

A is non-singular matrix. Therefore equations can be solve.

$$A^{-1} = \frac{adj\ A}{|A|}$$

$$= \frac{\begin{bmatrix} 4 & -6 \\ -3 & 5 \end{bmatrix}}{2}$$

$$= \frac{1}{2}\begin{bmatrix} 4 & -6 \\ -3 & 5 \end{bmatrix}$$

Now $A^{-1}B = \frac{1}{2}\begin{bmatrix} 4 & -6 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} 25 \\ 17 \end{bmatrix}$

$$= \frac{1}{2}\begin{bmatrix} (4)(25) + (-6)(17) \\ (-3)(25) + (5)(17) \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 100 - 102 \\ -75 + 85 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} -2 \\ 10 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-2}{2} \\ \frac{10}{2} \end{bmatrix}$$

$$A^{-1}B = \begin{bmatrix} -1 \\ 5 \end{bmatrix}$$

But $X = A^{-1}B$

Thus $X = \begin{bmatrix} -1 \\ 5 \end{bmatrix}$

or $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 5 \end{bmatrix}$

Hence, $x = -1$

$y = 5$

$S.S = \{(-1, 5)\}$

(v) $x + y = 2$

$y = 2 + x$

Sol: Write equations again

$x + y = 2$

$-x + y = 2$

In matrix form

$$\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

Let $A \ \ X = B$

$$A^{-1}AX = A^{-1}B$$

$$X = A^{-1}B$$

Now $$A = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix}$$

$$= (1)(1) - (1)(-1)$$

$$= 1 + 1$$

$$|A| = 2 \neq 0$$

A is non-singular matrix. Therefore equations can be solve.

$$A^{-1} = \frac{adj\ A}{|A|}$$

$$= \frac{\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}}{2} = \frac{1}{2}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

and $$A^{-1}B = \frac{1}{2}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

Now

$$= \frac{1}{2}\begin{bmatrix} (1)(2) + (-1)(2) \\ (1)(2) + (1)(2) \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 2 - 2 \\ 2 + 2 \end{bmatrix}$$

$$=\frac{1}{2}\begin{bmatrix}0\\4\end{bmatrix}$$

$$=\begin{bmatrix}\frac{0}{2}\\\frac{4}{2}\end{bmatrix}$$

$$A^{-1}B=\begin{bmatrix}0\\2\end{bmatrix}$$

while $X=A^{-1}B$

$$X=\begin{bmatrix}0\\2\end{bmatrix}$$

or $\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}0\\2\end{bmatrix}$

Hence, $x=0$

$y=2$

$S.S=\{(0,2)\}$

(vi) $\frac{x}{2}+\frac{y}{3}=1$ (i)

$-4x+y=14$ (ii)

Sol: Multiply (i) by 6.

$$6\left(\frac{x}{2}\right)+6\left(\frac{y}{3}\right)=6$$

$3x+2y=6$

(2nd condition)

$-4x+y='4$

$$\begin{bmatrix} 3 & 2 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 14 \end{bmatrix}$$ (In matrix form)

Let $A\ X = B$

$$A^{-1}A\ X = A^{-1}B$$

$$X = A^{-1}B$$

Now $A = \begin{bmatrix} 3 & 2 \\ -4 & 1 \end{bmatrix}$

$$|A| = \begin{bmatrix} 3 & 2 \\ -4 & 1 \end{bmatrix}$$

$$= (3)(1) - (2)(-4)$$

$$= 3 + 8$$

$$= 11 \neq 0$$

A is non-singular matrix. Therefore equations can be solve.

$$A^{-1} = \frac{adj\ A}{|A|}$$

$$= \frac{\begin{bmatrix} 1 & -2 \\ 4 & 3 \end{bmatrix}}{11} = \frac{1}{11}\begin{bmatrix} 1 & -2 \\ 4 & 3 \end{bmatrix}$$

and - now $A^{-1}B = \frac{1}{11}\begin{bmatrix} 1 & -2 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 6 \\ 14 \end{bmatrix}$

$$= \frac{1}{11}\begin{bmatrix} (1)(6) + (-2)(14) \\ (4)(6) + (3)(14) \end{bmatrix}$$

$$= \frac{1}{11}\begin{bmatrix} 6-28 \\ 24+42 \end{bmatrix}$$

$$= \frac{1}{11}\begin{bmatrix} -22 \\ 66 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-22}{11} \\ \frac{66}{11} \end{bmatrix}$$

$$A^{-1}B = \begin{bmatrix} -2 \\ 6 \end{bmatrix}$$

while $X = A^{-1}B$

$$X = \begin{bmatrix} -2 \\ 6 \end{bmatrix}$$

or $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 6 \end{bmatrix}$

Hence, $x = -2$

$y = 6$

$$S.S = \{(-2, 6)\}$$

3. Solve, using matrix inversion method

$$3x - y = 10$$

$$2x + 3y = 3$$

In matrix form

$$\begin{bmatrix} 3 & -1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 3 \end{bmatrix}$$

Let $A\ X = B$

and $A^{-1}A\ X = A^{-1}B$

$$X = A^{-1}B$$

Now $A = \begin{bmatrix} 3 & -1 \\ 2 & 3 \end{bmatrix}$

$$|A| = \begin{bmatrix} 3 & -1 \\ 2 & 3 \end{bmatrix}$$

$$= (3)(3) - (-1)(2)$$

$$= 9 + 2$$

$$= 11 \neq 0$$

A is non-singular matrix. Therefore equations can be solve.

$$A^{-1} = \frac{adj\ A}{|A|}$$

$$= \frac{\begin{bmatrix} 3 & 1 \\ -2 & 3 \end{bmatrix}}{11} = \frac{1}{11}\begin{bmatrix} 3 & 1 \\ -2 & 3 \end{bmatrix}$$

and now $A^{-1}B = \frac{1}{11}\begin{bmatrix} 3 & 1 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 10 \\ 3 \end{bmatrix}$

$$= \frac{1}{11}\begin{bmatrix} (3)(10) + (1)(3) \\ (-2)(10) + (3)(3) \end{bmatrix}$$

$$= \frac{1}{11}\begin{bmatrix} 30 + 3 \\ -20 + 9 \end{bmatrix}$$

$$= \frac{1}{11}\begin{bmatrix} 33 \\ -11 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{33}{11} \\ \frac{-11}{11} \end{bmatrix}$$

$$A^{-1}B = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

while $X = A^{-1}B$

$$X = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

or $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

Hence, $x = 3$

$y = -1$

$S.S = \{(3, -1)\}$

4. Use Cramer's rule to solve the simultaneous equations. Give the reason where solution is not possible.

(i) $x + 2y = 3$

$x + 3y = 5$

Remember that:

Cramer's Rule

If $a_1x + a_2y = b_1$

$$a_3x + a_4y = b_2$$

then

$$x = \frac{\begin{vmatrix} b_1 & a_2 \\ b_2 & a_4 \end{vmatrix}}{\begin{vmatrix} a_1 & a_2 \\ a_3 & a_4 \end{vmatrix}} = \frac{|D_1|}{|A|}$$

$$y = \frac{\begin{vmatrix} a_1 & b_1 \\ a_3 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & a_2 \\ a_3 & a_4 \end{vmatrix}} = \frac{|D_2|}{|A|}$$

Sol: $x + 2y = 3$

$$x + 3y = 5$$

$$\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$ In matrix form

$$AX = B$$

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} = (1)(3) - (1)(2) = 3 - 2 = 1$$

$$|D_1| = \begin{vmatrix} 3 & 2 \\ 5 & 3 \end{vmatrix} = (3)(3) - (2)(5) = 9 - 10 = -1$$

$$|D_2| = \begin{vmatrix} 1 & 3 \\ 1 & 5 \end{vmatrix} = (1)(5) - (3)(1) = 5 - 3 = 2$$

$$x = \frac{|D_1|}{|A|} = \frac{-1}{1} = -1$$

$$y=\frac{|D_2|}{|A|}=\frac{2}{1}=2$$

$$S.S=\{(-1,2)\}$$

(ii) $2x+y=1$

$5x+3y=2$

Sol: $\begin{bmatrix}2 & 1\\5 & 3\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}1\\2\end{bmatrix}$ In matrix form

$AX=B$

$$A=\begin{bmatrix}2 & 1\\5 & 3\end{bmatrix}$$

$$|A|=\begin{vmatrix}2 & 1\\5 & 3\end{vmatrix}$$

$$=(2)(3)-(1)(5)$$

$$=6-5=1$$

$$|D_1|=\begin{vmatrix}1 & 1\\2 & 3\end{vmatrix}$$

$$=(1)(3)-(1)(2)$$

$$=3-2=1$$

$$|D_2|=\begin{vmatrix}2 & 1\\5 & 2\end{vmatrix}=(2)(2)-(1)(5)$$

$$=4-5$$

$$=-1$$

$$x=\frac{|D_1|}{|A|}=\frac{1}{1}=1$$

$$y=\frac{|D_2|}{|A|}=\frac{-1}{1}=-1$$

S.S= $\{(1,-1)\}$

(iii) $x+3y=1$

$2x+8y=0$

Sol: $\begin{bmatrix}1 & 3\\ 2 & 8\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}1\\ 0\end{bmatrix}$ In matrix form

$AX=B$

$$A=\begin{bmatrix}1 & 3\\ 2 & 8\end{bmatrix}$$

$$|A|=\begin{vmatrix}1 & 3\\ 2 & 8\end{vmatrix}=(1)(8)-(3)(2)$$

$$=8-6=2$$

$$|D_1|=\begin{vmatrix}1 & 3\\ 0 & 8\end{vmatrix}=(1)(8)-(3)(0)$$

$$=8-0$$

$$=8$$

$$|D_2|=\begin{vmatrix}1 & 1\\ 2 & 0\end{vmatrix}=(1)(0)-(1)(2)$$

$$=0-2$$

$$=-2$$

$$x = \frac{|D_1|}{|A|} = \frac{8}{2} = 4$$

$$y = \frac{|D_2|}{|A|} = \frac{-2}{2} = -1$$

$$S.S = \{(4, -1)\}$$

(iv) $-2x + 6y = 5$

$x - 3y = -7$

Sol: $\begin{bmatrix} -2 & 6 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -7 \end{bmatrix}$ In matrix form

$AX = B$

$$A = \begin{bmatrix} -2 & 6 \\ 1 & -3 \end{bmatrix}$$

$$|A| = \begin{vmatrix} -2 & 6 \\ 1 & -3 \end{vmatrix}$$

$$= (-2)(-3) - (6)(1)$$

$$= 6 - 6 = 0$$

$A = \begin{bmatrix} -2 & 6 \\ 1 & -3 \end{bmatrix}$ is singular matrix, therefore S.S is not possible.

(v) $x - 3y = 5$

$2x - 5y = 9$

Sol: $\begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 9 \end{bmatrix}$ In matrix form

$$AX = B$$

$$A = \begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & -3 \\ 2 & -5 \end{vmatrix}$$

$$= (1)(-5) - (-3)(2)$$

$$= -5 + 6 = 1$$

$$|D_1| = \begin{vmatrix} 5 & -3 \\ 9 & -5 \end{vmatrix} = (5)(-5) - (-3)(9)$$

$$= -25 + 27$$

$$= 2$$

$$|D_2| = \begin{vmatrix} 1 & 5 \\ 2 & 9 \end{vmatrix}$$

$$= (1)(9) - (5)(2)$$

$$= 9 - 10 = -1$$

$$x = \frac{|D_1|}{|A|} = \frac{2}{1} = 2$$

$$y = \frac{|D_2|}{|A|} = \frac{-1}{1} = -1$$

$$\text{s.s} = \{(2, -1)\}$$

(vi) $5x + 2y = 13$

$2x + 5y = 17$

Sol: $\begin{bmatrix} 5 & 2 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 13 \\ 17 \end{bmatrix}$ In matrix form

$AX = B$

$A = \begin{bmatrix} 5 & 2 \\ 2 & 5 \end{bmatrix}$

$|A| = \begin{vmatrix} 5 & 2 \\ 2 & 5 \end{vmatrix} = (5)(5) - (2)(2)$

$= 25 - 4$

$= 21$

$|D_1| = \begin{vmatrix} 13 & 2 \\ 17 & 5 \end{vmatrix} = (13)(5) - (2)(17)$

$= 65 - 34$

$= 31$

$|D_2| = \begin{vmatrix} 5 & 13 \\ 2 & 17 \end{vmatrix} = (5)(17) - (13)(2)$

$= 85 - 26 = 59$

$x = \dfrac{|D_1|}{|A|} = \dfrac{31}{21}$

$y = \dfrac{|D_2|}{|A|} = \dfrac{59}{21}$

$\text{S.S} = \left\{\left(\dfrac{31}{21}, \dfrac{59}{21}\right)\right\}$

5. Write the following matrices in the form of linear equations.

(i) $\begin{bmatrix} 2 & -1 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

Sol: $\begin{bmatrix} (2)(x) + (-1)(y) \\ (5)(x) + (2)(y) \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

then $\begin{bmatrix} 2x - y \\ 5x + 2y \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

and $2x - y = 2$

$5x + 2y = 4$

(ii) $\begin{bmatrix} -5 & 2 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

Sol: $\begin{bmatrix} (-5)(x) + (2)(y) \\ (2)(x) + (-3)(y) \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

$\begin{bmatrix} -5x + 2y \\ 2x - 3y \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

thus, $-5x + 2y = 2$

and $2x - 3y = -1$

(iii) $\begin{bmatrix} -4 & 1 \\ 5 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Sol: $\begin{bmatrix} -4(x) + (1)(y) \\ 5(x) + (4)(y) \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\begin{bmatrix} -4x + y \\ 5x + 4y \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

thus, $-4x + y = 1$

and $5x + 4y = -1$

(iv) $$\begin{bmatrix} 0.8 & -0.6 \\ 0.6 & 0.8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Sol: $$\begin{bmatrix} (0.8)(x) + (-0.6)(y) \\ (0.6)(x) + (0.8)(y) \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 0.8x - 0.6y \\ 0.6x + 0.8y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

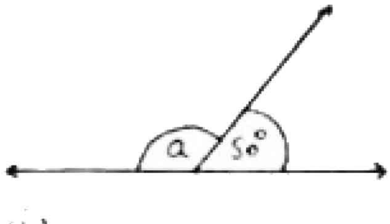
thus, $0.8x - 0.6y = 1$

and $0.6x + 0.8y = 2$

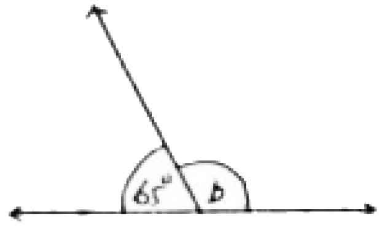
Exercise # 4

1. Write down the angles marked with letters. Write whether the angles are complimentary or supplementary?

Sol:



(i)



(i) $m\angle a + 50^\circ = 180^\circ$

$m\angle a = 180^\circ - 50^\circ$

$m\angle a = 130^\circ$

(Supplementary angles)

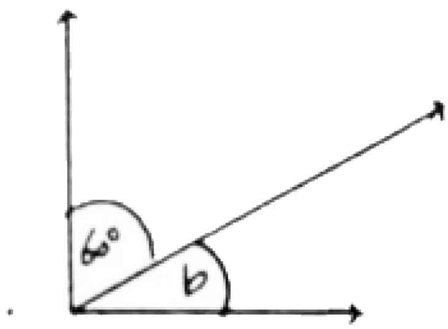
(ii) $m\angle b + 65^\circ = 180^\circ$

$m\angle b = 180^\circ - 65^\circ$

$m\angle b = 115^\circ$

(Supplementary angles)

(iii)



(iii) $m\angle c + 138^\circ = 180^\circ$

$m\angle c = 180^\circ - 138^\circ$

$m\angle c = 42^\circ$

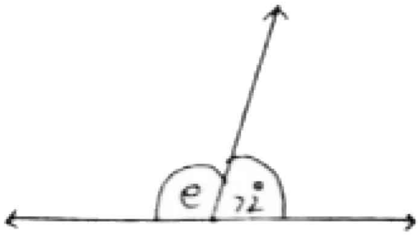
(Supplementary angles)

(iv) $m\angle b + 60^\circ = 90^\circ$

$m\angle b = 90^\circ - 60^\circ$

$m\angle b = 30^\circ$

(Complementary angles)

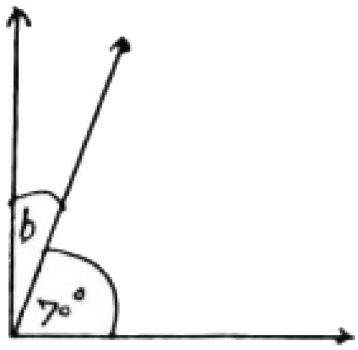


(v) $m\angle e + 72^\circ = 180^\circ$

$m\angle e = 180^\circ - 72^\circ$

$m\angle e = 108^\circ$

(Supplementary angles)



(vi) $m\angle b + 70^\circ = 90^\circ$

$m\angle b = 90^\circ - 70^\circ$

$m\angle b = 20^\circ$

(Complementary angles)

2. Two angles are supplementary and the greater exceeds the smaller by 30°. How many degrees are there in each angle?

Sol: The size of small angle $= x$ Let

The size of large angle $= x + 30^\circ$

According to statement

$$x + (x + 30^\circ) = 180^\circ$$

$$x + x + 30^\circ = 180^\circ$$

$$2x + 30^\circ = 180^\circ$$

$$2x = 180^\circ - 30^\circ$$

$$2x = 150^\circ$$

$$x = \frac{150^\circ}{2}$$

$$x = 75^\circ$$

The size of small gle $= 75^\circ$

The size of large angle $= x + 30^\circ = 75^\circ + 30^\circ = 105^\circ$

angles $= 75^\circ, 105^\circ$

3. If 40° is added to an angle, the resulting angle is equal to the supplement of the original angle. Find the original angle.

Sol: Let the required angle $= x$

size of the angle after adding $= x + 40^{\circ}$

The supplement of 1st angle $= 180^{\circ} - x$

According to statement

$$x + 40^{\circ} = 180^{\circ} - x$$

$$x + x = 180^{\circ} - 40^{\circ}$$

$$2x = 140^{\circ}$$

$$x = \frac{140^{\circ}}{2}$$

$$x = 70^{\circ}$$

4. The sum of two angles is 100°, and the difference between their supplements is 100°. Find the angles.

Let the size of 1st angle $= x^{\circ}$

The size of 2nd angle $= 100^{\circ} - x^{\circ}$

The supplement of $x^{\circ} = 180^{\circ} - x^{\circ}$

The supplement of $100^{\circ} - x^{\circ} = 180^{\circ} - (100^{\circ} - x^{\circ})$

$$= 180^{\circ} - 100^{\circ} + x^{\circ}$$

According to the statement

$$(180^{\circ} - 100^{\circ} + x^{\circ}) - (180^{\circ} - x^{\circ}) = 100^{\circ}$$

$$\cancel{180^{\circ}} - 100^{\circ} + x^{\circ} - \cancel{180^{\circ}} + x^{\circ} = 100^{\circ}$$

$$2x^{o} - 100^{o} = 100^{o}$$

$$2x = 100^{o} + 100^{o}$$

$$2x = 200^{o}$$

$$x = \frac{200^{o}}{2}$$

$$x = 100^{o}$$

The size of 1st angle $= 100^{o}$

The size of 2nd angle $= 100^{o} - 100^{o}$

$= 0^{o}$

5. The sum of two angles is 100^{o}, the supplement of the first angle exceeds the supplement of the second angle 40^{o}. Find the angles.

Sol: Let the size of 1[st] angle $= x^{o}$

The size of 2nd angle $= 100^{o} - x^{o}$

The supplement of 1st angle $= 180^{o} - x^{o}$

The supplement of 2nd angle $= 180^{o} - (100^{o} - x)$

$= 180^{o} - 100^{o} + x^{o}$

$= 80^{o} + x^{o}$

According to the statement

$$180^{o} - x^{o} - 40^{o} = 80^{o} + x^{o}$$

$$-x^{o} - x^{o} = 80^{o} + 40^{o} - 180^{o}$$

$$-2x^{o} = -60^{o}$$

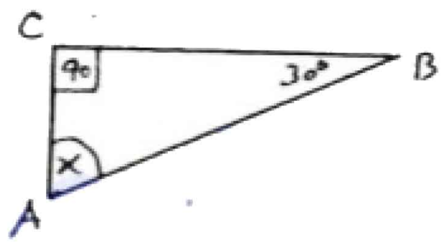
$$x^{0} = 30^{0}$$

$$1\text{st angle} = 30^{0}$$

$$2\text{nd angle} = 100^{0} - 30^{0}$$

$$= 70^{0}$$

6. Write the equation for the given triangle and solve it.



Sol: The sum of angles of any triangles $= 180^{0}$

The sum of angles of $\Delta ABC = x + 90^{0} + 30^{0}$

$= x + 120^{0}$

According to the statement

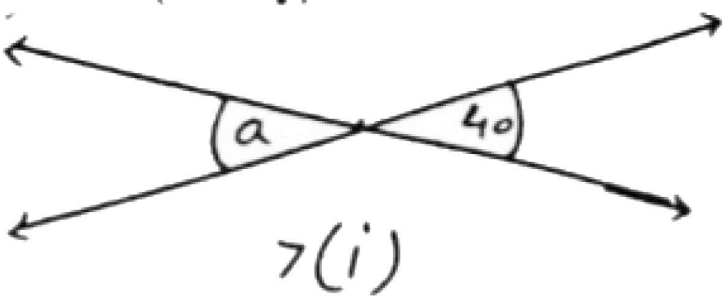
$$x + 120^{0} = 180^{0}$$

$$x = 180^{0} - 120^{0}$$

$$x = 60^{0}$$

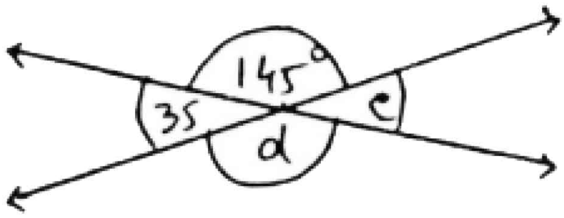
7. **Write down the angles marked with letters.**

7(i) $m\angle a = 40^{0}$ (vertically opp. angles)



7(ii) $m\angle c = 35^o$ (vertically opp. angles)

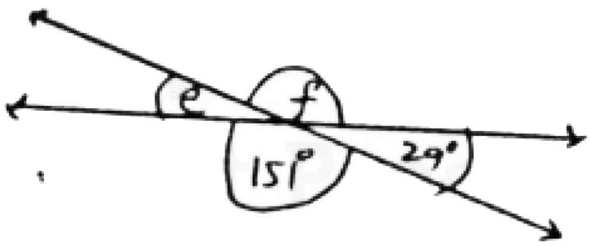
$m\angle d = 145^o$ (vertically opp. angles)



7(ii)

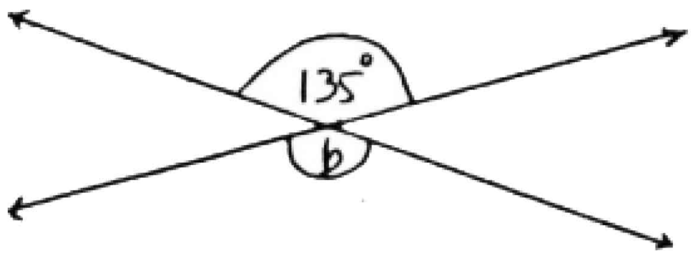
7(iii) $m\angle e = 29^o$ (vertically opp. angles)

$m\angle f = 151^o$ (vertically opp. angles)



7(iii)

7(iv) $m\angle b = 135^o$ (vertically opp. angles)

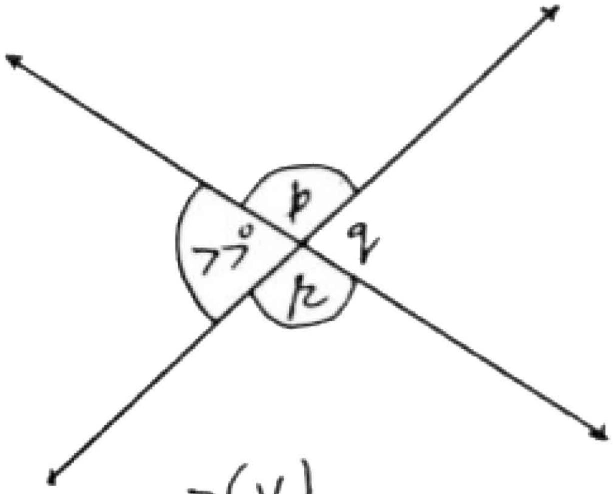


7(iv)

7(v) $m\angle q = 70^o$ (vertically opp. angles)

$m\angle p = 180 - 77^o$ (supplementary angles)

$= 103^o$



7(v)

$m\angle r = m\angle p$ (vertically opp. angles)

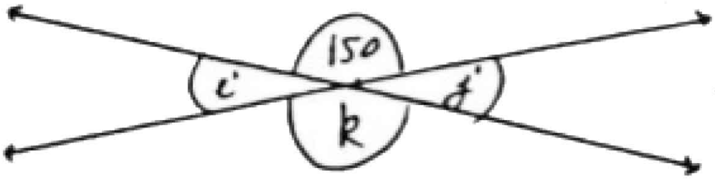
$= 103^\circ$ (i)

7(vi)

$m\angle k = 150^\circ$ (vertically opp. angles)

$m\angle i + 150^\circ = 180^\circ$ (supplementary angles)

$m\angle i = 180^\circ - 150^\circ$



7(vi)

(i) $m\angle i = 30^\circ$

and $m\angle j = m\angle i$ (vertically opp. angles)

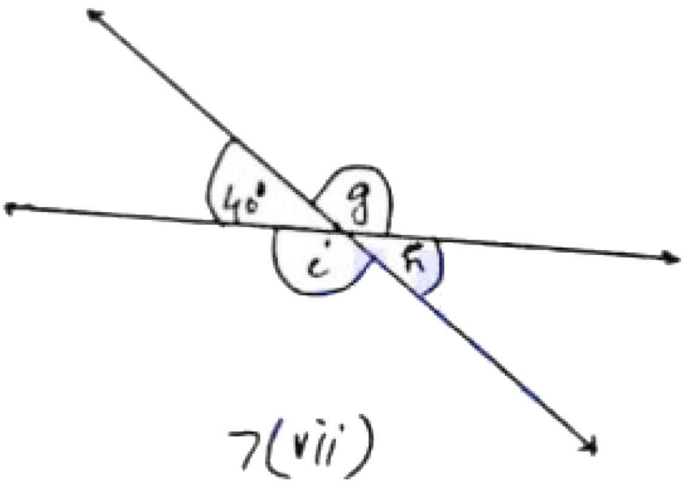
$m\angle j = 30^\circ$ from (i)

7(vii)

(vertically opp. angles) $m\angle h = 40''$

(supplementary angles) $m\angle 40 + \angle g = 180''$

$m\angle g = 180'' - 40$



$m\angle g = 140''$ (i)

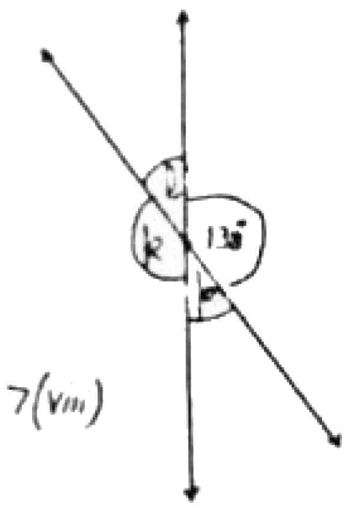
(vertically opp. angles) $m\angle i = m\angle g$ and

from (i) $= 140''$

(vertically opp. angles) $m\angle k = 138''$ **7(viii)**

(supplementary angles) $m\angle p + 138'' = 180''$

$m\angle p = 180'' - 138''$



$m\angle p = 42''$ (i)

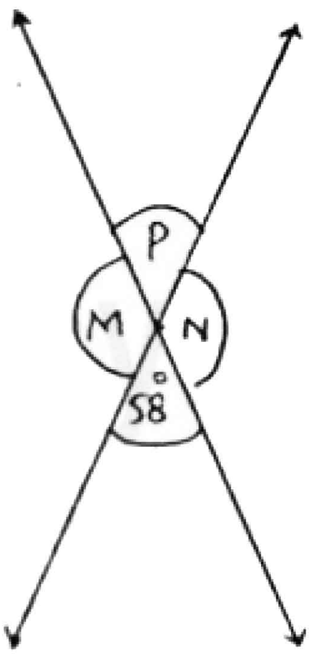
(vertically opp. angles) $m\angle l = m\angle p$ and

from (i) $m\angle l = 42''$

(vertically opp. angles) $m\angle P = 58''$ **7(ix)**

(supplementary angles) $m\angle N + 58'' = 180''$

$m\angle N = 180'' - 58$

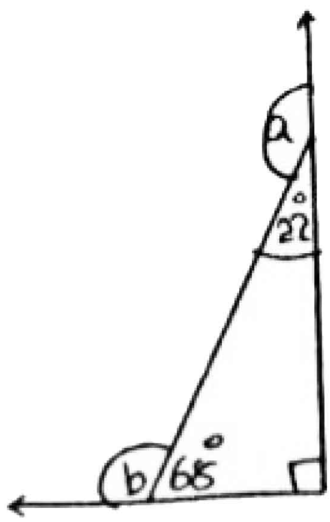


$m\angle N = 122''$ (i)

(vertically opp. angles) $m\angle M = m\angle N$ and

from (i) $= 122''$

7(x)



(supplementary angles) $m\angle a + 22^\circ = 180^\circ$

$$m\angle a = 180^\circ - 22^\circ$$

$$m\angle a = 158^\circ$$

(supplementary angles) $m\angle b + 68^\circ = 180^\circ$

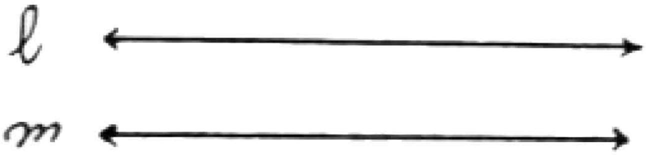
$$m\angle b = 180^\circ - 68$$

$$m\angle b = 112^\circ$$

(i) *PARALLEL LINES*

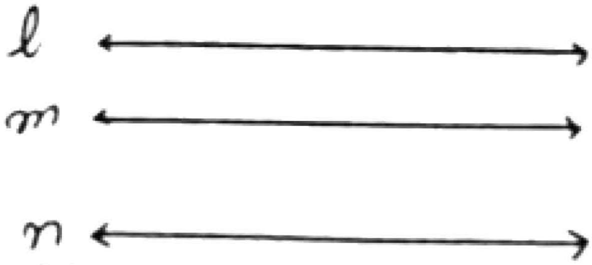
Parallel lines are two straight lines in the same plane which never meet.

The lines a and b are parallel, we write $a \parallel b$.



(ii) *Properties of Parallel Lines*

(a) Two lines parallel to a third are parallel to each other.



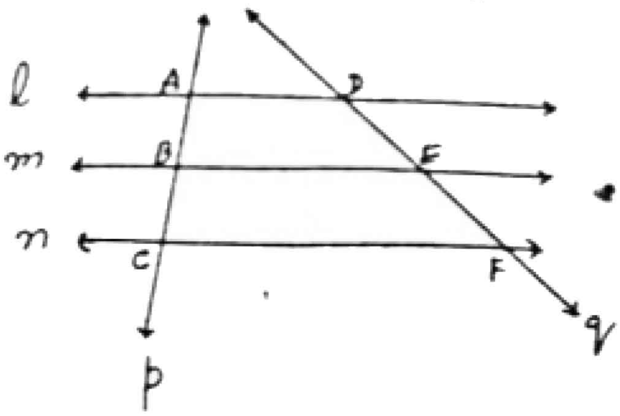
$l // n$

$m // n$

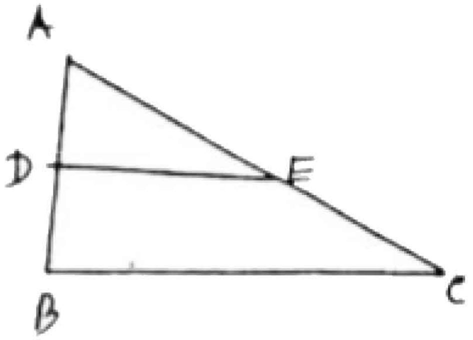
$l // m$

(b) If three parallel lines are intercepted by two transversals in such a way that the two intercepts on one transversal are equal to each other, the two intercepts on the second transversal are also equal.

i.e. if $\overleftrightarrow{AD} \parallel \overleftrightarrow{BE} \parallel \overleftrightarrow{CF}$



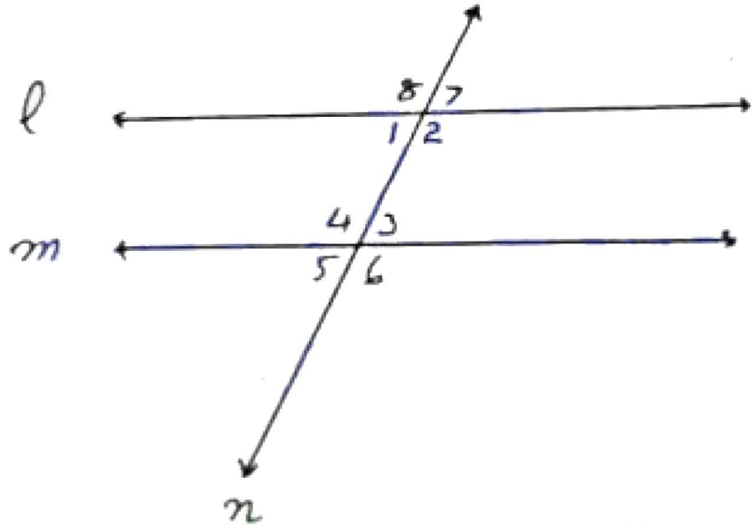
(c) If a line bisects one side of a triangle and is parallel to a second side, then it bisects the third side.



i.e. if ΔABC with $\overline{BD} \cong \overline{DA}$, $\overline{DE} \parallel \overline{BC}$ then $\overline{AE} \cong \overline{CE}$

Transversal

A transversal is a line that intersects two lines in differnt points.



If a transversal *"t"* intersects two parallel lines a and b, the angles formed are identified as follows:

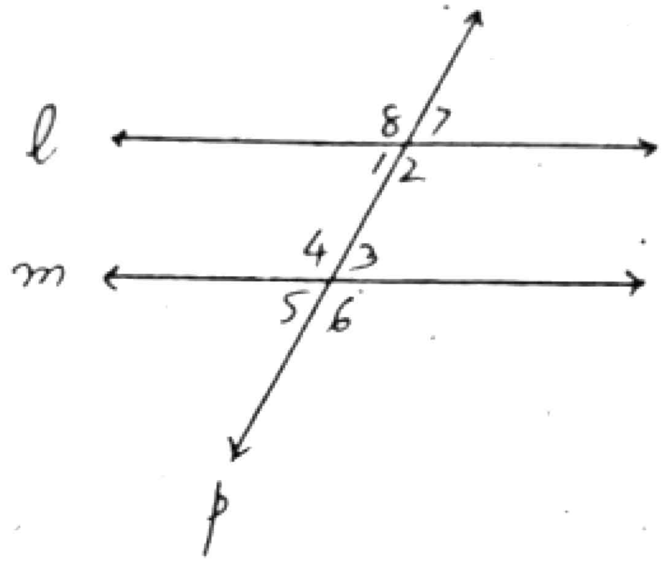
1. Four interior angles : $\angle 1, \angle 2, \angle 3, \angle 4$
2. Four exterior angles : $\angle 5, \angle 6, \angle 7, \angle 8$
3. Two pairs of alternate interior angles $\angle 1$ and $\angle 3$; and $\angle 4$
4. Two pairs of alternate exterior angles $\angle 5$ and $\angle 7$; $\angle 6$ and $\angle 8$
5. Two pairs of interior angles on the same side of the transversal: $\angle 2$ and $\angle 3$; $\angle 1$ and $\angle 4$.
6. Four pairs of corresponding angles: $\angle 3$ and $\angle 7$; $\angle 4$ and $\angle 8$; $\angle 2$ and $\angle 6$; $\angle 1$ and $\angle 5$.

Relation Between the Pairs of Angles

If two parallel lines are cut by a transversal, the corresponding angles are equal.

$[\angle 1 = \angle 2, \angle 2 = \angle 3, \angle 1 = \angle 3]$

d) If two parallel lines are cut by a transversal, the alternate interior angles are equal.



a || b, lines a and b are cut by the transversal c at point M and N to form the pairs of alternate interior angles.

$(\angle 1, \angle 2)$ *and* $(\angle 3, \angle 4)$

$\angle 1 = \angle 2, \angle 3 = \angle 4$

e) If two parallel lines are intercepted by a transversal, then pairs of interior angles on the same side of transversal are supplementary.

$AB \parallel CD$, lines are cut by the transversal t, angles a, b, c and d are formed.

(i) $m\angle 2 = m\angle 4$

$m\angle 1 = m\angle 3$

(ii) $m\angle 3 = m\angle 7$

$m\angle 4 = m\angle 8$

$m\angle 6 = m\angle 2$

$m\angle 5 = m\angle 1$

(iii) $m\angle 7 = m\angle 5$

$m\angle 6 = m\angle 8$

(iv) $m\angle 2 + m\angle 3 = 180^o$

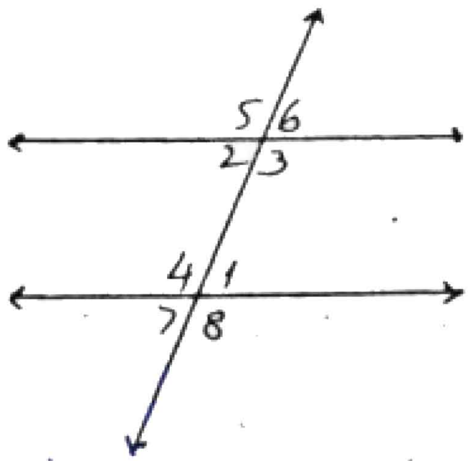
$m\angle 1 + m\angle 4 = 180^o$

(v) $m\angle 5 + m\angle 8 = 180^o$

$m\angle 6 + m\angle 7 = 180^o$

Exercise

1. **Look at the given figure and answer the following questions.**



(a) The pair of alternative angles
(b) The pair of corresponding angles
(c) The pair of complementary angles
(d) The pair of supplementary angles
(e) The pair of vertical angles

Answers:

a) The pair of alternative interior angles: $(\angle 1, \angle 2)$ *and* $(\angle 3, \angle 4)$

b) The pair of corresponding angles:

$\angle 1, \angle 6; \angle 3, \angle 8; \angle 2, \angle 7; \angle 5, \angle 4$

c) The pair of complementary angles: No one

d) The pair of supplementary angles:

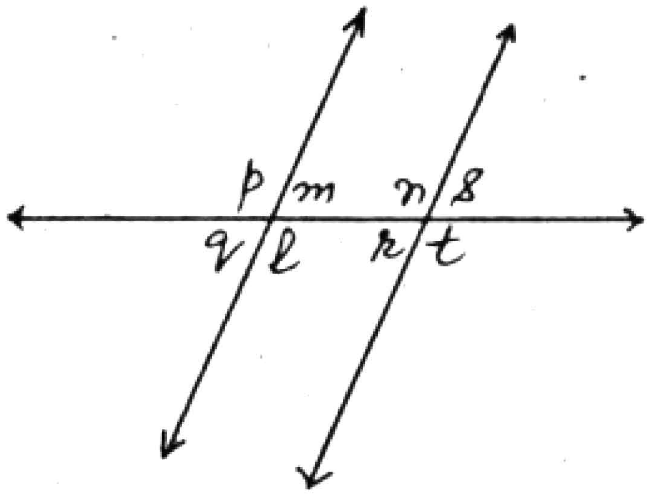
$(\angle 4, \angle 1); (\angle 4, \angle 7); (\angle 7, \angle 8); (\angle 8, \angle 1)$

$(\angle 5, \angle 6); (\angle 5, \angle 2); (\angle 2, \angle 3); (\angle 3, \angle 6)$

e) The pair of vertical angles:

$(\angle 5, \angle 3); (\angle 6, \angle 2); (\angle 4, \angle 6); (\angle 1, \angle 7)$

2. Look at the given figure and answer the following questions.



(a) The pair of alternative interior angles
(b) The pair of corresponding angles
(c) The pair of complementary angles
(d) The pair of supplementary angles
(e) The pair of vertical angles

Answers:

The alternate interior angles : $(\angle n, \angle 1), (\angle m, \angle r)$

The corresponding angles:

$$(\angle p, \angle n); (\angle m, \angle s); (\angle q, \angle r); (\angle l, \angle t)$$

The complementary angles: No one

The supplementary angles:

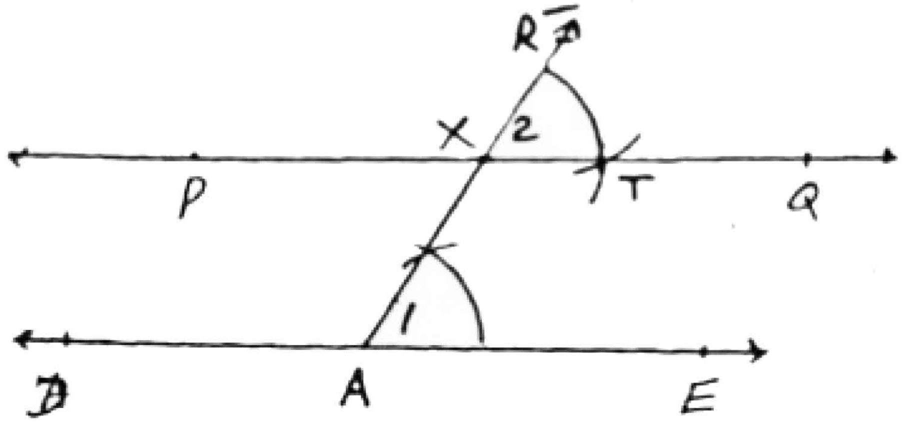
$$(\angle p, \angle m); (\angle p, \angle q); (\angle q, \angle l); (\angle l, \angle m)$$

$$(\angle n, \angle s); (\angle n, \angle r); (\angle r, \angle t); (\angle t, \angle s)$$

The vertical angles: $(\angle p, \angle l); (\angle m, \angle q); (\angle n, \angle t); (\angle r, \angle s)$

3. Take a point 'X' outside a line $\overleftrightarrow{DE}$. Draw a line through

'X' which cuts $\overleftrightarrow{DE}$ at some point. Making corresponding angles congruent draw a line parallel to $\overleftrightarrow{DE}$.



Steps of Construction:

(i) Draw a line $\overleftrightarrow{DE}$.

(ii) Take any point "X" which lay outside of $\overleftrightarrow{DE}$.

(iii) Take any point "A" on $\overleftrightarrow{DE}$.

(iv) Join "A" with "X" then extend it

(v) Draw corresponding angles $\angle 1$ & $\angle 2$ with the help of compasses.

(vi) Extend $\overline{XT}$ on both sides.

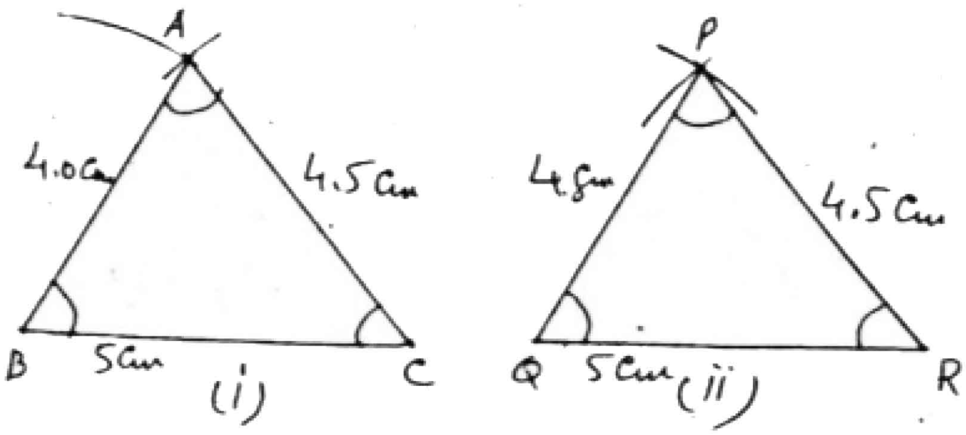
$\overline{PQ} \parallel \overline{DE}$ which passes through X.

CONGRUENT AND SIMILAR FIGURES

Congruent Figures

The word congruent comes from Latin meaning "together agree". Two geometrical figures which ahve the same size and shape are congruent.

One figure is congruent to the other. The symbol for congruent is $\cong$. Thus two segments are congruent when they have the same size.



In figure (i) and (ii)

$$m\overline{AB} = m\overline{PQ}$$

$$m\overline{BC} = m\overline{QR}$$

$$m\overline{CA} = m\overline{RP}$$

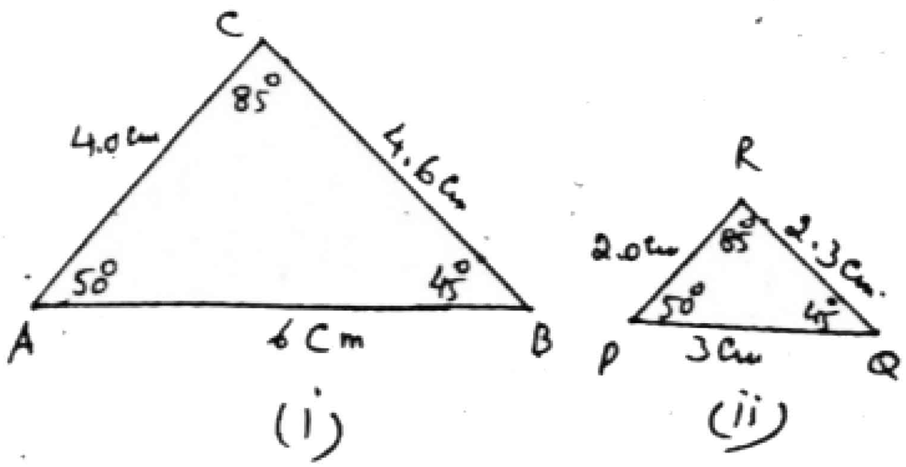
$$m\angle B = m\angle Q$$

$$m\angle C = m\angle R$$

$$m\angle A = m\angle P$$

Similar Figures

In the polygons below, the members of each pair are similar to each other.



In figure (i) and (ii)

$m\angle A = 50^o, \quad m\angle P = 50^o$

$m\angle B = 45^o, \quad m\angle Q = 45^o$

$m\angle C = 85^o, \quad m\angle R = 85^o$

$$\frac{m\overline{AC}}{m\overline{PR}} = \frac{4}{2} = \frac{2}{1}$$

$$\frac{m\overline{AB}}{m\overline{PQ}} = \frac{6}{3} = \frac{2}{1}$$

$$\frac{m\overline{BC}}{m\overline{QR}} = \frac{4.6}{2.3} = \frac{2}{1}$$

Exercise

Tell Whether or not the Figures in Question 1-3 are Similar:

1. All squares; Yes

all rectangles; No

all regular hexagons. Yes

2. Two rectangles with sides 8, 12, 10 and 15.

Ans. These are similar figures

$$\frac{10}{15} = \frac{8}{12}.$$

$$\frac{2}{3} = \frac{2}{3}$$

3. Two rhombuses with angles of 55^o and 125^o.

Ans. These are similar figures because the four.

4. The sides of a polygon are 5cm, 6cm, 7cm, 8cm, and 9cm. In a similar polygon the sides corresponding to 6cm is 12cm. Find the other sides of the second polygon.

Ans. According to the given condition the ratio among corresponding sides is $\frac{6}{12}=\frac{1}{2}$.

$$\frac{6}{12}=\frac{1}{2}=\frac{7}{a}=\frac{8}{b}=\frac{5}{c}=\frac{4}{d}=\frac{9}{e}$$

$\frac{7}{a}=\frac{1}{2} \Rightarrow \quad a=7\times 2=14$ cm

$\frac{8}{b}=\frac{1}{2} \Rightarrow \quad b=8\times 2=16$ cm

$\frac{5}{c}=\frac{1}{2} \Rightarrow \quad c=5\times 2=10$ cm

$\frac{4}{d}=\frac{1}{2} \Rightarrow \quad d=4\times 2=8$ cm

$\frac{9}{e}=\frac{1}{2} \Rightarrow \quad e=9\times 2=18$ cm

5. The sides of a quadrilateral are 2cm, 4cm, 6cm, and 7cm. The longest side of a similar quadrailateral is 21cm. Find the other sides.

Ans. The longest side of quadrilateral is 7cm. Thus, the ratio between the biggest sides $\frac{7}{21}=\frac{1}{3}$.

According to the statement the ratio between the corresponding sides is $\frac{1}{3}$.

$\frac{1}{3}=\frac{6}{a}=\frac{5}{b}=\frac{4}{c}=\frac{2}{d}$ Thus

$\frac{6}{a}=\frac{1}{3} \Rightarrow \quad a=6\times 3=18$ cm therefore

$\frac{5}{b}=\frac{1}{3} \Rightarrow \quad b=5\times 3=15$ cm

$\frac{4}{c}=\frac{1}{3} \Rightarrow \quad c=4\times 3=12$ cm

$$\frac{2}{d}=\frac{1}{3} \Rightarrow \quad d=2\times3=6\text{ cm}$$

While a, b, c, d are the rest sides of quadrilateral.

6. The sides of a polygon are 5cm, 2cm, 7cm, 3cm, 4cm. Find the sides of a similar polygon whose side corresponding to 2cm is 6cm. What is the ratio of the perimeters of these two polygons?

Sol: According to the statement the ratio between the correspondign sides is $\frac{2}{6}=\frac{1}{3}$.

$$\frac{1}{3}=\frac{5}{a}=\frac{7}{b}=\frac{3}{c}=\frac{4}{d} \qquad \text{thus}$$

$$\frac{5}{a}=\frac{1}{3} \Rightarrow \quad a=5\times3=15\text{ cm}$$

$$\frac{7}{b}=\frac{1}{3} \Rightarrow \quad b=7\times3=21\text{ cm}$$

$$\frac{3}{c}=\frac{1}{3} \Rightarrow \quad c=3\times3=9\text{ cm}$$

$$\frac{4}{d}=\frac{1}{3} \Rightarrow \quad d=4\times3=12\text{ cm}$$

While a, b, c, d are the rest sides of polygon.

The perimeter of 1st polygon = 5 + 2 + 7 + 3 + 4

= 21cm

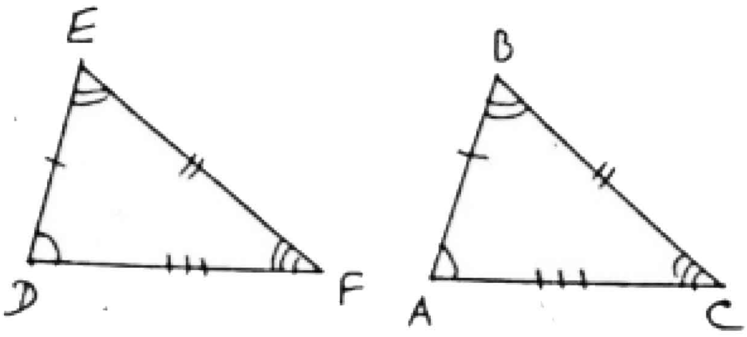
The perimeter of 2nd polygon = 15 + 21 + 9 + 12 + 6

= 63cm

The ratio between the perimeter $=\frac{21}{63}=\frac{1}{3}$ ؛ 1 : 3

7. What are the congruent pairs of corresponding sides and

corresponding angles?



$\overline{AB} \cong DE$	$\angle A \cong \angle D$
$\overline{AC} \cong \overline{DF}$	$\angle B \cong \angle E$
$\overline{BC} \cong \overline{FE}$	$\angle C \cong \angle F$

8. Are all similar figures congruent? Explain why?

Sol: All similar figures are equal in size and shape.
Therefore, similar figures are congruent.

9. Are all congruent figures similar? Explain why?

Sol: All congruent figures have same shape but differ in size.
Therefore, congruent figures are not similar.

Exercise 7.4

1. Fill in the blanks.

(a) If $\Delta ABC \cong \Delta FDE$, then

(i) $\overline{AB} =$ ________ (ii) $\overline{BC} =$ ________

(iii) $\overline{AC} =$ ________ (iv) $m\angle A =$ ________

(v) $m\angle B =$ ________ (vi) $m\angle C =$ ________

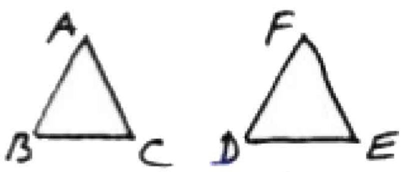
(b) In ΔPQR, the angle included between side PR and QR is ________

(c) In , the side included between $\angle E$ and $\angle F$ is ________

(d) If $\overline{AB} = \overline{QP}, m\angle B = m\angle P, \overline{BC} = \overline{PR}$, then by ________ condition. $\Delta ABC \cong \Delta QPR$

(e) If $m\angle A = m\angle R, m\angle B = m\angle P, \overline{AB} = \overline{RP}$ then by ________ congruence condition. $\Delta ABC \cong \Delta RPQ$.

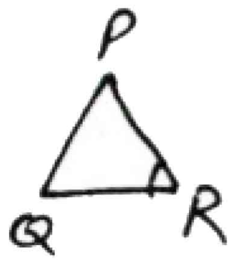
Answers:



(a)

(*i*)	$\overline{AB} \cong \overline{FD}$	(*ii*)	$\overline{BC} \cong \overline{DE}$
(*iii*)	$\overline{AC} \cong \overline{FE}$	(*iv*)	$\angle A \cong \angle F$
(*v*)	$\angle B \cong \angle D$	(*vi*)	$\angle C \cong \angle E$

mid angle $\angle R$



(b)

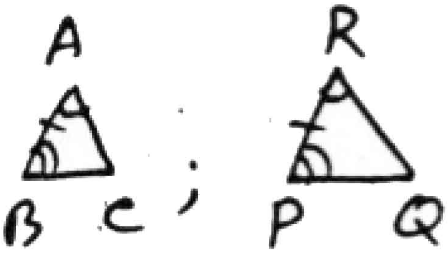
mid side $\overline{FE}$

(c)

$m\angle A = m\angle R$

$m\angle B = m\angle P$

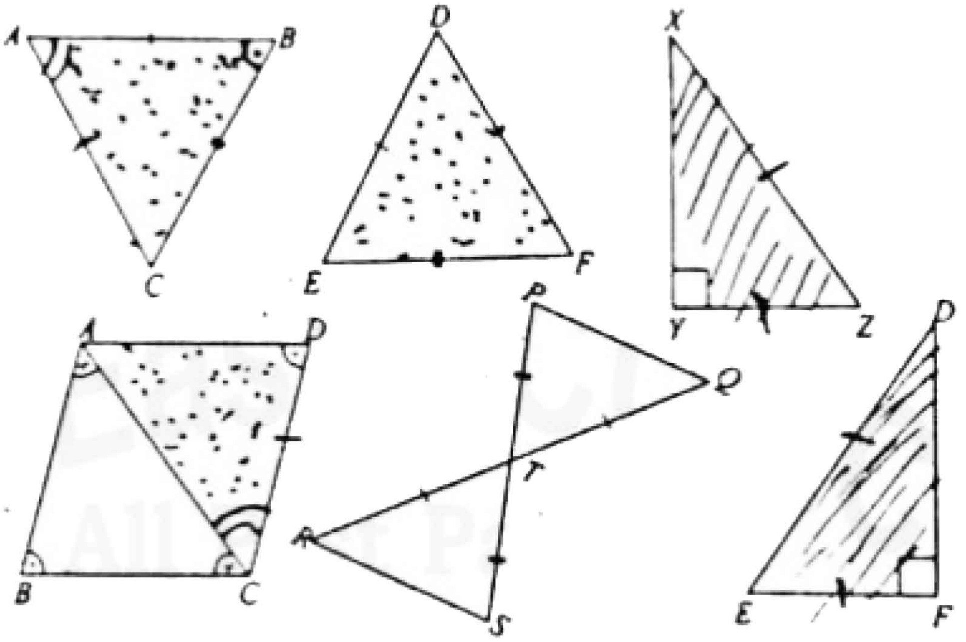
$\overline{AB} = \overline{RP}$



(d)

$\Delta ABC \cong \Delta RPQ$ ASA $\cong$ ASA

2. In figure, the pairs of corresponding equal parts in a pair of triangles are shown with similar markings. Specify the two triangles which become congruent. Also, write the congruence of two triangles in symbolic form.



Sol:

(*i*) $\Delta ABC \cong \Delta DEF$ SSS ≅ SSS

(*ii*) $\Delta XYZ \cong \Delta DFE$ RHS ≅ RHS

(*iii*) $\Delta ABC \cong \Delta CDA$ ASA ≅ ASA

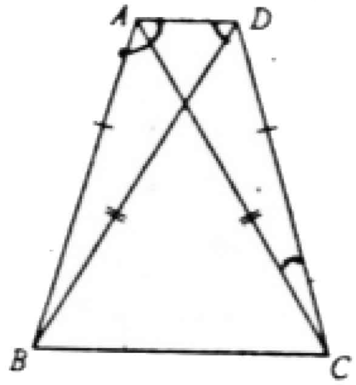
(*iv*) $\Delta PQT \cong \Delta SRT$ SAS ≅ SAS

3. In figure, ABC and DBC are two triangles on a common base $\overline{BC}$ such that $\overline{AB} = \overline{DC}$ and , where A and D lie on the same side of BC. In ΔADB and ΔDAC , state the corresponding parts so that $\Delta ADB = \Delta DAC$.

Which condition do you use to establish the congruence?

If $m\angle DCA = 40^{o}$ and $m\angle BAD = 100^{o}$.

Find $\angle ADB$.



Sol: Now ΔABC and ΔDBC

common $\overline{BC} \cong \overline{BC}$

given $\begin{cases} \overline{AB} \cong \overline{DC} \\ \overline{AC} \cong \overline{DB} \end{cases}$

Now ΔADB and ΔDAC

$\overline{DA} \cong \overline{AD}$

$\overline{DB} \cong \overline{AC}$

$\overline{BA} \cong \overline{CD}$

$\Delta ADB \cong \Delta DAC$

SSS $\cong$ SSS

$m\angle DCA = 40^{o}$ now

$m\angle BAD = 100^{o}$ and

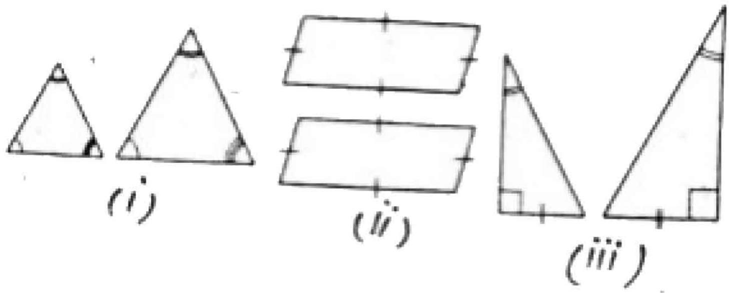
$m\angle ABD = 40^{o}$ therefore

$m\angle ADB = 180^{o} - 100^{o} - 40^{o}$

$= 180^{o} - 140^{o}$

$= 40^{o}$

4. Identify the following figure as congruent, similar or neither.



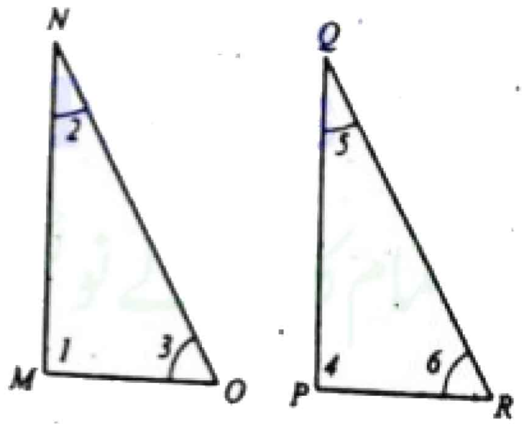
Sol: (i) congruent (ii) congruent

(iii) congruent

No one is similar

5. Identify the corresponding parts in ΔMNO and ΔPQR.

(*i*)	$\overline{MN}$	$\leftrightarrow$	☐
(*ii*)	$\overline{NO}$	$\leftrightarrow$	☐
(*iii*)	$\overline{PR}$	$\leftrightarrow$	☐
(*iv*)	$\angle$	$\leftrightarrow$	☐

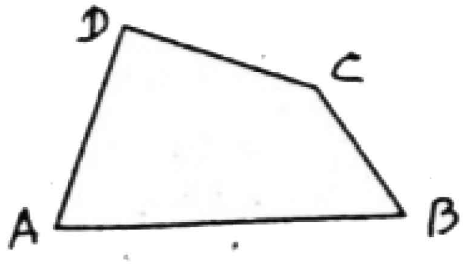


Answers:

(*i*)	$\overline{MN}$	$\leftrightarrow$	$\overline{PQ}$
(*ii*)	$\overline{NO}$	$\leftrightarrow$	$\overline{QR}$
(*iii*)	$\overline{PR}$	$\leftrightarrow$	$\overline{MO}$
(*iv*)	$\angle 1$	$\leftrightarrow$	$\angle 4$

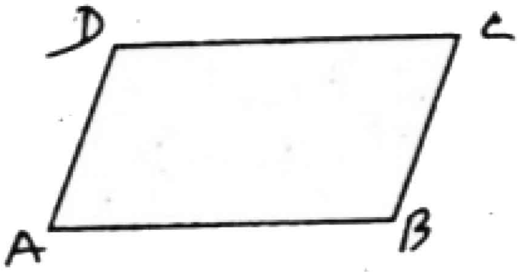
Quadrilaterals:

A quadrilateral is a polygon with four sides.



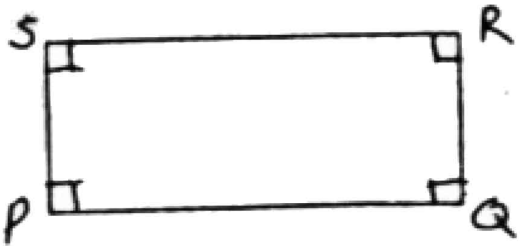
Parallelogram:

A parallelogram is a quadrilateral with two pairs of parallel sides.



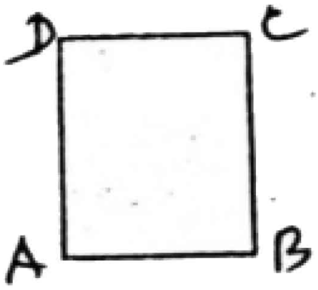
Rectangle

A rectangle is a parallelogram containing a right angle.



Square

A square is an equilateral rectangle.

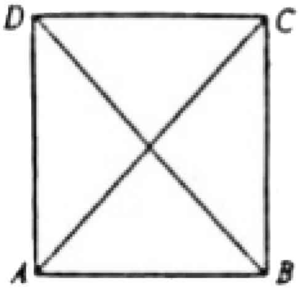


Properties of Congruency

Four Sides of a Square are Equal

ABCD is a square. Measure $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{DA}$. We find that.

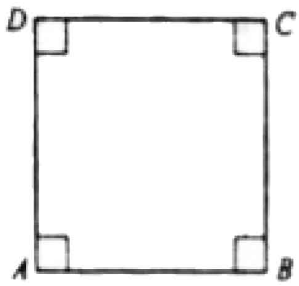
$m\overline{AB} = m\overline{BC} = m\overline{CD} = m\overline{DA} = 2.8\text{cm}.$



Four Angles of a Square are Right Angles

ABCD is a square. Measure angle A, B, C, D with protractor. We find that

$m\angle A = m\angle B = m\angle C = m\angle D = 90^0$

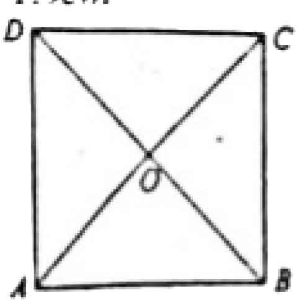


Diagonals of a Square Bisect Each Other:

Consider a square ABCD, the diagonals and intersect at 'O'. We find that

$m\overline{OA} = m\overline{OC} = 1.9cm$ *and*

$m\overline{OB} = m\overline{OD} = 1.9cm$



7.5.2 *Opposite Sides of a Rectangle are Equal*

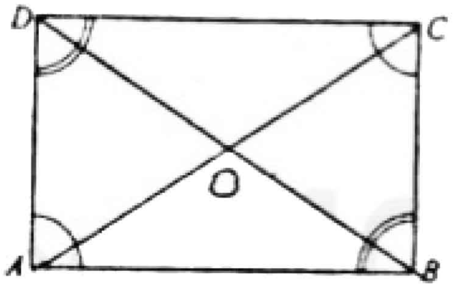
Consider Rectangle

Let us consider a rectangle $ABCD$.

$\overline{AB}, \overline{CD}$ and $\overline{AD}, \overline{BC}$ are opposite pairs of rectangle $ABCD$.

We find that $m\overline{AB} = m\overline{CD} = 4.5cm$ and

$m\overline{AD} = m\overline{BC} = 2.8cm$



$$m\overline{AB} = m\overline{DC} \quad \text{(i)}$$

$$m\overline{AD} = m\overline{BD} \text{ and}$$

$$m\angle A = m\angle B = m\angle C = m\angle D = 90^{o} \quad \text{(ii)}$$

(iii)

$$m\overline{OA} = m\overline{OC},$$

$$m\overline{OB} = m\overline{OD}$$

$$m\overline{OA} = m\overline{OC} = m\overline{OB} = m\overline{OD}$$

Properties of Parallelogram

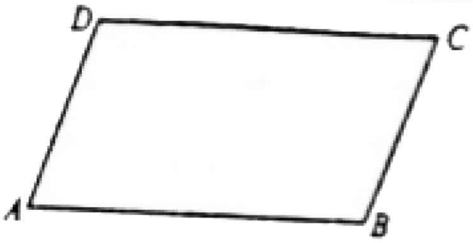
▶ ***The opposite sides of a parallelogram are equal.***

$ABCD$ is a parallelogram. $\overline{AB}, \overline{CD}$ and $\overline{AD}, \overline{BC}$ are pairs of opposite sides.

We find that

$m\overline{AB} = m\overline{CD} = 3.9cm$ *and*

$m\overline{AD} = m\overline{BC} = 2.0cm$



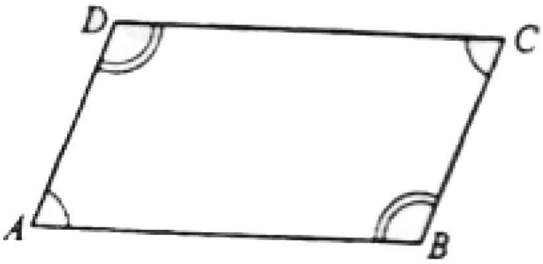
- ***The opposite angles of a parallelogram are equal.***

$ABCD$ is a parallelogram. $\angle A, \angle C$ and $\angle B, \angle D$ are pairs of opposite angles.

We find that

$m\angle A = m\angle C = 70^0$ *and*

$m\angle B = m\angle D = 110^0$

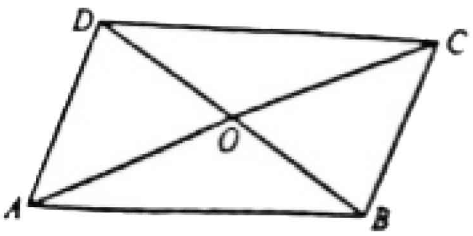


- ***The diagonals of a parallelogram bisect each other.***

A parallelogram $ABCD$, the diagonals $\overline{AC}$ and $\overline{BD}$ intersect at O. We find that

$$m\overline{OA} = m\overline{OC} = 2.5cm$$

and $$m\overline{OD} = m\overline{OB} = 2.5cm$$



Exercise 7.5

1. **Fill in the blanks:**

(i) A parallelogram that contains a right angle is ______.

(ii) An equilateral rectangle is a ______.

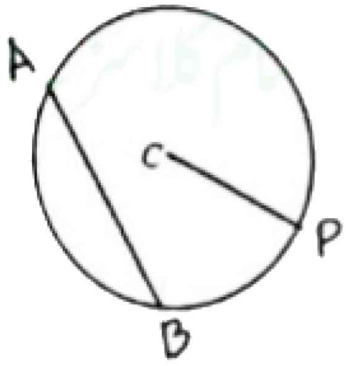
(iii) A polygon with four sides is a ______.

(iv) The diagonals of a parallelogram ______ each other.

(v) The opposite angles of a parallelogram are ______.

Answers:

(i)	Rectangle	(ii)	Square	(iii)	Quadrilateral
(iv)	Bisect	(v)	Congruent		



A circle is the set of points in a plane which are at a constant distance from a fixed point in the plane.

Centre

The fixed point C is called the centre of the circle.

Radial Segment

P is any point on the circumference of the circle with centre O. $\overline{OP}$ is called the radial segment of the circle.

Radius

A radius of a circle is the length of a segment joining the centre to any point on the circle. In the given figure is $m\overline{CP}$ the radius. Usually represented by 'r'.

Chord

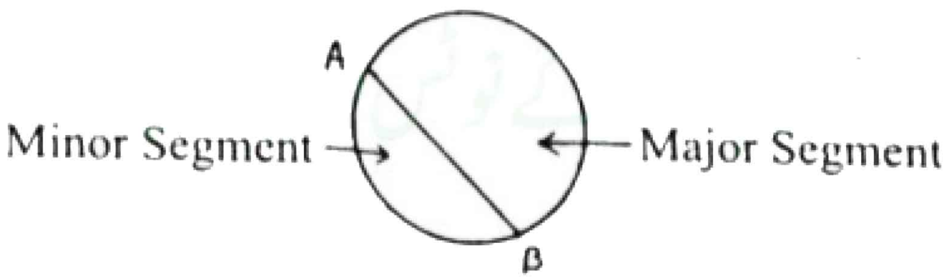
A chord of a circle is a segment connecting any two points on the circle. In the given figure $\overline{AB}$ is a chord.

Segment of a Circle

A chord $\overline{AB}$ of a circle divides the circle in two parts. These are called segment of the circle.

Minor Segment

The included area between minor arc and the chord is minor segment.



Major Segment

The included area between major arc and chord is called major segment.

Diameter

A diameter of a circle is a chord that passes through the centre. The length of a diameter of a circle is twice the length of the radius of the same circle.

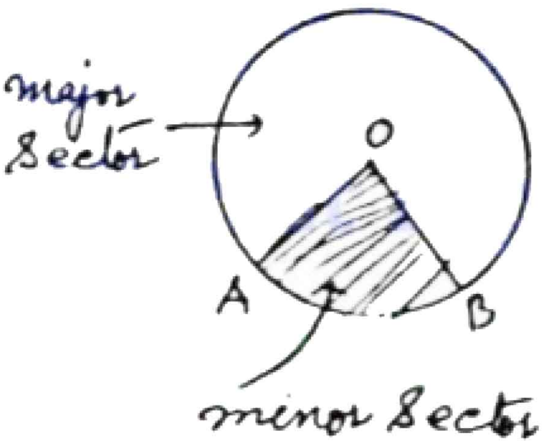
$$Diameter = 2 \times radius$$

Equal Circles

Equal circles are circles having equal radii or equal diameter

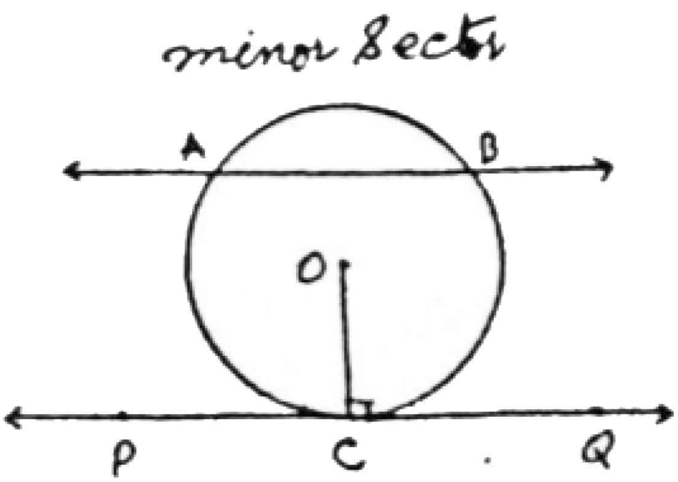
Sector

A sector is the circular region bounded by an arc of a circle and its two corresponding radial segments is called a sector of the circle. In the figure, region *AOB* is the sector of the circle with centre at *O*.



Secant

A secant is a line which intersects a circle in two points.



Tangent

A tangent to a circle is the line perpendicular to radius of the circle at its outer extremity.

The point on the circle at which the radius and tangent meet is knwon as the Point of Contant or Point of Tangency.

Angle in a Semi-Circle is a Right Angle

1- Draw a line-segment $\overline{AB}$ of any length. Mark the mid point of $\overline{AB}$ as O.

2- Draw a semi-circle on $\overline{AB}$ with radius $\overline{OA}$.

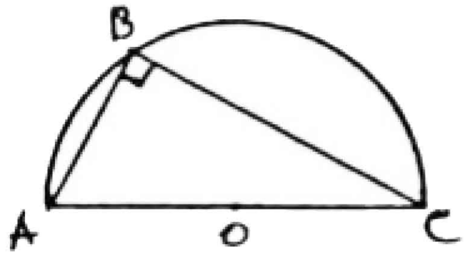
3- Take any point C on the semi-circle. Join A with C and B with C.

Thus, $\angle ACB$ is an angle in the semi-circle APB.

4- Now take a protractor and place it along $\overline{AC}$ so that the centre of the protractor falls on C.

We note that the measure of the $\angle ACB$ by looking at the marking on the protractor corresponding to arm $\overline{CB}$ of $m\angle ACB$ is of 90^0, i.e $m\angle ACB = 90^0$ or a right angle.

Thus, angle in a semi-circle is a right angle.



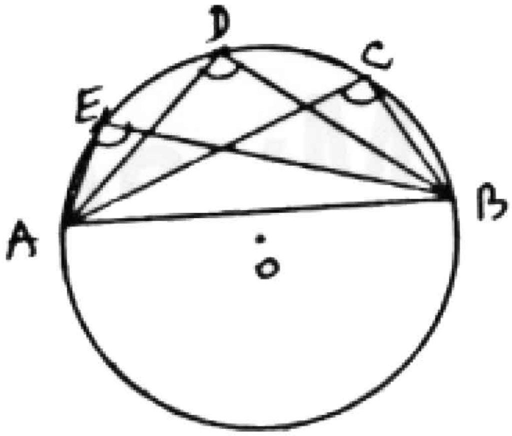
Angles in the Same Segment are Equal:

Draw a circle with centre 'O'. Take two points B and C on the circle and join them. $\overline{BC}$ divides the circle into two parts.

Draw angles, $\angle BAC$ and $\angle BDC$ in the same segment as shown in the figure. Take a sheet of tracing paper and make a trace copy of $\angle BAC$. Place the trace copy of $\angle BAC$ on $\angle BDC$.

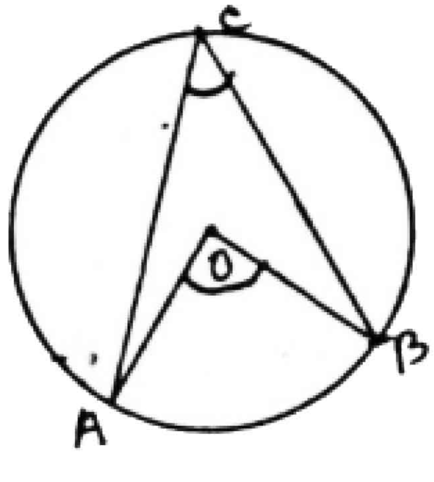
A falls on D and $\overline{AB}$ falls on $\overline{DC}$.

So that we observe that $\overline{BD}$ falls on $\overline{AC}$. Thus $\angle BAC = \angle BDC$, this shows that angles in the same segment are equal.



Central Angle

The central angle of a minor arc of a circle is double that the angle subtended by corresponding major arc.



$$m\angle AOB = m2\angle ACB$$

Exercise 6

1- **Fill in the blanks:**

(i) In a plane the set of points whose distance from a fixed point is same is called ______.

(ii) The distance of a point of a circle from its centre is called ______.

(iii) A line segment whose end points lie on the circle is called ______.

(iv) A chord that passes through the centre of the circle is called ______.

(v) Half of a circle is called ______.

(vi) An arc which is greater than a semicircle is called ______.

(vii) One and only one circle can be constructed with a given centre and given ______.

(viii) A region bounded by an arc and two of its radial segments is called ______.

(ix) A straight line that intersects a circle at two points is called ______.

(x) Angle in a semi-circle is a ______.

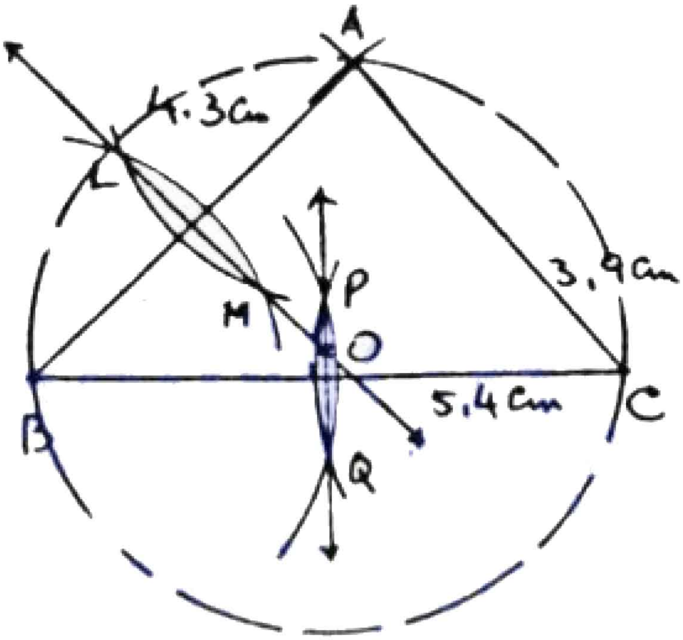
Answers:

(i)	circle	(ii)	radius
(iii)	chord	(iv)	diameter
(v)	semicircle	(vi)	major arc
(vii)	radius	(viii)	sector
(ix)	secant line	(x)	right angle

Exercise 8.4

Q.1. **Draw a triangle ABC in which** $m\overline{BC} = 5.4cm$, $m\overline{AB} = 4.3cm$ *and* $m\overline{AC} = 3.9cm$. **Find the in centre**

Sol.



Steps of Construction:

(i) Draw a line segment $\overline{BC} = 5.4cm$

(ii) With B as centre draw an arc of radius 4.3 cm.

(iii) With C as centre draw an arc of radius 3.9cm which intersect the first arc at A.

(iv) Join A with B and C.

ABC is the required triangle.

(v) Draw perpendicular bisectors $\overleftrightarrow{LM}$ and $\overleftrightarrow{PQ}$ of the sides $\overline{AB}$ and $\overline{BC}$ which intersect each other at O.

Point O is the required incentre.

UNIT 8 Practical Geometry

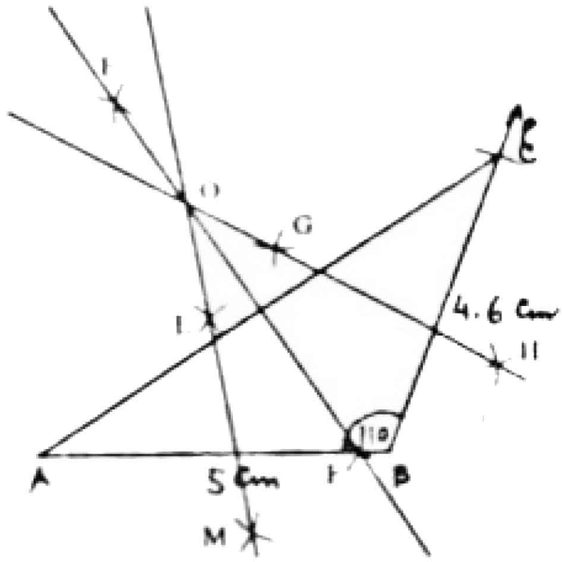
- **Construction of a Triangle**
- **Construction of a Quadrilateral**
- **Tangent to a Circle**

After completing of this unit, the students will be able to:

- construct a triangle having:
  - Two sides and the included angle.
  - One side and two of the angles.
  - Two of its sides and angle opposite to one of them with all the three possibilities).
- Draw:
  - Angle bisectors.
  - Altitudes.
  - Medians of a given triangle and verify their concourrency.
- Construct a rectangle when:
  - Two sides are given.
  - Diagonal and one side are given.
- Construct a square when its diagonal is given.
- Construct a parallelogram when two adjacent sides and the angle included between them is given.
- locate the centre of given circle.
- draw a circle passing through three given non-collinear points.
- draw a tangent to a given circle from a point P when P lies.
  - On the circumference,
  - Outside the circle.
- Draw:
  - Direct common tangent or external tangent.
  - Transverse common tangent or internal tangent to two equal circles.
- Draw a tangent to:
  - Two unequal touching circles.
  - Two unequal intersecting circles.

Q.2. **Construct a ABC in which** $m\overline{BC} = 4.6cm$, $\angle B = 110^{\circ}$ **and** $m\overline{AB} = 5cm$. **Draw the perpendicular bisectors of its sides.**

Sol.



Steps of Construction:

(i) Draw a line segment $\overline{AB}$ = 5cm

(ii) At point B, draw an angle 110^{O} with the help of compasses

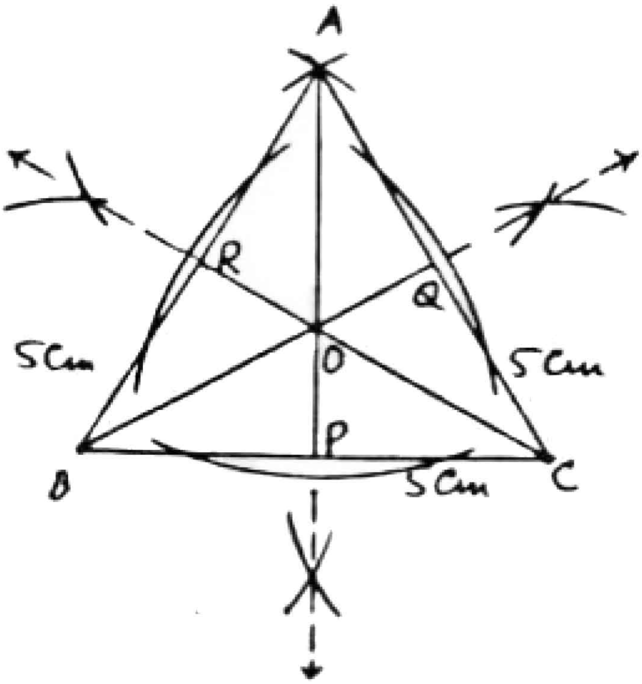
(iii) Cut $m\overline{BC} = 4.6cm$ at $\overline{BP}$.

(iv) Join "C" with A. ABC is the required triangle.

(v) Draw perpendiculars EF, GH and LM of the sides $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$ respectively. They meet each other at point "O".

Q.3. **Draw a equilateal Δ ABC in which $m\overline{AB} = m\overline{BC} = m\overline{AC} = 5cm$. Draw its altitudes and measure their lengths are they equal?**

Sol.



Steps of Construction:

(i) Draw a line segment $m\overline{BC} = 5cm$.

(ii) Draw arcs of radius 5cm with taking centre B and C, which intersect eachother at A.

(iii) Join A with B and C.

ABC is the required equilateral.

(iv) $AB \perp BC$, $BQ \perp CA$ and $AB \perp CR$

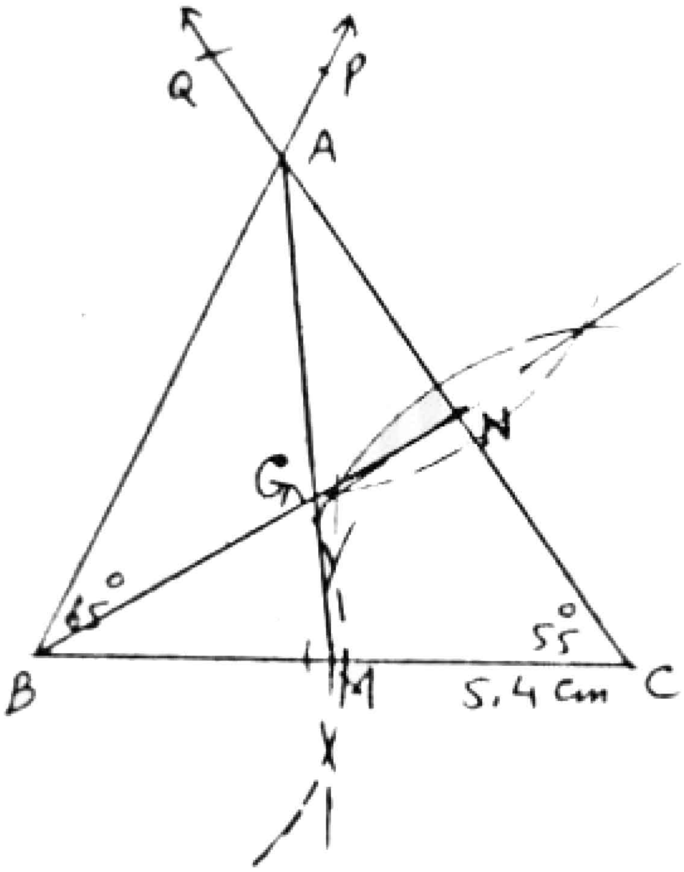
And

(v) $m\overline{AP} = m\overline{BC} = m\overline{CR} = 4.2cm$

All the altitudes are equal in lengths.

Q.4. **Construct a Δ ABC in which $m\overline{BC} = 5.4cm$, $\angle B = 65^{o}$ and $m\angle C = 55^{o}$. Find the centroid of the triangle.**

Sol.



Steps of Construction:

(i) Draw a line segment $\overline{BC}$ 5.4cm.

(ii) Now draw an angle of 65^{o} at point B & 55^{o} at point C. $\overrightarrow{BP}$ and $\overrightarrow{CQ}$ intersect eachother at point A.

(iii) ABC is the required triangle.

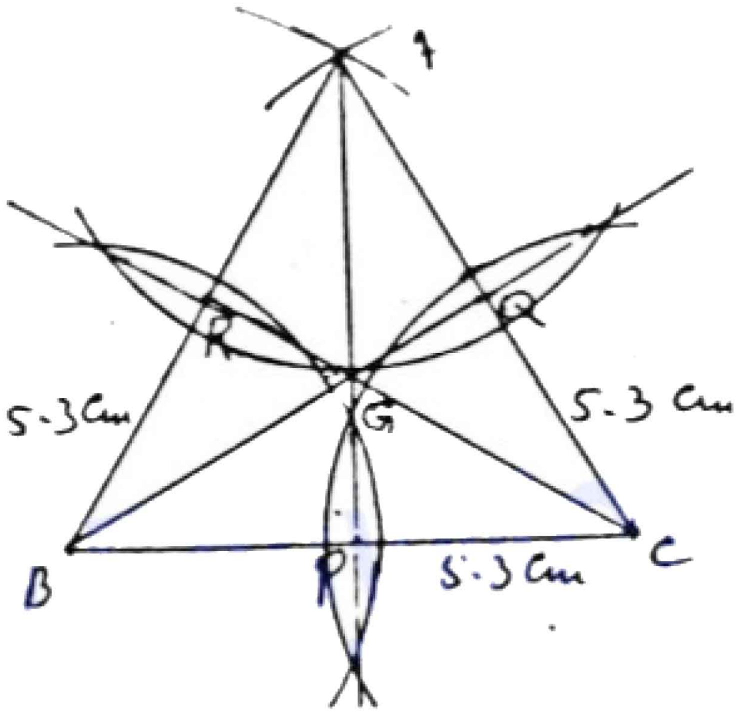
(iv) M and N are the mid points of $\overline{BC}$ and $\overline{AC}$.

(v) $\overline{AM}$ and $\overline{BN}$ are the medians which intersect eachother at point G.

Thus point "G" is the required centroid of triangle.

Q.5. **Draw an equilateral triangle each of whose sides is 5.3cm. Draw its medians. Are they equal?**

Sol.



Steps of Construction:

(i) Draw a line segment $\overline{BC}$ 5.3cm.

(ii) Taking B and C as centre draw two arcs which intersect eachother at point A.

(iii) Join point A with B and C.

ABC is the required equilateral.

(iv) Draw medians of sides $\overline{AB}$, $\overline{CA}$, $\overline{BC}$ at points P, Q & R.

(v) Join A with P, B with Q and C with R.

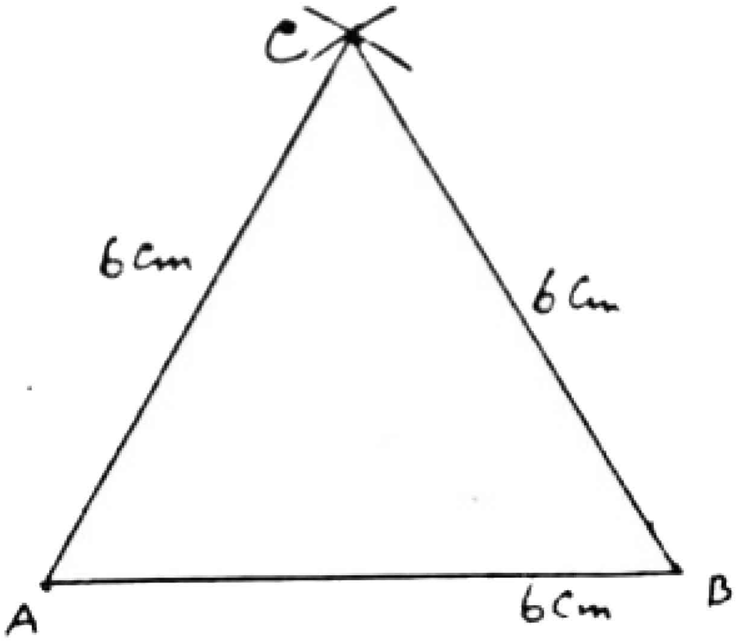
AP, BQ, CR are the required medians.

(vi) $m\overline{AP} = m\overline{BQ} = m\overline{CR} = 4.5cm$

So that the medians are equal in lengths.

Q.6. Draw an equilateral triangle with length of each side 6cm.

Sol.



Steps of Construction:

(i) Draw a line segment $\overline{AB}$ = 6cm.

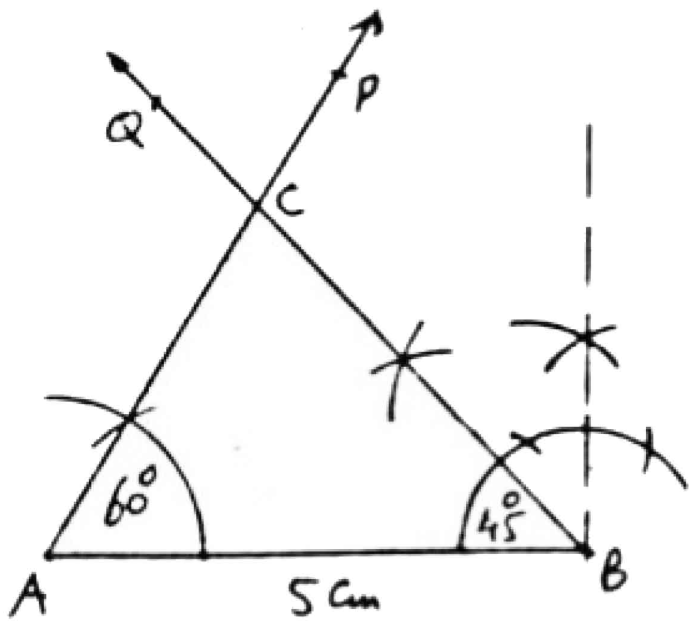
(ii) Taking A and B as centre draw two arcs of radius 6cm each. They intersect eachother at point C.

(iii) Join point C with A and B.

ABC is the required equilateral

Q.7. Construct a triangle ABC with base length 5cm and the angles at both ends of the base are 45^{0} and 60^{0} respectively.

Sol.



Steps of Construction:

(i) Draw a line segment $\overline{AB} = 5cm$.

(ii) Draw m $\angle BAP$ 60^0 at point A.

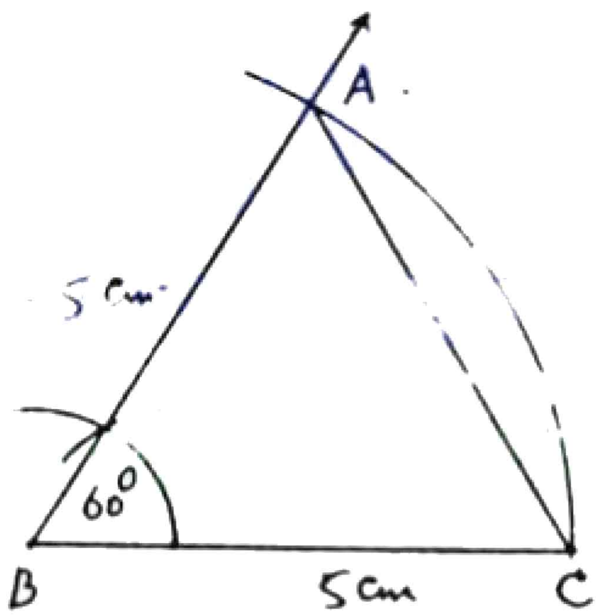
(iii) Draw an angle $m\angle ABQ = 45^0$ at point B.

(iv) $\overrightarrow{AP}$ and $\overrightarrow{BQ}$ intersect eachother at point C.

ABC is the required triangle.

Q.8. Draw an isosceles triangle with length of the equal sides 5cm and the angle included between them is 60^0.

Sol.



Steps of Construction:

(i) Draw a line segment $\overline{BC} = 5cm$.

(ii) At point B, draw $m\angle ABC = 60^0$ using compasses.

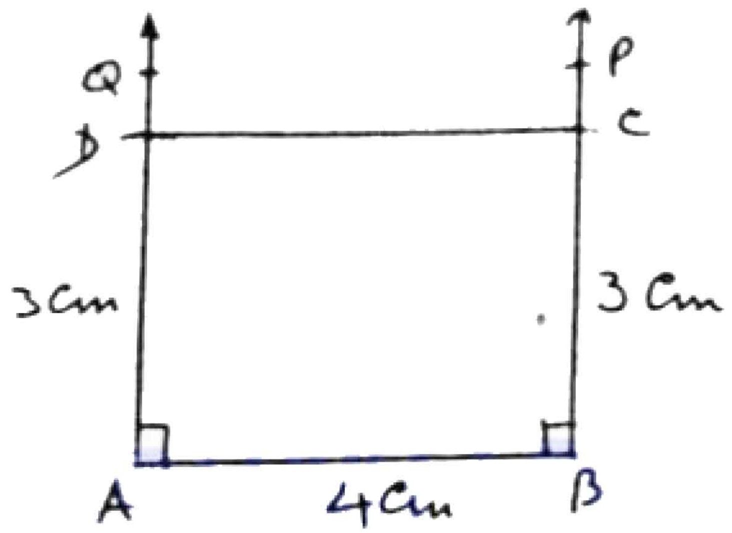
(iii) Cut $m\overline{BA} = 5cm$.

(iv) Join point A with C.

ABC is the required isosceles.

Q.9. **Construct a rectangle whose adjacent sides are 4cm and 3cm.**

Sol.



Steps of Construction:

(i) Draw a line segment $\overline{AB} = 4cm$.

(ii) At points A and B, draw right angles with the help of compasses.

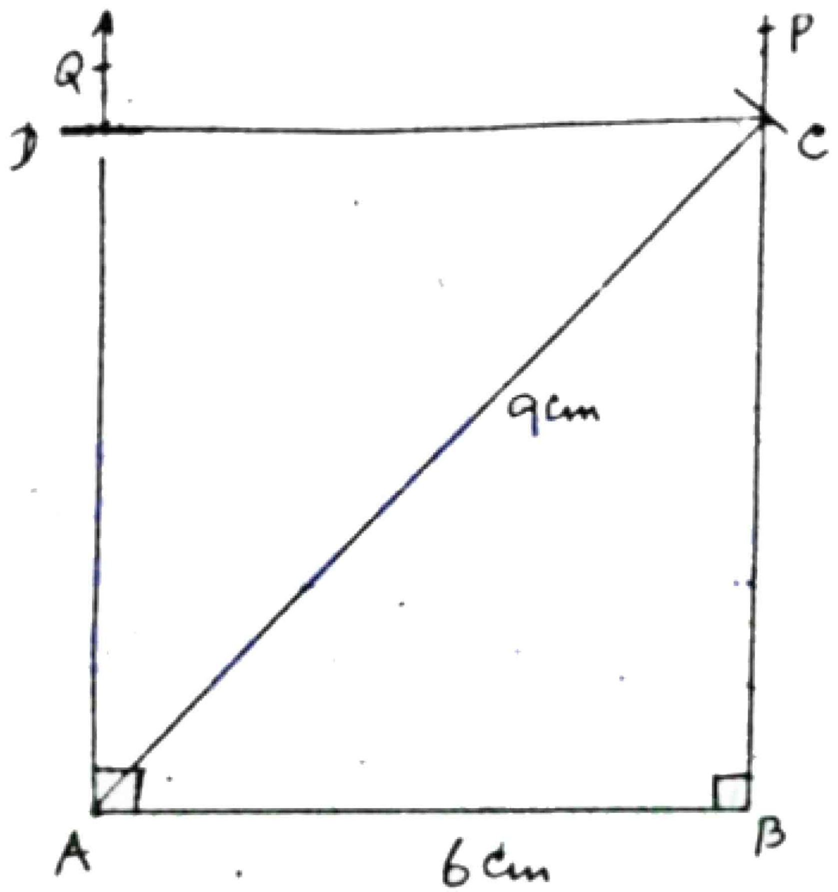
(iii) Cut $m\overline{AD} = m\overline{BC} = 3cm$.

(iv) Join point C with D.

ABCD is the required rectangle.

Q.10. Construct a rectangle whose one side is 6cm and an adjacent diagonal of 9cm.

Sol.



Steps of Construction:

(i) Draw a line segment $\overline{AB} = 6cm$.

(ii) Draw right angle at points A and B.

(iii) Taking centre as A draw on arc of radius 9cm which intersect $\overrightarrow{BP}$ at C.

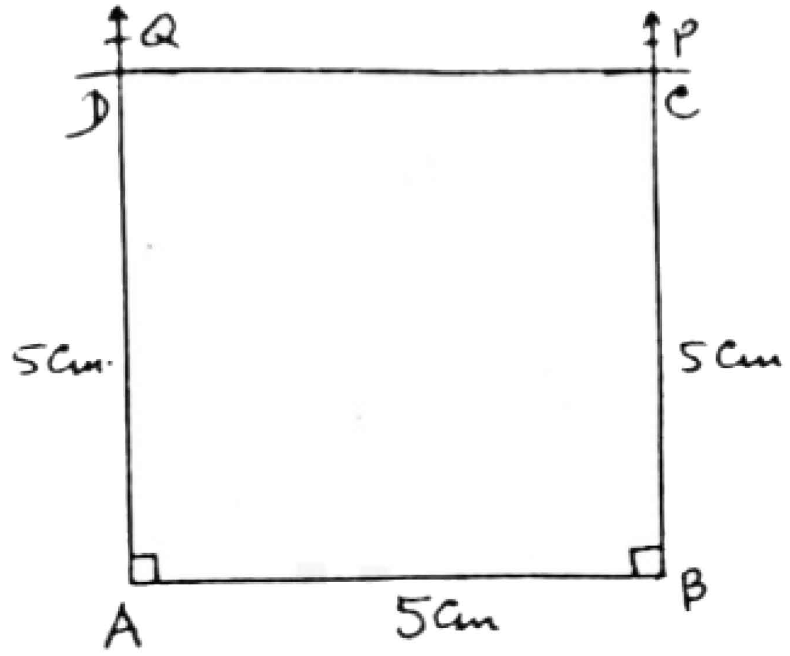
(iv) Cut $m\overline{BC}$, $m\overline{AD}$ at $\overrightarrow{AQ}$.

(v) Join point C with D.

$ABCD$ is the required rectangle.

Q.11. Construct a square whose one side is 5cm.

Sol.



Steps of Construction:

(i) Draw a line segment $\overline{AB} = 5cm$.

(ii) At points A and B, draw right angle with the help of compasses.

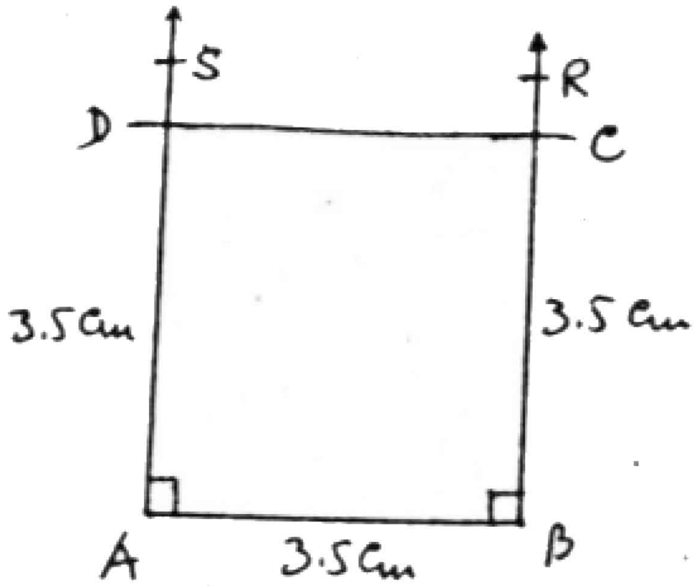
(iii) Cut $m\overline{BC} = m\overline{AD} = 5cm$ at $\overrightarrow{AQ}$ and $\overrightarrow{BP}$.

(iv) Join C with D.

$ABCD$ is the required square.

Q.12. Construct a square whose one side is 3.5cm.

Sol.



Steps of Construction:

(i) Draw a line segment $\overline{AB} = 3.5cm.$

(ii) At points A and B, draw right angle with the help of compasses.

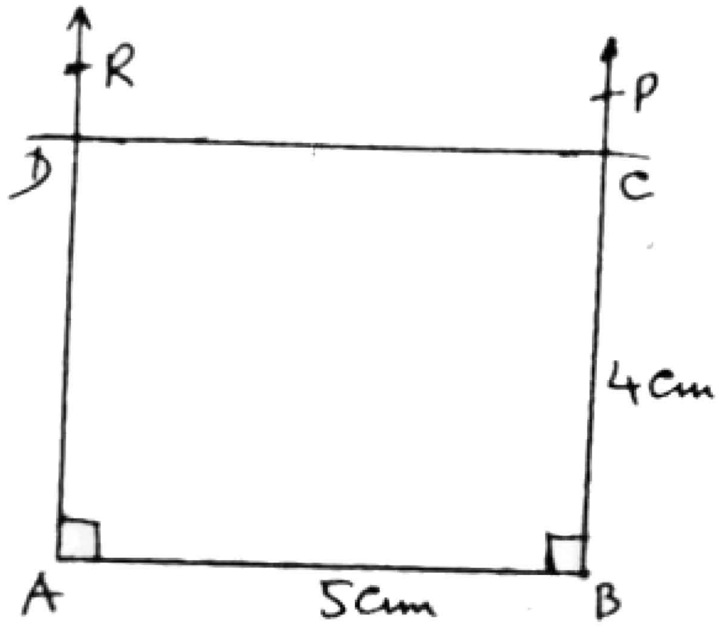
(iii) Cut $m\overline{BC} = m\overline{AD} = 3.5cm$ at $\overrightarrow{AQ}$ and $\overrightarrow{BR}$.

(iv) Join C with D.

$ABCD$ is the required square.

Q.13. **Construct a rectangle whose two adjacent sides measure 5cm and 4cm and their included angle is 90^0.**

Sol.



Steps of Construction:

(i) Draw a line segment $\overline{AB} = 5cm$.

(ii) At point A, draw right angle with the help of compasses.

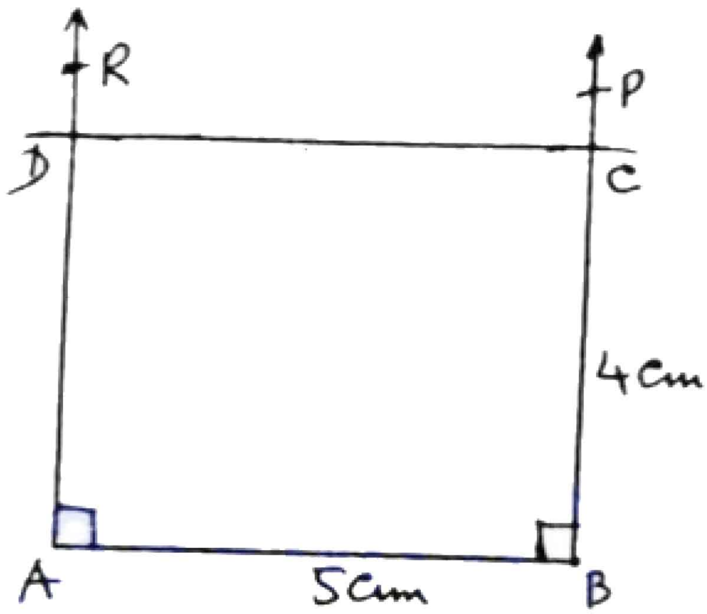
(iii) Cut $m\overline{BC} = m\overline{AD} = 4cm$ at $\overrightarrow{AR}$ and $\overrightarrow{BP}$.

(iv) Join C with D.

$ABCD$ is the required rectangle.

Q.13. Construct a rectangle whose two adjacent sides measure 5cm and 4cm and their included angle is 90°.

Sol.



Steps of Construction:

(i) Draw a line segment $\overline{AB} = 5cm$.

(ii) At point A, draw right angle with the help of compasses.

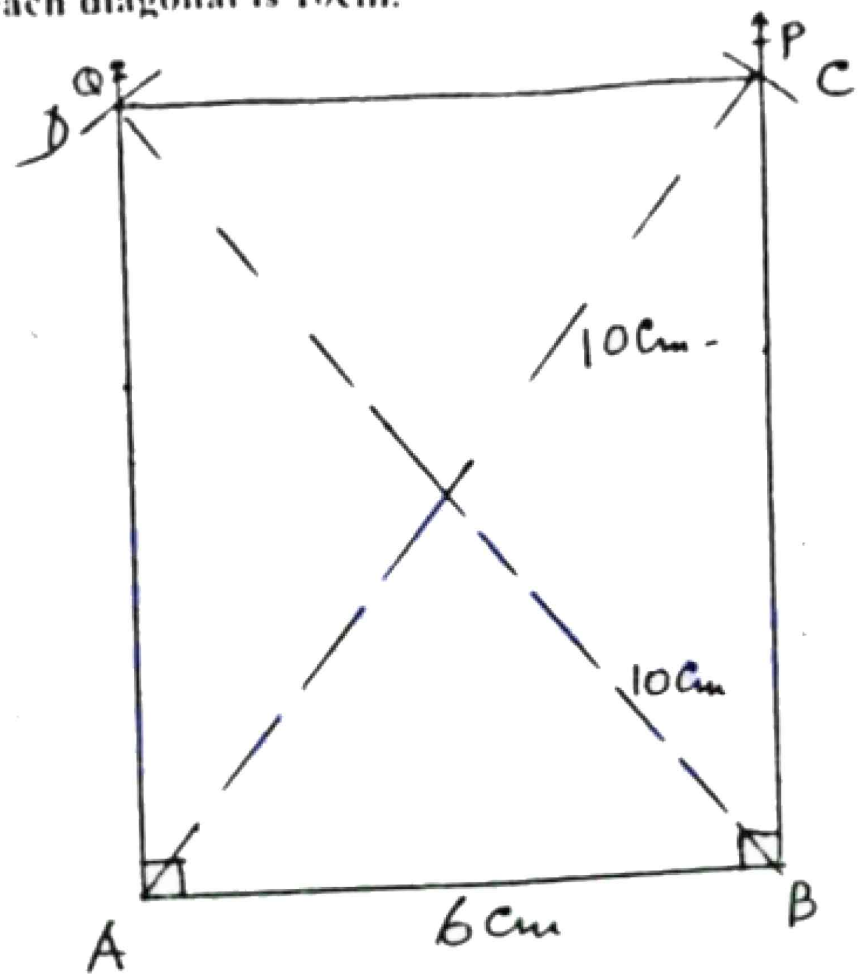
(iii) Cut $m\overline{BC} = m\overline{AD} = 4cm$ at $\overrightarrow{AR}$ and $\overrightarrow{BP}$.

(iv) Join C with D.

$ABCD$ is the required rectangle.

Q.14. Draw a rectangle whose one side is 8cm and the length of each diagonal is 10cm.

Sol.



Steps of Construction:

(i) Draw a line segment $\overline{AB} = 6cm$.

(ii) At point A and B, draw right angle with the help of compasses.

(iii) Draw an arc of radius 10cm with taking centre point "A" which intersect $\overline{BP}$ at point C.

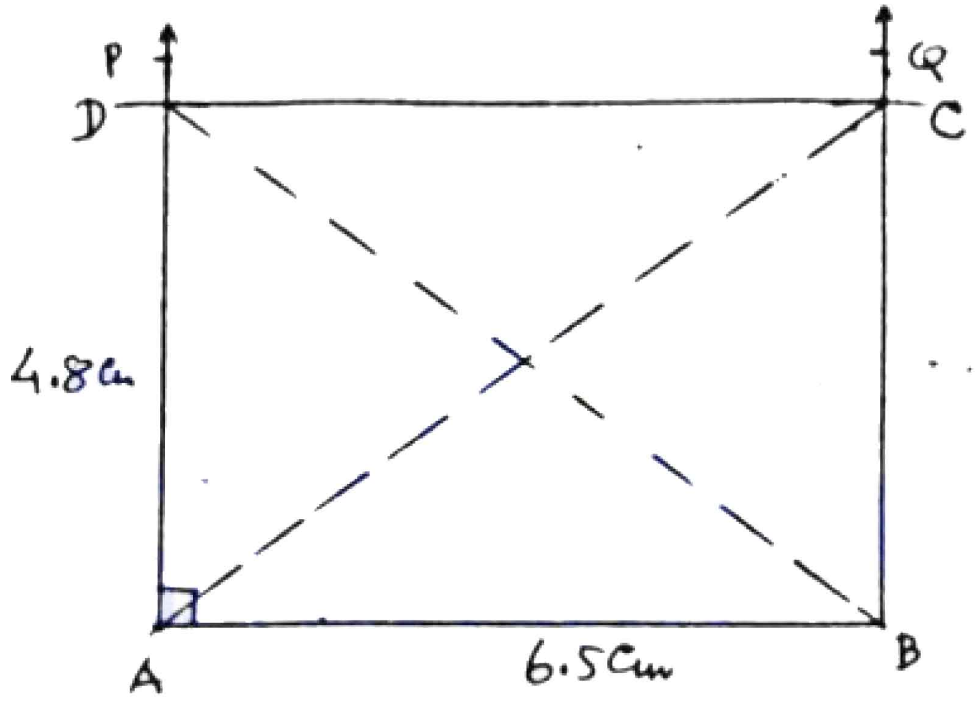
(iv) Now, draw an arc of radius cm again with taking centre at point B. Which intersect $\overrightarrow{AQ}$ at point D.

(v) Joint point C with D.

$ABCD$ is the required rectangle.

Q.15. **Draw a rectangle ABCD in which $m\overline{AB} = 6.5$cm and $m\overline{AD} = 4.8$cm and $m\angle BAD = 90^0$. Measure its diagonals.**

Sol.



$m\overline{AC} = m\overline{BD} = 8.1cm$

Steps of Construction:

(i) Draw a line segment $\overline{AB} = 6.5$cm.

(ii) At point A and B, draw right angle with the help of compasses.

(iii) Intersect $m\overline{BC} = m\overline{AD} = 4.8cm$ at $\overrightarrow{AP}$ and $\overrightarrow{BQ}$.

(iv) Join C with D.

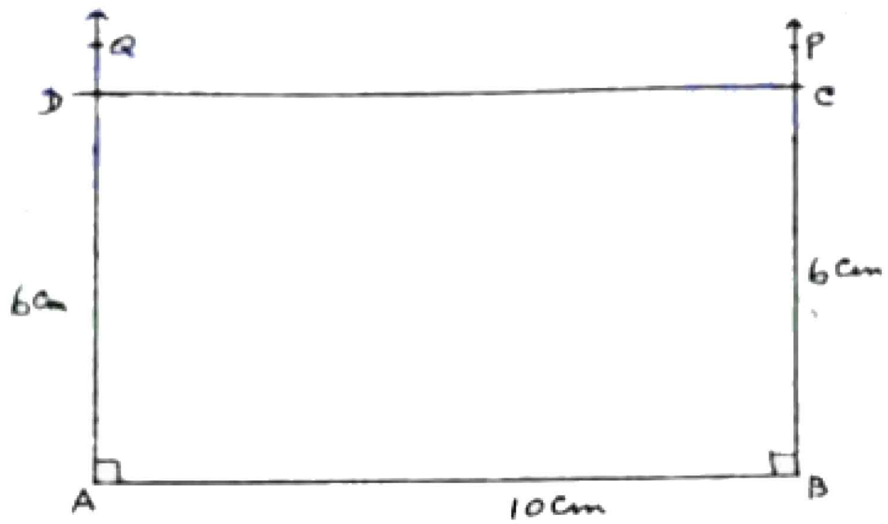
(v) $ABCD$ is the required rectangle.

Q.16. Name the following quadrilaterals when:

	Questions	Answers
(i)	The diagonals are equal and the adjacent sides are unequal.	Rectangle
(ii)	The diagonals are equal and the adjacent sides are equal.	Square
(iii)	All the sides are equal and one angle is 90°.	Square
(iv)	All the angles are equal and the adjacent sides are unequal	Rectangle

Q.17. Construct a rectangle with sides $10cm$ and $6cm$.

Sol.



Steps of Construction:

(i) Draw a line segment $\overline{AB} = 10cm$.

(ii) At point A and B, draw right angle with the help of compasses/

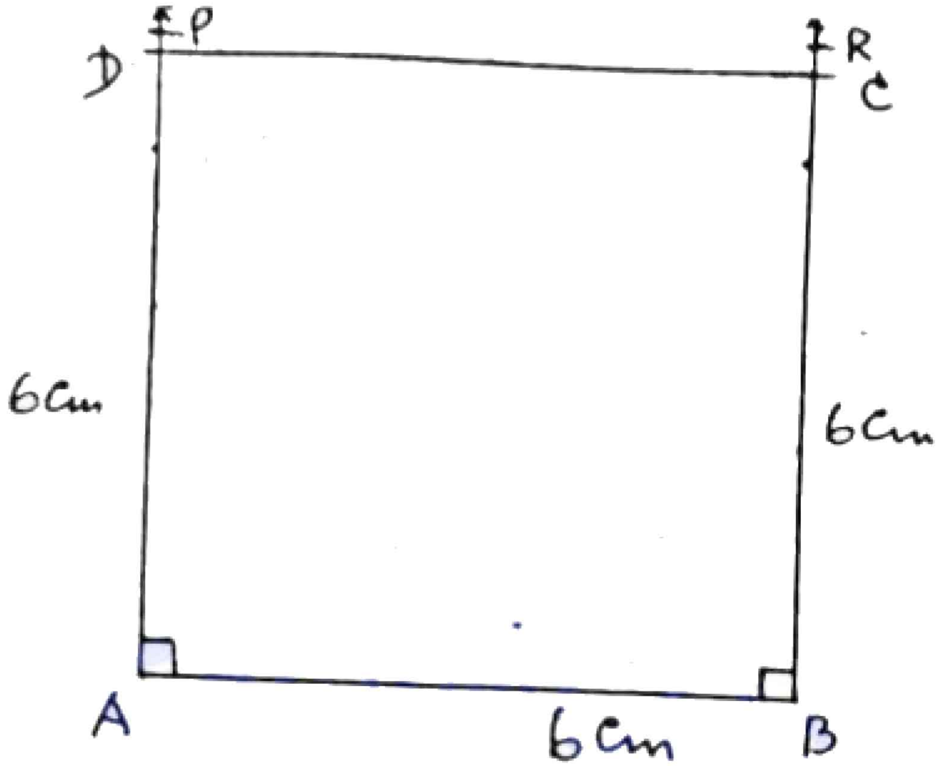
(iii) Intersect $m\overline{BC} = m\overline{AD} = 6cm$ at $\overrightarrow{AQ}$ and $\overrightarrow{BP}$.

(iv) Join C with D.

$ABCD$ is the required rectangle.

Q.18. Construct a square with side of length 6cm.

Sol.



Steps of Construction:

(i) Draw a line segment $\overline{AB}$ = 6cm.

(ii) Points A and B, draw right angle with the help of compasses.

(iii) Cut $\overrightarrow{AP} = \overrightarrow{BR}$ = 6cm at $\overline{AD}$ and $\overline{BC}$.

(iv) Join C with D.

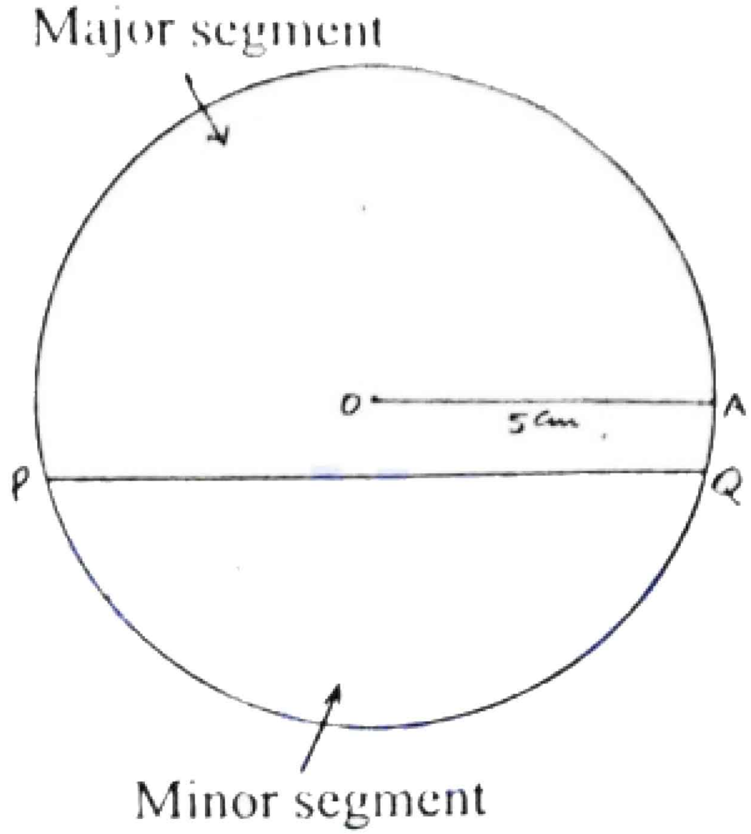
$ABCD$ is the required square.

Q.19. Name the following triangles.

Questions	Answers
(i) With all the three sides equal in length.	Equilateral triangle
(ii) With two sides equal in length.	Isosceles triangle
(iii) None of the sides is equal to the other.	Scalene triangle

Q.20. **Draw a circle with centre O and radius $5cm$. Explain the steps necessary to draw a segment of the circle.**

Sol.



Steps of Construction:

(i) Take any point O.

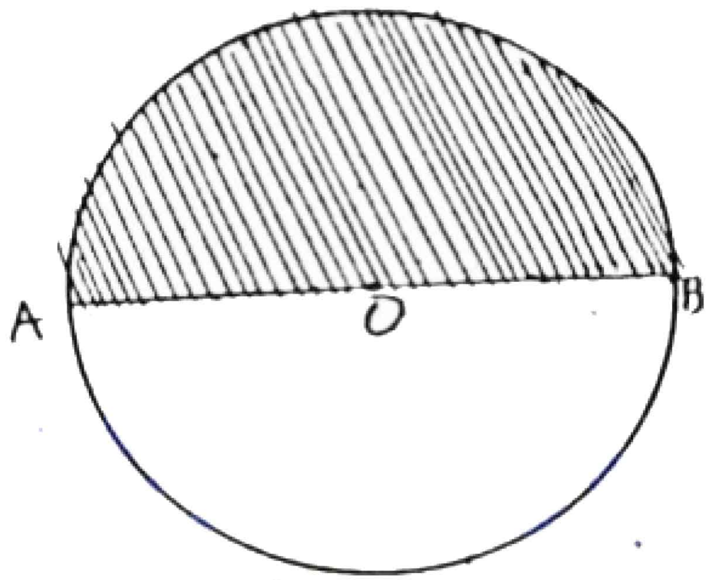
(ii) Taking centre with "O", draw an arc of radius 5cm.

(iii) Now, take any diameter $\overline{PQ}$.

Conclusions: $\overline{PQ}$ has divided the circle into two parts. The major segment part and minor segment part.

Q.21. Draw a circle with center O and any radius. Draw the diameter AB and shade one semicircular region.

Sol.



Steps of Construction:

(i) Take any point O.

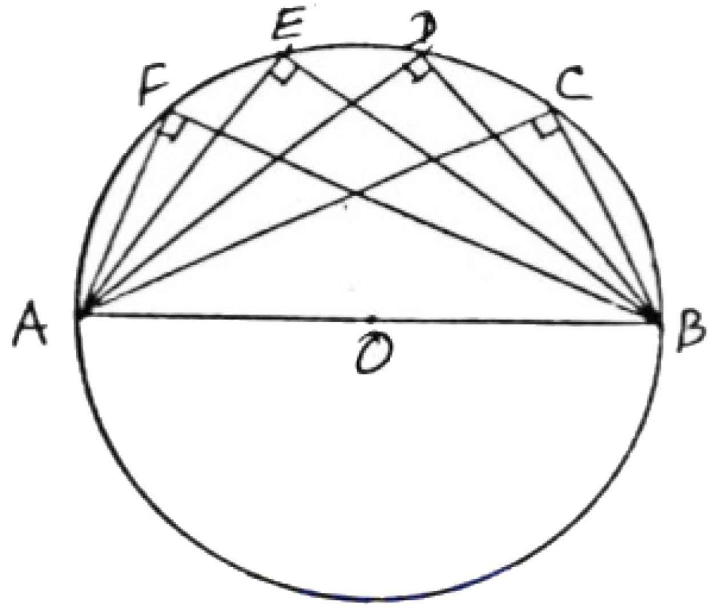
(ii) With taking "O" as centre, draw a circle with suitable radius.

(iii) Draw $\overline{AOB}$ as diameter.

Conclusion: The circle has divided into two parts. Now, shaded the half part.

Q.22. **Show four angles in a semi-circular region of question 21.**

Sol.



Steps of Construction:

(i) Draw a circle with suitable radius and marks its centre as "O".

(ii) Draw a diameter $\overline{AOB}$.

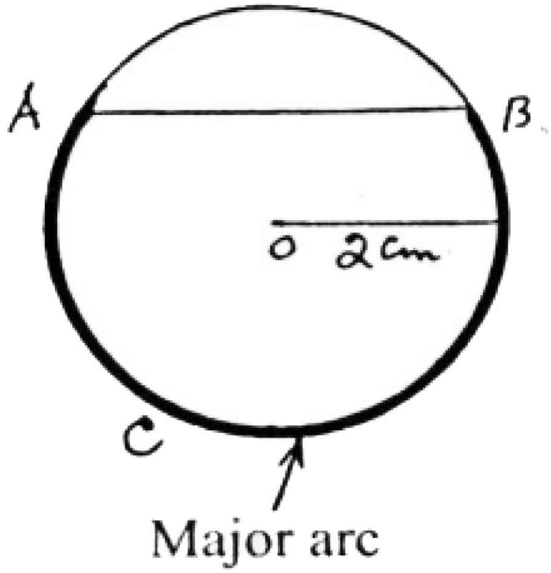
(iii) Take a point C, D, E, F at half curved area.

(iv) Join these points with A and B.

Conclusion: $\angle ACB, \angle ADB, \angle AEB, \angle AFB$ are the required four angles.

Q.23. Draw a circle of radius $2cm$ with center O. Draw a chrod and shade the portion showing major arc.

Sol.



Steps of Construction:

(i) Take any point O.

(ii) With taking "O" as centre, draw a circle with radius 2cm.

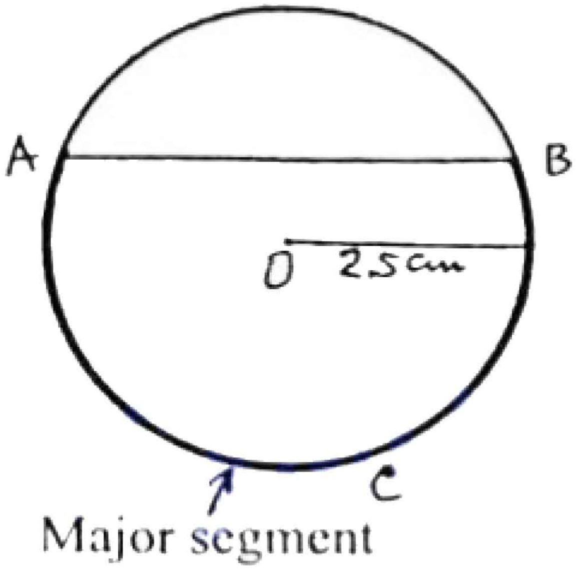
(iii) Take $\overline{AB}$ as chord.

Thus, $\overset{\frown}{ACB}$ is the major arc.

(iv) In figure $\overset{\frown}{ACB}$ (major arc) is quite prominent.

Q.24. Draw a circle of radius 2.5cm with center at *O*. Draw a chrod and shade the portion showing the minor arc of the circle.

Sol.



Steps of Construction:

(i) Take any point "*O*".

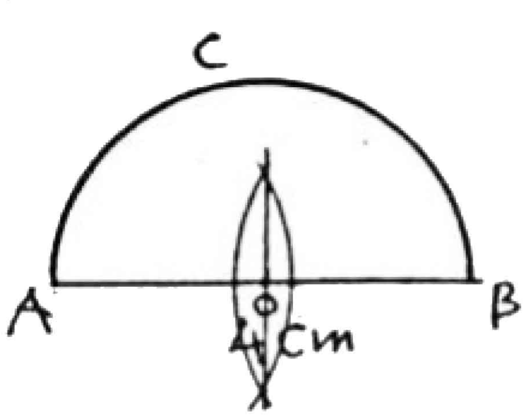
(ii) With taking "*O*" as centre, draw a circle with radius 2.5cm

(iii) Draw $\overline{AB}$ as chord.

(iv) The major arc $\overparen{ACB}$ is quite prominent in the figure.

Q.25. Draw a semi-circle with diameter 4cm and center at *O*.

Sol.



Steps of Construction:

(i) Draw a line segment $\overline{AB} = 4cm$.

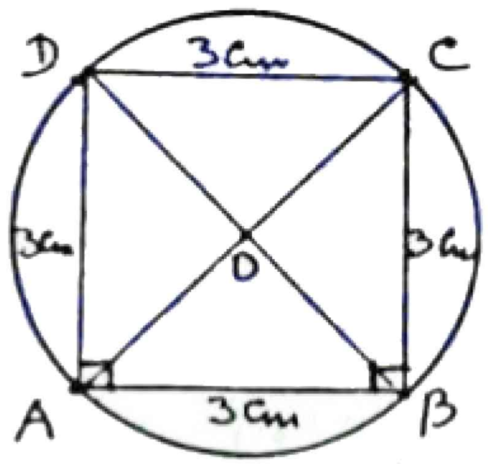
(ii) Now take "O" as centre, draw a semi-circle with radius $m\overline{AO}$ or $m\overline{OB}$.

(iii) With taking "O" as centre $m\overline{OA}$ or $m\overline{OB}$.

(iv) ACB is the required semi-circle.

Q.26. Draw a circle passing through the vertices of a square of side 3cm.

Sol.



Steps of Construction:

(i) Draw a line segment $\overline{AB}$ = 3cm.

(ii) At points A and B, draw right angles at each ($\because$ $m\overline{AD} = m\overline{BC} = 3cm$)

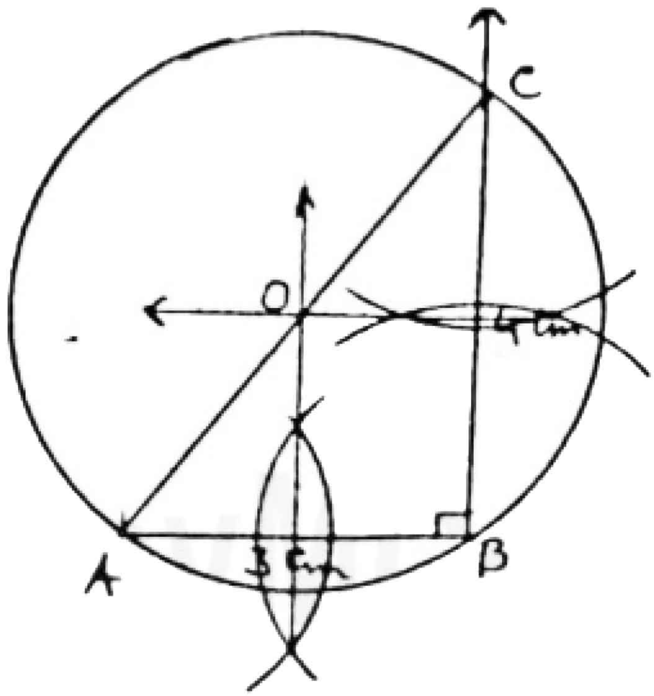
(iii) Join C with D.

(iv) Draw two diagonals $\overline{AC}$ and $\overline{BD}$ which intersect eachother at O.

(v) Taking "O" as centre, draw a circle with radius $m\overline{OB}$ or $m\overline{OA}$ or $m\overline{OC}$ or $m\overline{OD}$.

Q.27. In a right triangle ABC, $m\overline{AB} = 3cm$ and $m\overline{BC} = 4cm$ with right angle at B. Draw a circle through A, B and C.

Sol.



Steps of Construction:

(i) Draw a line segment $\overline{AB} = 3cm$.

(ii) At point B, draw right angle with the help of compasses.

(iii) Cut $m\overline{BC} = 4cm$.

(iv) Join C with A.

ABC is the required triangle.

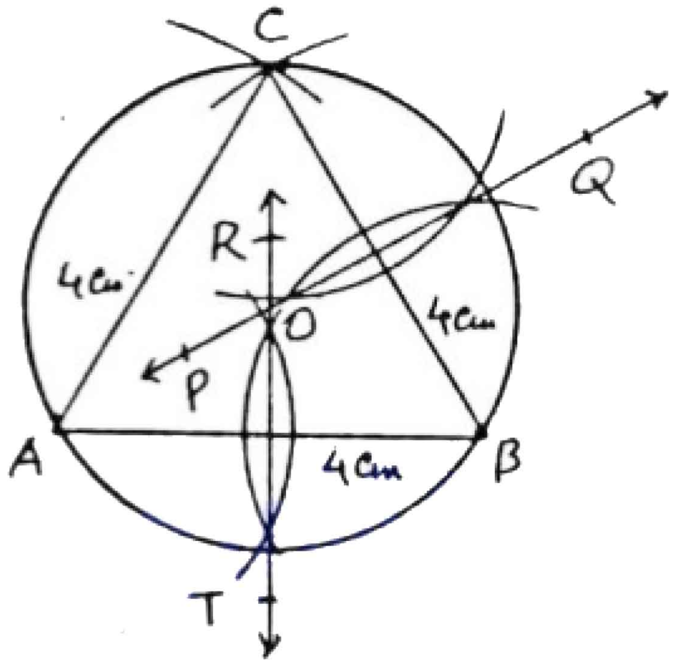
(v) Now, draw perpendicular bisector of sides $\overline{AB}$ and $\overline{BC}$ which cut eachother at point "O".

(vi) With taking "O" as centre draw a circle with radius $m\overline{OB}$ or $m\overline{OC}$ or $m\overline{OA}$ respectively.

Conclusion: The circle is passing through the vertices (A, B and C)

Q.28. Draw a circle passing through the three vertices of an equilateral triangle with length of each side $4cm$.

Sol.



Steps of Construction:

(i) Draw a line segment $m\overline{AB} = 4cm$.

(ii) Taking A and B as centre, draw two arcs of radius 4 cm each. They intersect eachother at point "C".

(iii) Join C with points A and B.

ABC is the equilateral triangle.

(iv) Now, draw a perpendicular bisectors $\overleftrightarrow{RT}$ and $\overleftrightarrow{PQ}$ of sides $\overline{AB}$ and $\overline{BC}$ respectively.

They meet eachother at point "O".

(v) With taking "O" as centre, draw a circle with radius $m\overline{OC}$ or $m\overline{OA}$ or $m\overline{OB}$ respectively.

Conclusion: The circle is passing through the vertices (A, B, C) of triangle.

Review Exercise 8:

I- *Encircle the correct answer:*

1- **The number of medians in a triangle is:**

(a)	1	(b)	2
(c)	3	(d)	4

2- **The number of altitudes in a triangle is:**

(a)	1	(b)	2
(c)	3	(d)	4

3- **The number of angle bisectors in a triangle is:**

(a)	1	(b)	2
(c)	3	(d)	4

4- **The number perpendicular bisectors of the side of a triangle is:**

(a)	1	(b)	2
(c)	3	(d)	4

5- **The angle bisectors of a triangle are:**

(a)	concurrent	(b)	collinear
(c)	perpendicular	(d)	non-concurrent

6- **The medians of a triangle are:**

(a)	concurrent	(b)	collinear
(c)	non-concurrent	(d)	4

7- **The altitudes of a triangle are:**

(a)	concurrent	(b)	collinear
(c)	non-collinear	(d)	5

8- **A line joining one vertex of a triangle to the mid point of its opposite sides is called:**

(a)	angle bisector	(b)	altitude
(c)	median	(d)	side bisector

9- **A line joining one vertex of a triangle and perpendicular to its opposite side is called:**

(a) angle bisector (b) median

(c) altitude (d) side bisector

10- **A line coplanar with a circle and intersecting the circle at one point only is called:**

(a) tangent line (b) median

(c) altitude (d) normal line

Answers:

1-	(c)	2-	(c)	3-	(c)	4-	(c)	5-	(a)
6-	(a)	7-	(a)	8-	(c)	9-	(c)	10-	(a)

II- **Fill in the blanks.**

1- The altitudes of a triangle are __________.

2- The medians of a triangle are __________.

3- The angle bisector of a triangle are __________.

4- The perpendicular bisector of the three sides of a triangle are __________.

5- The line joining one vertex of a triangle and perpendicular to its opposite side is called __________ of a triangle.

6- A line joining one vertex of a triangle to the midpoint of its opposite side is called __________ of a triangle.

7- A line bisecting the angle of a triangle is called the __________.

8- Every triangle has __________ altitudes.

9- Every triangle has __________ median.

10- Every triangle has __________ right bisectors.

Answers:

1-	concurrent	2-	concurrent	3-	concurrent	4-	concurrent	5-	altitude
6-	median	7-	angle bisector	8-	three	9-	three	10-	three

Exercise 9.1

1. ***Find the third side of each right triangle with legs a and b and hypotenuse c.***

(i) $a = 3, \quad b = 4, \quad c = ?$

(ii) $a = 5, \quad c = 13, \; b = ?$

(iii) $b = 5, \quad c = 61, \; a = ?$

Solution:

(i) $a = 3$

$b = 4$

$c = ?$

$c^2 = a^2 + b^2$ By Pythagoras theorem

$c^2 = (3)^2 + (4)^2$ Putting values of a, b

$c^2 = 9 + 16$

$c^2 = 25$

$\sqrt{c^2} = \sqrt{25}$ Taking square root

$\boxed{c = 5}$

(ii) $a = 5, \quad c = 13, \quad b = ?$

By Pythagoras theorem

$c^2 = a^2 + b^2$

$b^2 = c^2 - a^2$ or

Putting values of a, c

$b^2 = (13)^2 - (5)^2$

$b^2 = 169 - 25$

$b^2 = 144$

$\sqrt{b^2} = \sqrt{144}$ Taking square root

$\boxed{b = 12}$

(iii) b = 5

c = 61

a = ?

By Pythagoras theorem

$c^2 = a^2 + b^2$

$a^2 = c^2 - b^2$ therefore

Putting values of b, c

$$a^2 = (61)^2 - (5)^2$$

$$a^2 = 3721 - 25$$

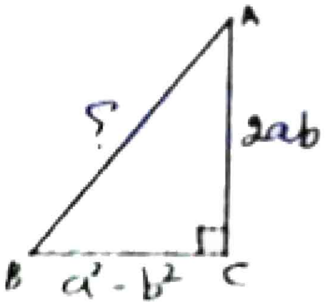
$$a^2 = 3696$$

$a^2 = 3696$ Taking square root

$$\sqrt{a^2} = \sqrt{16 \times 231}$$

$$\boxed{a = 4\sqrt{231}}$$

2 If the legs of a right triangle are 2ab and $a^2 - b^2$, prove that hypotenuse is $a^2 + b^2$.



Sol

one side = $2ab$

2nd side = $a^2 - b^2$

Hypotenuse = ?

By Pythagoras theorem

$$(\text{Hypotenuse})^2 = (2ab)^2 + (a^2 - b^2)^2$$

$$\text{Hypotenuse} = \sqrt{(a^2 - b^2)^2 + (2ab)^2}$$

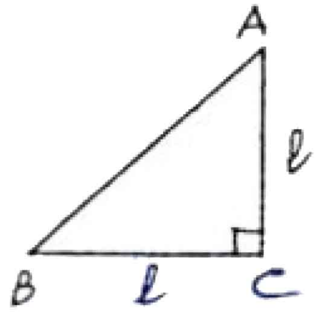
$$= \sqrt{a^4 + b^4 - 2a^2b^2 + 4a^2b^2}$$

$$= \sqrt{a^4 + b^4 + 2a^2b^2}$$

$$= \sqrt{(a^2 + b^2)^2}$$

$$\text{Hypotenuse} = (a^2 + b^2)$$

3. Find the hypotenuse of the right isosceles triangle each of whose legs is l.



Sol:

$$m\overline{AC} = l$$

$$m\overline{BC} = l$$

$$m\overline{AB} = ?$$

By Pythagoras theorem

$$\left(m\overline{AB}\right)^2 = \left(m\overline{AC}\right)^2 + \left(m\overline{BC}\right)^2$$

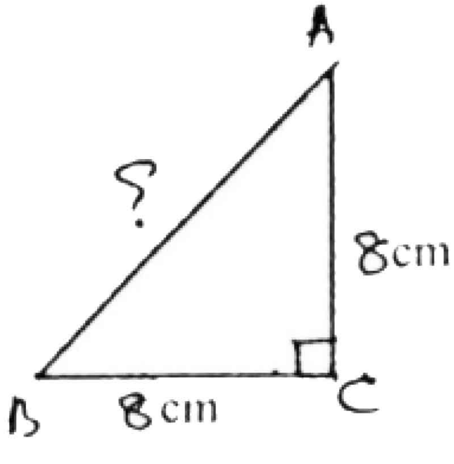
$$= l^2 + l^2$$

$$= 2l^2$$

$$\sqrt{m\overline{AB}^2} = \sqrt{2l^2} \quad \text{Taking square root}$$

$$m\overline{AB} = \sqrt{2}\, l \text{ units}$$

4. Find the hypotenuse of a right isosceles triangle whose legs are 8cm



Sol:

$a = 8$ cm here

$b = 8$ cm

$c = ?$

By Pythagoras theorem

$c^2 = a^2 + b^2$

Putting values of a, b

$c^2 = (8)^2 + (8)^2$

$c^2 = 64 + 64$

$c^2 = 128$

$\sqrt{c^2} = \sqrt{128}$ Taking square root

$c = \sqrt{8 \times 8 \times 2}$

$c = 8\sqrt{2}$ cm

5. If the numbers represent the lengths of the sides of a triangle, which triangles are right triangles?

(i) 3, 4, 5

(ii) 9, 17, 25

(iii) 11, 61, 60

Sol: Length of sides 3, 4, 5

We observer that, is the square of big side equal or not equal the square of the other to sides? Then these sides will be right angle Δ otherwise not.

$(5)^2 = 25$	$(3)^2 + (4)^2$
	$= 9 + 16$
	$= 25$

These sides are sides of right angle Δ.

(ii)

$(25)^2$	$(9)^2 + (17)^2$
$= 625$	$= 81 + 289$
	$= 370$

$$625 \neq 370$$

Sides 9, 17, 25 are not the sides of right angle triangle.

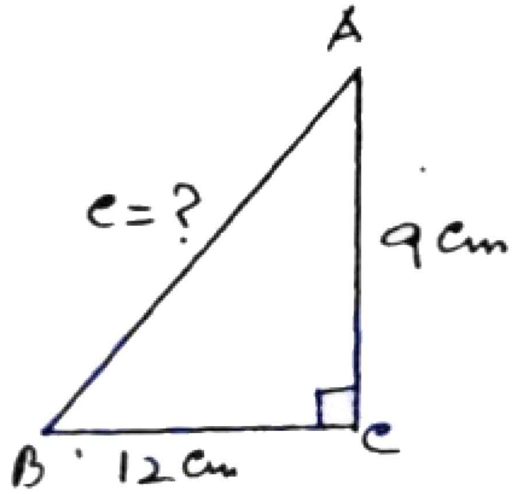
(iii)

$(61)^2$	$(11)^2 + (60)^2$
$= 3721$	$= 121 + 3600$
	$= 3721$

Sides 11, 61, 60 are the sides of right angle triangle.

6. ΔABC is right angled at C. If $m\overline{AC} = 9cm$ and $m\overline{BC} = 12cm$, find the length $\overline{AB}$, using Pythagoras theorem.

Sol: In ΔABC



b = 9 cm

a = 12 cm

c = ?

By Pythagoras theorem

$c^2 = a^2 + b^2$

Putting values of a, b

$c^2 = (12)^2 + (9)^2$

$= 144 + 81$

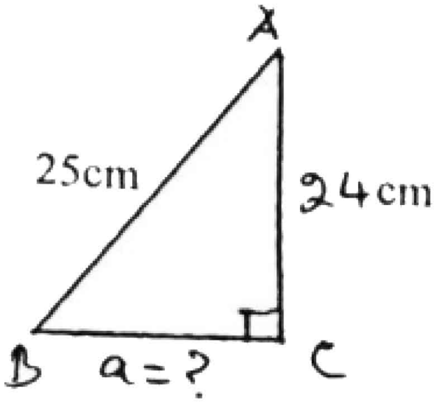
$c^2 = 225$

$\sqrt{c^2} = \sqrt{225}$ Taking square root

$\boxed{c = 15}$ cm

7. The hypotenuse of a right triangle is 25cm. If one of the sides are of length 24cm, find the length of the other side.

Sol:



$c = 25$ cm here

$b = 24$ cm

$a = ?$

By Pythagors theorem

$$c^2 = a^2 + b^2$$

$$a^2 = c^2 - b^2$$ or

Putting values of b, c

$$a^2 = (25)^2 - (24)^2$$

$$= 625 - 576$$

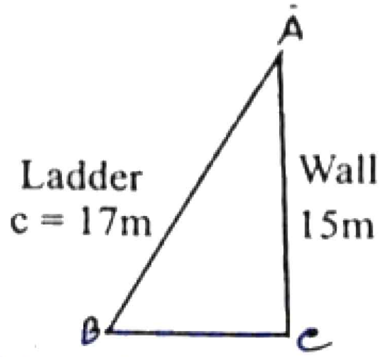
$$a^2 = 49$$

$$\sqrt{a^2} = \sqrt{49}$$ Taking square root

$$\boxed{a = 7cm}$$

8. A ladder 17m long when set against the wall of a house just reaches a window at a height of 15m from the ground. How far is the lower end of the ladder from the base of the wall?

Sol:



Lower end of the ladder from the wall = a

$b = 15\text{m}$ here

$c = 17\text{m}$

$a = ?$

By Pythagoras theorem

$$a^2 = c^2 - b^2$$

Putting values of c, b

$$a^2 = (17)^2 - (15)^2$$

$$= 289 - 225$$

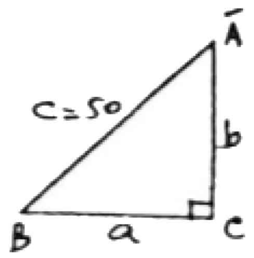
$$a^2 = 64$$

$\sqrt{a^2} = \sqrt{64}$ Taking square root

$\boxed{a = 8}$ m

9. The two legs of a right triangle are equal and the square of the hypotenuse is 50. Find the length of each leg.

Sol:



$c = 50$ here

$a = b$

By Pythagoras theorem

$c^2 = a^2 + b^2$

$c^2 = a^2 + a^2$ $(a = b)$

$c^2 = 2a^2$

$2a^2 = c^2$ or

Putting values of c^2

$2a^2 = (50)^2$

$a^2 = \frac{2500}{2}$

$a^2 = 1250$

$\sqrt{a^2} = \sqrt{1250}$ taking square root

$a = \sqrt{625 \times 2}$

$a = 25\sqrt{2}$

Length of each side $= 25\sqrt{2}$ units

10. The sides of a triangle are 15cm, 36cm and 39cm. Show that it is a right angled triangle.

Sol:

$$\text{The length of big side} = 39 \text{ cm}$$

$$\text{The length of small side} = 15\text{cm}, 36\text{cm}$$

$$(\text{Lengths of big side})^2 = (39)^2$$

$$= 1521 \qquad \text{(i)}$$

$$\text{The sum of the square of small sides} = (15)^2 + (36)^2$$

$$= 225 + 1296$$

$$\text{Now its prove that} = 1521 \qquad \text{(ii)}$$

These lengths are the lengths of right angle triangle.

Area

The surface inside the boundary of a shape is called area. We see that, Is the big side of square equal or not equal the square of the other two sides? Then these sides will be sides of right angle triangle otherwise not.

Area of a Triangle when all the three sides are given

A triangle ABC with sides a, b, c and

$$2S = a + b + c \Rightarrow \qquad S = \frac{a+b+c}{2},$$

where 'S' is half the perimeter of a triangle.

Then area of any triangle is $A = \sqrt{S(S-a)(S-b)(S-c)}$.

This is called ***Hero's Formula*** for finding the area of a triangle.

Exercise 4.5

1. A verandah 40m long, 15m wide is to be paved with stones each measuring 6cm by 5cm. Find the number of stones

Sol Length of verandah = 40m

Width of verandah = 15m

Area of veranda = 40 × 15

= 600 sq. m

Area of one stone = 6 × 5 Now

= 30 sq. m

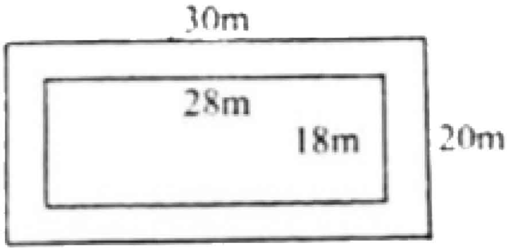
$$\text{Number of stones} = \frac{\textit{Total area}}{\textit{area of one stone}}$$

$$= \frac{600}{30}$$

Number of stones = 20 tiles

2. How many tiles of $40cm^2$ will be required to pave the footpath 1m wide carried round the outside of a grassy plot 28m by 18m?

Sol: Tiles fixed round the out side of a plot therefore,



External length = 28 + 1 + 1

$= 30\text{ m}$

External width $= 18 + 1 + 1$

$= 20\text{ m}$

Area of plot with path $= 30 \times 20$

$= 600\text{ sq. m}$

The area of plot $= 28 \times 18$

$= 504\text{ sq. m}$

Area of path $= 600 - 504$

$= 96\text{ sq. m}$

Area of one tile $= 40\text{ sq. m}$

$= \frac{40}{100 \times 100}\text{ sq. m}$

$$\text{Number of tiles} = \frac{\text{Area of Path}}{\text{area of one tile}}$$

$$= \frac{96}{\frac{40}{100 \times 100}}$$

$$= \frac{{}^{24}96 \times 100 \times 100}{40}$$

Number of tiles $= 24000$

3. Find the area of a room 5.49m long and 3.87m wide. What is the cost of carpeting the room if the rate of carpet is Rs. 10.50 per m^2?

Sol: Length of room $= 5.49\text{m}$

Width of room $= 3.87\text{m}$

Area of room $= \text{Length} \times \text{Width}$

$= 5.49 \times 3.87$

$= 21.2463$ sq. m

Cost of carpeting one square m = Rs. 10.50

Cost of carpeting 21.2463 sq.m = 10.50×21.2463

$= 223.08615$

Cost of carpeting = Rs. 223 Approx

4. The area of a rectangular rice field is 2.5 hectares and its sides are in the ratio 3 : 2. Find the perimeter of the field.

Area of field = 2.5 hectares

Area of field = 25000 sq. m

Length of filed = $3x$ m

Width of field = $2x$ m and

Area = $3x \times 2x$

Area = $6x^2$ sq. m

from (i) and (ii)

$$6x^2 = 25000$$

$$x^2 = \frac{25000}{6}$$

$x^2 = 4166.67$ Taking square root

$$x = 64.55$$

Length $3x$ = 193.65

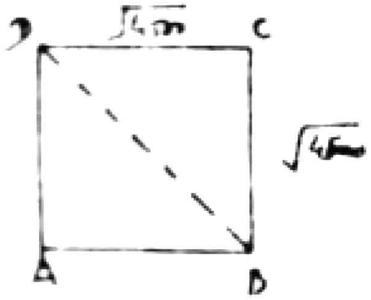
Width $2x$ = 129.10

Perimeter = $2(3x + 2x) = 2(193.65 + 129.10)$

$= 2(322.75)$

$= 645.50$ m

5. The area of a square playground is 4500 m^2. How long will a man take to cross it diagonally at the speed of 3km per hour?



$= 4500$ sq m

$= \sqrt{4500}$ m

Length of diagonal of a square $= m\overline{BD} = \sqrt{\left(\sqrt{4500}\right)^2 + \left(\sqrt{4500}\right)^2}$

By Pythagoras theorem $= 4500 + 4500$

$= \sqrt{9000}$

$m\overline{BD} = 94.87$ m

Time spent to cross 3km distane = 60min

Time spent to cross 3000 km distance = 60 min

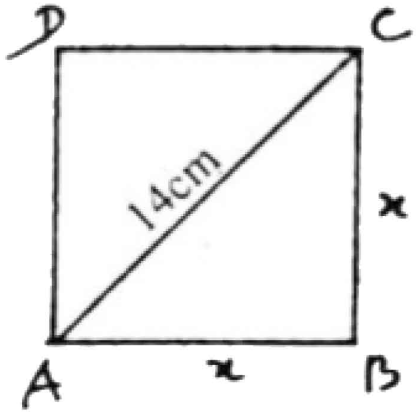
Tiem spent to cross 94.87m distance $= \frac{60}{3000} \times 94.87$ min

$= \frac{94.87}{50}$

$= 1.8974$ min

$= 1$ min 54 sec

6. The diagonal of a square is 14cm. Find its area.



Diagonal of square = 14cm

$$\text{Area of a square} = \frac{14 \times 14}{2}$$

$$= 14 \times 7$$

$$= 98 \text{ sq. cm}$$

2nd method:

Suppose that side of a square = x cm

By Pythagoras theorem

$$x^2 + x^2 = (14)^2$$

$$2x^2 = 14 \times 14$$

$$x^2 = \frac{14 \times 14}{2}$$

$$= 14 \times 7 = 98$$

and x^2 is area of a square.

Thus, Area of square = 98 sq.cm

7. Find the area of a triangle whose sides are.

(i) 120cm, 150cm and 200cm

(ii) 50dm, 78dm and 112dm

Sol: $a = 200$ here

$b = 150$

$c = 120$

$$S = \frac{a+b+c}{2}$$

$$S = \frac{200+150+120}{2}$$

$$S = \frac{470}{2}$$

$$S = 235$$

$$S - a = 235 - 200 = 35$$

$$S - b = 235 - 150 = 85$$

$$S - c = 235 - 120 = 115$$

Area of Δ $A = A = \sqrt{S(S-a)(S-b)(S-c)}$

$$= \sqrt{235 \times 35 \times 85 \times 115}$$

$$= \sqrt{5 \times 47 \times 5 \times 7 \times 5 \times 17 \times 5 \times 23}$$

$$= \sqrt{5^2 \times 47 \times 7 \times 5^2 \times 17 \times 23}$$

$$= 5 \times 5\sqrt{47 \times 7 \times 17 \times 23}$$

$$= 25\sqrt{128639}$$

$$= 25 \times 358.66$$

$= 8967$ sq. cm approx

(ii) $a = 112$ dm here

$b = 78$ dm

$c = 50$ dm

$$S = \frac{a+b+c}{2}$$

$$= \frac{112+78+50}{2}$$

$$= \frac{240}{2}$$

$$S = 120$$

$$\text{Area (A)} = A = \sqrt{S(S-a)(S-b)(S-c)}$$

$$= \sqrt{120(120-112)(120-78)(120-50)}$$

$$= \sqrt{120 \times 8 \times 42 \times 70}$$

$$A = \sqrt{3 \times 5 \times 8 \times 8 \times 7 \times 2 \times 3 \times 7 \times 5 \times 2}$$

$$= \sqrt{3^2 \times 5^2 \times 8^2 \times 7^2 \times 2^2}$$

$$= 3 \times 5 \times 8 \times 7 \times 2$$

$$= 1680 \text{ sq. dm}$$

8. The perimeter of a triangular field is 540m and its sides are in the ratio 25 : 17 : 12. Find the area of triangle.

Hint: Let the sides be $25x$, $17x$, $12x$ meters.

Then $25x + 17x + 12x = 540 \Rightarrow x = 10$

Sol: Let sides are $25x$, $17x$, $12x$

perimeter $= 25x + 17x + 12x$

$= 54x$

perimeter $= 540$ m but

then $540 = 54x$

$x = 10$

sides $25 \times 10, 17 \times 10, 12 \times 10$

$= 250, 170, 120$ m

$$S = \frac{a+b+c}{2}$$

$$= \frac{250+170+120}{2}$$

$$= \frac{540}{2} = 270$$

$S - a = 270 - 250 = 20$

$S - b = 270 - 170 = 100$

$S - c = 270 - 120 = 150$

Area (A) = $A = \sqrt{S(S-a)(S-b)(S-c)}$

$$= \sqrt{270 \times 20 \times 100 \times 150}$$

$$= \sqrt{27 \times 10 \times 2 \times 10 \times 2 \times 50 \times 3 \times 50}$$

$$= \sqrt{10^2 \times 2^2 \times 50^2 \times 3 \times 27}$$

$$= 10 \times 2 \times 50 \times \sqrt{3 \times 27}$$

$$= 1000 \times \sqrt{3 \times 3 \times 3 \times 3}$$

$$= 1000 \times 3 \times 3$$

$= 9000$ sq. meter

9. Find the area of a parallelogram if its two adjacent sides are 12cm and 14cm and diagonal is 18cm.

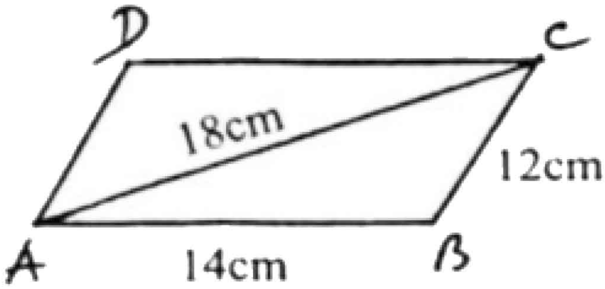
Hints:

Let ABCD is $a||^m$ in which .

Find area of ΔABC .

Area of $||^{m}$ = 2 (Area of ΔABC)

Sol: Find area of $\Delta\ ACB$



$a = 18$ cm

$b = 14$ cm

$c = 12$ cm

$$S = \frac{a+b+c}{2}$$

$$S = \frac{18+14+12}{2}$$

$$S = \frac{44}{2}$$

$$S = 22$$

$$S-a = 22-18 = 4$$

$$S-b = 22-14 = 8$$

$$S-c = 22-12 = 10$$

Area of $\Delta\ ABC = A = \sqrt{S(S-a)(S-b)(S-c)}$

$$= \sqrt{22\times4\times8\times10}$$

$$= \sqrt{11\times2\times2\times2\times2\times2\times2\times2\times5}$$

$$= \sqrt{11\times2^2\times2^2\times2^2\times2\times5}$$

$= 2 \times 2 \times 2\sqrt{110}$

$= 8\sqrt{110}$ sq. cm ....(i)

Area of ΔACD = Area of ΔABC and

$= 8\sqrt{110}$ sq. cm ....(ii)

Area of $||^{m}$ $ABCD$= (i) + (ii)

$= 8\sqrt{110} + 8\sqrt{110}$

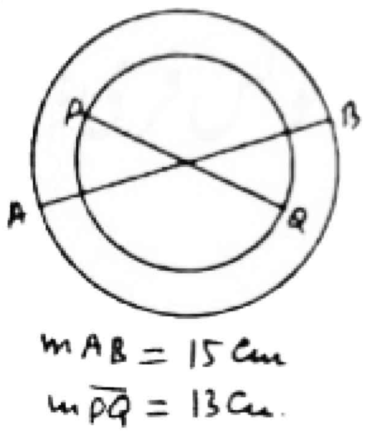
$= (8+8)\sqrt{110}$

$= 16\sqrt{110}$ sq. cm

10. Find the area of the following washers whose external and interanl diameters are:

(i) 15cm and 13cm (ii) 1.2m and 0.9m

(iii) 40mm and 33mm.



Sol: External diameter = $\frac{15}{2}cm$

Internal diameter = $\frac{13}{2}cm$

Total area = πr^2

$= \pi\left(\frac{15}{2}\right)^2$

Internal area $= \pi r^2$

$$= \pi\left(\frac{13}{2}\right)^2$$

Area of washers $= \pi\left(\frac{15}{2}\right)^2 - \pi\left(\frac{13}{2}\right)^2$

$$= \pi\left[\left(\frac{15}{2}\right)^2 - \left(\frac{13}{2}\right)^2\right]$$

$$= \frac{22}{7}\left[\frac{225}{4} - \frac{169}{4}\right]$$

$$= \frac{22}{7}\left(\frac{225-169}{4}\right)$$

$$= \frac{22}{7}\left(\frac{56}{4}\right)$$

$= 44$ sq cm

Sol. External diameter $= \frac{1.2}{2} = .6\,m = \frac{6}{10}\,m$

Internal diameter $= \frac{0.9}{2} = \frac{9}{10 \times 2}$

$$= \frac{9}{20}\,m$$

Total area $= \pi r^2$

$$= \pi\left(\frac{6}{10}\right)^2$$

Internal area $= \pi r^2$

$$= \pi\left(\frac{9}{20}\right)^2$$

$$\text{Area of washers} = \pi\left(\frac{6}{10}\right)^2 - \pi\left(\frac{9}{20}\right)^2$$

$$= \pi\left[\left(\frac{6}{10}\right)^2 - \left(\frac{9}{20}\right)^2\right]$$

$$= \frac{22}{7}\left(\frac{36}{100} - \frac{81}{400}\right)$$

$$= \frac{22}{7}\left[\frac{144 - 81}{400}\right]$$

$$= \frac{22}{\cancel{7}_1}\left(\frac{^{9}\cancel{63}}{400}\right)$$

$$= \frac{198}{400}$$

$$= 0.495 \text{ sq. mm}$$

Sol:

$$\text{Diameter of big half circle} = \frac{40}{2} = 20mm$$

$$\text{Internal diameter} = \frac{33}{2}mm$$

$$\text{Total area} = \pi r^2$$

$$= \pi(20)^2$$

$$\text{Internal area} = \pi r^2$$

$$= \pi\left(\frac{33}{2}\right)^2$$

$$\text{Area of washers} = \pi(20)^2 - \pi\left(\frac{33}{2}\right)^2$$

$$= \pi\left[(20)^2 - \left(\frac{33}{2}\right)^2\right]$$

$$= \frac{22}{7}\left[400 - \frac{1089}{4}\right]$$

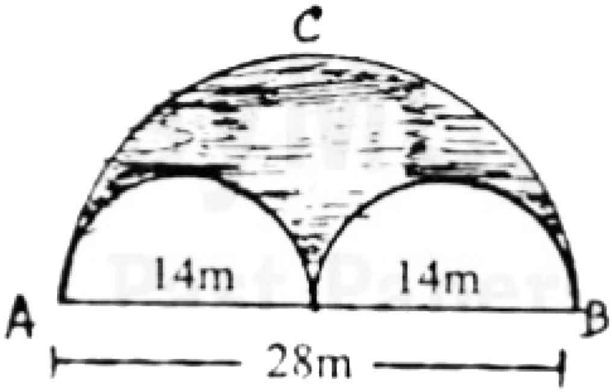
$$= \frac{22}{7}\left[\frac{1600 - 1089}{4}\right]$$

$$= \frac{22}{7}\left(\frac{511^{73}}{4_2}\right)$$

$$= \frac{11 \times 73}{2}$$

$= 401.5$ sq. mm

11. Find the area of the shaded region.



Diameter of big half circle $= \frac{28}{2} = 14\ m$

Area of big half circle $= \frac{1}{2}\pi r^2$

$$= \frac{1}{2}\pi(14)^2$$

$$= \frac{1}{2} \times \frac{22}{7} \times 14^7 \times 14$$

$= 308$ sq. m

Diameter of every small half circle $= \frac{14}{2} = 7$ m

Total area of both $= \frac{1}{2}\pi r^2$

$$= \frac{1}{2} \times \frac{22}{7} \times (7)^2$$

$$= \frac{1}{2} \times \frac{22^{11}}{7} \times 7 \times 7$$

$$= 77 \text{ sq. m}$$

small circles $= 2 \times 77$

$= 154$ sq. m

Area of shaded region $= 308 - 154$

$= 154$ sq. m

12. Find the area of an equilateral triangle whose side is 8m.

Sol. $a = b = c = 8\text{m}$

$$S = \frac{a+b+c}{2} = \frac{8+8+8}{2} = \frac{24}{2} = 12$$

$S - a = 12 - 8 = 4$

$S - b = 12 - 8 = 4$

$S - c = 12 - 8 = 4$

Area of Δ $= A = \sqrt{S(S-a)(S-b)(S-c)}$

$$= \sqrt{12 \times 4 \times 4 \times 4}$$

$$= \sqrt{3 \times 4 \times 4 \times 4 \times 4}$$

$$= 4 \times 4\sqrt{3}$$

$$= 16\sqrt{3} \text{ sq. m}$$

13. The side of an equilateral triangle is 6cm. Find its area.

$a = b = c = 6$ here

$$S = \frac{a+b+c}{2} = \frac{6+6+6}{2} = \frac{18}{2} = 9$$

$S - a = 9 - 6 = 3$

$S - b = 9 - 6 = 3$

$S - c = 9 - 6 = 3$

Area of Δ $= A = \sqrt{S(S-a)(S-b)(S-c)}$

$= \sqrt{9 \times 3 \times 3 \times 3}$

$= \sqrt{3 \times 3 \times 3 \times 3 \times 3}$

$= 3 \times 3\sqrt{3} = 9\sqrt{3}$ sq. cm

14. Find the area of the right triangle with legs 12cm and 35cm.

Length of one side $= 12$ cm

Length of 2nd side $= 35$ cm

Area of right angles $= \dfrac{base \times altitude}{2}$

$= \dfrac{35 \times 12}{2}$

$= 35 \times 6$

$= 210$ sq.cm

15. The base of a rectangle is three times its altitude. The area is $147cm^2$. Find the dimensions of the rectangle.

Sol Let altitude $= x$

Length of base $= 3x$ cm

Area $= (x)(3x)$

$= 3x^2$

$$\text{Area} = 147 \text{ sq. cm} \quad \text{but}$$

$$3x^2 = 147 \quad \text{therefore}$$

$$x^2 = \frac{147}{3}$$

$$x^2 = 49$$

$$\sqrt{x^2} = \sqrt{49} \quad \text{Taking square root}$$

$$x = 7$$

altitude = 7 cm

Base = $3x$

$= 3 \times 7$

Base = 21 cm

16. Find the base of the parallelogram whose attitude is 18cm and whose area is $3m^2$.

Length of altitude = 18 cm

Area = 3 sq.m

Area = $3 \times 100 \times 100$ sq.m

$$\text{Length of base} = \frac{area}{altitude}$$

$$= \frac{3 \times 100 \times 100}{18}$$

$$= \frac{5000}{3}$$

Length of base = 1666.67 cm

17. The area of a parallelogram is $144cm^2$. Find the altitude if the base is 2m long.

Area of a parallelogram = 144 sq. m

Length of base = 2 cm

Length of altitude = $\frac{area}{base}$

$= \frac{144}{2}$

Length of altitude = 72 cm

18. Find the area of the rectangle 2m long and 18cm wide.

Length of a rectangle = 2m

= 200 cm

Width = 18 cm

Area = Length × Width

= 200 × 18

= 3600 sq.cm

19. The area of an equilateral triangle is $4\sqrt{2}cm^2$. Find the length of a side.

Sol: If side of an any equilateral triangle are "a" units then its area will be $\frac{\sqrt{3}\, a^2}{4}$.

therefore $\frac{\sqrt{3}\, a^2}{4} = 4\sqrt{3}$

$$a^2 = \left(4\sqrt{3}\right)\left(\frac{4}{\sqrt{3}}\right)$$

$$a^2 = 4 \times 4$$

$\sqrt{a^2} = \sqrt{4 \times 4}$ Taking square root

$a = 4$ cm

Volume of a cube $= V = l \times l \times l$

$= (l)^3$

Volume of a cuboid of length l, breadth b and height h is

Volume of cuboid $= V = l \times b \times h$

Volume of a cuboid of length l, breadth b and height h

Volume of cylinder $= \pi r^2 h$

where "r" is radius of the base and "h" is height.

Volume of cylinder $= \frac{1}{3} \pi r^2 h$

where 'r' is radius of base and "h" is height.

Volume of sphere $= \frac{4}{3} \pi r^3$

where 'r' is radius of sphere.

Remember that:

1- As $1\text{cm} = 10\text{mm}$,

therefore, $1\text{cm}^3 = 10 \times 10 \times 10\text{mm}^3$

$1\text{cm}^3 = 1000\text{mm}^3$

2- $1\text{m}^3 = 100 \times 100 \times 100\text{cm}^3$

$= 1000000 \text{ cm}^3$

$$\boxed{1m^3 = 10^6 cm^3}$$

also $1\text{m}^3 = 1000 \times 1000 \times 1000 \text{ mm}^3$

$$\boxed{1m^3 = 10^9\ mm^3}$$

3- For measurement of volumes of liquids, we use the terms liters (l) and milliliters (ml).

$$1cm^3 = 1ml$$

$$1000cm^3 = 1l$$

$$\text{and} \quad 1m^3 = 1000000\ cm^3 = 1000\ l$$

$$\boxed{1m^3 = 1kl \text{ (1 kiloliter)}}$$

Exercise 9.[illegible]

Find the Volume of the Solids

1. A cube of a side 4cm

Sol: A cube of a side = 4

Volume of cube = $(l)^3$

$= (4)^3$

$= 64$ cubic cm

2. A cube whose total area is $96cm^2$.

Total area of cube = 96

areas of cube = 6

area of 1 face $= \dfrac{96}{6}$

$= 16$ sq. cm

Length of edge $= \sqrt{16}$

$= 4$ cm

volume of cube $= (l)^3$

$= (4)^3$

$= 64 \text{ cm}^3$

3. A rectangular box with length $4m$ breadth $3m$ and height $2m$.

length of rectangular $= 4\ m$

Breadth of rectangular $= 3$ m

Height of rectangular $= 2$ m

volume of cuboid $= l \times b \times h$

$= 4 \times 3 \times 2$

$= 24\ (\text{m})^3$

4. Right cylinder, with radius of base 4cm, altitude 10cm, use $\pi = \frac{22}{7}$.

Sol: Radius of base $= (r) = 4$ cm

altitude $= (h) = 10$ cm

volume of cylinder $= \pi r^2 h$

$= \frac{22}{7} \times (4)^2 (10)$

$= \frac{22}{7} \times 16 \times 10$

$= \frac{3520}{7}$

$$= 502.86\ cm^3$$

5. Circular cone, with radius of base 3cm, altitude 10cm.

Radius of circular base $= r = 3$ cm

Height of altitude $= h = 10$ cm

$$\text{Volume of circular cone} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\left(\frac{22}{7}\right)(3)^2(10)$$

$$= \frac{1}{\cancel{3}}\times\frac{22}{7}\times\cancel{3}\times 3\times 10$$

$$= \frac{660}{7}$$

$$= 94.3\ cm^3$$

6. Sphere, with radius 3cm.

Radius of sphere $= r = 3$ cm

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3}\times\frac{22}{7}\times(3)^3$$

$$= \frac{4}{\cancel{3}}\times\frac{22}{7}\times\cancel{3}\times 3\times 3$$

$$= \frac{4\times 22\times 3\times 3}{7}$$

$$\text{Volume of sphere} = \frac{792}{7}$$

$$= 113.14\ cm^3$$

7. Right circular cylinder, with circumferences of base 4cm, altitude 1m.

Sol. Circumference of base = 4cm

Circumference of base = $2\pi r$

then $2\pi r = 4$

$$r = \frac{4}{2\pi}$$

$$= \frac{2}{\pi}\text{ cm} \qquad \text{(i)}$$

Length of cylinder (h) = 1m

= 100 cm

Volume of cylinder = $\pi r^2 \times h$

from (i) $= \pi\left(\frac{2}{\pi}\right)^2 (100)$

$$= \pi \times \frac{2\times 2\times 100}{\pi \times \pi}$$

$$= \frac{7\times 2\times 2\times 100}{22_{11}}$$

$$= \frac{1400}{11}$$

$= 127.3\text{cm}^3$ approx

8. Cone with altitude 9cm, radius of base 6cm.

Sol: Cone with altitude = h = 9 cm

Cone with radius of base = r = 6 cm

Volume of cone = $\frac{1}{3}\pi r^2 \times h$

$$= \frac{1}{3}\left(\frac{22}{7}\right)(6)^2 \times 9$$

$$= \frac{1}{\cancel{3}} \times \frac{22}{7} \times \cancel{36}^{12} \times 9$$

$$= \frac{22 \times 12 \times 9}{7}$$

$$= \frac{2376}{7}$$

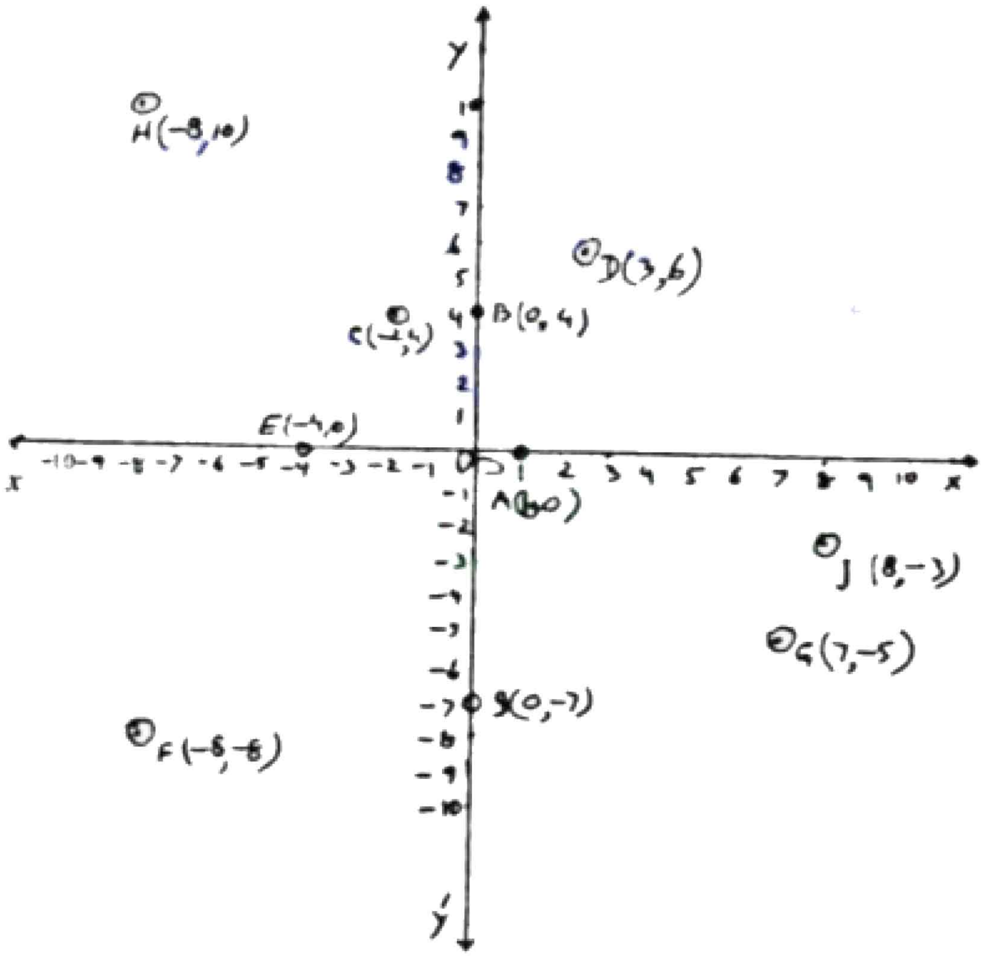
$$= 339.4 \text{ cm}^3$$

Exercise 10.1

Q.1. Describe the location of these points on the number plane.

(i)	A(1, 0)	(ii)	B(0,4)	(iii)	C(-2, 4)	(iv)	D(3, 6)
(v)	E(-4, 0)	(vi)	F(-8, -8)	(vii)	G(7, -5)	(viii)	H(-8, 10)
(ix)	I(0, -7)	(x)	J(8, -3)				

Solution:



The points are represented on the graph paper.

Q.2. Find the distance between the following pairs of points.

(i)	$(2,1), (-4,3)$,	(ii)	$(-1,3), (-2,-1)$
(iii)	$(7,-2), (-2, 3)$	(iv)	$(a,-b), (b,-a)$

(i) Let $P(x_1, y_1) = (2, 1)$:حل

and $Q(x_2, y_2) = (-4, 3)$

distance formula $m\overline{PQ} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$= \sqrt{(2+4)^2 + (1-3)^2}$

$= \sqrt{(6)^2 + (-2)^2}$

$= \sqrt{36+4}$

$= \sqrt{40}$

$= \sqrt{2 \times 2 \times 10}$

$= 2\sqrt{10}$ units

(ii) Let $P(x_1, y_1) = (-1, 3)$

and $Q(x_2, y_2) = (-2, -1)$

distance formula $m\overline{PQ} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$= \sqrt{(-1+2)^2 + (3+1)^2}$

$= \sqrt{(1)^2 + (4)^2}$

$= \sqrt{1+16}$

$= \sqrt{17}$ units

(iii) Let $P(x_1, y_1) = (7, -2)$

and $Q(x_2, y_2) = (-2, 3)$

distance formula $m\overline{PQ} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$$= \sqrt{(7+2)^2 + (-2-3)^2}$$

$$= \sqrt{(9)^2 + (-5)^2}$$

$$= \sqrt{81 + 25}$$

$$= \sqrt{106} \text{ units}$$

(iv) Let $P(x_1, y_1) = (a, -b)$

and $Q(x_2, y_2) = (b, -a)$

$$m\overline{PQ} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$= \sqrt{(a-b)^2 + (-b+a)^2}$$

$$= \sqrt{(a-b)^2 + (a-b)^2}$$

$$= \sqrt{2(a-b)^2} = (a-b)\sqrt{2} \text{ units}$$

Q.3. Express by an equation, the fact, that the point $P(x,y)$ is aquidistant from A(2, 4) and B(6, 8).

Sol. $P(x, y)$; $A(2, 4)$

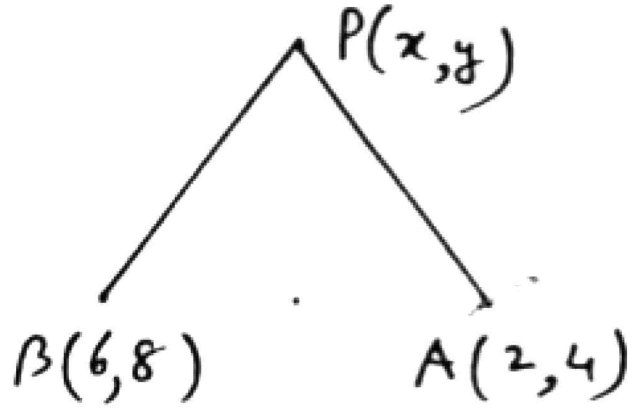
Applying the distance formula

$$m\overline{PA} = \sqrt{(x-2)^2 + (y-4)^2}$$

and $P(x, y)$; $B(6, 8)$

Applying the distance formula

$$m\overline{PB} = \sqrt{(x-6)^2 + (y-8)^2}$$



According to the statement $m\overline{PA} = m\overline{PB}$

Therefore

$$\sqrt{(x-2)^2 + (y-4)^2} = \sqrt{(x-6)^2 + (y-8)^2}$$

By taking square

$$(x-2)^2 + (y-4)^2 = (x-6)^2 + (y-8)^2$$

$$\cancel{x^2} - 4x + 4 + \cancel{y^2} - 8y + 16 = \cancel{x^2} - 12x + 36 + \cancel{y^2} - 16y + 64$$

$$-4x + 12x - 8y + 16y + 4 + 16 - 36 - 64 = 0$$

(Divided by 8)

$$8x + 8y - 80 = 0$$

or

$$x + y - 10 = 0$$

Q.4. Show that the points A(5, 4), B(4, −3), C(−2, 5) are aquidistant from D(1, 1).

Sol. $A(5,4),\ B(4,-3),\ C(-2,5),\ D(1,1)$

(i) $D(1,1);\ A(5,4)$ Let find $m\overline{DA}$

Suppose that $D(x_1, y_1);\ A(x_2, y_2)$

Distance formula

$$m\overline{DA} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$= \sqrt{(1-5)^2 + (1-4)^2}$$

$$= \sqrt{(-4)^2 + (-3)^2}$$

$$= \sqrt{(16+9)} = \sqrt{25} = 5 \quad (P)$$

(ii) **Now find** $m\overline{DB}$

$D(1,1);\ B(4,-3)$

Let $D(x_1, y_1);\ B(x_2, y_2)$

Distance formula

$$m\overline{DB} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$= \sqrt{(1-4)^2 + (1+3)^2}$$

$$= \sqrt{(-3)^2 + (4)^2}$$

$$= \sqrt{(9+16)} = \sqrt{25} = 5 \quad (Q)$$

(iii) **Now find** $m\overline{DC}$

$D(1,1);\ C(-2,5)$

Suppose that: $D(x_1, y_1);\ C(x_2, y_2)$

Distance formula:

$$m\overline{DC} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$= \sqrt{(1\times 2)^2 + (1-5)^2}$$

$$= \sqrt{(3)^2 + (-4)^2}$$

$$= \sqrt{(9 + 16)} = \sqrt{25} = 5 \quad (R)$$

From R, Q, P

Points A, B, C are equidistant from "D".

Q.5. **Find the point on the x-axis which is equidistant from (2, 4) and (6, 8).**

(Hint: call the point $(x, 0)$. Find x.)

Sol. Y ordinate is "0" on x-axis.

Let we suppose the required point is $P(x, 0)$.

Let we have supposed that points A (2, 4), B(6, 8)

Now we find $m\overline{PA}$ and $m\overline{PB}$.

Distance formula:

$$m\overline{PA} = \sqrt{(x-2)^2 + (0-4)^2}$$

$$= \sqrt{x^2 - 4x + 4 + 16}$$

$$= \sqrt{x^2 - 4x + 20}$$

$$m\overline{PB} = \sqrt{(x-6)^2 + (0-8)^2} \quad \text{and}$$

$$= \sqrt{x^2 - 12x + 36 + 64}$$

$$m\overline{PB} = \sqrt{x^2 - 12x + 100}$$

$$m\overline{PA} = m\overline{PB}$$

Now, according to the statement

$$\sqrt{x^2-4x+20}=\sqrt{x^2-12x+100}$$

$$x^2-4x+20=x^2-12x+100$$

By taking square

$$-4x+12x=100-20$$

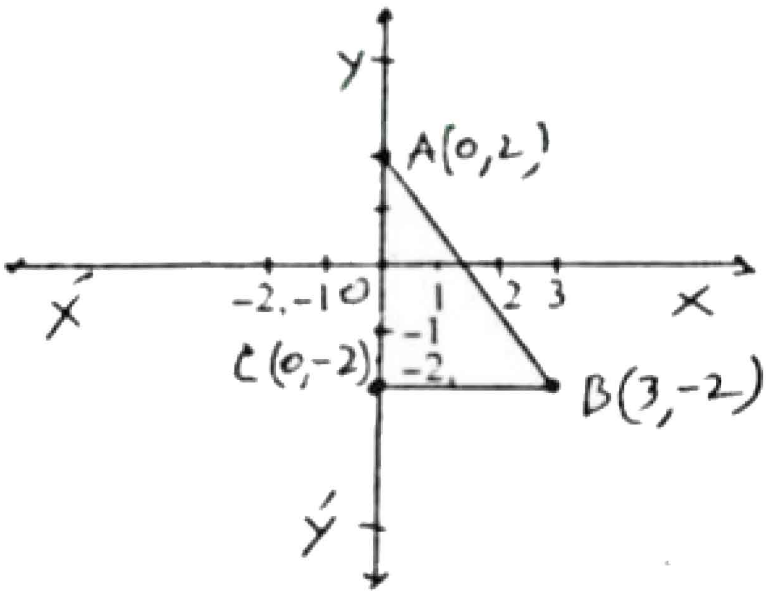
$$8x=80$$

$$x=\frac{80}{8}=10$$

Required points $(10, 0)$

Q.6. Show that the points A(0, 2), B(3 −2) and C(0, −2) are vertices of a right triangle.

Sol. Find the lengths of sides.



Distance formula:

A(0, 2), B(3, −2)

$$m\overline{AB}=\sqrt{(0-3)^2+(2+2)^2}$$

$$=\sqrt{(-3)^2+(4)^2}$$

$$= \sqrt{9+16}$$

$$= \sqrt{25}$$

$$= 5 \quad \text{units}$$

$B(3, -2)$ and $C(0, -2)$

$$m\overline{BC} = \sqrt{(3-0)^2 + (-2+2)^2}$$

$$= \sqrt{(3)^2 + (0)^2}$$

$$= \sqrt{9}$$

$$= 3 \quad \text{units}$$

$C(0, -2)$ and $A(0, 2)$

$$m\overline{CA} = \sqrt{(0-0)^2 + (-2-2)^2}$$

$$= \sqrt{0 + (-4)^2}$$

$$= \sqrt{16}$$

$$= 4 \quad \text{units}$$

$$\left(m\overline{BC}\right)^2 + \left(m\overline{CA}\right)^2 = (3)^2 + (4)^2$$

$$= 9 + 16$$

$$= 25 \text{ ....(i)}$$

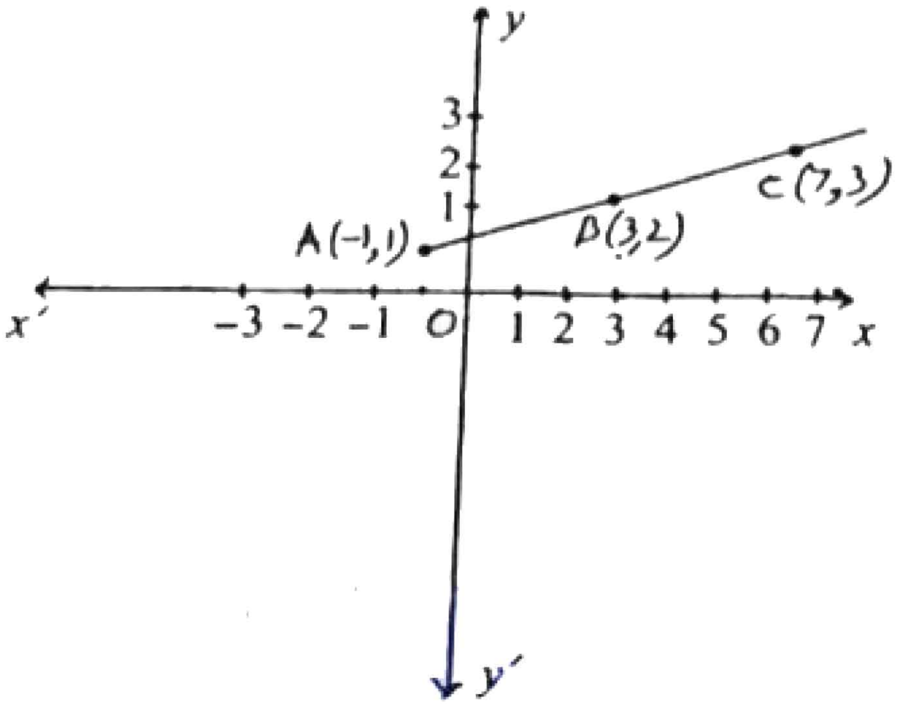
$$\left(m\overline{AB}\right)^2 = (5)^2 \text{ and}$$

$$= 25 \text{..........(ii)}$$

From (i) and (ii); It is proved that the points A, B and C are the vertices of a right angle triangle.

Q.7. **Show that the points A(−1, 1), B(3, 2), C(7, 3) are collinear.**

Sol.



Let we find the mid distance of points A(−1, 1). B(3, 2), C(7, 3)

$$A(-1,1) \text{ ,,, } B(3,2)$$

$$m\overline{AB} = \sqrt{(-1-3)^2+(1-2)^2}$$

$$= \sqrt{(-4)^2+(-1)^2}$$

$$= \sqrt{16+1} = \sqrt{17}$$

$B(3, 2)$ and $C(7, 3)$

$$m\overline{BC} = \sqrt{(3-7)^2+(2-3)^2}$$

$$= \sqrt{(-4)^2+(-1)^2}$$

$$m\overline{BC} = \sqrt{16+1} = \sqrt{17}$$

$A(-1, 1)$ and $C(7, 3)$

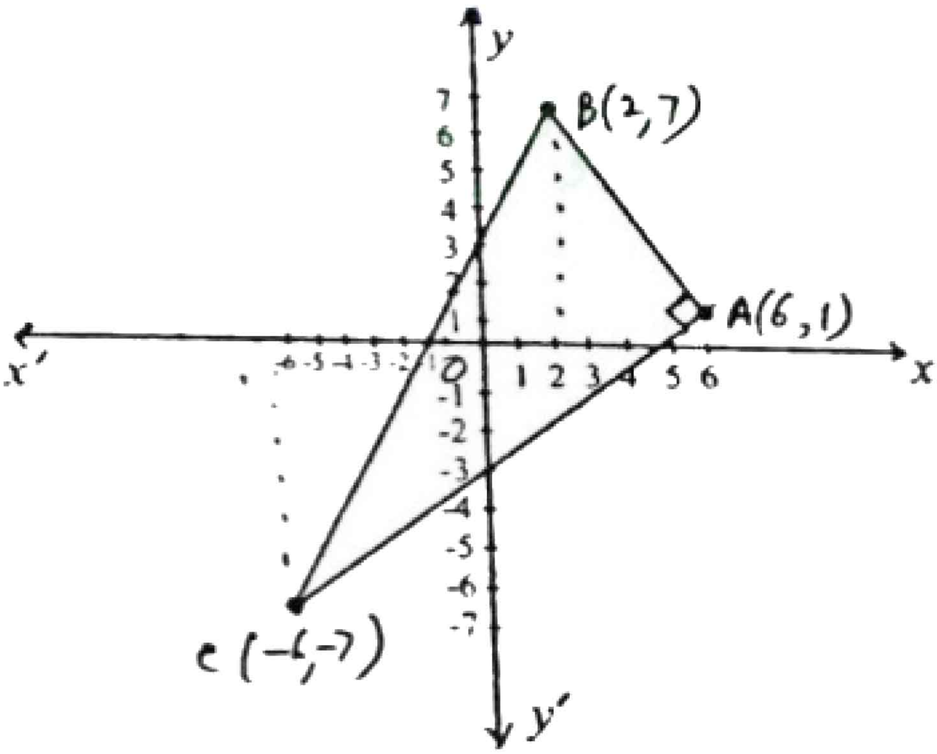
$$m\overline{AC} = \sqrt{(-1-7)^2+(1-3)^2}$$
$$= \sqrt{(-8)^2+(-2)^2}$$
$$= \sqrt{64+4}$$
$$= \sqrt{68}$$
$$= \sqrt{4\times17} = 2\sqrt{17}$$

Now

$$m\overline{AB} + m\overline{BC} = \sqrt{17}+\sqrt{17}$$
$$= 2\sqrt{17} = m\overline{AC}$$

Points A, B and C are collinear.

Q.8. Show that the points A(6, 1), B(2, 7) and C(−6, −7) are vertices of a right triangle.



Let we find the square of mid distances $A(6, 1)$, $B(2, 7)$, $C(-6,-7)$

Using the distance formula

$A(6, 1)$, $B(2, 7)$

$$\left(m\overline{AB}\right)^2 = (6-2)^2 + (1-7)^2$$

$$= (4)^2 + (-6)^2$$

$$= 16 + 36 = 52$$

$B(2, 7), C(-6, -7)$

$$\left(m\overline{BC}\right)^2 = (2+6)^2 + (7+7)^2$$

$$= (8)^2 + (14)^2$$

$$= 64 + 196 = 260$$

$A(6, 1), C(-6, -7)$

$$\left(m\overline{CA}\right)^2 = (-6-6)^2 + (-7-1)^2$$

$$= (-12)^2 + (-8)^2$$

$$= 144 + 64 = 208$$

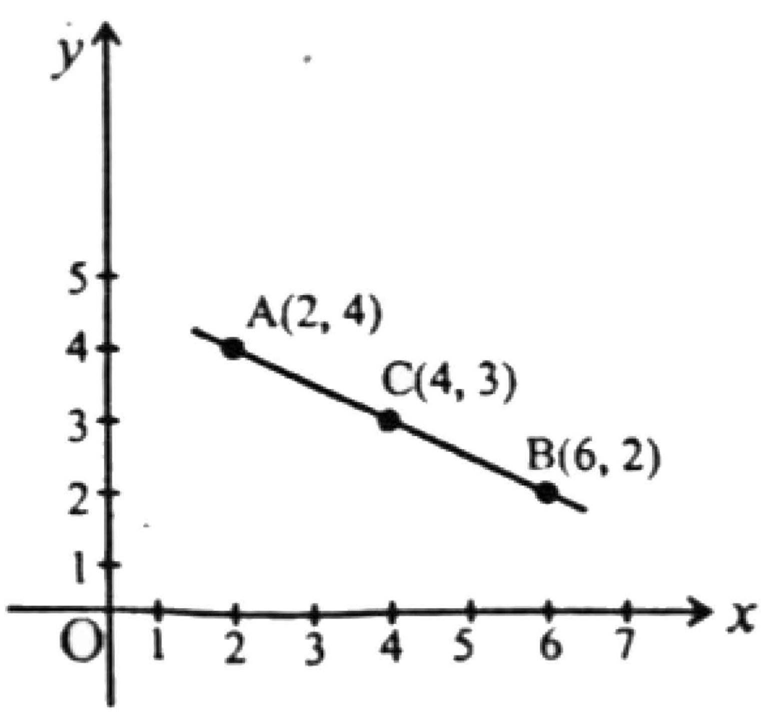
Now $\left(m\overline{AB}\right)^2 + \left(m\overline{CA}\right)^2 = 52 + 208$

$$= 260 = \left(m\overline{BC}\right)^2$$

The sum of two lengths is equal to the length of third side.
Points A, B, C are vertices of right angled.

Q.9. Show that the points A(2, 4), B(6, 2), C(4, 3) are collinear.

Sol:



Let we find the mid distance of A(2, 4), B(6, 2), C(4, 3)

$A(2, 4), B(6, 2)$

$$m\overline{AB} = \sqrt{(2-6)^2 + (4-2)^2}$$

$$= \sqrt{(-4)^2 + (2)^2}$$

$$= \sqrt{16+4} = \sqrt{20} = 2\sqrt{5} \quad \text{units}$$

$A(2,4), C(4,3)$

$$m\overline{AC} = \sqrt{(2-4)^2 + (4-3)^2}$$

$$= \sqrt{(-2)^2 + (1)^2}$$

$$= \sqrt{4+1}$$

$$= \sqrt{5} \quad \text{units}$$

$C(4, 3), B(6, 2)$

$$m\overline{CB} = \sqrt{(4-6)^2 + (3-2)^2}$$

$$= \sqrt{(-2)^2 + (1)^2}$$

$$= \sqrt{4+1}$$

$$= \sqrt{5} \quad \text{units}$$

$$m\overline{AC} + m\overline{CB} = \sqrt{5} + \sqrt{5}$$

$$= 2\sqrt{5} \quad \text{units}$$

$$= m\overline{AB}$$

Thus, A, B, C are collinear.

$$= \sqrt{16+196}$$

$$= \sqrt{212}$$

$$= 2\sqrt{53} \quad \text{units}$$

$B(-6, -3)$, $C(4, -9)$

$$m\overline{BC} = \sqrt{(-6-4)^2 + (-3+9)^2}$$

$$= \sqrt{(-10)^2 + (6)^2}$$

$$= \sqrt{100+36}$$

$$= \sqrt{136}$$

$$= \sqrt{4 \times 34}$$

$$= 2\sqrt{34} \quad \text{units}$$

$C(4, -9)$, $A(-2, 11)$

$$m\overline{CA} = \sqrt{(4+2)^2 + (-9-11)^2}$$

$$= \sqrt{(6)^2 + (-20)^2}$$

$$= \sqrt{36+400}$$

$$= \sqrt{436}$$

$$= \sqrt{4 \times 109}$$

$$= 2\sqrt{109} \quad \text{units}$$

From (i), (ii), (iii)

$$m\overline{AB} = 2\sqrt{53}$$

$$m\overline{BC} = 2\sqrt{34}$$

$$m\overline{CA} = 2\sqrt{109}$$

Now $\quad m\overline{AB} \neq m\overline{BC} \neq m\overline{CA}$

The points A(−2, 11) B(−6, −3) and C(4, −9) are vertices of scalene.

Q.12. Show that the points A(6, 1), B(2, 7) and C(−6, 7) are of a scalene triangle.

Sol. Let we find the mid distance of $A(6, 1)$, $B(2, 7)$, $C(-6, 7)$

$A(6, 1)$, $B(2, 7)$

$$m\overline{AB} = \sqrt{(6-2)^2 + (1-7)^2}$$

$$= \sqrt{(4)^2 + (-6)^2}$$

$$= \sqrt{16 + 36}$$

$$= \sqrt{52}$$

$$= \sqrt{4 \times 13}$$

$$= 2\sqrt{13} \text{ ....(i)}$$

$B(2,7)$, $C(-6,7)$

$$m\overline{BC} = \sqrt{(2+6)^2 + (7-7)^2}$$

$$= \sqrt{(8)^2 + (0)^2}$$

$$= \sqrt{64}$$

$$= 8 \text{ ....(ii)}$$

$C(-6, 7)$, $A(6, 1)$

$$m\overline{CA} = \sqrt{(-6-6)^2 + (7-1)^2}$$

$$= \sqrt{(-12)^2 + (6)^2}$$

$$m\overline{CA} = \sqrt{144 + 36} \quad \ldots\ldots\ldots \text{(iii)}$$

$$= \sqrt{180}$$

$$= \sqrt{9 \times 4 \times 5}$$

$$= 3 \times 2\sqrt{5}$$

$$= 6\sqrt{5} \quad \text{units}$$

From (i), (ii), (iii)

$$m\overline{AB} \neq m\overline{BC} \neq m\overline{CA}$$

Thus pints A(6, 1) $B(2,7)$ and $C(-6,7)$ are vertices of scalene triangle.

Q.13. Show that the points A(2, −5), B(−4, −3) and C(−1, 5) are of a scalene triangle.

Sol. We find the distance between the points $A(2-5), B(-4, -3)$, $C(-1, 5)$

$$A(2-5),\ B(-4, -3)$$

$$m\overline{AB} = \sqrt{(2+4)^2 + (-5+3)^2}$$

$$= \sqrt{(6)^2 + (-2)^2}$$

$$= \sqrt{36 + 4}$$

$$= \sqrt{40} = 2\sqrt{10}$$

$$B(-4, -3),\ C(-1, 5)$$

$$m\overline{BC} = \sqrt{(-4+1)^2 + (-3-5)^2}$$

$$= \sqrt{(-3)^2 + (-8)^2}$$

$$= \sqrt{9+64}$$

$$= \sqrt{73} \quad \text{.........................} \quad \text{(ii)}$$

$$C(-1, 5), A(2, -5)$$

$$m\overline{CA} = \sqrt{(-1-2)^2 + (5+5)^2}$$

$$= \sqrt{(-3)^2 + (10)^2}$$

$$= \sqrt{9+100}$$

$$= \sqrt{109} \quad \text{.......................} \text{(iii)}$$

$$m\overline{AB} = 2\sqrt{10}$$

$$m\overline{BC} = \sqrt{73}$$

$$m\overline{CA} = \sqrt{109}$$

$$m\overline{AB} \neq m\overline{BC} \neq m\overline{CA}$$

Thus, A, B, C arc vertices of scalene triangle.