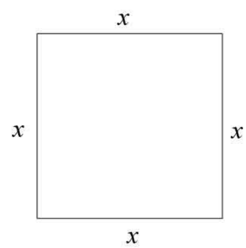
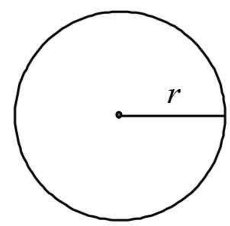
Function and Limits

Concept of Functions:

Historically, the term function was first used by German mathematician Leibnitz (1646-1716) in 1673 to denote the dependence of one quantity on another e.g.



1) The area "A" of a square of side "x" is given by the formula $A=x^2$. As area depends on its side x, so we say that A is a function of x.



2) The area "A" of a circular disc of radius "r" is given by the formula $A=\pi r^2$ As area depends on its radius r, so we say that A is a function of r.

3) The volume "V" of a sphere of radius "r" is given by the formula $V=\frac{4}{3}\pi r^3$. As volume V of a sphere depends on its radius r, so we say that V is a function of r.

The Swiss mathematician, Leonard Euler conceived the idea of denoting function written as $y=f(x)$ and read as y is equal to f of x. $f(x)$ is called the value of f at x or image of x under f.

The variable x is called independent variable and the variable y is called dependent variable of f.

If x and y are real numbers then f is called real valued function of real numbers.

Domain of the function:

If the independent variable of a function is restricted to lie in some set, then this set is called the domain of the function e.g.
Dom of $f = \{0 \le x \le 5\}$

Range of the function:

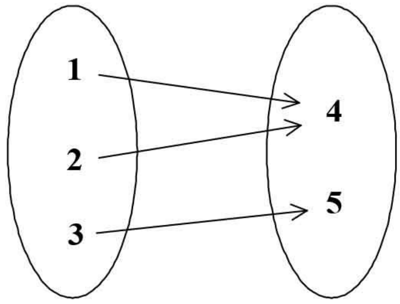
The set of all possible values of $f(x)$ as x varies over the domain of f is called the range of f e.g. $y = 100 - 4x^2$.
As x varies over the domain [0,5] the values of $y = 100 - 4x^2$ vary between $y=0$ (when $x=5$) and $y = 100$ (when $x=0$)
Range of $f = \{0 \le y \le 100\}$

Definition:

A function is a rule by which we relate two sets A and B (say) in such a way that each element of A is assigned with one and only one element of B. For example

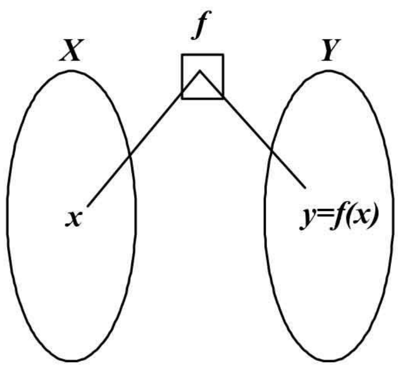
is a function from A to B.

its Domain = {1,2,3} and Range = {4,5}



In general:

A function f from a set 'X' to a set 'Y' is a rule that assigns to each element x in X one and only one element y in Y.(a unique element y in Y)



(f is function from X to Y)

If an element "y, of Y is associated with an element "x, of X, then we write y=f (x) &read as y" is equal to f of x. Here f(x) is called image of f at x or value of f at x .

Or if a quantity y depends on a quantity x in such a way that each value of x determines exactly one value of y. Then we say that y is a function of x.

The set x is called Domain of f. The set of corresponding elements y in y is called Range of f. we say that y is a function of x.

Exercise 1.1

Q1. (a) Given that $f(x) = x^2 - x$

i. $f(-2) = (-2)^2 - (-2) = 4 + 2 = 6$

ii. $f(0) = (0)^2 - (0) = 0$

iii. $f(x-1) = (x-1)^2 - (x-1) = x^2 - 2x + 1 - x + 1 = x^2 - 3x + 2$

iv. $f(x^2+4) = (x^2+4)^2 - (x^2+4) = x^4 + 8x^2 + 16 - x^2 - 4 = x^4 + 7x^2 + 12$

(b) Given that $f(x) = \sqrt{x+4}$

$i)\ f(-2)=\sqrt{-2+4}=\sqrt{2}$

$ii)\ f(0)=\sqrt{0+4}=\sqrt{4}=2$

$iii)\ f(x-1)=\sqrt{x-1+4}=\sqrt{x+3}$

$iv)\ f(x^2+4)=\sqrt{x^2+4+4}=\sqrt{x^2+8}$

Q2. *Given that*

i) $f(x)=6x-9$

$f(a+h)=6(a+h)-9=6a+6h-9$

$f(a)=6a-9$

Now $\dfrac{f(a+h)-f(a)}{h}=\dfrac{(6a+6h-9)-(6a-9)}{h}$

$=\dfrac{6a+6h-9-6a+9}{h}=\dfrac{6h}{h}=6$

ii) $f(x)=\sin x$ *given*

$\because$ $\sin\theta-\sin\varphi=2\cos\left(\dfrac{\theta+\varphi}{2}\right)\sin\left(\dfrac{\theta+\varphi}{2}\right)$

$f(a+h)=\sin(a+h)$ *and* $f(a)=\sin a$

Now $\dfrac{f(a+h)-f(a)}{h}=\dfrac{\sin(a+h)-\sin a}{h}$

$=\dfrac{1}{h}\left[\sin(a+h)-\sin a\right]$

$=\dfrac{1}{h}\left[2\cos\left(\dfrac{a+h+a}{2}\right)\sin\left(\dfrac{a+h-a}{2}\right)\right]=\dfrac{1}{h}\left[2\cos\left(\dfrac{2a+h}{2}\right)\sin\left(\dfrac{h}{2}\right)\right]$

$=\dfrac{1}{h}\left[2\cos\left(\dfrac{2a}{2}+\dfrac{h}{2}\right)\sin\left(\dfrac{h}{2}\right)\right]=\dfrac{2}{h}\cos\left(a+\dfrac{h}{2}\right)\sin\left(\dfrac{h}{2}\right)$

iii) Given that $f(x) = x^3 + 2x^2 - 1$

$$f(a+h) = (a+h)^3 + 2(a+h)^2 - 1 = a^3 + h^3 + 3ah(a+h) + 2(a^2 + 2ah + h^2) - 1$$

$$= a^3 + h^3 + 3a^2h + 3ah^2 + 2a^2 + 4ah + 2h^2 - 1$$

$$f(a) = a^3 + 2a^2 - 1$$

Now $f(a+h) - f(a)$

$$= \frac{a^3 + h^3 + 3a^2h + 3ah^2 + 2a^2 + 4ah + 2h^2 - 1 - (a^3 + 2a^2 - 1)}{h}$$

$$= \frac{1}{h}\left[a^3 + h^3 + 3a^2h + 3ah^2 + 2a^2 + 4ah + 2h^2 - 1 - a^3 - 2a^2 + 1\right]$$

$$= \frac{1}{h}\left[h^3 + 3a^2h + 3ah^2 + 4ah + 2h^2\right] = \frac{h}{h}\left[h^2 + 3a^2 + 3ah + 4a + 2h\right]$$

$$= h^2 + 3a^2 + 3ah + 4a + 2h = h^2 + 3ah + 2h + 3a^2 + 4a = h^2 + (3a+2)h + 3a^2 + 4a$$

iv) Given that $f(x) = \cos x$

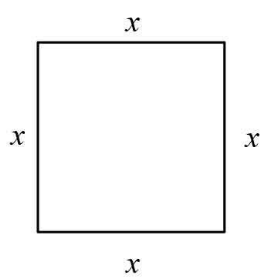
so $f(a+h) = \cos(a+h)$

and $f(a) = \cos a$

Now $\dfrac{f(a+h) - f(a)}{h}$

$$= \frac{\cos(a+h) - \cos a}{h} = \frac{1}{h}\left[-2\sin\left(\frac{2a+h}{2}\right)\sin\left(\frac{h}{2}\right)\right] = \frac{-2}{h}\sin\left(a + \frac{h}{2}\right)\sin\left(\frac{h}{2}\right)$$

Q3. (a) If 'x' unit be the side of square.



Then its perimeter P = $x + x + x + x = 4x$ ……………….. (1)

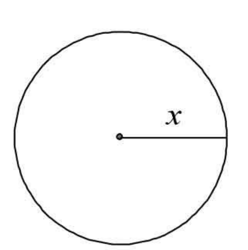
A = Area = $x \,.\, x = x^2$ …………… (2)

From (2) $x = \sqrt{A}$ putting in (1)

P = $4\sqrt{A}$

∴ P is expressed as Area

(b) Let x units be the radius of circle



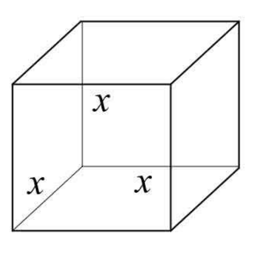
Then Area = A = πx^2 ……………….. (1)

Circumference = C = $2\pi x$ ……………….. (2)

From (2) $x = \dfrac{C}{2\pi}$ Putting in (1)

$$A = \pi\left(\frac{c}{2\pi}\right)^2 = \pi\left(\frac{c^2}{4\pi^2}\right) = \frac{c^2}{4\pi}$$

A = $\dfrac{c^2}{4\pi}$ ∵ *Area is a function of Circumference*



(c) Let x unit be each side of cube.

The Volume of Cube = $x \,.\, x \,.\, x = x^3$ ……………….. (1)

Area of base = A = x^2 ……………….. (2)

From (2) $x = \sqrt{A}$ Putting in (1)

$V = (\sqrt{A})^3 = (A)^{3/2}$

$Q5. \quad f(x) = x^3 - ax^2 + bx + 1$

If $f(2) = -3$ *and* $f(-1) = 0$

$(2)^3 - a(2)^2 + b(2) + 1 = -3$

$8 - 4a + 2b + 1 = -3$

$9 - 4a + 2b = -3$

$12 - 4a + 2b = 0$

Dividing by -2

$-6 + 2a - b = 0$................. (1)

$(-1)^3 - a(-1)^2 + (-1) + 1 = 0$

$-1 - a - b + 1 = 0$

$-a - b = 0$

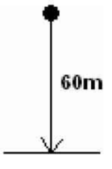
$a + b = 0$ .... (2)

Solving (1) *and* (2)

$$\begin{array}{r} 2a - b - 6 = 0 \\ a + b = 0 \\ \hline 3a - 6 = 0 \end{array}$$

$a = 2$ *and* $(2) \Rightarrow b = -a \Rightarrow b = -2$

$Q6. \quad h(x) = 40 - 10x^2$



$(a) \quad x = 1\sec$

$h(1) = 40 - 10(1)^2$

$= 30m$

$(b) \quad x = 1.5\sec$

$h(1.5) = 40 - 10(1.5)^2$

$= 40 - 10(2.25) = 40 - 22.5 = 17.5m$

$(c) \quad x = 1.7\sec$

$h(1.7) = 40 - 10(1.7)^2$

$= 40 - 10(2.89) = 40 - 28.9 = 11.1m$

ii) Does the stone strike the ground = ?

$h(x) = 0$

$40 - 10x^2 = 0$

$-10x^2 = -40 \Rightarrow x^2 = 4$

$x = \pm 2$

Stone strike the ground after 2 sec.

Graphs of Function

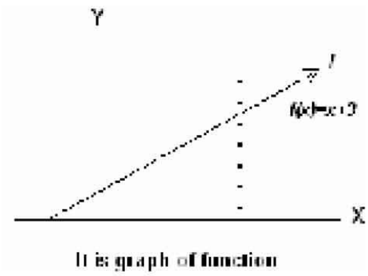
Definition:

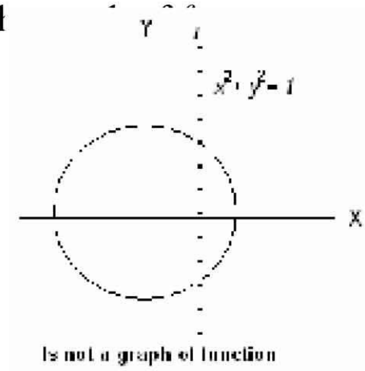
The graph of a function *f* is the graph of the equation $y = f(x)$. It consists of the points in the Cartesian plane chose co-ordinates (x , y) are input - output pairs for *f*
.

Note that not every curve we draw in the graph of a function. A function *f* can have only one value *f(x)* for each *x* in its domain.

Vertical Line Test

No vertical line can intersect the graph of a function more than once. Thus, a circle cannot be the graph of a function. Since some vertical lines intersect the circle Twice. If 'a' is the domain of the function *f* , then the vertical line x = a will intersect th
in the single point (a , *f(a)*).





Types of Function

ALGEBRAIC FUNCTIONS

Those functions which are defined by algebraic expressions.

1) Polynomial Functions:

$P(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x + a_0$ **Is a Polynomial Function for all x where $a_0 , a_1 , a_2 .... a_n$ are real numbers, and exponents are non-negative integer . a_n is called leading coefft of p(x) of degree n, Where $a_n \neq 0$**

$\Rightarrow$ *Degree of polynomial function is the* max*imum power of x in equation*

$P(x) = 2x^4 - 3x^3 + 2x - 1$ $\qquad$ $\deg ree = 4$

2) Linear Function: if the degree of polynomial fn is '1, is called linear function .i.e. p(x)=ax+b

or $\Rightarrow$ Degree of polynomial function is one.

$f(x) = ax + b \qquad a \neq 0$

$\because \qquad y = 5x + b$

3) Identity Function: For any set X, a function I: $X \to x$ of the form y = x or *f(x)* = x. Domain and range of I is x. Note. I (x)= ax +b be a linear fn if a=1,b=0 then I(x)=x or y=x is called identity fn

4) Constant Function:

$C : X \to y$ *defined by* | **or** $f : X \to y$ If f (x)=c, (const) then f is called constant fn

$C(x) = a \qquad \forall x \in X \text{ and } a \in y$

e.g. $\quad C : R \to R$

$C(x) = 2 \text{ or } y = 2 \quad \forall x \in R$

eg y=5

5) Rational Function:

$$R(x)=\frac{P(x)}{Q(x)}$$

Both $P(x)$ *and* $Q(x)$ *are polynomial and* $Q(x)\neq 0$

e.g. $R(x)=\dfrac{3x^2+4x+1}{5x^3+2x^2+1}$

Domain of rational function is the set of all real numbers for which $Q(x)\neq 0$

6) Exponential Function:

A function in which the variable appears as exponent (power) is called an exponential function.

i) $y=a^x \;\therefore\; x\in R \qquad a>0$

ii) $y=e^x \;\therefore\; x\in R \text{ and } e=2.178$

iii) $y=2^x \text{ or } y=e^{xh}$

are some exponential functions.

7) Logarithmic Function:

If $x=a^y$ *then* $y=\log_a^x$ $x>0$

$\because \quad a>0 \quad a\neq 1$

'a' is called the base of Logarithemic function

Then $y=\log_a^x$ *is Logarithmic function of base 'a'*

i) *If base* $=10$ *then* $y=\log_{10}^x$

is called common Logarithm of x

ii) *If base* $=e=2.718$

$y=\log_e^x=\ln x$ *is called natural* log

8) Hyperbolic Function:

We define as

i) $y=\sinh(x)=\dfrac{e^x-e^{-x}}{2}$ Sine hyperbolic function or hyperbolic sine function

$Dom=\{x/x\in R\}$ *and* $Range=\{y/y\in R\}$

ii) $y=\cosh(x)=\dfrac{e^x+e^{-x}}{2}$ *is called hyperbolic cosine function* $\Rightarrow$ $x\in R, y\in[1,\infty)$

iii) $y=Tanhx=\dfrac{e^x-e^{-x}}{e^x+e^{-x}}=\dfrac{\sinh x}{\cosh x}$ iv) $y=\coth x=\dfrac{\cosh x}{\sinh x}$

v) $y=\sec hx=\dfrac{1}{\cosh x}=\dfrac{2}{e^x+e^{-x}}$ $x\in R$

vi) $y=\cos echx=\dfrac{1}{\sinh x}=\dfrac{2}{e^x-e^{-x}}$ $Dom=\{x\neq 0 : x\in R$

9) Inverse Hyperbolic Function: (Study in B.Sc level)

i) $y=\sinh^{-1}x=\ln\left(x+\sqrt{x^2+1}\right)$ *for* $\forall x\in R$

ii) $y=\cosh^{-1}x=\ln\left(x+\sqrt{x^2-1}\right)$ *for* $\forall x\in R$ *and* $x>1$

iii) $y=Tanh^{-1}x=\frac{1}{2}\ln\left|\frac{1+x}{1-x}\right|$ $x\neq 1$ *and* $|x|<1$

iv) $y=\sec h^{-1}x=\ln\left(\frac{1}{x}+\frac{\sqrt{1-x^2}}{x}\right)$ $0<x\leq 1$

v) $y=\coth^{-1}x=\frac{1}{2}\left|\frac{x+1}{x-1}\right|$ $\because$ $|x|>1$

vi) $y=\cos ech^{-1}x=\ln\left(\frac{1}{x}+\frac{\sqrt{1+x^2}}{|x|}\right)$ $x\neq 0$

10) Trigonometric Function:

Functions	Domain(x)	Range(y)
i) $y=\sin x$	*All real numbers* $\because -\infty<x+\infty$	$-1\leq y\leq 1$
ii) $y=\cos x$	*All real numbers* $\because -\infty<x<\infty$	$-1\leq y\leq 1$
iii) $y=\tan x$	$x\in R-(2k+1)\frac{\pi}{2}$ $k\in Z$	$\because$ 'R' *all real numbers*
iv) $y=\cot x$	$x\in R-k\pi$ $k\in Z$	R
v) $y=\sec x$	$x\in R-(2k+1)\frac{\pi}{2}$ $k\in Z$	$R-(-1,1)$ *or* $R-(-1<y<1)$
vi) $y=\cos ecx$	$x\in R-(k\pi)$ $k\in Z$	$R-(-1<y<1)$

11) Inverse Trigonometric Functions:

Function	Dom(x)	Range(y)
$y=\sin^{-1}x\Leftrightarrow x=\sin y$	$-1\leq x\leq 1$	$-\frac{\pi}{2}\leq y\leq\frac{\pi}{2}$
$y=\cos^{-1}x\Leftrightarrow x=\cos y$	$-1\leq x\leq 1$	$0\leq y\leq\pi$

$y = Tan^{-1}x \Leftrightarrow x = Tany$	$x \in R$ $or\ -\infty < x < \infty$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
$y = Sec^{-1}x \Leftrightarrow x = \sec y$	$x \in R-(-1,1)$	$y \in [0,\pi] - \left\{\frac{\pi}{2}\right\}$
$y = Co\sec^{-1} x \Leftrightarrow x = \cos ecy$	$x \in R-(-1,1)$	$y \in \left[-\frac{\pi}{2},\frac{\pi}{2}\right] - \{0\}$
$y = Cot^{-1}x \Leftrightarrow x = \cot y$	$x \in R$	$0 < y < \pi$

12) Explicit Function:

If y is easily expressed in terms of x, then y is called an explicit function of x.

$\Rightarrow y = f(x)$ *e.g.* $y = x^3 + x + 1$ *etc.*

13) Implicit Function:

If x and y are so mixed up and y cannot be expressed in term of the independent variable x, Then y is called an implicit function of x. It can be written as. $f(x, y) = 0$

e.g. $x^2 + xy + y^2 = 2$ etc.

14) Parametric Function:

For a function y $= f(x)$ if both x & y are expressed in another variable say 't' or θ which is called a parameter of the given curve.

Such as:

i) $x = at^2$ Parametric parabola
$y = 2at$

ii) $x = a\cos t$ *Parametric equation of circle* $y^2 = 4a$

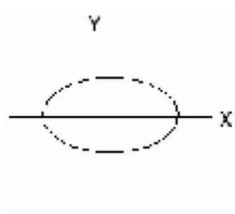
$y = a\sin t$

$x^2 + y^2 = a^2$

iii) $x = a\cos\theta$ *Parametric equation of Ellipse*

$y = b\sin\theta$

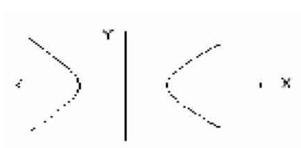
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



vi) $x = a\sec\theta$ *Parametric equation of hyperbola*

$y = b\tan\theta$

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$



Exercise 1.1

$Q7.\quad Parabola \quad \Rightarrow \quad y^2 = 4ax$ ....(1)

$x = at^2$ ....(i)

$y = 2at$ ....(ii)

$To\ eliminating\ 't'\ from\ (ii) \quad t = \dfrac{y}{2a} \quad putting\ (i)$

$$x = a\left(\frac{y}{2a}\right)^2 \Rightarrow x = a\left(\frac{y^2}{4a^2}\right) \Rightarrow x = \frac{y^2}{4a}$$

$\Rightarrow \quad y^2 = 4ax \quad which\ is\ same\ as\ (1)$

$which\ is\ equation\ of\ parabola.$

$ii) \quad x = a\cos\theta, \quad y = b\sin\theta$

$\Rightarrow \quad \dfrac{x}{a} = \cos\theta$..........$(i)$ $\quad and \quad \dfrac{y}{b} = \sin\theta$..............$(ii)$ $\quad To\ eliminating\ \theta\ from\ (i)\ and\ (ii)$

$Squaring\ and\ adding\ (i)\ and\ (ii)$

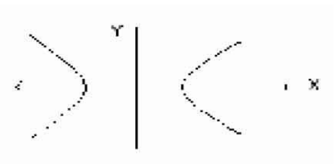
$$\left(\frac{x}{a}\right)^2 + \left(\frac{y^2}{b}\right) = 1 \quad represent\ a\ Ellipse$$

$iii) \quad x = a\sec\theta, \quad y = b\tan\theta$

$\dfrac{x}{a} = \sec\theta$...............$(i)$ $\quad \dfrac{y}{b} = \tan\theta$..................$(ii)$

$Squaring\ and\ Subtracting \quad (i)\ and\ (ii)$

$$\left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2 = \sec^2\theta - \tan^2\theta \Rightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 + \tan^2\theta - \tan^2\theta \Rightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



Which is equation of hyperbola

$Q8.\quad (i) \quad \sinh 2x = 2\sinh x\cosh x$

$$R.H.S = 2\sinh x\cosh x = 2\left(\frac{e^x - e^{-x}}{2}\right)\left(\frac{e^x + e^{-x}}{2}\right) = 2\left(\frac{e^{2x} - e^{-2x}}{4}\right) = \frac{e^{2x} - e^{-2x}}{2}$$

$= \sinh 2x = L.H.S$

$ii) \quad \sec^2 hx = 1 - \tan^2 hx$

$$R.H.S. = 1 - \tan^2 hx = 1 - \left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right) = 1 - \left(\frac{e^{2x} + e^{-2x} - 2}{e^{2x} + e^{-2x} + 2}\right)$$

$$= \frac{e^{2x} + e^{-2x} + 2 - e^{2x} - e^{-2x} + 2}{e^{2x} + e^{-2x} + 2} = \frac{4}{\left(e^x + e^{-x}\right)^2} = \frac{1}{\left(e^x + e^{-x}/2\right)^2}$$

$$= \frac{1}{\cosh^2 x} = \sec h^2 x = L.H.S$$

iii) $\cos eh^2 x = \coth^2 x - 1$

$$R.H.S = \coth^2 x - 1 = \left(\frac{e^x + e^{-x}}{e^x - e^{-x}}\right)^2 - 1 = \frac{\left(e^x + e^{-x}\right)^2 - \left(e^x - e^{-x}\right)^2}{\left(e^x - e^{-x}\right)^2} = \frac{\left(e^{2x} + e^{-2x} + 2\right) - \left(e^{2x} + e^{-2x} - 2\right)}{\left(e^x - e^{-x}\right)^2}$$

$$= \frac{e^{2x} + e^{-2x} + 2 - e^{2x} - e^{-2x} + 2}{\left(e^x - e^{-x}\right)^2} = \frac{4}{\left(e^x - e^{-x}\right)^2} = \frac{1}{\left(e^x - e^{-x}/2\right)^2} = \frac{1}{\sinh^2 x} = \cos ech 2x = L.H.S$$

Q9. $f(x) = x^3 + x$

replace x by $-x$

$$f(-x) = (-x)^3 + (-x) = -x^3 - x = -\left[x^3 + x\right] = -f(x)$$

$\Rightarrow$ $f(x) = x^3 + x$ *is odd function*

ii) $f(x) = (x+2)^2$

replace x by $-x$

$$f(-x) = (-x+2)^2 \neq \pm f(x)$$

$f(x) = (x+2)^2$ *is neither even nor odd*

iii) $f(x) = x\sqrt{x^2+5}$

replace x by $-x$

$$f(-x) = (-x)\sqrt{(-x)^2+5} = -\left[x\sqrt{x^2+5}\right] = -f(x)$$ $f(x)$ *is odd function.*

iv) $f(x) = \frac{x-1}{x+1}$

replace x by $-x$

$$f(-x) = \frac{-x-1}{-x+1} = \frac{-(x+1)}{-(x-1)} = \frac{x+1}{x-1} \neq \pm f(x)$$

$f(x)$ *is neither even nor odd function.*

v) $f(x) = x^{\frac{2}{3}} + 6$

replace x by $-x$

$$f(-x) = (-x)^{\frac{2}{3}} + 6 = \left[(-x)^2\right]^{\frac{1}{3}} + 6 = x^{\frac{2}{3}} + 6 = f(x)$$

$f(x)$ *is an even function.*

$$= \frac{x+1}{x-1} \neq \pm f(x)$$

$\therefore$ f(x) is neither even nor odd function.

(v) $\quad$ **f(x) = $x^{2/3} + 6$**

$$f(-x) = (-x)^{2/3} + 6$$
$$= [(-x)^2]^{1/3} + 6$$
$$= (x^2)^{1/3} + 6$$
$$= x^{2/3} + 6$$
$$= f(x)$$

$\therefore$ f(x) is an even function.

(vi) $\quad$ **f(x) = $\frac{x^3 - x}{x^2 + 1}$**

$$f(-x) = \frac{(-x)^3 - (-x)}{(-x)^2 + 1}$$
$$= \frac{-x^3 + x}{x^2 + 1}$$
$$= \frac{-(x^3 - x)}{x^2 + 1}$$
$$= -f(x)$$

$\therefore$ f(x) is an odd function.

Composition of Functions:

Let f be a function from set X to set Y and g be a function from set Y to set Z. The composition of f and g is a function, denoted by gof, from X to Z and is defined by.

$$(gof)(x) = g(f(x)) = gf(x) \text{ for all } x \in X$$

Inverse of a Function:

Let f be one-one function from X onto Y. The inverse function of f, denoted by f^{-1}, is a function from Y onto X and is defined by.

$$x = f^{-1}(y), \ \forall\, y \in Y \text{ if and only if } y = f(x), \forall\, x \in X$$

EXERCISE 1.2

Q.1 $\quad$ **The real valued functions f and g are defined below. Find**

(a) fog (x) $\quad$ **(b)** gof (x) $\quad$ **(c)** fof (x) $\quad$ **(d)** gog (x)

(i) $\quad$ **f(x) = 2x + 1 ; g(x) = $\frac{3}{x-1}$, x ≠ 1**