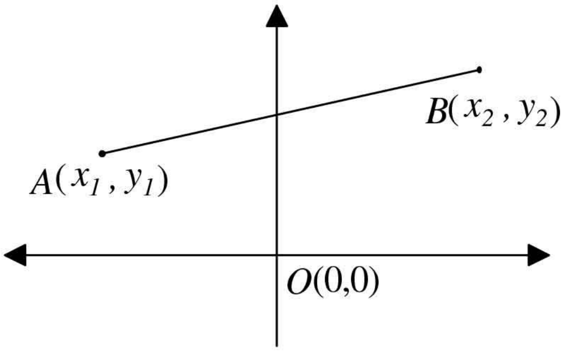
Distance Formula

Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be two points in a plane and d be a distance between A and B then

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

or $$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

See proof on book at page181

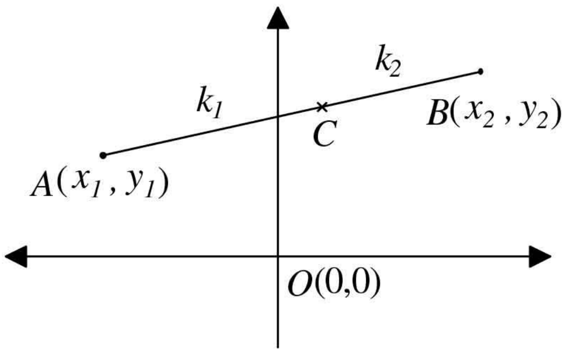


Ratio Formula

Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be two points in a plane. The coordinates of the point C dividing the line segment AB in the ratio $k_1 : k_2$ are

$$\left(\frac{k_1x_2 + k_2x_1}{k_1 + k_2}, \frac{k_1y_2 + k_2y_1}{k_1 + k_2}\right)$$

See proof on book at page 182



If C be the midpoint of AB i.e. $k_1 : k_2 = 1:1$ then coordinate of C becomes

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Question # 1

Describe the location in the plane of the point $P(x, y)$ for which

(i) $x > 0$ (ii) $x > 0$ and $y > 0$ (iii) $x = 0$
(iv) $y = 0$ (v) $x < 0$ and $y \geq 0$ (vi) $x = y$
(vii) $|x| = -|y|$ (viii) $|x| \geq 3$ (ix) $x > 2$ and $y = 2$
(x) x and y have opposite signs.

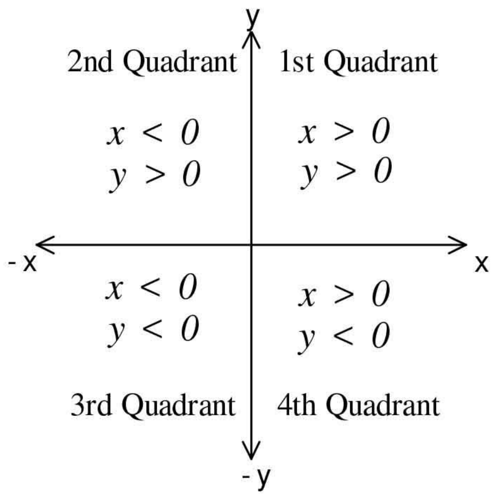
Solution

(i) $x > 0$
Right half plane

(ii) $x > 0$ and $y > 0$
The 1st quadrant.

(iii) $x = 0$
y-axis

(iv) $y = 0$
x-axis



(v) $x<0$ and $y\geq 0$

2^{nd} quadrant & negative x-axis

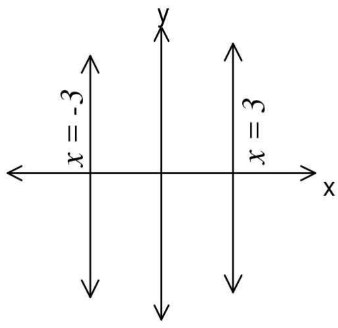
(vi) $x=y$

It is a line bisecting 1^{st} and 3^{rd} quadrant.

(vii) $|x|=-|y|$

A positive value can't equal to a negative value, except number zero, so origin,

$(0,0)$, is the only point which satisfies $|x|=-|y|$



(viii) $|x|\geq 3$

$\Rightarrow\ \pm x\geq 3 \quad \Rightarrow\ x\geq 3$ or $-x\geq 3$

$\Rightarrow\ x\geq 3$ or $x\leq -3$

which is the set of points lying on right side of the line $x=3$ and the points lying on left side of the line $x=-3$.

(ix) $x>2$ and $y=2$

The set of all points on the line $y=2$ for which $x>2$.

(x) x and y have opposite signs.

It is the set of points lying in 2^{nd} and 4^{th} quadrant.

Question # 2

Find each of the following

(i)the distance between the two given points

(ii)Midpoint of the line segment joining the two points

(a) $A(3,1):B(-2,-4)$ (b) $A(-8,3);B(2,-1)$ (c) $A\left(-\sqrt{5},-\frac{1}{3}\right);B\left(-3\sqrt{5},5\right)$

Solution

(a) $A(3,1)$; $B(-2,-4)$

(i) $|AB|=\sqrt{(-2-3)^2+(-4-1)^2}=\sqrt{(-5)^2+(-5)^2}$

$=\sqrt{25+25}=\sqrt{50}=\sqrt{25\times 2}=5\sqrt{2}$

(ii) Midpoint of AB = $\left(\frac{3-2}{2},\frac{1-4}{2}\right)=\left(\frac{1}{2},\frac{-3}{2}\right)$

> Review:
> The midpoint of $A(x_1,y_1)$ and $B(x_2,y_2)$ is $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$.

(b) $A(-8,3)$; $B(2,-1)$

Do yourself as above.

(c) $A\left(-\sqrt{5}, -\frac{1}{3}\right)$; $B\left(-3\sqrt{5}, 5\right)$

(i) $$|AB| = \sqrt{\left(-3\sqrt{5}+\sqrt{5}\right)^2 + \left(5+\frac{1}{3}\right)^2} = \sqrt{\left(2\sqrt{5}\right)^2 + \left(\frac{16}{3}\right)^2}$$
$$= \sqrt{20+\frac{256}{9}} = \sqrt{\frac{436}{9}} = \sqrt{\frac{4\times 109}{9}} = \frac{2\sqrt{109}}{3}$$

(ii) Midpoint of $AB = \left(\frac{-\sqrt{5}-3\sqrt{5}}{2}, \frac{-\frac{1}{3}+5}{2}\right) = \left(\frac{-4\sqrt{5}}{2}, \frac{\frac{14}{3}}{2}\right) = \left(-2\sqrt{5}, \frac{7}{3}\right)$

Question # 3

Which of the following points are at a distance of 15 units from the origin?

(a) $\left(\sqrt{176}, 7\right)$ (b) $(10, -10)$ (c) $(1, 15)$ (d) $\left(\frac{15}{2}, \frac{15}{2}\right)$

Solution

(a) Distance of $\left(\sqrt{176}, 7\right)$ from origin $= \sqrt{\left(\sqrt{176}-0\right)^2 + (7-0)^2}$
$$= \sqrt{(176)+(49)}$$
$$= \sqrt{(176)+(49)} = \sqrt{225} = 15$$

$\Rightarrow$ the point $\left(\sqrt{176}, 7\right)$ is at 15 unit away from origin.

(b) Distance of $(10, -10)$ from origin $= \sqrt{(10-0)^2 + (-10-0)^2}$
$$= \sqrt{100+100} = \sqrt{200}$$
$$= \sqrt{100\times 2} = 10\sqrt{2} \neq 15$$

$\Rightarrow$ the point $(10, -10)$ is not at distance of 15 unit from origin.

(c) *Do yourself as above*

(d) Distance of $\left(\frac{15}{2}, \frac{15}{2}\right)$ from origin $= \sqrt{\left(\frac{15}{2}-0\right)^2 + \left(\frac{15}{2}-0\right)^2}$
$$= \sqrt{\frac{225}{4}+\frac{225}{4}} = \sqrt{\frac{225}{2}} = \frac{15}{\sqrt{2}} \neq 15$$

Hence the point $\left(\frac{15}{2}, \frac{15}{2}\right)$ is not at distance of 15 unit from origin.

Question # 4

Show that

(i) the point $A(0,2)$, $B(\sqrt{3}, -1)$ and $C(0, -2)$ are vertices of a right triangle.

(ii) the point $A(3,1)$, $B(-2,-3)$ and $C(2,2)$ are vertices of an isosceles triangle.

(iii) the point $A(3,1)$, $B(-2,-3)$ and $C(2,2)$ and $\mathrm{D}(4,-5)$ are vertices of a parallelogram. Is the parallelogram a square?

Solution

(i) Given: $A(0,2)$, $B(\sqrt{3},-1)$ and $C(0,-2)$

$$|AB|=\sqrt{\left(\sqrt{3}-0\right)^2+(-1-2)^2}=\sqrt{\left(\sqrt{3}\right)^2+(-3)^2}$$

$$=\sqrt{3+9}=\sqrt{12} \qquad \Rightarrow |AB|^2=12$$

$$|BC|=\sqrt{\left(0-\sqrt{3}\right)^2+(-2+1)^2}=\sqrt{\left(-\sqrt{3}\right)^2+(-1)^2}$$

$$=\sqrt{3+1}=\sqrt{4}=2 \qquad \Rightarrow |BC|^2=4$$

$$|CA|=\sqrt{(0-0)^2+(2+2)^2}=\sqrt{0+(4)^2}$$

$$=\sqrt{16}=4 \qquad \Rightarrow |CA|^2=16$$

$\because\ |AB|^2+|BC|^2=12+4=16=|CA|^2$

$\therefore$ by Pythagoras theorem A, B & C are vertices of a right triangle.

(ii) Given: $A(3,1)$, $B(-2,-3)$ and $C(2,2)$

$$|AB|=\sqrt{(-2-3))^2+(-3-1)^2}=\sqrt{(-5)^2+(-4)^2}=\sqrt{25+16}=\sqrt{41}$$

$$|BC|=\sqrt{(2-(-2))^2+(2-(-3))^2}=\sqrt{(4)^2+(5)^2}=\sqrt{16+25}=\sqrt{41}$$

$$|CA|=\sqrt{(3-2)^2+(1-2)^2}=\sqrt{(1)^2+(-1)^2}$$

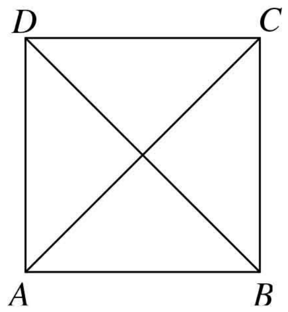
$$=\sqrt{1+1}=\sqrt{2}$$

$\because\ |AB|=|BC| \quad \Rightarrow\ A, B$ & C are vertices of an isosceles triangle.

(iii) Given: $A(5,2)$, $B(-2,3)$ & $C(-3,-4)$ and $D(4,-5)$

$$|AB|=\sqrt{(-2-5)^2+(3-2)^2}=\sqrt{(-7)^2+(1)^2}$$

$$=\sqrt{49+1}=\sqrt{50}=5\sqrt{2}$$



$$|BC|=\sqrt{(-3+2)^2+(-4-3)^2}=\sqrt{(-1)^2+(-7)^2}$$

$$=\sqrt{1+49}=\sqrt{50}=5\sqrt{2}$$

$$|CD|=\sqrt{(4+3)^2+(-5+4)^2}=\sqrt{(7)^2+(-1)^2}$$

$$=\sqrt{49+1}=\sqrt{50}=5\sqrt{2}$$

$$|DA|=\sqrt{(5-4)^2+(2+5)^2}=\sqrt{(1)^2+(7)^2}$$

$$=\sqrt{1+49}=\sqrt{50}=5\sqrt{2}$$

$\because\ |AB|=|CD|$ and $|BC|=|DA| \quad \Rightarrow\ A, B, C$ and D are vertices of parallelogram.

Now $|AC|=\sqrt{(-3-5)^2+(-4-2)^2}=\sqrt{(-8)^2+(-6)^2}$
$=\sqrt{64+36}=\sqrt{100}=10$
$|BD|=\sqrt{(4+2)^2+(-5-3)^2}=\sqrt{(6)^2+(-8)^2}$
$=\sqrt{36+64}=\sqrt{100}=10$

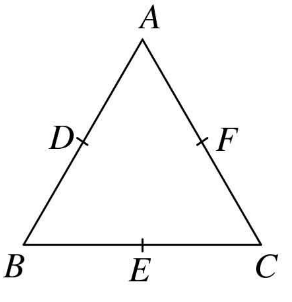
Since all sides are equals and also both diagonals are equal therefore A,B,C,D are vertices of a square.

Question # 5

The midpoints of the sides of a triangle are $(1,-1)$,$(-4,-3)$ and $(-1,1)$. Find coordinates of the vertices of the triangle.

Solution

Let $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$ are vertices of triangle ABC, and let $D(1,-1)$, $E(-4,-3)$ and $F(-1,1)$ are midpoints of sides AB, BC and CA respectively. Then



$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right) = (1,-1)$$

$\Rightarrow\ x_1+x_2=2\ \ldots$ (i) and $y_1+y_2=-2\ \ldots$ (ii)

$$\left(\frac{x_2+x_3}{2},\frac{y_2+y_3}{2}\right) = (-4,-3)$$

$\Rightarrow\ x_2+x_3=-8\ldots$ (iii) and $y_2+y_3=-6\ \ldots$ (iv)

$$\left(\frac{x_3+x_1}{2},\frac{y_3+y_1}{2}\right) = (-1,1)$$

$\Rightarrow\ x_1+x_3=-2\ \ldots$ (v), and $y_1+y_3=2\ \ldots$ (vi)

Subtracting (i) and (iii)

$x_1+x_2 = 2$
$\underline{-\ x_2 \pm x_3 = -8}\ \ldots$ (vii)
$x_1 - x_3 = 10$

Adding (v) and (vii)

$x_1+x_3=-2$
$\underline{x_1-x_3=10}$
$2x_1 = 8 \Rightarrow \boxed{x_1=4}$

Putting value of x_1 in (i)

$4+x_2=2$
$\Rightarrow\ x_2=2-4 \Rightarrow \boxed{x_2=-2}$

Putting value of x_1 in (v)

$4+x_3=-2$
$\Rightarrow\ x_3=-2-4 \Rightarrow \boxed{x_3=-6}$

Subtracting (ii) and (iv)

$y_1+y_2 = -2$
$\underline{-\ y_2 \pm y_3 = \mp 6}\ \ldots$ (viii)
$y_1 - y_3 = 4$

Adding (vi) and (viii)

$y_1+y_3 = 2$
$\underline{y_1-y_3 = 4}$
$2y_1 = 6 \Rightarrow \boxed{y_1=3}$

Putting value of y_1 in (ii)

$3+y_2=-2$
$\Rightarrow\ y_2=-2-3 \Rightarrow \boxed{y_2=-5}$

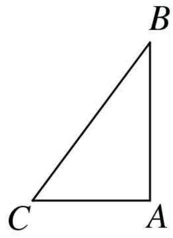
Putting value of y_1 in (v)

$3+y_3=2$
$\Rightarrow\ y_3=2-3 \Rightarrow \boxed{y_3=-1}$

Hence vertices of triangle are $(4,3),(-2,-5)$ & $(-6,-1)$.

Question # 6

Find h such that the point $A\left(\sqrt{3},-1\right)$, $B(0,2)$ and $C(h,-2)$ are vertices of a right angle with right angle at the vertex A.



Solution

Since ABC is a right triangle therefore by Pythagoras theorem

$$|AB|^2+|CA|^2=|BC|^2$$

$$\Rightarrow \left[\left(0-\sqrt{3}\right)^2+(2+1)^2\right]+\left[\left(\sqrt{3}-h\right)^2+(-1+2)^2\right]=(h-0)^2+(-2-2)^2$$

$$\Rightarrow [3+9]+\left[3-2\sqrt{3}h+h^2+1\right]=h^2+16$$

$$\Rightarrow 12+4-2\sqrt{3}\,h+h^2=h^2+16$$

$$\Rightarrow -2\sqrt{3}\,h=h^2+16-12-4-h^2 \quad \Rightarrow -2\sqrt{3}\,h=0 \quad \Rightarrow \boxed{h=0}.$$

Question # 7

Find h such that $A(-1,h),B(3,2)$ and $C(7,3)$ are collinear.

Solution

Points (x_1,y_1), (x_2,y_2) and (x_3,y_3) are collinear if

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}=0$$

Since given points are collinear therefore

$$\begin{vmatrix} -1 & h & 1 \\ 3 & 2 & 1 \\ 7 & 3 & 1 \end{vmatrix}=0$$

$$\Rightarrow -1(2-3)-h(3-7)+1(9-14)=0 \quad \Rightarrow -1(-1)-h(-4)+1(-5)=0$$

$$\Rightarrow 1+4h-5=0 \quad \Rightarrow 4h-4=0 \quad \Rightarrow 4h=4 \quad \Rightarrow \boxed{h=1}$$

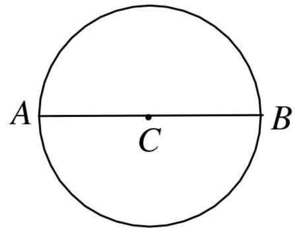
Question # 8

The points $A(-5,-2)$ and $B(5,-4)$ are end of a diameter of a circle. Find the centre and radius of the circle.

Solution

The centre of the circle is mid point of AB

i.e. centre 'C' $=\left(\dfrac{-5+5}{2},\dfrac{-2-4}{2}\right)=\left(\dfrac{0}{2},\dfrac{-6}{2}\right)=(0,-3)$



Now radius $=|AC|$

$$=\sqrt{(0+5)^2+(-3+2)^2}$$

$$=\sqrt{25+1} \quad =\sqrt{26}$$

Question # 9

Find h such that the points $A(h,1)$, $B(2,7)$ and $C(-6,-7)$ are vertices of a right triangle with right angle at the vertex A

Solution

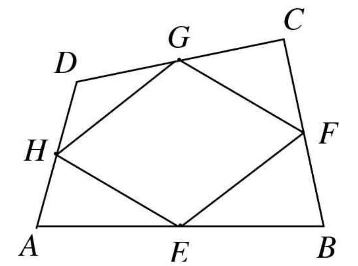
Do yourself as Question # 6

Hint: you will get a equation $h^2+4h-60=0$

Solve this quadratic equation to get two values of h.

Question # 10

A quadrilateral has the points $A(9,3), B(-7,7), C(-3,-7)$ and $D(-5,5)$ as its vertices. Find the midpoints of its sides. Show that the figure formed by joining the midpoints consecutively is a parallelogram.



Solution

Given: $A(9,3)$, $B(-7,7)$, $C(-3,-7)$ and $D(5,-5)$

Let E, F, G and H be the mid-points of sides of quadrilateral

Coordinate of E $= \left(\frac{9-7}{2},\frac{3+7}{2}\right) = \left(\frac{2}{2},\frac{10}{2}\right) = (1,5)$

Coordinate of F $= \left(\frac{-7-3}{2},\frac{7-7}{2}\right) = \left(\frac{-10}{2},\frac{0}{2}\right) = (-5,0)$

Coordinate of G $= \left(\frac{-3+5}{2},\frac{-7-5}{2}\right) = \left(\frac{2}{2},\frac{-12}{2}\right) = (1,-6)$

Coordinate of H $= \left(\frac{9+5}{2},\frac{3-5}{2}\right) = \left(\frac{14}{2},\frac{-2}{2}\right) = (7,-1)$

Now $|EF| = \sqrt{(-5-1)^2+(0-5)^2} = \sqrt{36+25} = \sqrt{61}$

$|FG| = \sqrt{(1+5)^2+(-6-0)^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$

$|GH| = \sqrt{(7-1)^2+(-1+6)^2} = \sqrt{36+25} = \sqrt{61}$

$|HE| = \sqrt{(1-7)^2+(5+1)^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$

Since $|EF| = |GH|$ and $|FG| = |HE|$

Therefore $EFGH$ is a parallelogram.

Question # 11

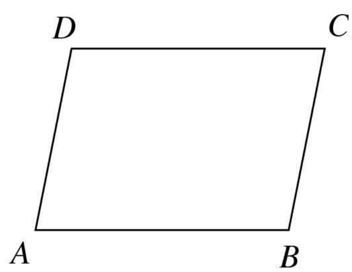
Find h such that the quadrilateral with vertices $A(-3,0), B(1,-2,)C(5,0)$ and $D(1,h)$ is parallelogram. Is it a square?

Solution

Given: $A(-3,0)$, $B(1,-2)$, $C(5,0)$, $D(1,h)$

Quadrilateral $ABCD$ is a parallelogram if

$|AB| = |CD|$ & $|BC| = |AD|$



when $|AB| = |CD|$

$\Rightarrow \sqrt{(1+3)^2+(-2-0)^2} = \sqrt{(1-5)^2+(h-0)^2}$

$\Rightarrow \sqrt{16+4} = \sqrt{16+h^2} \quad \Rightarrow \sqrt{20} = \sqrt{16+h^2}$

On squaring

$$20 = 16 + h^2 \quad \Rightarrow h^2 = 20 - 16 \quad \Rightarrow h^2 = 4 \quad \Rightarrow h = \pm 2$$

When $h = 2$, then $D(1,h) = D(1,2)$

Then $|AB| = \sqrt{(1+3)^2 + (-2-0)^2} = \sqrt{16+4} = \sqrt{20}$

$|BC| = \sqrt{(5-1)^2 + (0+2)^2} = \sqrt{16+4} = \sqrt{20}$

$|CA| = \sqrt{(1-5)^2 + (2-0)^2} = \sqrt{16+4} = \sqrt{20}$

$|DA| = \sqrt{(-3-1)^2 + (-0-2)^2} = \sqrt{16+4} = \sqrt{20}$

Now for diagonals

$|AC| = \sqrt{(5+3)^2 + (0-0)^2} = \sqrt{64+0} = 8$

$|BD| = \sqrt{(1-1)^2 + (2+2)^2} = \sqrt{0+16} = 4$

Since all sides are equal but diagonals $|AC| \neq |BD|$

Therefore $ABCD$ is not a square.

Now when $h = -2$, then $D(1,h) = D(1,-2)$ but we also have $B(1,-2)$

i.e. B and D represents the same point, which can not happened in quadrilateral so we can not take $h = -2$.

Question # 12

If two vertices of an equilateral triangle are $A(-3,0)$ and $B(3,0)$, find the third vertex. How many of these triangles are possible?

Solution

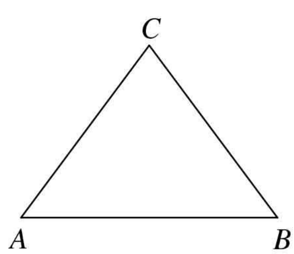
Given: $A(-3,0)$, $B(3,0)$

Let $C(x,y)$ be a third vertex of an equilateral triangle ABC.

Then $|AB| = |BC| = |CA|$

$\Rightarrow \sqrt{(3+3)^2 + (0-0)^2} = \sqrt{(x-3)^2 + (y-0)^2} = \sqrt{(x+3)^2 + (y-0)^2}$

$\Rightarrow \sqrt{36+0} = \sqrt{x^2 - 6x + 9 + y^2} = \sqrt{x^2 + 6x + 9 + y^2}$



On squaring

$$36 = x^2 + y^2 - 6x + 9 = x^2 + y^2 + 6x + 9 \quad \text{............}(i)$$

From equation (i)

$$x^2 + y^2 - 6x + 9 = x^2 + y^2 + 6x + 9$$

$\Rightarrow x^2 + y^2 - 6x + 9 - x^2 - y^2 - 6x - 9 = 0$

$\Rightarrow -12x = 0 \quad \Rightarrow x = 0$

Again from equation (i)

$$36 = x^2 + y^2 - 6x + 9$$

$\Rightarrow 36 = (0)^2 + y^2 - 6(0) + 9 \qquad \because x = 0$

$\Rightarrow 36 = y^2 + 9 \quad \Rightarrow y^2 = 36 - 9 = 27 \quad \Rightarrow y = \pm 3\sqrt{3}$

so coordinate of C is $\left(0, 3\sqrt{3}\right)$ or $\left(0, -3\sqrt{3}\right)$.

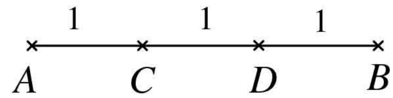
And hence two triangle can be formed with vertices $A(-3,0), B(3,0), C\left(0,3\sqrt{3}\right)$ and $A(-3,0), B(3,0), C\left(0,-3\sqrt{3}\right)$.

Question # 13

Find the points trisecting the join of $A(-1,4)$ and $B(6,2)$.

Solution

Given: $A(-1,4),\ B(6,2)$



Let C and D be points trisecting A and B

Then $AC:CB = 1:2$

So coordinate of $C = \left(\dfrac{1(6)+2(-1)}{1+2}, \dfrac{1(2)+2(4)}{1+2}\right)$

$= \left(\dfrac{6-2}{3}, \dfrac{2+8}{3}\right) = \left(\dfrac{4}{3}, \dfrac{10}{3}\right)$

Also $AD:DB = 2:1$

So coordinate of $D = \left(\dfrac{2(6)+1(-1)}{2+1}, \dfrac{2(2)+1(4)}{2+1}\right)$

$= \left(\dfrac{12-1}{3}, \dfrac{4+4}{3}\right) = \left(\dfrac{11}{3}, \dfrac{8}{3}\right)$

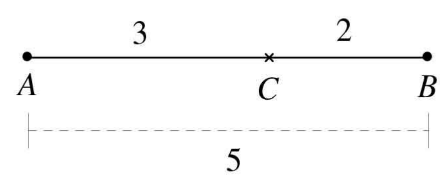
Hence $\left(\dfrac{4}{3}, \dfrac{10}{3}\right)$ and $\left(\dfrac{11}{3}, \dfrac{8}{3}\right)$ are points trisecting A and B.

Question # 14

Find the point three-fifth of the way along the line segment from $A(-5,8)$ to $B(5,3)$.

Solution

Given: $A(-5,8),\ B(5,3)$



Let $C(x, y)$ be a required point

$\because\ AC:CB = 3:2$

$\therefore$ Co-ordinate of $C = \left(\dfrac{3(5)+2(-5)}{3+2}, \dfrac{3(3)+2(8)}{3+2}\right)$

$= \left(\dfrac{15-10}{5}, \dfrac{9+16}{5}\right) = \left(\dfrac{5}{5}, \dfrac{25}{5}\right) = (1, 5)$

Question # 15

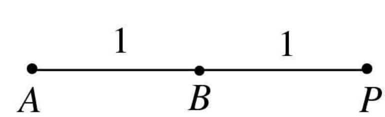
Find the point P on the joint of $A(1,4)$ and $B(5,6)$ that is twice as far from A as B is from A and lies

(i) on the same side of A as B does. (ii) on the opposite side of A as B does.

Solution

Given: $A(1,4)$, $B(5,6)$

(i) Let $P(x, y)$ be required point, then



$$AB:AP = 1:2$$

$\Rightarrow AB:BP = 1:1$ i.e. B is midpoint of AP

Then $B(5,6) = \left(\dfrac{1+x}{2},\dfrac{4+y}{2}\right)$

$$\Rightarrow 5 = \frac{1+x}{2} \quad \text{and} \quad 6 = \frac{4+y}{2}$$

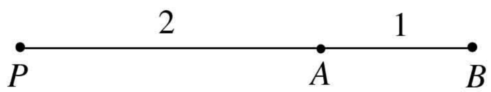
$$\Rightarrow 10 = 1+x \quad \text{and} \quad 12 = 4+y$$

$$\Rightarrow x = 10-1 \quad , \quad y = 12-4$$

$$=9 \quad , \quad =8$$

Hence $P(9,8)$ is required point.

(ii) Since $PA:AB = 2:1$



$$\Rightarrow A(1,4) = \left(\frac{2(5)+1(x)}{2+1},\frac{2(6)+1(y)}{2+1}\right)$$

$$= \left(\frac{10+x}{3},\frac{12+y}{3}\right)$$

$$\Rightarrow 1 = \frac{10+x}{3} \quad \text{and} \quad 4 = \frac{12+y}{3}$$

$$\Rightarrow 3 = 10+x \quad \text{and} \quad 12 = 12+y$$

$$\Rightarrow x = 3-10 \quad \text{and} \quad y = 12-12$$

$$= -7 \quad , \quad = 0$$

Hence $P(-7,0)$ is required point.

Question # 16

Find the point which is equidistant from the points $A(5,3), B(2,-2)$ and $C(4,2)$. What is the radius of the circumcircle of the ΔABC ?

Solution

Given: $A(5,3)$, $B(-2,2)$ and $C(4,2)$

Let $D(x,y)$ be a point equidistance from A , B and C then

$$\left|\overline{DA}\right| = \left|\overline{DB}\right| = \left|\overline{DC}\right|$$

$$\Rightarrow \left|\overline{DA}\right|^2 = \left|\overline{DB}\right|^2 = \left|\overline{DC}\right|^2$$

$$\Rightarrow (x-5)^2+(y-3)^2 = (x+2)^2+(y-2)^2 = (x-4)^2+(y-2)^2 \quad \ldots\ldots\ldots \text{(i)}$$

From eq. (i)

$$(x-5)^2+(y-3)^2 = (x+2)^2+(y-2)^2$$

$$\Rightarrow x^2-10x+25+y^2-6y+9 = x^2+4x+4+y^2-4y+4$$

$$\Rightarrow x^2-10x+25+y^2-6y+9-x^2-4x-4-y^2+4y-4 = 0$$

$$\Rightarrow -14x-2y+26 = 0 \quad \Rightarrow 7x+y-13 = 0 \quad \ldots\ldots\ldots \text{(ii)}$$

Again from equation (i)

$$(x+2)^2+(y-2)^2 = (x-4)^2+(y-2)^2$$

$\Rightarrow\ x^2+4x+4+y^2-4y+4 = x^2-8x+16+y^2-4y+4$

$\Rightarrow\ 12x-12 = 0 \quad \Rightarrow\ 12x = 12 \quad \Rightarrow\ x = 1$

Put $x=1$ in eq. (ii)

$7(1)+y-13 = 0 \quad \Rightarrow\ y-6 = 0 \quad \Rightarrow\ y = 6$

Hence $(1,6)$ is required point.

Now radius of circumcircle $= \left|\overline{DA}\right|$

$= \sqrt{(5-1)^2+(3-6)^2} = \sqrt{16+9} = \sqrt{25} = 5$ units

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Intersection of Median

Let $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$ are vertices of triangle.

Intersection of median is called centroid of triangle and can be determined as

$$\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$$

See proof at page 184

Centre of In-Circle (In-Centre)

Let $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$ are vertices of triangle.

And $|AB| = c$, $|BC| = a$, $|CA| = b$

Then in-centre of triangle $= \left(\dfrac{ax_1+bx_2+cx_3}{a+b+c},\dfrac{ay_1+by_2+cy_3}{a+b+c}\right)$

See proof at page 184

Question # 17

The points $(4,-2),(-2,4)$ and $(5,5)$ are the vertices of a triangle. Find in-centre of the triangle.

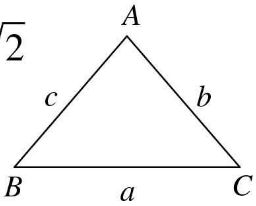
Solution

Let $A(4,-2)$, $B(-2,4)$, $C(5,5)$ are vertices of triangle then

$a = |BC| = \sqrt{(5+2)^2+(5-4)^2} = \sqrt{49+1} = \sqrt{50} = 5\sqrt{2}$

$b = |CA| = \sqrt{(4-5)^2+(-2-5)^2} = \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$

$c = |AB| = \sqrt{(-2-4)^2+(4+2)^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$



Now

In-centre $= \left(\dfrac{ax_1+bx_2+cx_3}{a+b+c},\dfrac{ay_1+by_2+cy_3}{a+b+c}\right)$

$= \left(\dfrac{5\sqrt{2}(4)+5\sqrt{2}(-2)+6\sqrt{2}(5)}{5\sqrt{2}+5\sqrt{2}+6\sqrt{2}},\dfrac{5\sqrt{2}(-2)+5\sqrt{2}(4)+6\sqrt{2}(5)}{5\sqrt{2}+5\sqrt{2}+6\sqrt{2}}\right)$

$= \left(\dfrac{20\sqrt{2}-10\sqrt{2}+30\sqrt{2}}{16\sqrt{2}},\dfrac{-10\sqrt{2}+20\sqrt{2}+30\sqrt{2}}{16\sqrt{2}}\right)$

$= \left(\dfrac{40\sqrt{2}}{16\sqrt{2}},\dfrac{40\sqrt{2}}{16\sqrt{2}}\right) = \left(\dfrac{5}{2},\dfrac{5}{2}\right)$

Question # 18

Find the points that divide the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ into four equal parts.

Solution

Given: $A(x_1, y_1)$, $B(x_2, y_2)$

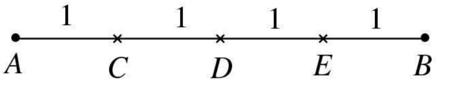
Let C, D and E are points dividing AB into four equal parts.

$\because\ AC:CB = 1:3$

$$\Rightarrow \text{Co-ordinates of } C = \left(\frac{1(x_2)+3(x_1)}{1+3}, \frac{1(y_2)+3(y_1)}{1+3}\right) = \left(\frac{3x_1+x_2}{4}, \frac{3y_1+y_2}{4}\right)$$

Now $AD:DB = 2:2$

$= 1:1$ i.e. D is midpoint of AB.



$$\Rightarrow \text{Co-ordinates of } D = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

Now $AE:EB = 3:1$

$$\Rightarrow \text{Co-ordinates of } E = \left(\frac{3(x_2)+1(x_1)}{3+1}, \frac{3(y_2)+1(y_1)}{3+1}\right) = \left(\frac{x_1+3x_2}{4}, \frac{y_1+3y_2}{4}\right)$$

Hence $\left(\frac{3x_1+x_2}{4}, \frac{3y_1+y_2}{4}\right)$, $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ and $\left(\frac{x_1+3x_2}{4}, \frac{y_1+3y_2}{4}\right)$ are the points dividing AB into four equal parts.