EXERCISE 3.5

Q.1 $\int \frac{3x+1}{x^2-x-6}\,dx$

Solution:

$$\int \frac{3x+1}{x^2-x-6}\,dx$$

$$= \int \frac{3x+1 \qquad dx}{x^2-3x+2x-6}$$

$$= \int \frac{3x+1}{x(x-3)+2(x-3)}\,dx$$

$$= \int \frac{3x+1}{(x+2)(x-3)}\,dx$$

Let

$$\frac{3x+1}{(x+2)(x-3)} = \frac{A}{x+2} + \frac{B}{x-3} \quad \text{\_\_\_\_ (1)}$$

Where A & B are constant of partial fractions which are to be determined.

Multiplying $(x+2)(x-3)$ on both sides in (1)

$$3x+1 = A(x-3) + B(x+2) \qquad \text{\_\_\_\_\_\_\_\_} \qquad (2)$$

<u>To find A</u>

Put $x+2=0$

$x=-2$ in (2)

$$3(-2)+1 = A(-2-3)$$

$$-6+1 = A(-5)$$

$$-5 = -5A$$

$$A = \frac{-5}{-5}$$

$$\boxed{A = 1}$$

<u>To find B</u>

Put $x-3=0$

$x = 3$ in (2)

$$3(3) + 1 = B(3 + 2)$$

$$9 + 1 = 5B$$

$$5B = 10$$

$$B = \frac{10}{5}$$

∴ From eq. (1)

$$\frac{3x + 1}{(x + 2)(x - 3)} = \frac{1}{x + 2} + \frac{2}{x - 3}$$

$$\int \frac{3x + 1}{(x + 2)(x - 3)} dx = \int \frac{dx}{x + 2} + 2\int \frac{dx}{x - 3}$$

$$= \boxed{ln\,|x + 2| + 2ln\,|x - 3| + c} \qquad \text{Ans.}$$

Q.2 $\displaystyle \int \frac{\mathbf{5x + 8}}{\mathbf{(x + 3)(2x - 1)}}\, \mathbf{dx}$

Solution:

$$\int \frac{5x + 8}{(x + 3)(2x - 1)} dx$$

Let

$$\frac{5x + 8}{(x + 3)(2x - 1)} = \frac{A}{x + 3} + \frac{B}{2x - 1} \qquad \text{———} \qquad (1)$$

Where A & B are constant of partial fractions which are to be determined.

Multiplying $(x + 3)(2x - 1)$ on both sides in (1)

$$5x + 8 = A(2x - 1) + B(x + 3) \qquad \text{———} \qquad (2)$$

To find A

Put $x + 3 = 0$

$x = -3$ in (2)

$$5(-3) + 8 = A[2(-3) - 1]$$

$$-15 + 8 = A(-6 - 1)$$

$$-7 = -7A$$

$$A = \frac{-7}{-7}$$

$$\boxed{A = 1}$$

To find B

Put $2x - 1 = 0$

$2x = 1$

$$x \quad = \frac{1}{2} \text{ in } (2)$$

$$5\left(\frac{1}{2}\right) + 8 \quad = B\,(\frac{1}{2} + 3)$$

$$\frac{5}{2} + 8 \;=\; B\left(\frac{1+6}{2}\right)$$

$$\frac{5+16}{2} \;=\; \left(\frac{7}{2}\right) B$$

$$\frac{21}{2} \;=\; \frac{7}{2}\, B$$

$$\frac{21}{2} \times \frac{2}{7} = B$$

$$\boxed{B \;=\; 3}$$

∴ From eq. (1)

$$\frac{5x+8}{(x+3)\,(2x-1)} = \frac{1}{x+3} + \frac{3}{2x-1}$$

Integrate

$$\int \frac{5x+8 \qquad dx}{(x+3)\,(2x-1)} \;=\; \int \frac{dx}{x+3} + \frac{3}{2} \int \frac{2dx}{2x-1}$$

$$= \quad \boxed{ln\,|x+3| + \frac{3}{2}\, ln\,|2x-1| + c} \qquad \text{Ans.}$$

Q.3 $\displaystyle \int \frac{\mathbf{x^2 + 3x - 34}}{\mathbf{x^2 + 2x - 15}}\ \mathbf{dx}$ **(Guj. Board 2006)**

Solution:

$$\int \frac{x^2 + 3x - 34}{x^2 + 2x - 15}\ dx$$

$$= \quad \int \left(1 + \frac{x - 19}{x^2 + 2x - 15}\right) dx$$

$$\begin{array}{r} 1 \\ x^2 + 2x - 15 \,\overline{\big)\, x^2 + 3x - 34} \\ \underline{\pm x^2 \pm 2x \mp 15} \\ x - 19 \end{array}$$

$$= \quad \int dx + \int \frac{x - 19}{x^2 + 5x - 3x - 15}\ dx$$

$$= \quad x + \int \frac{x - 19}{x(x+5) - 3\,(x+5)}\ dx$$

$$= \quad x + \int \frac{x - 19}{(x-3)\,(x+5)}\ dx$$

$$= \quad x + I \quad \text{———} \quad (1)$$

$$I = \int \frac{x-19}{(x-3)(x+5)}\,dx$$

Let

$$\frac{x-19}{(x-3)(x+5)} = \frac{A}{x-3} + \frac{B}{x+5} \quad \text{———} \quad (2)$$

Where A & B are constant of potential fractions which are to be determined.

Multiplying (x – 3) (x + 5) on both sides in (2)

$$X - 19 = A(x+5) + B(x-3) \quad \text{———} \quad (3)$$

<u>To find A</u>

Put $x - 3 = 0$

$x = 3$ in (3)

$3 - 19 = A(3+5)$

$-16 = 8A$

$$A = \frac{-16}{8}$$

$$\boxed{A = -2}$$

<u>To find B</u>

Put $x + 5 = 0$

$x = -5$ in (3)

$-5 - 19 = B(-5-3)$

$-24 = -8B$

$$B = \frac{-24}{-8}$$

$$\boxed{B = 3}$$

∴ From eq. (2)

$$\frac{x-19}{(x-3)(x+5)} = \frac{-2}{x-3} + \frac{3}{x+5}$$

Integrate

$$\int \frac{x-19}{(x-3)(x+5)}\,dx = 2\int \frac{dx}{x-3} + 3\int \frac{dx}{x+5}$$

$$I = -2\,ln\,|x-3| + 3\,ln\,|x+5| + c$$

∴ From eq. (1)

$$= \boxed{x - 2ln\,|x-3| + 3ln\,|x+5| + c} \quad \text{Ans.}$$

Q.4 $\displaystyle\int \frac{\mathbf{(a-b)\,x}}{\mathbf{(x-a)\,(x-b)}}\,\mathbf{dx}$

Solution:

$$\int \frac{(a - b)\,x}{(x-a)\,(x-b)}\,dx$$

Let

$$\frac{(a-b)x}{(x-a)\,(x-b)} = \frac{A}{x-a} + \frac{B}{x-b} \quad \text{———} \quad (1)$$

Where A & B are constant of partial fractions which are to be determined.

Multiplying $(x-a)\,(x-b)$ on both sides in (1)

$$(a-b)\,x = A\,(x-b) + B\,(x-a) \quad \text{———} \quad (2)$$

To find A

Put $x - a = 0$

$x = a$ in (2)

$(a-b)\,a = A\,(a-b)$

$\dfrac{(a-b)a}{a-b} = A$

$$\boxed{A = a}$$

To find B

Put $x - b = 0$

$x = b$ in eq. (2)

$(a-b)\,b = B\,(b-a)$

$\dfrac{(a-b)\,b}{b-a} = B$

$B = -\dfrac{(b-a)\,b}{b-a}$

$$\boxed{B = -b}$$

From eq. (1)

$$\frac{(a-b)\,x}{(x-a)\,(x-b)} = \frac{a}{x-a} + \frac{-b}{x-b}$$

Integrate

$$\int \frac{(a-b)\,x}{(x-a)\,(x-b)}\,dx = a\int\frac{dx}{x-a} - b\int\frac{dx}{x-b}$$

$$= \boxed{a\,ln\,|x-a| - b\,ln\,|x-b| + c} \qquad \text{Ans.}$$

Q.5 $\displaystyle\int \frac{3-x}{1-x-6x^2}\,dx$

Solution:

$$\int \frac{3-x}{1-x-6x^2}\,dx$$

$$= \int \frac{3-x}{1-3x+2x-6x^2}\,dx$$

$$= \int \frac{3-x}{(1-3x)+2x(1-3x)}\,dx$$

$$= \int \frac{3-x}{(1+2x)(1-3x)}\,dx$$

Let

$$\frac{3-x}{(1+2x)(1-3x)} = \frac{A}{1+2x} + \frac{B}{1-3x} \quad \text{———} \quad (1)$$

Where A and B are constant of partial fractions which are to be determined

Multiplying $(1+2x)(1-3x)$ on both sides in (1)

$$3-x = A(1-3x) + B(1+2x) \quad \text{———} \quad (2)$$

<u>To find A</u>

Put $1+2x = 0$

$2x = -1$

$x = \frac{-1}{2}$ in (2)

$$3-\frac{-1}{2} = A\left[1-3\left(\frac{-1}{2}\right)\right]$$

$$3+\frac{1}{2} = A\left(1+\frac{3}{2}\right)$$

$$\frac{6+1}{2} = A\left(\frac{2+3}{2}\right)$$

$$\frac{7}{2} = \left(\frac{5}{2}\right)A$$

$$\frac{7\times 2}{2\times 5} = A$$

$$\boxed{A = \frac{7}{5}}$$

To find B

Put $1 - 3x = 0$

$$3x = 1$$

$$x = \frac{1}{3} \text{ in } (2)$$

$$3 - \frac{1}{3} = B\left(1 + 2\left(\frac{1}{3}\right)\right)$$

$$\frac{9-1}{3} = B\left(1 + \frac{2}{3}\right)$$

$$\frac{8}{3} = B\left(\frac{3+2}{3}\right)$$

$$\frac{8}{3} = \left(\frac{5}{3}\right)B$$

$$\frac{8}{3} \times \frac{3}{5} = B$$

$$\boxed{B = \frac{8}{5}}$$

∴ From eq. (1)

$$\frac{3-x}{(1+2x)(1-3x)} = \frac{\frac{7}{5}}{1+2x} + \frac{\frac{8}{5}}{1-3x}$$

Integrate

$$\int \frac{3-x}{(1+2x)(1-3x)}\, dx = \frac{7}{5}\int \frac{dx}{1+2x} + \frac{8}{5}\int \frac{dx}{1-3x}$$

$$= \frac{7}{5.2}\int \frac{2dx}{1+2x} - \frac{8}{5 \times 3}\int \frac{-3}{1-3x}\, dx$$

$$= \boxed{\frac{7}{10} \ln|1+2x| - \frac{8}{5} \ln|1-3x| + c} \qquad \text{Ans.}$$

Q.6 $\displaystyle\int \frac{2x}{x^2 - a^2}\, dx$

Solution:

$$\int \frac{2x}{x^2 - a^2}\, dx$$

$$= \int \frac{2x}{(x+a)(x-a)}\, dx$$

Let

$$\frac{2x}{(x+a)(x-a)} = \frac{A}{x+a} + \frac{B}{x-a} \text{ ———— } (1)$$

Where A and B are constant of partial fractions which are to be determined

Multiplying $(x + a)(x - a)$ on both sides in (1)

$$2x = A(x-a) + B(x+a) \text{ ———— } (2)$$

To find A

Put $x + a = 0$

$x = -a$ in (2)

$$2(-a) = A(-a-a)$$

$$-2a = 2a\,A$$

$$A = \frac{-2a}{-2a}$$

$$\boxed{A = 1}$$

To find B

Put $x - a = 0$

$x = a$ in (2)

$$2a = B(a+a)$$

$$2a = 2a\,B$$

$$B = \frac{2a}{2a}$$

$$\boxed{B = 1}$$

∴ From eq. (1)

$$\frac{2x}{(x+a)(x-a)} = \frac{1}{x+a} + \frac{1}{x-a}$$

Integrate

$$\int \frac{2x}{(x+a)(x-a)}\,dx = \int \frac{dx}{x+a} + \int \frac{dx}{x-a}$$

$$= \boxed{\ln|x+a| + \ln|x-a| + c} \quad \text{Ans.}$$

Q.7 $\displaystyle\int \frac{1}{6x^2 - 5x - 4}\,dx$

Solution:

$$\int \frac{1}{6x^2 - 5x - 4}\,dx$$

$$= \int \frac{1}{x^2 + 8x - 3x - 4}\,dx$$

$$= \int \frac{1}{2x\,(3x + 4) - 1\,(3x + 4)}\,dx$$

$$= \int \frac{1}{(2x - 1)\,(3x + 4\,)}\,dx$$

Let

$$\frac{1}{(2x - 1)\,(3x + 4)} = \frac{A}{2x - 1} + \frac{B}{3x + 4} \text{ ———— } (1)$$

Where A and B are constant of partial fractions which are to be determined.

Multiplying $(2x - 1)\,(3x + 4)$ on both sides in (1)

$$1 = A\,(3x + 4) + B\,(2x - 1) \text{ ———— } (2)$$

To find A

Put $2x - 1 = 0$

$2x = 1$

$x = \frac{1}{2}$ in eq. (2)

$$1 = A\left(\frac{3}{2} + 4\right)$$

$$1 = A\left(\frac{3 + 8}{2}\right)$$

$$2 = A\,(11)$$

$$A = \frac{2}{11}$$

To find B

Put $3x + 4 = 0$

$3x = -4$

$x = \frac{-4}{3}$ in eq. (2)

$$1 = B\left[\,2\left(\frac{-4}{3}\right) - 1\,\right]$$

$$1 = B\left(\frac{-8}{3} - 1\right)$$

$$1 = B\left(\frac{-8-3}{3}\right)$$

$$3 = -11B$$

$$\boxed{B = \frac{-3}{11}}$$

$\therefore$ From eq. (1)

$$\frac{1}{(2x - 1)(3x + 4)} = \frac{\frac{2}{11}}{2x - 1} + \frac{\frac{-3}{11}}{3x + 4}$$

$$\int \frac{dx}{(2x - 1)(3x + 4)} = \frac{1}{11}\int \frac{2dx}{2x - 1} - \frac{1}{11}\int \frac{3}{3x + 4}\, dx$$

$$= \boxed{\frac{1}{11} ln\,|2x - 1| - \frac{1}{11} ln\,|3x + 4| + c} \qquad \text{Ans.}$$

Q.8 $\displaystyle \int \frac{\mathbf{2x^3 - 3x^2 - x - 7}}{\mathbf{2x^2 - 3x - 2}}\,\mathbf{dx}$

Solution:

$$\int \frac{2x^3 - 3x^2 - x - 7}{2x^2 - 3x - 2}\, dx$$

$$\because \begin{array}{r} x \\ 2x^2 - 3x - 2 \,\overline{)\, 2x^3 - 3x^2 - x - 7} \\ \underline{\underset{-}{}2x^3 \mp 3x^2 \mp 2x} \\ x - 7 \end{array}$$

$$= \int \left(x + \frac{x - 7}{2x^2 - 3x - 2}\right) dx$$

$$= \int x dx + \int \frac{x - 7}{2x^2 - 4x + x - 2}\, dx$$

$$= \frac{x^2}{2} + \int \frac{x - 7}{2x(x - 2) + 1(x - 2)}\, dx$$

$$= \frac{x^2}{2} + \int \frac{x - 7}{(2x + 1)(x - 2)}\, dx$$

$$= \frac{x^2}{2} + I \quad \text{———} \quad (1)$$

Where

$$I = \int \frac{x - 7}{(2x + 1)(x - 2)}\, dx$$

$$\frac{x - 7}{(2x + 1)(x - 2)} = \frac{A}{2x + 1} + \frac{B}{x - 2} \quad \text{———} \quad (2)$$

Where A & B are constant of partial fractions which are to be determined.

Multiplying (2x + 1) (x – 2) on both sides in (2)

$$x - 7 = A(x - 2) + B(2x + 1) \quad \text{———} \quad (3)$$

To find A

$$\text{Put} \quad 2x + 1 = 0$$

$$2x = -1$$

$$x = \frac{-1}{2} \text{ in } (3)$$

$$\frac{-1}{2} - 7 = A\left(\frac{-1}{2} - 2\right)$$

$$\frac{-1-14}{2} = A\left(\frac{-1-4}{2}\right)$$

$$\frac{-15}{2} = A\left(\frac{-5}{2}\right)$$

$$\frac{-15}{2} \times \frac{-2}{2} = A$$

$$\boxed{A = 3}$$

To find B

$$\text{Put} \quad x - 2 = 0$$

$$x = 2 \text{ in } (3)$$

$$2 - 7 = B[2(2) + 1]$$

$$-5 = B(4 + 1)$$

$$-5 = 5B$$

$$B = \frac{-5}{5}$$

$$\boxed{B = -1}$$

∴ From eq. (2)

$$\frac{x - 7}{(2x + 1)(x - 2)} = \frac{3}{2x + 1} + \frac{-1}{x - 2}$$

$$\int \frac{x - 7 \quad dx}{(2x + 1)(x - 2)} = \frac{3}{2} \int \frac{2dx}{2x + 1} - \int \frac{dx}{x - 2}$$

$$= \boxed{\frac{3}{2} \ln|2x + 1| - \ln|x - 2| + c} \quad \text{Ans.}$$

Q.9 $\displaystyle\int \frac{\mathbf{3x^2 - 12x + 11}}{\mathbf{(x - 1)(x - 2)(x - 3)}} \, \mathbf{dx}$

Solution:

$$\int \frac{3x^2 - 12x + 11}{(x-1)(x-2)(x-3)} dx$$

Let $\frac{3x^2 - 12x + 11}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$ ——— (1)

Where A, B and C are constant of partial fractions which are to be determined.

Multiplying (x – 1) (x – 2) (x – 3) on both sides in (1)

$3x^2 - 12x + 11 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$ ——(2)

To find A

Put $x - 1 = 0$

$x = 1$ in (2)

$3(1)^2 - 12(1) + 11 = A(1-2)(1-3)$

$3 - 12 + 11 = A(-1)(-2)$

$2 = 2A$

$A = \frac{2}{2}$

$\boxed{A = 1}$

To find B

Put $x - 2 = 0$

$x = 2$ in (2)

$3(2)^2 - 12(2) + 11 = B(2-1)(2-3)$

$3(4) - 24 + 11 = B(1)(-1)$

$12 - 24 + 11 = -B$

$-1 = -B$

$\boxed{B = 1}$

To find C

∴ From eq. (1)

$$\frac{3x^2 - 12x + 11}{(x-1)(x-2)(x-3)} = \frac{1}{x-1} + \frac{1}{x-2} + \frac{1}{x-3}$$

Integrate

$$\int \frac{3x^2 - 12x + 11 \quad dx}{(x-1)(x-2)(x-3)} = \int \frac{dx}{x-1} + \int \frac{dx}{x-2} + \int \frac{dx}{x-3}$$

$$= \boxed{ln|x-1| + ln|x-2| + ln|x-3| + c}$$ Ans.

Q.10 $\displaystyle\int \frac{2x-1}{x(x-1)(x-3)}\,dx$

Solution:

$$\int \frac{2x-1}{x(x-1)(x-3)}\,dx$$

Let

$$\frac{2x-1}{x(x-1)(x-3)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x-3} \text{ ——— } (1)$$

Where A, B and C are constant of partial fractions which are to be determined.

Multiplying x(x – 1) (x – 3) on both sides in (1)

$$2x-1 = A(x-1)(x-3) + Bx(x-3) + Cx(x-1) \text{ ——}(2)$$

<u>To find A</u>

Put $x = 0$ in eq (2)

$$2(0)-1 = A(0-1)(0-3)$$

$$-1 = A(-1)(-3)$$

$$-1 = 3A$$

$$\boxed{A = \frac{-1}{3}}$$

<u>To find B</u>

Put $x-1 = 0$

$x = 1$ in (2)

$$2(1)-1 = B(1)(1-3)$$

$$2-1 = B(-2)$$

$$1 = -2B$$

$$\boxed{B = \frac{-1}{2}}$$

<u>To find C</u>

Put $x-3 = 0$

$x = 3$ in (2)

$$2(3)-1 = C(3)(3-1)$$

$$6-1 = C(3)(2)$$

$$5 = 6C$$

$$\boxed{C = \frac{5}{6}}$$

∴ From eq. (1)

$$\frac{2x-1}{x(x-1)(x-3)} = \frac{\frac{-1}{3}}{x} + \frac{\frac{-1}{2}}{x-1} + \frac{\frac{5}{6}}{x-3}$$

Integrate $\int \frac{2x-1}{x(x-1)(x-3)}\,dx = \frac{-1}{3}\int \frac{dx}{x} - \frac{1}{2}\int \frac{dx}{x-1} + \frac{5}{6}\int \frac{dx}{x-3}$

$$= \boxed{\frac{-1}{3}\ln|x| - \frac{1}{2}\ln|x-1| + \frac{5}{6}\ln|x-3| + c} \quad \text{Ans.}$$

Q.11 $\displaystyle\int \frac{5x^2+9x+6}{(x^2-1)(2x+3)}\,dx$

Solution:

$$\int \frac{5x^2+9x+6}{(x^2-1)(2x+3)}\,dx$$

$$= \int \frac{5x^2+9x+6}{(x+1)(x-1)(2x+3)}\,dx$$

Let

$$\frac{5x^2+9x+6}{(x+1)(x-1)(2x+3)} = \frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{2x+3} \quad \text{——— (1)}$$

Where A, B and C are constant of partial fractions which are to be determined.

Multiplying (x + 1) (x – 1) (2x + 3) on both sides in (1)

$$5x^2+9x+6 = A(x-1)(2x+3) + B(x+1)(2x+3) + C(x-1)(x-1) \text{— (2)}$$

To find A

Put $x + 1 = 0$

$x = 1$ in (2)

$5(-1)^2 + 9(-1) + 6 = A(-1-1)(-2+3)$

$5 - 9 + 6 = A(-2)(1)$

$2 = -2A$

$A = \frac{2}{-2}$

$\boxed{A = -1}$

To find B

Put $x - 1 = 0$

$x = 1$ in (2)

$$5(1)^2 + 9(1) + 6 = B\,(1 + 1)\,(2 + 3)$$
$$5 + 9 + 6 = B\,(2\,)\,(5)$$
$$20 = 10\,B$$
$$B = \frac{20}{10}$$
$$\boxed{B = 2}$$

<u>To find C</u>

$$\text{Put} \quad 2x + 3 = 0$$
$$2x = -3$$
$$x = \frac{-3}{2} \text{ in (2)}$$
$$5\left(\frac{-3}{2}\right)^2 + 9\left(\frac{-3}{2}\right) + 6 = C\left(\frac{-3}{2} + 1\right)\left(\frac{-3}{2} - 1\right)$$
$$5\left(\frac{9}{4}\right) - \frac{27}{2} + 6 = C\left(\frac{-3+2}{2}\right)\left(\frac{-3-2}{2}\right)$$
$$\frac{45}{4} - \frac{27}{2} + 6 = C\left(\frac{-1}{2}\right)\left(\frac{-5}{2}\right)$$
$$\frac{45 - 54 + 24}{4} = \frac{5}{4}\,C$$
$$\frac{15}{4} \times \frac{4}{5} = C$$
$$\boxed{C = 3}$$

∴ From eq. (1)

$$\frac{5x^2 + 9x + 6}{(x + 1)\,(x - 1)\,(2x + 3)} = \frac{-1}{x + 1} + \frac{2}{x - 1} + \frac{3}{2x + 3}$$

Integrate

$$\int \frac{5x^2 + 9x + 6}{(x + 1)\,(x - 1)\,(2x + 3)}\,dx = -\int \frac{dx}{x + 1} + 1\int \frac{dx}{x - 1} + \frac{3}{2}\int \frac{2}{2x + 3}\,dx$$

$$= \boxed{ln\,|x+1| + 2ln\,|x - 1| + \frac{3}{2}\,ln\,|2x + 3| \; + \; c} \quad \text{Ans.}$$

Q.12 $\displaystyle\int \frac{\mathbf{4 + 7x}}{\mathbf{(1 + x)^2\,(2 + 3x)}}\,\mathbf{dx}$

Solution:

$$\int \frac{4 + 7x}{(1 + x)^2 (2 + 3x)} dx$$

Let

$$\frac{4 + 7x}{(2 + 3x)(1 + x)^2} = \frac{A}{2 + 3x} + \frac{B}{1 + x} + \frac{C}{(1 + x)^2} \quad \text{———} \quad (1)$$

Where A, B and C are constant of partial fractions which are to be determined.

Multiplying $(2 + 3x)(1 + x)^2$ on both sides in (1)

$$4 + 7x = A(1 + x)^2 + B(2 + 3x)(1 + x) + C(2 + 3x) \text{ ——}(2)$$

$$4 + 7x = A(1 + x^2 + 2x) + B(2 + 5x + 3x^2) + 2C + 3Cx$$

$$4 + 7x = (A + 3B)x^2 + (2A + 5B + 3C)x + (A + 2B + 2C) \text{ ——} \quad (3)$$

<u>To find A</u>

Put $2 + 3x = 0$

$3x = -2$

$x = \frac{-2}{3}$ in (3)

$$4 + 7\left(\frac{-2}{3}\right) = A\left(1 - \frac{2}{3}\right)^2$$

$$4 - \frac{14}{3} = A\left(\frac{3\text{-} 2}{3}\right)^2$$

$$\frac{12 - 14}{3} = A\left(\frac{1}{3}\right)^2$$

$$\frac{-2}{3} = \frac{1}{9}A$$

$$\frac{-2 \times 9}{3} = A$$

$$\boxed{A = -6}$$

<u>To find C</u>

Put $1 + x = 0$

$x = -1$ in (2)

$$4 + 7(-1) = C(2 - 3)$$

$$4 - 7 = C(-1)$$

$$-3 = -C$$

$$\boxed{C = 3}$$

To find B comparing the coefficient of x^2 in (3)

$$A + 3B = 0$$