**Theorem on Anti-Derivatives**

i) $\int cf(x)dx = c\int f(x)dx$ where $c$ is constant.

ii) $\int[f(x)\pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$

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**Important Integral**

Since $\frac{d}{dx}x^{n+1} = (n+1)x^n$

Taking integral w.r.t $x$

$$\int \frac{d}{dx}x^{n+1}dx = \int (n+1)x^n dx$$

$$\Rightarrow\ x^{n+1} = (n+1)\int x^n dx$$

$$\Rightarrow\ \boxed{\int x^n\ dx = \frac{x^{n+1}}{n+1}} \quad \text{where } n \neq -1$$

If $n=-1$ then

$$\int x^{-1}dx = \int \frac{1}{x}dx \quad (\text{here } x \neq 0)$$

Since $\frac{d}{dx}\ln x = \frac{1}{x}$

Therefore $\boxed{\int \frac{1}{x}dx = \ln|x| + c}$

**Note:** Since log of zero and negative numbers does not exist therefore in above formula mod assure that we are taking a log of +ive quantity.

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**Question # 1**

Evaluate the following indefinite integrals.

(i) $\int(3x^2-2x+1)dx$

(ii) $\int\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)dx, (x>0)$

(iii) $\int x(\sqrt{x}+1)dx, (x>0)$

(iv) $\int(2x+3)^{\frac{1}{2}}dx$

(v) $\int(\sqrt{x}+1)^2 dx, (x>0)$

(vi) $\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 dx, (x>0)$

(vii) $\int\frac{3x+2}{\sqrt{x}}dx, (x>0)$

(viii) $\int\frac{\sqrt{y}(y+1)}{y}dy, (y>0)$

(ix) $\int\frac{(\sqrt{\theta}-1)^2}{\sqrt{\theta}}d\theta, (\theta>0)$

(x) $\int\frac{(1-\sqrt{x})^2}{\sqrt{x}}dx, (x>0)$

(xi) $\int\frac{e^{2x}+e^x}{e^x}dx$

***Solution***

(i) $\int(3x^2-2x+1)dx = 3\int x^2dx - 2\int xdx + \int dx$

$$= 3\cdot\frac{x^{2+1}}{2+1} - 2\cdot\frac{x^{1+1}}{1+1} + x + c$$

$$= 3\cdot\frac{x^3}{3} - 2\cdot\frac{x^2}{2} + x + c$$

$$= x^3 - x^2 + x + c$$

(ii) $\int\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)dx = \int\left(x^{\frac{1}{2}}+x^{-\frac{1}{2}}\right)dx$

$$= \int x^{\frac{1}{2}}dx + \int x^{-\frac{1}{2}}dx$$

$$= \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + c$$

$$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= \frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + c$$

(iii) $\int x(\sqrt{x}+1)dx = \int x\left(x^{\frac{1}{2}}+1\right)dx$

$$= \int\left(x^{\frac{3}{2}}+x\right)dx$$

$$= \int x^{\frac{3}{2}}dx + \int x\,dx$$

$$= \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + \frac{x^{1+1}}{1+1} + c$$

$$= \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + \frac{x^2}{2} + c$$

**Important Integral**

Since $\frac{d}{dx}(ax+b)^{n+1} = (n+1)(ax+b)^n\cdot a$

Taking integral

$$\int \frac{d}{dx}\left(ax+b\right)^{n+1} dx = \int (n+1)\left(ax+b\right)^{n} \cdot a\ dx$$

$$\Rightarrow\ \left(ax+b\right)^{n+1} = (n+1)\cdot a\int \left(ax+b\right)^{n}\ dx$$

$$\Rightarrow\ \boxed{\int \left(ax+b\right)^{n}\ dx = \frac{\left(ax+b\right)^{n+1}}{(n+1)\cdot a}}$$

(iv) $$\int (2x+3)^{\frac{1}{2}}\,dx\ = \frac{(2x+3)^{\frac{1}{2}+1}}{\left(\frac{1}{2}+1\right)\cdot 2}+c$$

$$=\frac{(2x+3)^{\frac{3}{2}}}{\left(\frac{3}{2}\right)\cdot 2}+c$$

$$=\frac{1}{3}(2x+3)^{\frac{3}{2}}+c$$

(v) $$\int \left(\sqrt{x}+1\right)^{2} dx\ = \int \left((\sqrt{x})^{2}+2\sqrt{x}+1\right)dx$$

$$=\int \left(x+2(x)^{\frac{1}{2}}+1\right)dx$$

$$=\int x\,dx+2\int (x)^{\frac{1}{2}}\,dx+\int dx$$

$$=\frac{x^{1+1}}{1+1}+2\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}+x+c$$

$$=\frac{x^{2}}{2}+2\frac{x^{\frac{3}{2}}}{{}^{3}\!/_{2}}+x+c$$

$$=\frac{x^{2}}{2}+\frac{4x^{\frac{3}{2}}}{3}+x+c$$

(vi) $$\int \left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^{2} dx\ = \int \left(x+\frac{1}{x}-2\right)dx$$

$$=\int x dx+\int \frac{1}{x}\,dx-2\int dx$$

$$=\frac{x^{2}}{2}+\ln|x|-2x+c$$

(vii) $$\int \frac{3x+2}{\sqrt{x}}\,dx\ = \int \frac{3x+2}{x^{1/2}}\,dx$$

$$=\int \frac{3x}{x^{1/2}}+\frac{2}{x^{1/2}}\,dx$$

$$=\int \left(3x^{1/2}+2x^{-1/2}\right)dx$$

$$=3\int x^{1/2}dx+2\int x^{-1/2}dx$$

*Now do yourself.*

(viii) $$\int \frac{\sqrt{y}\,(y+1)}{y}\,dy$$

$$=\int \frac{\sqrt{y}\,(y+1)}{\left(\sqrt{y}\right)^{2}}\,dy\ = \int \frac{(y+1)}{\sqrt{y}}\,dy$$

$$=\int \left(\frac{y}{\sqrt{y}}+\frac{1}{\sqrt{y}}\right)dy\ = \int \left(y^{\frac{1}{2}}+y^{-\frac{1}{2}}\right)dy$$

$$=\int y^{\frac{1}{2}}dy+\int y^{-\frac{1}{2}}dy$$

$$=\frac{y^{\frac{1}{2}+1}}{\frac{1}{2}+1}+\frac{y^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}+c\ \ =\frac{y^{\frac{3}{2}}}{\frac{3}{2}}+\frac{y^{\frac{1}{2}}}{\frac{1}{2}}+c$$

$$=\frac{2}{3}y^{\frac{3}{2}}+2y^{\frac{1}{2}}+c$$

(ix) $$\int \frac{\left(\sqrt{\theta}-1\right)^{2}}{\sqrt{\theta}}\ d\theta = \int \frac{\theta-2\sqrt{\theta}+1}{\sqrt{\theta}}\ d\theta$$

$$=\int \left(\frac{\theta}{\sqrt{\theta}}-\frac{2\sqrt{\theta}}{\sqrt{\theta}}+\frac{1}{\sqrt{\theta}}\right)d\theta$$

$$=\int \left(\theta^{\frac{1}{2}}-2+\theta^{-\frac{1}{2}}\right)d\theta$$

$$=\frac{\theta^{\frac{1}{2}+1}}{\frac{1}{2}+1}-2\theta+\frac{\theta^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}+c$$

$$=\frac{\theta^{\frac{3}{2}}}{{}^{3}\!/_{2}}-2\theta+\frac{\theta^{\frac{1}{2}}}{{}^{1}\!/_{2}}+c$$

$$=\frac{2}{3}\theta^{\frac{3}{2}}-2\theta+2\theta^{\frac{1}{2}}+c\quad Ans$$

(x) $$\int \frac{\left(1-\sqrt{x}\right)^{2}}{\sqrt{x}}\ dx\ = \int \frac{1-2\sqrt{x}+x}{\sqrt{x}}\ dx$$

$$=\int \left(\frac{1}{\sqrt{x}}-\frac{2\sqrt{x}}{\sqrt{x}}+\frac{x}{\sqrt{x}}\right)dx$$

$$=\int \left(x^{-\frac{1}{2}}-2+x^{\frac{1}{2}}\right)dx$$

$$=\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}-2x+\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}+c$$

$$=\frac{x^{\frac{1}{2}}}{{}^{1}\!/_{2}}-3\theta+\frac{x^{\frac{3}{2}}}{{}^{3}\!/_{2}}+c$$

$$=2x^{\frac{1}{2}}-2x+\frac{2}{3}x^{\frac{3}{2}}+c \quad Ans$$

**Important Integral**

We know $\dfrac{d}{dx}e^{ax}=a\cdot e^{ax}$

Taking integral

$$\int\frac{d}{dx}e^{ax}\,dx=\int a\cdot e^{ax}dx$$

$$\Rightarrow\ e^{ax}=a\int e^{ax}dx$$

$$\Rightarrow\ \boxed{\int e^{ax}dx=\frac{e^{ax}}{a}}$$

Also note that $\displaystyle\int e^{(ax+b)}dx=\frac{e^{(ax+b)}}{a}$

(xi) $\displaystyle\int\frac{e^{2x}+e^{x}}{e^{x}}\,dx=\int\left(\frac{e^{2x}}{e^{x}}+\frac{e^{x}}{e^{x}}\right)dx$

$$=\int\left(e^{x}+1\right)dx$$

$$=\int e^{x}dx+\int dx$$

$$=e^{x}+x+c \qquad \underline{Ans}$$

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**Question # 2**

Evaluate

(i) $\displaystyle\int\frac{dx}{\sqrt{x+a}+\sqrt{x+b}}\begin{pmatrix}x+a>0\\x+b>0\end{pmatrix}$

(ii) $\displaystyle\int\frac{1-x^{2}}{1+x^{2}}dx$

(iii) $\displaystyle\int\frac{dx}{\sqrt{x+a}+\sqrt{x}},\left(x>0,a>0\right)$

(iv) $\displaystyle\int\left(a-2x\right)^{\frac{3}{2}}dx$

(v) $\displaystyle\int\frac{\left(1+e^{x}\right)^{3}}{e^{x}}dx$

(vi) $\displaystyle\int\sin(a+b)x\,dx$

(vii) $\displaystyle\int\sqrt{1-\cos 2x}\,dx,\left(1-\cos 2x>0\right)$

(viii) $\displaystyle\int\ln x\times\frac{1}{x}\,dx,\left(x>0\right)$

(ix) $\displaystyle\int\sin^{2}x\,dx$

(x) $\displaystyle\int\frac{1}{1+\cos x}dx,\left(-\frac{\pi}{2}<x<\frac{\pi}{2}\right)$

(xi) $\displaystyle\int\frac{ax+b}{ax^{2}+2bx+c}dx$

(xii) $\displaystyle\int\cos 3x\sin 2x\,dx$

(xiii) $\displaystyle\int\frac{\cos 2x-1}{1+\cos 2x}\,dx,\left(1+\cos 2x\neq 0\right)$

(xiv) $\displaystyle\int\tan^{2}x\,dx$

***Solution***

(i) $\displaystyle\int\frac{dx}{\sqrt{x+a}+\sqrt{x+b}}$

$$=\int\frac{dx}{\sqrt{x+a}+\sqrt{x+b}}\cdot\frac{\sqrt{x+a}-\sqrt{x+b}}{\sqrt{x+a}-\sqrt{x+b}}$$

$$=\int\frac{\sqrt{x+a}-\sqrt{x+b}}{x+a-x-b}\,dx$$

$$=\int\frac{\left(x+a\right)^{\frac{1}{2}}-\left(x+b\right)^{\frac{1}{2}}}{a-b}\,dx$$

$$=\frac{1}{a-b}\left[\int\left(x+a\right)^{\frac{1}{2}}dx-\int\left(x+b\right)^{\frac{1}{2}}dx\right]$$

$$=\frac{1}{a-b}\left[\frac{\left(x+a\right)^{\frac{1}{2}+1}}{\frac{1}{2}+1}-\frac{\left(x+b\right)^{\frac{1}{2}}}{\frac{1}{2}+1}\right]+c$$

$$=\frac{1}{a-b}\left[\frac{\left(x+a\right)^{\frac{3}{2}}}{\frac{3}{2}}-\frac{\left(x+b\right)^{\frac{3}{2}}}{\frac{3}{2}}\right]+c$$

$$=\frac{2}{3\left(a-b\right)}\left[\left(x+a\right)^{\frac{3}{2}}-\left(x+b\right)^{\frac{3}{2}}\right]+c \quad Ans.$$

**Important Integral**

Since $\dfrac{d}{dx}Tan^{-1}x=\dfrac{1}{1+x^{2}}$

Also $\dfrac{d}{dx}\left(-Cot^{-1}x\right)=\dfrac{1}{1+x^{2}}$

Therefore $\displaystyle\int\frac{1}{1+x^{2}}dx=Tan^{-1}x$ or $-Cot^{-1}x$

Similarly $\displaystyle\int\frac{1}{\sqrt{1-x^{2}}}dx=Sin^{-1}x$ or $-Cos^{-1}x$

$\displaystyle\int\frac{1}{x\sqrt{x^{2}-1}}dx=Sec^{-1}x$ or $-Csc^{-1}x$

(ii) $\displaystyle\int\frac{1-x^{2}}{1+x^{2}}dx$

$$\begin{array}{r} -1 \\ 1+x^{2}\overline{\smash{)}1-x^{2}} \\ \underset{+\quad +}{-1-x^{2}} \\ \hline 2 \end{array}$$

$$=\int\left(-1+\frac{2}{1+x^{2}}\right)dx$$

$$=-\int dx+2\int\frac{1}{1+x^{2}}dx$$

$$=-x+2Tan^{-1}x+c$$

(iii) $\displaystyle\int\frac{dx}{\sqrt{x+a}+\sqrt{x}}$

$$=\int\frac{dx}{\sqrt{x+a}+\sqrt{x}}\cdot\frac{\sqrt{x+a}-\sqrt{x}}{\sqrt{x+a}-\sqrt{x}}$$

$$=\int\frac{\sqrt{x+a}-\sqrt{x}}{x+a-x}\,dx$$

$$=\int\frac{(x+a)^{\frac{1}{2}}-(x)^{\frac{1}{2}}}{a}\,dx$$

$$=\frac{1}{a}\left[\int(x+a)^{\frac{1}{2}}\,dx-\int(x)^{\frac{1}{2}}dx\right]$$

$$=\frac{1}{a}\left[\frac{(x+a)^{\frac{1}{2}+1}}{\frac{1}{2}+1}-\frac{(x)^{\frac{1}{2}}}{\frac{1}{2}+1}\right]+c$$

$$=\frac{1}{a}\left[\frac{(x+a)^{\frac{3}{2}}}{\frac{3}{2}}-\frac{(x)^{\frac{3}{2}}}{\frac{3}{2}}\right]+c$$

$$=\frac{2}{3a}\left[(x+a)^{\frac{3}{2}}-x^{\frac{3}{2}}\right]+c \quad \textit{Ans.}$$

(iv) $\int(a-2x)^{\frac{3}{2}}\,dx$

$$=\frac{(a-2x)^{\frac{3}{2}+1}}{\left(\frac{3}{2}+1\right)\cdot(-2)}+c$$

$$=\frac{(a-2x)^{\frac{5}{2}}}{\left(\frac{5}{2}\right)\cdot(-2)}+c$$

$$=-\frac{(a-2x)^{\frac{5}{2}}}{5}+c$$

(v) $\int\frac{\left(1+e^{x}\right)^{3}}{e^{x}}dx=\int\frac{\left(1+3e^{x}+3e^{2x}+e^{3x}\right)}{e^{x}}dx$

$$=\int\left(\frac{1}{e^{x}}+\frac{3e^{x}}{e^{x}}+\frac{3e^{2x}}{e^{x}}+\frac{e^{3x}}{e^{x}}\right)dx$$

$$=\int\left(e^{-x}+3+3e^{x}+e^{2x}\right)dx$$

$$=\frac{e^{-x}}{-1}+3x+3e^{x}+\frac{e^{2x}}{2}+c$$

$$=-e^{-x}+3x+3e^{x}+\frac{1}{2}e^{2x}+c$$

**Important Integrals**

We know $\frac{d}{dx}\cos ax=-a\sin ax$

Taking integral

$$\int\frac{d}{dx}\cos ax\,dx=-\int a\sin ax\,dx$$

$$\Rightarrow\ \cos ax=-a\int\sin ax\ dx$$

$$\Rightarrow\ \boxed{\int\sin ax\,dx=-\frac{\cos ax}{a}}$$

Also $\frac{d}{dx}\sin ax=a\cdot\cos ax$

$$\therefore\ \boxed{\int\cos ax\,dx=\frac{\sin ax}{a}}$$

Similarly

$$\boxed{\int\sec^{2}ax\ dx=\frac{\tan ax}{a}}$$

$$\boxed{\int\text{cosec}^{2}ax\ dx=-\frac{\cot ax}{a}}$$

$$\boxed{\int\sec ax\tan ax\ dx=\frac{\sec ax}{a}}$$

$$\boxed{\int\csc ax\cot ax\ dx=-\frac{\csc ax}{a}}$$

Also note that

$$\int\sin(ax+b)\,dx=-\frac{\cos(ax+b)}{a}$$

$$\int\cos(ax+b)\,dx=\frac{\sin(ax+b)}{a}\quad\text{and so on.}$$

(vi) $\int\sin(a+b)x\,dx=-\frac{\cos(a+b)x}{a+b}+c$

*Do yourself*

(vii) $\int\sqrt{1-\cos 2x}\,dx$

$$=\int\sqrt{2\sin^{2}x}\,dx\quad\because\ \sin^{2}x=\frac{1-\cos 2x}{2}$$

$$=\sqrt{2}\int\sin x\,dx\quad=\sqrt{2}(-\cos x)+c$$

$$=-\sqrt{2}\cos x+c$$

**Important Formula**

$$\because\ \frac{d}{dx}[f(x)]^{n+1}=(n+1)[f(x)]^{n}\frac{d}{dx}f(x)$$

$$\Rightarrow\ \frac{d}{dx}[f(x)]^{n+1}=(n+1)[f(x)]^{n}f'(x)$$

Taking integral

$$\int\frac{d}{dx}[f(x)]^{n+1}dx=\int(n+1)[f(x)]^{n}f'(x)dx$$

$$\Rightarrow\ [f(x)]^{n+1}=(n+1)\int[f(x)]^{n}f'(x)dx$$

$$\Rightarrow\ \boxed{\int[f(x)]^{n}f'(x)dx=\frac{[f(x)]^{n+1}}{(n+1)}}\quad;\ n\neq-1$$

Also $\frac{d}{dx}\ln|f(x)|=\frac{1}{f(x)}\cdot f'(x)$

Taking integral

$$\ln|f(x)|=\int\frac{f'(x)}{f(x)}dx$$

i.e. $\boxed{\int \frac{f'(x)}{f(x)}dx = \ln\left|f(x)\right| + c}$

(viii) Let $I = \int \ln x \times \frac{1}{x}\,dx$

Put $f(x) = \ln x \;\Rightarrow\; f'(x) = \frac{1}{x}$

So $I = \int \left[f(x)\right] f'(x)\,dx$

$$= \frac{\left[f(x)\right]^{1+1}}{1+1} + c \;= \frac{\left[f(x)\right]^{2}}{2} + c$$

$$= \frac{\left(\ln x\right)^2}{2} + c$$

(ix) $\int \sin^2 x\,dx = \int \left(\frac{1-\cos 2x}{2}\right) dx$

$$= \int \left(\frac{1}{2} - \frac{1}{2}\cos 2x\right) dx$$

$$= \frac{1}{2}\int dx - \frac{1}{2}\int \cos 2x\,dx$$

$$= \frac{1}{2}x - \frac{1}{2}\,\frac{\sin 2x}{2} + c$$

$$= \frac{1}{2}x - \frac{1}{4}\sin 2x + c$$

(x) $\int \frac{1}{1+\cos x}dx$

$= \int \frac{1}{2\cos^2\frac{x}{2}}dx \qquad \because\; \cos^2\frac{x}{2} = \frac{1+\cos x}{2}$

$$= \frac{1}{2}\int \sec^2\frac{x}{2}\,dx \quad = \frac{1}{2}\,\frac{\tan\frac{x}{2}}{1/2} + c \;= \tan\frac{x}{2} + c$$

**Alternative**

$$\int \frac{1}{1+\cos x}dx \;= \int \frac{1}{1+\cos x} \times \frac{1-\cos x}{1-\cos x}dx$$

$$= \int \frac{1-\cos x}{1-\cos^2 x}dx$$

$$= \int \frac{1-\cos x}{\sin^2 x}dx$$

$$= \int \left(\frac{1}{\sin^2 x} - \frac{\cos x}{\sin^2 x}\right)dx$$

$$= \int \left(\operatorname{cosec}^2 x - \frac{\cos x}{\sin x \cdot \sin x}\right)dx$$

$$= \int \operatorname{cosec}^2 x\,dx - \int \operatorname{cosec} x \cot x\,dx$$

$$= -\cot x - \left(-\operatorname{cosec} x\right) + c$$

$$= \operatorname{cosec} x - \cot x + c$$

(xi) Let $I = \int \frac{ax+b}{ax^2+2bx+c}dx$

Put $f(x) = ax^2 + 2bx + c$

$\Rightarrow\; f'(x) = 2ax + 2b$

$\Rightarrow\; f'(x) = 2\left(ax+b\right) \;\Rightarrow\; \frac{1}{2}f'(x) = ax + b$

So $I = \int \frac{\frac{1}{2}f'(x)}{f(x)}dx$

$$= \frac{1}{2}\int \frac{f'(x)}{f(x)}dx \;= \frac{1}{2}\ln\left|f(x)\right| + c_1$$

$$= \frac{1}{2}\ln\left|\,ax^2 + 2bx + c\,\right| + c_1$$

**Review**

- $2\sin\alpha\cos\beta = \sin\left(\alpha+\beta\right) + \sin\left(\alpha-\beta\right)$
- $2\cos\alpha\sin\beta = \sin\left(\alpha+\beta\right) - \sin\left(\alpha-\beta\right)$
- $2\cos\alpha\cos\beta = \cos\left(\alpha+\beta\right) + \cos\left(\alpha-\beta\right)$
- $-2\sin\alpha\sin\beta = \cos\left(\alpha+\beta\right) - \cos\left(\alpha-\beta\right)$

(xii) $\int \cos 3x \sin 2x\,dx$

$$= \frac{1}{2}\int 2\cos 3x \sin 2x\,dx$$

$$= \frac{1}{2}\int \left[\sin(3x+2x) - \sin(3x-2x)\right] dx$$

$$= \frac{1}{2}\int \left[\sin 5x - \sin x\right] dx$$

$$= \frac{1}{2}\left[-\frac{\cos 5x}{5} - \left(-\cos x\right)\right] + c$$

$$= -\frac{1}{2}\left[\frac{\cos 5x}{5} - \cos x\right] + c$$

(xiii) $\int \frac{\cos 2x - 1}{1+\cos 2x}dx$

$= -\int \frac{1-\cos 2x}{1+\cos 2x}dx \qquad \because\; \sin^2 x = \frac{1-\cos 2x}{2}$

$= -\int \frac{2\sin^2 x}{2\cos^2 x}dx \qquad \cos^2 x = \frac{1+\cos 2x}{2}$

$$= -\int \tan^2 x\,dx \;= -\int \left(\sec^2 x - 1\right) dx$$

$$= -\int \sec^2 x\,dx + \int dx$$

$$= -\tan x + x + c$$

(xiv) $\int \tan^2 x\,dx \;= \int \left(\sec^2 x - 1\right) dx$

$$= \int \sec^2 x\,dx - \int dx$$

$$= \tan x - x + c$$

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## Important Integral

Since $\frac{d}{dx}\ln|ax+b| = \frac{1}{ax+b}\cdot\frac{d}{dx}(ax+b)$

$$\Rightarrow \frac{d}{dx}\ln|ax+b| = \frac{1}{ax+b}\cdot a$$

On Integrating

$$\Rightarrow \ln|ax+b| = a\int\frac{1}{ax+b}\,dx$$

$$\Rightarrow \boxed{\int\frac{1}{ax+b}\,dx = \frac{\ln|ax+b|}{a}}$$