Chapter 3

MOTION AND FORCE

LEARNING OBJECTIVES

At the end of this chapter the students will be able to:

- Understand displacement from its definition and illustration.
- Understand velocity, average velocity and instantaneous velocity.
- Understand acceleration, average acceleration & instantaneous acceleration.
- Understand the significance of area under velocity-time graph.
- Recall Newton's Laws of motion.
- Describe Newton's second law of motion as rate of change of momentum.
- Define impulse as a product of impulsive force and time.
- Describe law of conservation of momentum.
- Describe the force produced due to flow of water.
- Understand the process of rocket propulsion (simple treatment).
- Understand projectile motion in a non-resistive medium.
- Derive time of flight, maximum height and horizontal range of projectile motion.

Q.1 Define motion and rest.

Ans. **MOTION**

If a body is changing its position with respect to its surroundings then the body is said to be in motion.

REST

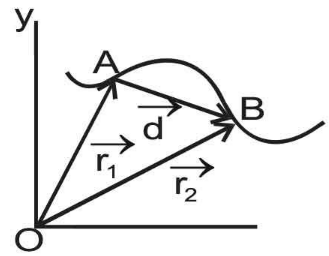
If a body is not changing its position with respect to some observer then the body is said to be at rest.

Q.2 Define displacement and distance.

Ans. **DISPLACEMENT**

The displacement is a change in the position of body from its initial position to its final position, or the shortest distance between two points is called displacement.

The displacement can be represented as a vector that describes how far and in what direction the body has been displaced from its initial position. The tail of displacement vector is located at the position where the displacement started and its tip is located at final position where displacement ended. If a body is moving along a curve as shown with A as its initial position and B as its final position then the displacement $\vec{d}$ of the body is represented by AB.



If $\vec{r}_1$ is position vector of A and $\vec{r}_2$ that of B then by head to tail rule

$$\vec{r}_1 + \vec{d} = \vec{r}_2$$

$$\therefore \quad \vec{d} = \vec{r}_2 - \vec{r}_1$$

It is a vector quantity and its SI unit is metre (m).

Distance

It is the separation between the two points. It is a scalar quantity and its SI unit is metre (m).

Q.3 Define velocity and types of velocity.

Ans. **VELOCITY**

The rate of change of displacement is known as velocity. Its direction is along the direction of displacement. So if $\vec{d}$ is the total displacement of the body in time t, then its average velocity during the interval t is defines as

$$\vec{V}_{av} = \frac{\vec{d}}{t}$$

It is a vector quantity and SI unit is m/s.

Dimensions

$$[\vec{V}] = m/s$$
$$= L/T$$
$$= [LT^{-1}]$$

Types of Velocity

There are three types of velocity:

(i) Uniform velocity (ii) Variable velocity (iii) Instantaneous velocity

(i) Uniform Velocity

If a body covers equal displacements in equal interval of times, however small may be interval the velocity is said to be uniform velocity.

(ii) Variable Velocity

If a body covers equal displacement in unequal interval of times however small may be the interval then it is said to be variable velocity. And its motion is non-uniform.

(iii) Instantaneous Velocity

Velocity of a body at any instant is called instantaneous velocity. **(OR)** The instantaneous velocity is also defined as the limiting value of $\frac{\Delta \vec{d}}{\Delta t}$ as the time interval Δt following the time Δt approaches to zero.

Mathematically

$$\vec{V}_{ins} = \lim_{\Delta t \to 0} \frac{\Delta \vec{d}}{\Delta t}$$

Note: *If the instantaneous velocity does not change the body is said to be moving with uniform velocity.*

Q.4 Define acceleration with its units.

Ans. **ACCELERATION**

The time rate of change of velocity of a body is called acceleration. As velocity is a vector so any change in velocity may be due to change in its magnitude or change in its direction or both. Consider a body whose velocity $\vec{V}_1$ at any time t changes to $\vec{V}_2$ in small time interval Δt, therefore the change in velocity $\Delta\vec{V}$ is

$$\Delta\vec{V} = \vec{V}_2 - \vec{V}_1$$

The average acceleration during time interval Δt is given by

$$\vec{a}_{ave} = \frac{\vec{V}_2 - \vec{V}_1}{\Delta t} = \frac{\Delta\vec{V}}{\Delta t}$$

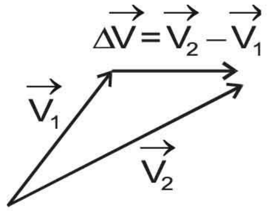
If the velocity of the body is increasing then its acceleration is positive while if the velocity of the body is decreasing then its acceleration is negative. The SI unit of acceleration is m/s^2.

For Your Information

Typical Speeds

Speed, ms^{-1}	Motion
300 000 000	Light, radio waves, x-rays, microwaves (in vacuum)
210 000	Earth-Sun travel around the galaxy
29 600	Earth around the Sun
1 000	Moon around the Earth
980	SR-71 reconnaissance jet
333	Sound (in air)
267	Commercial jet airliner
62	Commercial automobile (max.)
37	Falcon in a dive
29	Running cheetah
10	100-metres dash (max.)
9	Porpoise swimming
5	Flying bee
4	Human running
2	Human swimming

0.01	Walking ant



Dimensions

$$[\vec{a}] = m/s^2$$

$$= L/T^2 = [LT^{-2}]$$

Instantaneous Acceleration

Acceleration of a body at a particular instant is known as instantaneous acceleration.

$$\vec{a}_{ins} = \lim_{\Delta t \to 0} \frac{\Delta \vec{V}}{\Delta t}$$

Note: *For a body moving with uniform acceleration, its average acceleration is equal to the instantaneously accelerated.*

Q.5 Explain velocity-time graph.

Ans. VELOCITY-TIME GRAPH

Graphs which show the variation of velocity of an object with time are called velocity-time graphs. In such graphs, the time is taken along positive x-axis because it is the independent quantity.

When velocity of car is constant

When velocity of car is constant, its velocity–time graph is a horizontal straight line as shown in Fig. (i).

As the distance covered by the object is

$$S = vt$$

This distance moved by an object can also be found by using its velocity-time graph by calculating area under this graph. This area is shown shaded in Fig. (i).

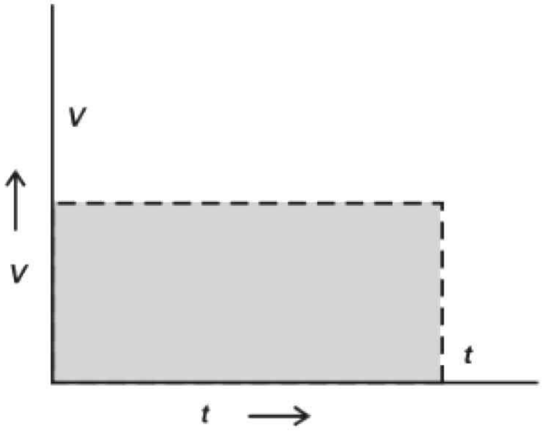


Fig. (i)

As it is a rectangle

$\therefore$ Area under the graph = Height x Width

$$H \times W = vt$$

$$A = vt$$

Hence distance covered = Area under V – t graph.

When car moves with constant acceleration

When the car moves with constant acceleration, the velocity-time graph is a straight line which rises the same height for equal intervals of time as shown in Fig. (ii).

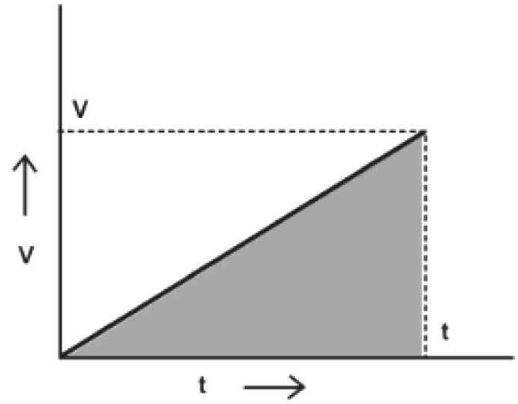


Fig. (ii)

Here the velocity of the object increased uniformly from O to V in time "t". Therefore

$$\therefore \quad V_{av} = \frac{0 + V}{2}$$

$$= \frac{1}{2} V$$

$$\therefore \quad S = V_{av}\, t$$

$$S = \frac{1}{2} V t$$

Now we calculate area under velocity-time graph which is equal to the area of the triangle shaded as shown in Fig. (ii).

$$\text{Area of } \Delta = \frac{1}{2} (\text{Base}) (\text{Height})$$

$$= \frac{1}{2} (t) (v)$$

$$= \frac{1}{2} v t$$

Hence distance covered = area under V–t graph

"In this case average acceleration of an object can be found by finding the slope of its velocity–time graph".

Note: *The area between the velocity–time graph and the time–axis is numerically equal to the distance covered by the object.*

When the car moves with increasing acceleration

When the car moves with increasing acceleration (non–uniform velocity) the velocity-time graph is a curve as shown in Fig. (iii).

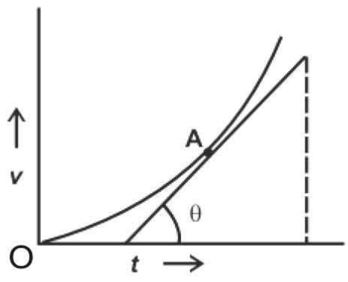


Fig. (iii)

The point A on the graph corresponds to time t. The magnitude of the Instantaneous acceleration at this instant is numerically equal to the slope of the tangent at point A on the velocity-time graph of the object as shown in Fig. (iii).

REVIEW OF EQUATIONS OF UNIFORMLY ACCELERATED MOTION

Suppose an object is moving with uniform acceleration 'a' along a straight line. If initial velocity of the object is 'V_1' and final velocity 'V_f' after a time interval t. And 'S' is distance covered then we have

$$V_f = V_i + at \qquad \ldots\ldots (1)$$

$$S = \left(\frac{V_f + V_i}{2}\right) \times t \quad \ldots\ldots (2)$$

$$S = V_i t + \frac{1}{2} a t^2 \quad \ldots\ldots (3)$$

$$V_f^2 = V_i^2 + 2aS \quad \ldots\ldots (4)$$

These equations are useful only for linear motion with uniform acceleration.

When the object moves along the straight line, the direction of motion does not change. In this case all the vector can be manipulated like scalars. In such problems the direction of initial is taken as positive. A negative sign is assigned to quantities where direction is opposite to that of initial velocity.

In the absence of air resistance, all objects near the surface of earth, moves towards the earth with a uniform acceleration. This acceleration, is known as acceleration due to gravity. It is denoted by 'g'. Its average value near the earth surface is taken as 9.8 ms^{-2} in the down ward direction.

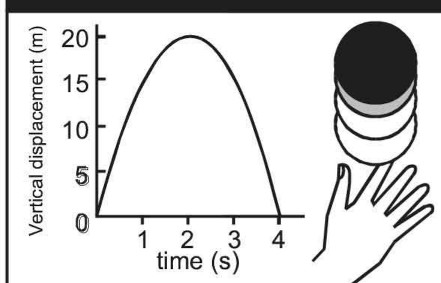
Note: *The equations for uniformly accelerated motion can also be applied to free fall motion of the objects by replacing 'a' by 'g'.*

Q.6 State and explain Newton's laws of motion.

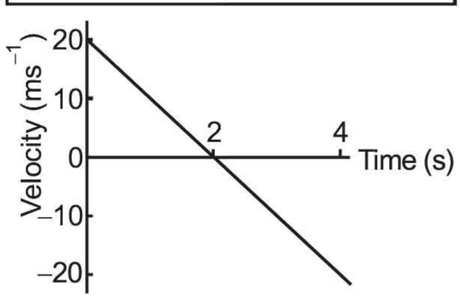
Ans. **NEWTON'S LAWS OF MOTION**

Newton's laws are empirical laws deduced from experiments. They were clearly stated for the 1st time by Sir Isaac Newton who published them in 1687 in his famous book called "Principia". Newton's laws are applicable only for speed which is negligible compared to speed of light. For very fast moving objects, such as atomic particle in an accelerator, relativistic mechanics developed by Einstein is applicable.

Do You Know?



How the displacement of a vertically thrown ball varies with time.



How the velocity of a vertically thrown ball varies with time? Velocity is upwards positive.

NEWTON'S FIRST LAW OF MOTION

A body at rest will remain at rest and a body moving with uniform velocity will continue to do so, unless acted upon by some unbalanced external force. This is also known as Law of "Inertia".

Inertia

The property of an object tending to maintain to the state of rest or state of uniform motion is known as object's inertia. The mass of the object is a quantitative measure of its inertia.

Frame of Reference

The space bounded by three mutually perpendicular lines is known as frame of reference. There are two types:

(i) Inertial frame of reference

The frame of reference in which Newton's laws of motions holds is known as inertial frame of reference. It is non–accelerated frame of reference.

(ii) Non-inertial frame of reference

A frame of reference in which Newton's laws of motions does not hold is known as non-inertial frame of reference. It is accelerated frame of reference.

e.g., A frame of reference stationed on earth is approximately an inertial frame of reference.

NEWTON'S SECOND LAW OF MOTION

A force applied on a body produces acceleration in its own direction. The acceleration produced is directly proportional with the applied force and inversely proportional with the mass of the body.

Mathematically, it is expressed as

$$\therefore \quad \vec{a} \propto \vec{F} \qquad \text{.........(i)}$$

$$\vec{a} \propto \frac{1}{m} \qquad \text{.........(ii)}$$

Combining (i) and (ii)

$$\vec{a} \propto \frac{\vec{F}}{m}$$

$$\vec{a} = k\frac{\vec{F}}{m}$$

where K = constant of proportionality.

If F = 1 N , m = 1 kg

a = 1 m/s^2

then K = 1

If S.I. units are used then

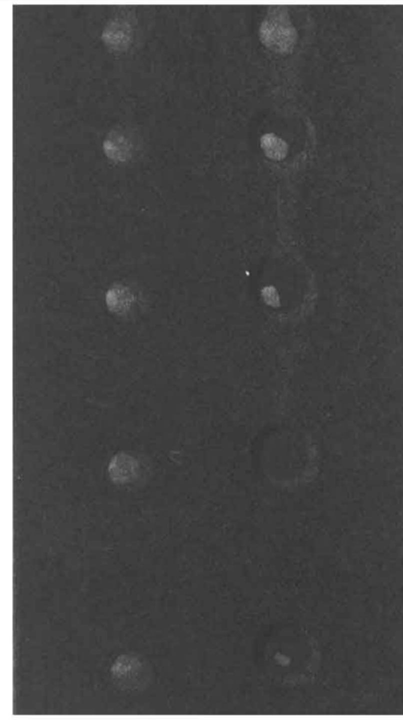
$$\therefore \quad \vec{F} = m\vec{a}$$

NEWTON'S THIRD LAW OF MOTION

Action and reaction are equal but in opposite direction.

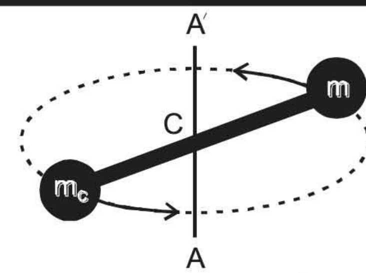
For example, whenever an interaction occurs between two objects, each object exerts the same force on the other, but in the opposite direction and for the same length of time. Each force in action-reaction pair acts only on one of the two bodies, the action and reaction forces never act on the same body.

Do You Know?



At the surface of the Earth, in situations where air friction is negligible, objects fall with the same acceleration regardless of their weights.

For Your Information



A measurement of mass independent of gravity. The unknown mass m and a calibrated mass m_c are mounted on a light weight rod. If the masses are equal, the rod will rotate without wobble about its centre.

Point to Ponder

A car accelerates along a road. Which force actually moves the car?
Ans: Force of friction

Q.7 What is the linear momentum? Also define its units.

Ans. MOMENTUM (LINEAR MOMENTUM)

It is defined as the "the product of mass and velocity of the object."

It is denoted by "P".

Mathematically

$$\vec{P} = m\vec{V}$$

or The quantity of motion in a moving body is called linear momentum.

Linear momentum is a vector quantity and has the direction in direction of velocity.

The magnitude of momentum depends upon the mass of body and velocity of the body.

Unit

The SI unit of momentum is kg m/s. It is also Ns.

Dimensions

$$[P] = \text{Kg m/s}$$
$$= ML/T$$
$$= [MLT^{-1}]$$

Q. Show that kg m/s is equal to Ns?

Ans. Kg m/s = Ns

As L.H.S. = Kg m/s

Multiple and divide by s

$$= \text{Kg m/s} \times \frac{s}{s}$$
$$= [\text{Kg m/s}^2] \times s$$
$$= Ns$$
$$= R.H.S$$

Q.8 How force and linear momentum are related? (OR) State Newton's second law of motion in terms of momentum.

Ans. MOMENTUM AND NEWTON'S SECOND LAW OF MOTION

Consider a body of mass 'm' moving with an initial velocity $\vec{V_i}$. Suppose an external force $\vec{F}$ acts upon if for time 't' after which velocity becomes $\vec{V_f}$.

As,
$$\vec{V_f} = \vec{V_i} + \vec{a}\,t$$
$$\vec{a}\,t = \vec{V_f} - \vec{V_i}$$
$$\vec{a} = \frac{\vec{V_f} - \vec{V_i}}{t} \quad \text{......... (1)}$$

Interesting Information

Throwing a package onto shore from a boat that was previously at rest causes the boat to move outward from shore (Newton's third law).

From Newton's 2nd Law

$$\vec{F} = m\vec{a}$$

$$\vec{a} = \frac{\vec{F}}{m} \qquad \text{.........} (2)$$

Point to Ponder

Which will be more effective in knocking a bear down.
i. a rubber bullet or
Ii. a lead bullet of the same Momentum

Ans. Rubber bullet will be more effective in knocking a bear down because its rate of change of momentum will be greater than that of lead bullet.

From equation (1) and (2)

$$\frac{\vec{F}}{m} = \frac{\vec{V_f} - \vec{V_i}}{t}$$

$$\vec{F} = \frac{m\vec{V_f} - \vec{V_i}}{t}$$

$$\vec{F} = \frac{\vec{P_f} - \vec{P_i}}{t}$$

$$\vec{F} = \frac{\Delta\vec{P}}{t}$$

Hence second law of motion in term of momentum can also be stated as "the time rate of change of momentum of a body equals the applied force."

Q.9 Define impulse and show that it is change in momentum.

Ans. IMPULSE

When a very large force acts on a body for a very short interval of time the momentum of the body changes. The product of such a force and time is called the impulse. It is denoted by I and it is a vector quantity.

$$\therefore \text{Impulse} = \vec{I} = \vec{F} \times t$$

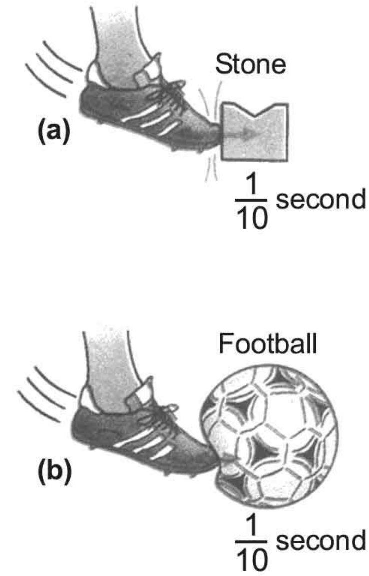
As
$$\vec{F} = \frac{m\vec{V}_f - m\vec{V}_i}{t}$$

$$\vec{F} \times t = m\vec{V}_f - m\vec{V}_i$$

$$\therefore \quad \text{Impulse} = \text{change in momentum } (\Delta\vec{P})$$

Unit: Its unit is $Kg\ ms^{-1}$ or Ns.

Point to Ponder



Which hurt you in the above situations (a) or (b) and think why?

Point to Ponder

Does a moving object have impulse?

Ans. There are two possibilities:

i. If a body is moving with constant velocity then change in momentum will be zero therefore impulse will be zero but if a body moves with variable velocity then there will be change in momentum and then the moving body will have impulse.

Do You Know?

Your hair acts like a crumple zone on your skull. A force of 5 N might be enough to fracture your naked skull (cranium), but with a covering of skin and hair, a force of 50 N would be needed.

Q.10 State and explain law of conservation of linear momentum.

Ans. LAW OF CONSERVATION OF MOMENTUM

Isolated System

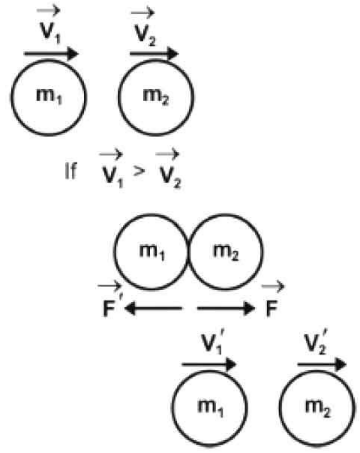
It is a system on which no external agency exerts any force. e.g., The molecules of a gas enclosed in a glass vessel at constant temperature constitute an isolated system. The molecules can collide with one another because of their random motion but, no external force can exert on them.

Statement

This law states that the total linear momentum of an isolated system remains constant.

Explanation

Consider an isolated system of two smooth hard interacting balls of masses m_1 and m_2, moving along the same straight line, in the same direction, with velocities $\vec{V_1}$ and $\vec{V_2}$ respectively. Both the balls collide and after collision, the ball of mass m_1 moves with velocity $\vec{V_1}'$ and m_2 moves with velocity $\vec{V_2}'$ in the same direction as shown in figure.



To find the change in momentum we use

$$\vec{F} \times t = m\vec{V_f} - m\vec{V_1}$$

For mass m_1

$$\vec{F} \times t = m_1\vec{V_1}' - m_1\vec{V_1} \quad \ldots\ldots\ldots (1)$$

Similarly for mass m_2

$$\vec{F}' \times t = m_2\vec{V_2}' - m_2\vec{V_2} \quad \ldots\ldots\ldots (2)$$

Adding (1) and (2)

$$\vec{F} \times t + \vec{F}' \times t = m_1\vec{V_1}' - m_1\vec{V_1} + m_2\vec{V_2}' - m_2\vec{V_2}$$

$$(\vec{F} + \vec{F}')\, t = m_1\vec{V_1}' - m_1\vec{V_1} + m_2\vec{V_2}' - m_2\vec{V_2}$$

Since the action F is equal and opposite to the reaction force F′

i.e., $$\vec{F}' = -\vec{F}$$

$$\therefore \quad t(\vec{F} - \vec{F}) = m_1\vec{V_1}' - m_1\vec{V_1} + m_2\vec{V_2}' - m_2\vec{V_2}$$

$$0 = m_1\vec{V_1}' - m_1\vec{V_1} + m_2\vec{V_2}' - m_2\vec{V_2}$$

$$m_1\vec{V_1} + m_2\vec{V_2} = m_1\vec{V_1}' + m_2\vec{V_2}'$$

which means that total initial momentum of the system before collision is equal to the final momentum of the system after collision Consequently the total change in momentum of the isolated two ball system is zero.

Point to Ponder

What is the effect on the speed of a fighter plane chasing another when it opens fire? What happens to the speed of pursued plane when it returns the fire?

Ans. **The speed of fighter plane chasing another will decrease due to law of conservation of momentum. While the speed of pursued plane will increase.**

Do You Wear Seat Belts?

When a moving car stops quickly, the passengers move forward towards the windshield. Seat belts change the forces of motion and prevent the passengers from moving. Thus the chance of injury is greatly reduced.

Do You Know?

A motorcycle's safety helmet is padded so as to extend the time of any collision to prevent serious injury.

Q.11 Define elastic and inelastic collision.

Ans. ELASTIC AND INELASTIC COLLISIONS

Collision

When two or more object come close enough so that there is some sort of interaction between them, with or without the presence of external force, we say a collision has been taken place between the objects.

There are two types of collision:

1. Head-on collision, such a collision in which after collision balls move in same direction as they move before collision.
2. Oblique collision (direction of balls changes after collision).

Elastic Collision

In the ideal case when no K.E is lost, the collision is said to be perfectly elastic.

For example, when a hard ball is dropped on to a marble floor, it rebounds to very nearly the initial height. It loses negligible amount of energy in the collision with the floor.

Inelastic Collision

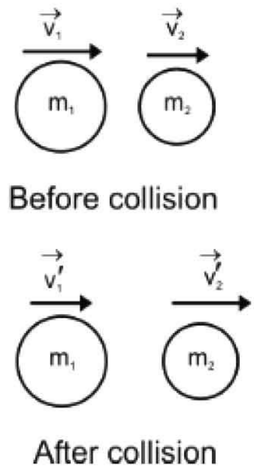
A collision in which the Kinetic Energy of the system is not conserved is called Inelastic Collision.

When two tennis balls collide then after collision, they will rebound with velocities less than the velocities before the impact. During this process, a portion of K.E. is lost, partly due to friction as the molecules in the ball move past one another when the balls distort and partly due to its change into heat and sound energies.

Note: Momentum and total energy are conserved in all types of collisions.

Q.12 Discuss elastic collision in one dimension and prove that speed of approach speed of released. (OR) Derive the formula of final velocities of two balls after an elastic collision in one dimensions.

Ans. ELASTIC COLLISION IN ONE DIMENSION



Consider two smooth, non–rotating balls of masses m_1 and m_2 moving initially with velocities $\vec{V}_1$ and $\vec{V}_2$ respectively, in the same direction. They collide and after collision, they move along the same straight line without rotation. Let their velocities after collision be $\vec{V}_1'$ and $\vec{V}_2'$ respectively, as shown in figure.

Consider direction of the velocity and momentum to the right.

Since the collision is elastic therefore both momentum and K.E. are conserved.

By Appling Law of conservation of momentum

$$m_1\vec{V}_1 + m_2\vec{V}_2 = m_1\vec{V}_1' + m_2\vec{V}_2' \qquad \text{......... (1)}$$

$$m_1\vec{V}_1 - m_1\vec{V}_1' = m_2\vec{V}_2' - m_2\vec{V}_2$$

$$m_1(\vec{V}_1 - \vec{V}_1') = m_2(\vec{V}_2' - \vec{V}_2) \qquad \text{......... (2)}$$

Using law of conservation of K.E.

$$\frac{1}{2} m_1 \vec{V}_1^2 + \frac{1}{2} m_2 \vec{V}_2^2 = \frac{1}{2} m_1 \vec{V}_1'^2 + \frac{1}{2} m_2 \vec{V}_2'^2$$

$$\frac{1}{2}\left(m_1 \vec{V}_1^2 + m_2 \vec{V}_2^2\right) = \frac{1}{2}\left(m_1 \vec{V}_1'^2 + m_2 \vec{V}_2'^2\right)$$

$$m_1 \vec{V}_1^2 + m_2 \vec{V}_2^2 = m_1 \vec{V}_1'^2 + m_2 \vec{V}_2'^2$$

$$m_1 \vec{V}_1^2 - m_1 \vec{V}_1'^2 = m_2 \vec{V}_2'^2 - m_2 \vec{V}_2^2$$

$$m_1\left(\vec{V}_1^2 - \vec{V}_1'^2\right) = m_2\left(\vec{V}_2'^2 - \vec{V}_2^2\right)$$

$$m_1(\vec{V}_1 - \vec{V}_1')(\vec{V}_1 + \vec{V}_2') = m_2(\vec{V}_2' - \vec{V}_2)(\vec{V}_2' + \vec{V}_2) \qquad \text{......... (3)}$$

Dividing equation (3) by equation (2)

$$\frac{m_1(\vec{V}_1 - \vec{V}_1')(\vec{V}_1 + \vec{V}_1')}{m_1(\vec{V}_1 - \vec{V}_1')} = \frac{m_2(\vec{V}_2' - \vec{V}_2)(\vec{V}_2' + \vec{V}_2)}{m_2(\vec{V}_2' - \vec{V}_2)}$$

$$\vec{V}_1 + \vec{V}_1' = \vec{V}_2' + \vec{V}_2 \qquad \ldots\ldots\ldots (4)$$

$$\vec{V}_1 - \vec{V}_2 = \vec{V}_2' - \vec{V}_1'$$

$$\vec{V}_1 - \vec{V}_2 = -(\vec{V}_1' - \vec{V}_2')$$

$$\vec{V}_{rel} = -\vec{V}'_{rel}$$

Before collision $(\vec{V}_1 - \vec{V}_2)$ is the velocity of first ball relative to second ball. Similarly $(\vec{V}_2' - \vec{V}_1')$ is the velocity of second ball relative to first ball after collision. It means that relative velocities before and after the collision has the same magnitude but are reversed after the collision. In other words, the magnitude of relative velocity of approach is equal to the magnitude of relative velocity of separation. i.e.,

$$\left\{\begin{matrix}\text{Magnitude of relative} \\ \text{velocity of approach}\end{matrix}\right\} = \left\{\begin{matrix}\text{Magnitude of relative} \\ \text{velocity of separation}\end{matrix}\right\}$$

Calculation of Velocity $\vec{V}_1'$ and $\vec{V}_2'$:

From equation (4)

$$\vec{V}_1 + \vec{V}_1' = \vec{V}_2' + \vec{V}_2$$

$$\vec{V}_2' = \vec{V}_1 + \vec{V}_1' - \vec{V}_2$$

Put this value in equation (1)

$$m_1\vec{V}_1 + m_2\vec{V}_2 = m_1\vec{V}_1' + m_2\left(\vec{V}_1 + \vec{V}_1' - \vec{V}_2\right)$$

$$m_1\vec{V}_1 + m_2\vec{V}_2 = m_1\vec{V}_1' + m_2\vec{V}_1 + m_2\vec{V}_1' - m_2\vec{V}_2$$

$$m_1\vec{V}_1 + m_2\vec{V}_2 + m_2\vec{V}_2 - m_2\vec{V}_1 = (m_1 + m_2)\,\vec{V}_1'$$

Dividing both sides by $(m_1 + m_2)$

$$\frac{(m_1 - m_2)\,\vec{V}_1 + 2m_2\vec{V}_2}{(m_1 + m_2)} = \frac{(m_1 + m_2)\,\vec{V}_1'}{(m_1 + m_2)}$$

$$\vec{V}_1' = \frac{(m_1 - m_2)\,\vec{V}_1}{(m_1 + m_2)} + \frac{2m_2\vec{V}_2}{(m_1 + m_2)} \qquad \ldots\ldots\ldots (5)$$

From equation (4)

$$\vec{V}_1' = \vec{V}_2' + \vec{V}_2 - \vec{V}_1$$

Do You Know?

If another car crashes into back of yours, the head-rest of the car seat can save you from serious neck injury. It helps to accelerate your head forward with the same rate as the rest of your body.

Put this value in equation (1)

$$\therefore \quad m_1\vec{V}_1 + m_2\vec{V}_2 = m_1\vec{V}_1' + m_2\vec{V}_2'$$

$$m_1\vec{V}_1 + m_2\vec{V}_2 = m_1\left(\vec{V}_2' + \vec{V}_2 - \vec{V}_1\right) + m_2\vec{V}_2'$$

$$m_1\vec{V}_1 + m_2\vec{V}_2 = m_1\vec{V}_2' + m_1\vec{V}_2 - m_1\vec{V}_2 + m_2\vec{V}_2'$$

$$m_1\vec{V}_1 + m_2\vec{V}_2 + m_1\vec{V}_1 - m_1\vec{V}_1 = (m_1 + m_2)\vec{V}_2'$$

$$2m_1\vec{V}_1 + (m_2 - m_1)\vec{V}_2 = (m_1 + m_2)\ \vec{V}_2'$$

Dividing both sides by $(m_1 + m_2)$

$$\therefore \quad \vec{V}_2' = \frac{2m_1\vec{V}_1}{m_1 + m_2} + \frac{(m_2 - m_1)\ \vec{V}_2}{m_1 + m_2} \quad \text{......... (6)}$$

Point to Ponder

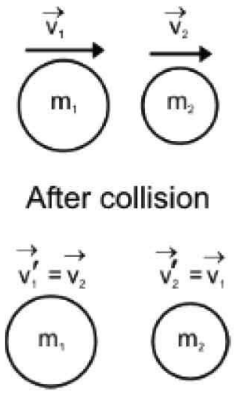
In thrill machine rides at amusement parks, there can be an acceleration of 3g or more. But without head rests, acceleration like this would not be safe. Think why not?

Q.13 Discuss the various cases of elastic collision in dimensions.

Ans. **SPECIAL CASES:**

Case-I: When $m_1 = m_2 = m$

Putting this in equation (5) and equation (6)



$$\therefore \quad \vec{V}_1' = \frac{(m - m)\vec{V}_1}{m + m} + \frac{2m_2\vec{V}_2}{m + m}$$

$$= 0 + \frac{2m\vec{V}_2}{2m} = \frac{2m\vec{V}_2}{2m}$$

$$\vec{V}_1' = \vec{V}_2$$

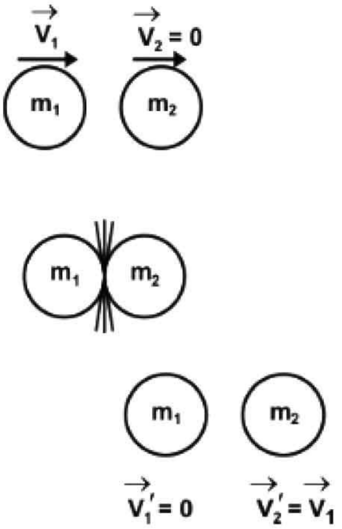
Now $$\vec{V}_2' = \frac{2m\vec{V}_1}{m + m} + \frac{(m - m)\vec{V}_2}{m + m}$$

$$= \frac{2m\vec{V}_1}{2m} + 0$$

$$\vec{V}_2' = \vec{V}_1$$

It means that when two balls of equal mass collide elastically, they simply exchange their velocities.

Case-II: When $m_1 = m_2 = m$ and $\vec{V}_2 = 0$ i.e., target ball at rest



Put this value in eq. (5) and (6)

$$\therefore \quad \vec{V}_1' = \frac{(m-m)\vec{V}_1}{m+m} + \frac{2m\,(0)}{m+m}$$

$$= 0 + 0$$

$$\vec{V}_1' = 0$$

$$\text{Now,} \quad \vec{V}_2' = \frac{2m\vec{V}_1}{m+m} + \frac{(m-m)\,(0)}{m+m}$$

$$= \frac{2m\vec{V}_1}{2m} + 0$$

$$\vec{V}_2' = \vec{V}$$

In this case the ball m_1 comes to rest after collision while ball m_2 that was at rest began to move with $\vec{V}_1$.

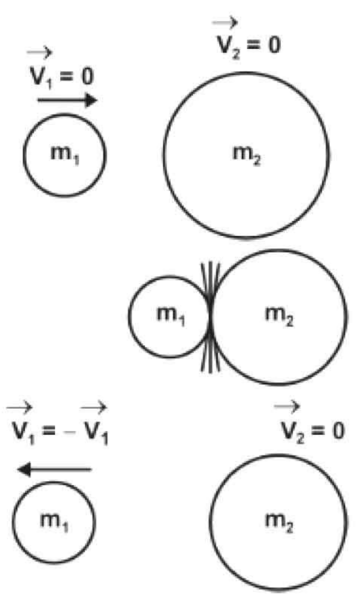
Case-III: When a light body collides with a massive body which is at rest.

i.e., $\vec{V}_2 = 0$

also $m_2 >> m_1$

i.e., $m_1 \simeq 0$

Putting this value in equation (5) and equation (6).



$$\therefore \quad \vec{V}_1' = \frac{0 - m_2}{0 + m_2}\vec{V}_1 + \frac{2m_2}{0 + m_2}(0)$$

$$= -\frac{m_2}{m_2}\vec{V}_1$$

$$\vec{V}_1' = -\vec{V}_1$$

$$\text{Also} \quad \vec{V}_2' = \frac{m_2 - m_1}{m_1 + m_2}\vec{V}_2 + \frac{2m_1\vec{V}_1}{m_1 + m_2}$$

$$= 0 + \frac{2\,(0)\,\vec{V}_1}{0 + m_2}$$

$$= 0 + 0$$

$$\vec{V}_2' = 0$$

This means that m_1 will bounce back with same velocity while m_2 remains stationary.

Case-IV: When a massive body collides with a lighter body at rest.