

Question # 1

The two points P and O' are given in xy -coordinates system. Find the XY -coordinates of P referred to the translated axes $O'X$ and $O'Y$

- (i) $P(3, 2); O'(1, 3)$
- (ii) $P(-2, 6); O'(-3, 2)$
- (iii) $P(-6, -8); O'(-4, -6)$
- (iv) $P\left(\frac{3}{2}, \frac{5}{2}\right); O'\left(-\frac{1}{2}, \frac{7}{2}\right)$

Solution

(i) Since $P(x, y) = P(3, 2)$

i.e. $x = 3$ and $y = 2$

$$O'(h, k) = O'(1, 3)$$

i.e. $h = 1$ and $k = 3$

$$\begin{aligned} \therefore X &= x - h \\ &= 3 - 1 = 2 \end{aligned}$$

Also $Y = y - k$

$$= 2 - 3 = -1$$

Hence $(2, -1)$ is point P in XY -coordinates.

(ii) *Do yourself*

(iii) *Do yourself*

(iv) Since $P(x, y) = P\left(\frac{3}{2}, \frac{5}{2}\right)$

i.e. $x = \frac{3}{2}$ and $y = \frac{5}{2}$

$$O'(h, k) = O'\left(-\frac{1}{2}, \frac{7}{2}\right)$$

i.e. $h = -\frac{1}{2}$ and $k = \frac{7}{2}$

$$\therefore X = x - h$$

$$= \frac{3}{2} - \left(-\frac{1}{2}\right) = 2$$

And $Y = y - k$

$$= \frac{5}{2} - \frac{7}{2} = -1$$

Hence $(2, -1)$ are coordinates of P in XY -axes.

Question # 2

The xy -coordinates axes are translated through the point O' whose coordinates are given in xy -coordinates. The coordinates of P are given in the

XY -coordinates system. Find the coordinates of P in xy -coordinates system.

- (i) $P(8, 10); O'(3, 4)$
- (ii) $P(-5, -3); O'(-2, -6)$
- (iii) $P\left(-\frac{3}{4}, -\frac{7}{6}\right); O'\left(\frac{1}{4}, -\frac{1}{6}\right)$
- (iv) $P(4, -3); O'(-2, 3)$

Solution

(i) $\because P(X, Y) = P(8, 10)$
 $\Rightarrow X = 8$ and $Y = 10$

$$O'(h, k) = O'(3, 4)$$

$$\Rightarrow h = 3 \text{ and } k = 4$$

$$\therefore X = x - h$$

$$\Rightarrow 8 = x - 3$$

$$\Rightarrow x = 8 + 3 \Rightarrow x = 11$$

Also $Y = y - k$

$$\Rightarrow 10 = y - 4$$

$$\Rightarrow y = 10 + 4 \Rightarrow y = 14$$

Hence $(11, 14)$ are coordinates of P in xy -axes.

(ii) *Do yourself*

(iii) $\because P(X, Y) = P\left(-\frac{3}{4}, -\frac{7}{6}\right)$

$$\Rightarrow X = -\frac{3}{4} \text{ and } Y = -\frac{7}{6}$$

$$O'(h, k) = O'\left(\frac{1}{4}, -\frac{1}{6}\right)$$

$$\Rightarrow h = \frac{1}{4} \text{ and } k = -\frac{1}{6}$$

$$\therefore X = x - h$$

$$\Rightarrow -\frac{3}{4} = x - \frac{1}{4}$$

$$\Rightarrow x = -\frac{3}{4} + \frac{1}{4} \Rightarrow x = -\frac{1}{2}$$

Also $Y = y - k$

$$\Rightarrow -\frac{7}{6} = y + \frac{1}{6}$$

$$\Rightarrow y = -\frac{7}{6} - \frac{1}{6} \Rightarrow y = -\frac{4}{3}$$

Hence $\left(-\frac{1}{2}, -\frac{4}{3}\right)$ is the required point.

(iv) *Do yourself*

Rotation of Axes

Let (x, y) be the coordinates of point P in xy -coordinate system. If the axes are rotated through an angle of θ and (X, Y) are coordinates of P in new XY -coordinate system then

$$X = x \cos \theta + y \sin \theta$$

$$Y = y \cos \theta - x \sin \theta$$

Question # 3

The xy -coordinates axes are rotated about the origin through the indicated angle. The new axes are OX and OY . Find the XY -coordinates of the point P with the given xy -coordinates.

- (i) $P(5,3); \theta = 45^\circ$
- (ii) $P(3,-7); \theta = 30^\circ$
- (iii) $P(11,-15); \theta = 60^\circ$
- (iv) $P(15,10); \theta = \arctan \frac{1}{3}$

Solution

$$(i) \quad \because P(x, y) = P(5, 3)$$

$$\Rightarrow x = 5 \quad \& \quad y = 3, \quad \theta = 45^\circ$$

$$\text{Since } X = x \cos \theta + y \sin \theta$$

$$= 5 \cos 45^\circ + 3 \sin 45^\circ$$

$$= 5\left(\frac{1}{\sqrt{2}}\right) + 3\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}}(5+3)$$

$$= \frac{8}{\sqrt{2}} = \frac{4 \times 2}{\sqrt{2}} = 4\sqrt{2}$$

$$\text{Now } Y = y \cos \theta - x \sin \theta$$

$$= 3 \cos 45^\circ - 5 \sin 45^\circ$$

$$= 3\left(\frac{1}{\sqrt{2}}\right) - 5\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}}(3-5)$$

$$= -2\left(\frac{1}{\sqrt{2}}\right) = -\sqrt{2}$$

Hence the required point is $(4\sqrt{2}, -\sqrt{2})$.

$$(ii) \quad \because P(x, y) = P(3, -7)$$

$$\Rightarrow x = 3 \quad \& \quad y = -7, \quad \theta = 30^\circ$$

$$\text{Since } X = x \cos \theta + y \sin \theta$$

$$= 3 \cos 30^\circ - 7 \sin 30^\circ$$

$$\Rightarrow X = 3\left(\frac{\sqrt{3}}{2}\right) - 7\left(\frac{1}{2}\right)$$

$$= \frac{3\sqrt{3}-7}{2}$$

$$\text{Now } Y = y \cos \theta - x \sin \theta$$

$$= -7 \cos 30^\circ - 3 \sin 30^\circ$$

$$= -7\left(\frac{\sqrt{3}}{2}\right) - 3\left(\frac{1}{2}\right) = \frac{-7\sqrt{3}-3}{2}$$

Hence the required point is

$$\left(\frac{3\sqrt{3}-7}{2}, \frac{-7\sqrt{3}-3}{2}\right).$$

(iv) *Do yourself*

$$(iv) \quad \because P(x, y) = P(15, 10)$$

$$\Rightarrow x = 15 \quad \& \quad y = 10$$

$\text{Also } \theta = \tan^{-1}\left(\frac{1}{3}\right)$ $\Rightarrow \tan \theta = \frac{1}{3}$ $\Rightarrow \frac{\sin \theta}{\cos \theta} = \frac{1/\sqrt{10}}{3/\sqrt{10}}$ $\Rightarrow \sin \theta = \frac{1}{\sqrt{10}}, \quad \cos \theta = \frac{3}{\sqrt{10}}$	$\tan \theta = \frac{y}{x} = \frac{1}{3}$ $\Rightarrow x = 3, \quad y = 1$ $r = \sqrt{x^2 + y^2}$ $= \sqrt{3^2 + 1^2}$ $= \sqrt{10}$
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$$\Rightarrow \sin \theta = \frac{1}{\sqrt{10}}, \quad \cos \theta = \frac{3}{\sqrt{10}}$$

$$\text{Now } X = x \cos \theta + y \sin \theta$$

$$= 15\left(\frac{3}{\sqrt{10}}\right) + 10\left(\frac{1}{\sqrt{10}}\right)$$

$$= \frac{1}{\sqrt{10}}(45+10) = \frac{55}{\sqrt{10}}$$

$$Y = y \cos \theta - x \sin \theta$$

$$= 10\left(\frac{3}{\sqrt{10}}\right) - 15\left(\frac{1}{\sqrt{10}}\right)$$

$$= \frac{1}{\sqrt{10}}(30-15) = \frac{15}{\sqrt{10}}$$

Hence the required point is $\left(\frac{55}{\sqrt{10}}, \frac{15}{\sqrt{10}}\right)$.

(iv)

$$\therefore P(x, y) = P(15, 10)$$

$$\Rightarrow x = 15 \quad \& \quad y = 10$$

$$\text{Also } \theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$\Rightarrow \tan \theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{\sin \theta}{\cos \theta} = \frac{1/2}{\sqrt{3}/2}$$

$$\Rightarrow \sin \theta = \frac{1}{2}, \quad \cos \theta = \frac{\sqrt{3}}{2}$$

$$\text{Now } X = x \cos \theta + y \sin \theta$$

$$= 15\left(\frac{\sqrt{3}}{2}\right) + 10\left(\frac{1}{2}\right)$$

$$= \frac{15\sqrt{3} + 10}{2}$$

$$Y = y \cos \theta - x \sin \theta$$

$$= 10\left(\frac{\sqrt{3}}{2}\right) - 15\left(\frac{1}{2}\right)$$

$$= \frac{10\sqrt{3} - 15}{2}$$

Hence the required point is

$$\left(= \frac{15\sqrt{3} + 10}{2}, \frac{10\sqrt{3} - 15}{2}\right).$$

Question # 4

The xy -coordinates axes are rotated about the origin through the indicated angle and the new axes are OX and OY , Find the xy -coordinates of P with the given XY -coordinates.

- (i) $P(-5, 3); \theta = 30^\circ$
- (ii) $P(-7\sqrt{2}, 5\sqrt{2}); \theta = 45^\circ$

Solution

$$(i) \quad \because P(X, Y)$$

$$\Rightarrow X = -5 \quad \& \quad Y$$

$$\begin{aligned} \tan \theta &= \frac{y}{x} = \frac{1}{\sqrt{3}} \\ \Rightarrow x &= \sqrt{3}, \quad y = 1 \\ r &= \sqrt{x^2 + y^2} \\ &= \sqrt{(\sqrt{3})^2 + 1^2} \\ &= \sqrt{4} = 2 \end{aligned}$$

Also $\theta = 30^\circ$

$$\text{Therefore } \sin \theta = \frac{1}{2} \quad \& \quad \cos \theta = \frac{\sqrt{3}}{2}$$

$$\text{Now } X = x \cos \theta + y \sin \theta$$

$$\Rightarrow -5 = x\left(\frac{\sqrt{3}}{2}\right) + y\left(\frac{1}{2}\right)$$

$$\Rightarrow \sqrt{3}x + y = -10 \quad \dots\dots\dots (i)$$

$$\text{Also } Y = y \cos \theta - x \sin \theta$$

$$\Rightarrow 3 = y\left(\frac{\sqrt{3}}{2}\right) - x\left(\frac{1}{2}\right)$$

$$\Rightarrow 6 = \sqrt{3}y - x$$

$$\Rightarrow x = \sqrt{3}y - 6 \quad \dots\dots\dots (ii)$$

Putting value of x in (i)

$$\sqrt{3}(\sqrt{3}y - 6) + y = -10$$

$$\Rightarrow 3y - 6\sqrt{3} + y = -10$$

$$\Rightarrow 4y = -10 + 6\sqrt{3}$$

$$\Rightarrow y = \frac{-10 + 6\sqrt{3}}{4}$$

$$= \frac{-5 + 3\sqrt{3}}{2}$$

Putting value of y in (ii)

$$x = \sqrt{3}\left(\frac{-5 + 3\sqrt{3}}{2}\right) - 6$$

$$= \frac{-5\sqrt{3} + 9}{2} - 6 = \frac{-5\sqrt{3} + 9 - 12}{2}$$

$$= \frac{-5\sqrt{3} - 3}{2}$$

Hence $\left(\frac{-5\sqrt{3} - 3}{2}, \frac{-5 + 3\sqrt{3}}{2}\right)$ is required point.

$$(ii) \quad \text{Do yourself}$$