Point of intersection of lines

Let $l_1:\ a_1x+b_1y+c_1=0$

$l_2:\ a_2x+b_2y+c_2=0$ be non-parallel lines.

Let $P(x_1,y_1)$ be the point of intersection of l_1 and l_2. Then

$$a_1x_1+b_1y_1+c_1=0 \ldots\ldots\ldots\ldots(i)$$
$$a_2x_1+b_2y_1+c_2=0 \ldots\ldots\ldots\ldots(ii)$$

Solving (i) and (ii) simultaneously, we have

$$\frac{x_1}{b_1c_2-b_2c_1}=\frac{-y_1}{a_1c_2-a_2c_1}=\frac{1}{a_1b_2-a_2b_1}$$

$$\Rightarrow \frac{x_1}{b_1c_2-b_2c_1}=\frac{1}{a_1b_2-a_2b_1} \text{ and } \frac{-y_1}{a_1c_2-a_2c_1}=\frac{1}{a_1b_2-a_2b_1}$$

$$\Rightarrow x_1=\frac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1} \text{ and } y_1=-\frac{a_1c_2-a_2c_1}{a_1b_2-a_2b_1}$$

Hence $\left(\dfrac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1},-\dfrac{a_1c_2-a_2c_1}{a_1b_2-a_2b_1}\right)$ is the point of intersection of l_1 and l_2.

Equation of line passing through the point of intersection.

Let $l_1:\ a_1x+b_1y+c_1=0$

$l_2:\ a_2x+b_2y+c_2=0$

Then equation of line passing through the point of intersection of l_1 and l_2 is

$$l_1+k\,l_2=0 \ , \text{ where } k \text{ is constant.}$$

i.e. $a_1x+b_1y+c_{11}+k\left(a_2x+b_2y+c_2\right)=0$

Question # 1

Find the point of intersection of the lines

(i) $x-2y+1=0 \quad 2x-y+2=0$ (ii) $3x+y+12=0 \quad x+2y-1=0$

(iii) $x+4y-12=0 \quad x-3y+3=0$

Solution

(i) $l_1:\ x-2y+1=0$

$l_2:\ 2x-y+2=0$

Slope of $l_1=m_1=-\dfrac{1}{-2}=\dfrac{1}{2}$

Slope of $l_2=m_2=-\dfrac{2}{-1}=2$

$\because\ m_1\neq m_2$, therefore lines are intersecting.

Now if (x,y) is the point of intersection of l_1 and l_2 then

$$\frac{x}{(-2)(2)-(-1)(1)}=\frac{-y}{(1)(2)-(2)(1)}=\frac{1}{(1)(-1)-(2)(-2)}$$

$$\Rightarrow \frac{x}{-4+1}=\frac{-y}{2-2}=\frac{1}{-1+4}$$

$$\Rightarrow \frac{x}{-3}=\frac{-y}{0}=\frac{1}{3}$$

$$\Rightarrow \frac{x}{-3}=\frac{1}{3} \quad \text{and} \quad \frac{-y}{0}=\frac{1}{3}$$

$$\Rightarrow x=\frac{-3}{3} \quad \text{and} \quad y=-\frac{0}{3}$$

$$\Rightarrow x=-1 \quad \text{and} \quad y=0$$

Hence $(-1,0)$ is the point of intersection.

(ii) $l_1 : 3x+y+12=0$

$l_2 : x+2y-1=0$

Slope of $l_1 = m_1 = -\frac{3}{1} = -3$

Slope of $l_2 = m_2 = -\frac{1}{2}$

$\because m_1 \neq m_2$, therefore lines are intersecting.

Now if (x, y) is the point of intersection of l_1 and l_2 then

$$\frac{x}{-1-24}=\frac{-y}{-3-12}=\frac{1}{6-1}$$

$$\Rightarrow \frac{x}{-25}=\frac{-y}{-15}=\frac{1}{5}$$

$$\Rightarrow \frac{x}{-25}=\frac{1}{5} \quad \text{and} \quad \frac{-y}{-15}=\frac{1}{5}$$

$$\Rightarrow x=\frac{-25}{5}=-5 \quad \text{and} \quad y=\frac{15}{5}=3$$

Hence $(-5,3)$ is the point of intersection.

(iii) *Do yourself as above.*

Question # 2

Find an equation of the line through

(i) the point $(2,-9)$ and the intersection of the lines $2x+5y-8=0$ and $3x-4y-6=0$

(ii) the intersection of the lines

$x-y-4=0$ $\quad$ $7x+y+20=0$ $\quad$ $6x+y-14=0$

(a) Parallel $\quad$ (ii) Perpendicular

to the line $6x+y-14=0$

(iii) through the intersection of the lines $x+2y+3=0,\ 3x+4y+7=0$

And making equal intercepts on the axes.

Solution

(i) Let $l_1:\ 2x+5y-8=0$

$l_2:\ 3x-4y-6=0$

Equation of line passing through point of intersection of l_1 and l_2 is

$$2x+5y-8+k(3x-4y-6)=0 \ldots \text{ (i)}$$

Since $(2,-9)$ lies on (i) therefore put $x=2$ and $y=-9$ in (i)

$$2(2)+5(-9)-8+k(3(2)-4(-9)-6)=0$$
$$\Rightarrow 4-45-8+k(6+36-6)=0$$
$$\Rightarrow -49+36k=0$$
$$\Rightarrow 36k=49 \qquad \Rightarrow k=\frac{49}{36}$$

Putting value of k in (i)

$$2x+5y-8+\frac{49}{36}(3x-4y-6)=0$$
$$\Rightarrow 72x+180y-288+49(3x-4y-6)=0 \qquad \times\text{ing by } 36$$
$$\Rightarrow 72x+180y-288+147x-196y-294=0$$
$$\Rightarrow 219x-16y-582=0 \quad \text{is the required equation.}$$

(ii) Let $l_1:\ x-y-4=0$

$l_2:\ 7x+y+20=0$

$l_3:\ 6x+y-14=0$

Let l_4 be a line passing through point of intersection of l_1 and l_2, then

$$l_4:\ l_1+kl_2=0$$
$$\Rightarrow x-y-4+k(7x+y+20)=0 \ldots \text{ (i)}$$
$$\Rightarrow (1+7k)x+(-1+k)y+(-4+20k)=0$$

Slope of $l_4 = m_1 = -\dfrac{1+7k}{-1+k}$

Slope of $l_3 = m_2 = -\dfrac{6}{1} = -6$

(a) If l_3 and l_4 are parallel then

$$m_1=m_2$$
$$\Rightarrow -\frac{1+7k}{-1+k}=-6$$
$$\Rightarrow 1+7k=6(-1+k) \qquad \Rightarrow 1+7k=-6+6k$$
$$\Rightarrow 7k-6k=-6-1 \qquad \Rightarrow k=-7$$

Putting value of k in (i)

$$x-y-4-7(7x+y+20)=0$$
$$\Rightarrow x-y-4-49x-7y-140=0$$

$\Rightarrow\ -48x-8y-144=0$
$\Rightarrow\ 6x+y+18=0$
is the required equation

(b) If l_3 and l_4 are $\perp$ then

$$m_1m_2=-1$$
$$\Rightarrow \left(-\frac{1+7k}{-1+k}\right)(-6)=-1$$
$$\Rightarrow 6(1+7k)=-(-1+k) \qquad \Rightarrow 6+42k=1-k$$
$$\Rightarrow 42k+k=1-6 \qquad \Rightarrow 43k=-5 \qquad \Rightarrow k=-\frac{5}{43}$$

Putting in (i) we have

$$x-y-4-\frac{5}{43}(7x+y+20)=0$$
$$\Rightarrow 43x-43y-172-5(7x+y+20)=0$$
$$\Rightarrow 43x-43y-172-35x-5y-100=0$$
$$\Rightarrow 8x-48y-272=0$$
$$\Rightarrow x-6y-34=0 \text{ is the required equation.}$$

(iii) Suppose $l_1:\ x+2y+3=0$
$l_2:\ 3x+4y+7=0$

Equation of line passing through the intersection of l_1 and l_2 is given by:

$$x+2y+3+k(3x+4y+7)=0 \text{ .... (i)}$$
$$\Rightarrow (1+3k)x+(2+4k)y+(3+7k)=0$$
$$\Rightarrow (1+3k)x+(2+4k)y=-(3+7k)$$
$$\Rightarrow \frac{(1+3k)x}{-(3+7k)}+\frac{(2+4k)y}{-(3+7k)}=1$$
$$\Rightarrow \frac{x}{-(3+7k)/(1+3k)}+\frac{y}{-(3+7k)/(2+4k)}=1$$

Which is two-intercept form of equation of line with

$$x-\text{intercept}=\frac{-(3+7k)}{(1+3k)} \quad \text{and} \quad y-\text{intercept}=\frac{-(3+7k)}{(2+4k)}$$

We have given

$$x-\text{intercept} = y-\text{intercept}$$
$$\Rightarrow \frac{-(3+7k)}{(1+3k)}=\frac{-(3+7k)}{(2+4k)}$$
$$\Rightarrow \frac{1}{(1+3k)}=\frac{1}{(2+4k)} \qquad \Rightarrow (2+4k) = (1+3k)$$
$$\Rightarrow 4k-3k = 1-2 \quad \Rightarrow k=-1$$

Putting value of k in (i)

$$x+2y+3-1(3x+4y+7) = 0$$
$$\Rightarrow x+2y+3-3x-4y-7 = 0 \quad \Rightarrow -2x-2y-4 = 0$$
$$\Rightarrow x+y+2 = 0$$
is the required equation.

Question # 3

Find an equation of the line through the intersection of $16x-10y-33=0$; $12x+14y+29=0$ and the intersection of $x-y+4=0$; $x-7y+2=0$

Solution

Let $l_1:\ 16x-10y-33=0$
$l_2:\ 12x+14y+29=0$
$l_3:\ x-y+4=0$
$l_4:\ x-7y+2=0$

For point of intersection of l_1 and l_2

$$\frac{x}{-290+462}=\frac{-y}{464+396}=\frac{1}{224+120}$$
$$\Rightarrow \frac{x}{172}=\frac{-y}{860}=\frac{1}{334}$$
$$\Rightarrow \frac{x}{172}=\frac{1}{334} \quad \text{and} \quad \frac{-y}{860}=\frac{1}{334}$$
$$\Rightarrow x=\frac{172}{334}=\frac{1}{2} \quad \text{and} \quad y=-\frac{860}{334}=-\frac{5}{2}$$
$$\Rightarrow \left(\frac{1}{2},-\frac{5}{2}\right) \text{ is a point of intersection of } l_1 \text{ and } l_2.$$

For point of intersection of l_3 and l_4.

$$\frac{x}{-2+28}=\frac{-y}{2-4}=\frac{1}{-7+1}$$
$$\Rightarrow \frac{x}{26}=\frac{-y}{-2}=\frac{1}{-6}$$
$$\Rightarrow \frac{x}{26}=\frac{1}{-6} \quad \text{and} \quad \frac{-y}{-2}=\frac{1}{-6}$$
$$\Rightarrow x=\frac{26}{-6}=-\frac{13}{3} \quad \text{and} \quad y=\frac{2}{-6}=-\frac{1}{3}$$
$$\Rightarrow \left(-\frac{13}{3},-\frac{1}{3}\right) \text{ is a point of intersection of } l_3 \text{ and } l_4.$$

Now equation of line passing through $\left(\frac{1}{2},-\frac{5}{2}\right)$ and $\left(-\frac{13}{3},-\frac{1}{3}\right)$

$$y+\frac{5}{2}=\frac{-1/3+5/2}{-13/3-1/2}\left(x-\frac{1}{2}\right)$$

$$\Rightarrow\ y+\frac{5}{2}=\frac{13/6}{-29/6}\left(x-\frac{1}{2}\right)\quad\Rightarrow\ y+\frac{5}{2}=-\frac{13}{29}\left(x-\frac{1}{2}\right)$$

$$\Rightarrow\ 29y+\frac{145}{2}=-13x+\frac{13}{2}\qquad\Rightarrow\ 13x-\frac{13}{2}+29y+\frac{145}{2}=0$$

$$\Rightarrow\ 13x+29y+66=0$$

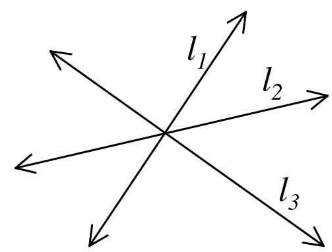
is the required equation.

Three Concurrent Lines

Suppose $l_1:\ a_1x+b_1y+c_1=0$

$l_2:\ a_2x+b_2y+c_2=0$

$l_3:\ a_3x+b_3y+c_3=0$



If l_1, l_2 and l_3 are concurrent (intersect at one point) then

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}=0$$

See proof on book at page 208

Question # 4

Find the condition that the lines $y=m_1x+c_1$; $y=m_2x+c_2$ and $y=m_3x+c_3$ are concurrent.

Solution

Assume that $l_1:\ y=m_1x+c_1$

$$\Rightarrow\ m_1x-y+c_1=0$$

$l_2:\ y=m_2x+c_2$

$$\Rightarrow\ m_2x-y+c_2=0$$

$l_3:\ y=m_3x+c_3$

$$\Rightarrow\ m_3x-y+c_3=0$$

If l_1, l_2 and l_3 are concurrent then

$$\begin{vmatrix} m_1 & -1 & c_1 \\ m_2 & -1 & c_2 \\ m_3 & -1 & c_3 \end{vmatrix}=0$$

$$\Rightarrow\ \begin{vmatrix} m_1 & -1 & c_1 \\ m_2-m_1 & 0 & c_2-c_1 \\ m_3-m_1 & 0 & c_3-c_1 \end{vmatrix}=0 \quad \text{by } \begin{matrix} R_2-R_1 \\ R_3-R_1 \end{matrix}$$

Expanding by C_2

$$-(-1)\left[(m_2-m_1)(c_3-c_1)-(m_3-m_1)(c_2-c_1)\right]+0-0=0$$

$$\Rightarrow\ \left[(m_2-m_1)(c_3-c_1)-(m_3-m_1)(c_2-c_1)\right]=0$$

$$\Rightarrow\ (m_2-m_1)(c_3-c_1)=(m_3-m_1)(c_2-c_1)$$

is the required condition.

Question # 5
Determine the value of P such that the lines $2x-3y-1=0$, $3x-y-5=0$ and $3x+py+8=0$ meet at a point .
Solution
Let $l_1:\ 2x-3y-1=0$
$l_2:\ 3x-y-5=0$
$l_3:\ 3x+py+8=0$
Since l_1, l_2 and l_3 meets at a point i.e. concurrent therefore

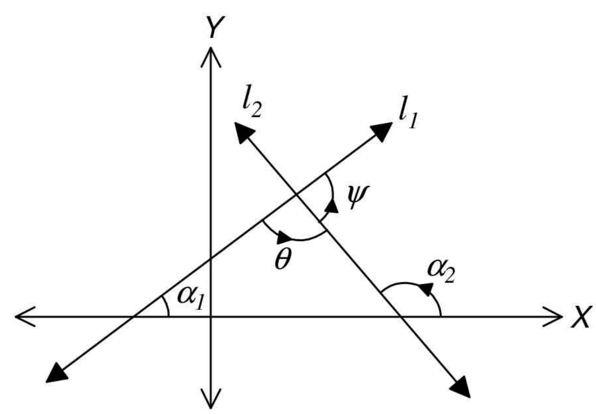
$$\begin{vmatrix} 2 & -3 & -1 \\ 3 & -1 & -5 \\ 3 & p & 8 \end{vmatrix}=0$$

$\Rightarrow\ 2(-8+5p)+3(24+15)-1(3p+3)=0$
$\Rightarrow\ -16+10p+72+45-3p-3=0$
$\Rightarrow\ 7p+98=0 \quad \Rightarrow\ 7p=-98$
$\Rightarrow\ p=-\dfrac{98}{7} \quad \Rightarrow\ \boxed{p=-14}$

Angle between lines
Let l_1 and l_2 be two lines. If α_1 and α_2 be inclinations and m_1 and m_2 be slopes of lines l_1 and l_2 respectively, Let θ be a angle from line l_1 to l_2 then θ is given by

$$\tan\theta=\frac{m_2-m_1}{1+m_1m_2}$$

See proof on book at page 219



Question # 6
Show that the lines $4x-3y-8=0$, $3x-4y-6=0$ and $x-y-2=0$ are concurrent and the third-line bisects the angle formed by the first two lines.
Solution
Let $l_1:\ 4x-3y-8=0$
$l_2:\ 3x-4y-6=0$
$l_3:\ x-y-2=0$
To check l_1, l_2 and l_3 are concurrent, let

$$\begin{vmatrix} 4 & -3 & -8 \\ 3 & -4 & -6 \\ 1 & -1 & -2 \end{vmatrix}$$

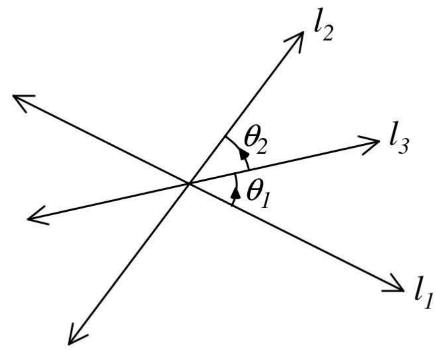
$=4(8-6)+3(-6+6)-8(-3+4)$
$=4(2)+3(0)-8(1)$
$=8+0-8=0$

Hence l_1, l_2 and l_3 are concurrent.

Slope of $l_1 = m_1 = -\dfrac{4}{-3} = \dfrac{4}{3}$

Slope of $l_2 = m_2 = -\dfrac{3}{-4} = \dfrac{3}{4}$

Slope of $l_3 = m_3 = -\dfrac{1}{-1} = 1$



Now let θ_1 be angle from l_1 to l_3 and θ_2 be a angle from l_3 to l_2. Then

$$\tan\theta_1 = \frac{m_3 - m_1}{1 + m_3 m_1} = \frac{1 - \frac{4}{3}}{1 + (1)\left(\frac{4}{3}\right)} = \frac{-\frac{1}{3}}{\frac{7}{3}} = -\frac{1}{7} \quad \ldots\ldots\ldots\ldots (i)$$

And
$$\tan\theta_2 = \frac{m_2 - m_3}{1 + m_2 m_3} = \frac{\frac{3}{4} - 1}{1 + \left(\frac{3}{4}\right)(1)} = \frac{-\frac{1}{4}}{\frac{7}{4}} = -\frac{1}{7} \quad \ldots\ldots\ldots\ldots (ii)$$

From (i) and (ii)

$$\tan\theta_1 = \tan\theta_2 \quad \Rightarrow \theta_1 = \theta_2$$

$\Rightarrow$ l_3 bisect the angle formed by the first two lines.

Question # 7

The vertices of a triangle are $A(-2,3)$, $B(-4,1)$ and $C(3,5)$.

Find coordinates of the

(i) centroid (ii) orthocentre (iii) circumcentre of the triangle

Are these three points are collinear?

Solution

Given vertices of triangles are $A(-2,3)$, $B(-4,1)$ and $C(3,5)$.

(i) Centroid of triangle is the intersection of medians and is given by

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

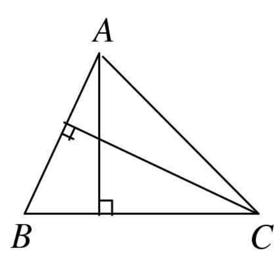
$$= \left(\frac{-2 - 4 + 3}{3}, \frac{3 + 1 + 5}{3}\right) = \left(\frac{-3}{3}, \frac{9}{3}\right) = (-1,3)$$

Hence $(-1,3)$ is the centroid of the triangle.

(ii) Orthocentre is the point of intersection of altitudes.

Slope of $\overline{AB} = m_1 = \dfrac{1-3}{-4+2} = \dfrac{-2}{-2} = 1$

Slope of $\overline{BC} = m_2 = \dfrac{5-1}{3+4} = \dfrac{4}{7}$



Since altitudes are $\perp$ to sides therefore

Slope of altitude on $\overline{AB} = -\dfrac{1}{m_1} = -\dfrac{1}{1} = -1$

Slope of altitude on $\overline{BC} = -\dfrac{1}{m_2} = -\dfrac{1}{4/7} = -\dfrac{7}{4}$

Equation of altitude on $\overline{AB}$ with slope -1 from $C(3,5)$

$$y-5=-1(x-3)$$
$$\Rightarrow\ y-5=-x+3 \quad \Rightarrow\ x-3+y-5=0$$
$$\Rightarrow\ x+y-8=0 \ \ldots\ldots\ldots.. \text{(i)}$$

Now equation of altitude on $\overline{BC}$ with slope $-\dfrac{7}{4}$ from $A(-2,3)$

$$y-3=-\frac{7}{4}(x+2)$$
$$\Rightarrow\ 4y-12=-7x-14 \quad \Rightarrow\ 7x+14+4y-12=0$$
$$\Rightarrow\ 7x+4y+2=0 \ \ldots\ldots\ldots\ldots.. \text{(ii)}$$

For point of intersection of (i) and (ii)

$$\frac{x}{2+32}=\frac{-y}{2+56}=\frac{1}{4-7}$$
$$\Rightarrow\ \frac{x}{34}=\frac{-y}{58}=\frac{1}{-3}$$
$$\Rightarrow\ \frac{x}{34}=\frac{1}{-3} \quad \text{and} \quad \frac{-y}{58}=\frac{1}{-3}$$
$$\Rightarrow\ x=-\frac{34}{3} \quad \text{and} \quad y=-\frac{58}{-3}=\frac{58}{3}$$

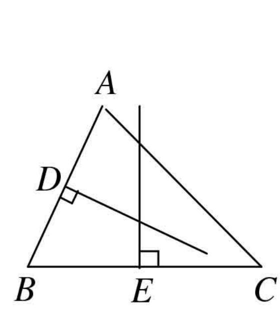
Hence $\left(-\dfrac{34}{3},\dfrac{58}{3}\right)$ is orthocentre of triangle ABC.

(iii) Circumcentre of the triangle is the point of intersection of perpendicular bisector.

Let D and E are midpoints of side $\overline{AB}$ and $\overline{BC}$ respectively.

Then coordinate of $D=\left(\dfrac{-4-2}{2},\dfrac{1+3}{2}\right)=\left(\dfrac{-6}{2},\dfrac{4}{2}\right)=(-3,2)$

Coordinate of $E=\left(\dfrac{-4+3}{2},\dfrac{1+5}{2}\right)=\left(\dfrac{-1}{2},\dfrac{6}{2}\right)=\left(-\dfrac{1}{2},3\right)$



Slope of $\overline{AB}=m_1=\dfrac{1-3}{-4+2}=\dfrac{-2}{-2}=1$

Slope of $\overline{BC}=m_2=\dfrac{5-1}{3+4}=\dfrac{4}{7}$

Slope of $\perp$ bisector on $\overline{AB}=-\dfrac{1}{m_1}=-\dfrac{1}{1}=-1$

Slope of $\perp$ bisector on $\overline{BC}=-\dfrac{1}{m_2}=-\dfrac{1}{4/7}=-\dfrac{7}{4}$

Now equation of ⊥ bisector having slope −1 through $D(-3,2)$

$$y-2=-1(x+3)$$
$$\Rightarrow\ y-2=-x-3 \quad \Rightarrow\ x+3+y-2=0$$
$$\Rightarrow\ x+y+1=0 \ \ldots\ldots\ldots\ldots \text{(iii)}$$

Now equation of ⊥ bisector having slope $-\frac{7}{4}$ through $E\left(-\frac{1}{2},3\right)$

$$y-3=-\frac{7}{4}\left(x+\frac{1}{2}\right) \quad \Rightarrow 4y-12=-7x-\frac{7}{2}$$
$$\Rightarrow\ 7x+\frac{7}{2}+4y-12=0 \quad \Rightarrow\ 7x+4y-\frac{17}{2}=0$$
$$\Rightarrow\ 14x+8y-17=0 \ \ldots\ldots\ldots\ldots\ldots \text{(iv)}$$

For point of intersection of (iii) and (iv)

$$\frac{x}{-17-8}=\frac{-y}{-17-14}=\frac{1}{8-14}$$
$$\Rightarrow\ \frac{x}{-25}=\frac{-y}{-31}=\frac{1}{-6}$$
$$\Rightarrow\ \frac{x}{-25}=\frac{1}{-6} \quad \text{and} \quad \frac{-y}{-31}=\frac{1}{-6}$$
$$\Rightarrow\ x=\frac{-25}{-6}=\frac{25}{6} \quad \text{and} \quad y=-\frac{31}{6}$$

Hence $\left(\frac{25}{6},-\frac{31}{6}\right)$ is the circumcentre of the triangle.

Now to check $(-1,3),\left(-\frac{34}{3},\frac{58}{3}\right)$ and $\left(\frac{25}{6},-\frac{31}{6}\right)$ are collinear, let

$$\begin{vmatrix} -1 & 3 & 1 \\ -34/3 & 58/3 & 1 \\ 25/6 & -31/6 & 1 \end{vmatrix}$$
$$=-1\left(\frac{58}{3}+\frac{31}{6}\right)-3\left(-\frac{34}{3}-\frac{25}{6}\right)+1\left(\frac{1054}{18}-\frac{1450}{18}\right)$$
$$=-1\left(\frac{49}{2}\right)-3\left(-\frac{31}{2}\right)+1(-22)$$
$$=-\frac{49}{2}+\frac{93}{2}-22=0$$

Hence centroid, orthocentre and circumcentre of triangle are collinear.

Question # 8

Check whether the lines $4x-3y-8=0;\ 3x-4y-6=0;\ x-y-2=0$ are concurrent. If so, find the point where they meet.

Solution

Let $l_1: \ 4x-3y-8=0$

$l_2: \ 3x-4y-6=0$

$l_3: \ x-y-2=0$

To check lines are concurrent, let

$$\begin{vmatrix} 4 & -3 & -8 \\ 3 & -4 & -6 \\ 1 & -1 & -2 \end{vmatrix}$$

$$=4(8-6)+3(-6+6)-8(-3+4)$$

$$=4(2)+3(0)-8(1) \quad =8+0-8 \ =0$$

Hence l_1 , l_2 and l_3 are concurrent.

For point of concurrency, we find intersection of l_1 and l_2 (You may choose any two lines)

$$\frac{x}{18-32}=\frac{-y}{-24+24}=\frac{1}{-16+9}$$

$$\Rightarrow \frac{x}{-14}=\frac{-y}{0}=\frac{1}{-7}$$

$$\Rightarrow \frac{x}{-14}=\frac{1}{-7} \quad \text{and} \quad \frac{-y}{0}=\frac{1}{-7}$$

$$\Rightarrow x=\frac{-14}{-7}=2 \quad \text{and} \quad y=-\frac{0}{-7}=0$$

Hence $(2,0)$ is the point of concurrency.

Question # 9

Find the coordinates of the vertices of the triangle formed by the lines $x-2y-6=0; \quad 3x-y+3=0; \quad 2x+y-4=0$. Also find measures of the angles of the triangle.

Solution

Let $l_1: \ x-2y-6=0$

$l_2: \ 3x-y+3=0$

$l_3: \ 2x+y-4=0$

For point of intersection of l_1 and l_2

$$\frac{x}{-6-6}=\frac{-y}{3+18}=\frac{1}{-1+6}$$

$$\Rightarrow \frac{x}{-12}=\frac{-y}{21}=\frac{1}{5}$$

$$\Rightarrow \frac{x}{-12}=\frac{1}{5} \quad \text{and} \quad \frac{-y}{21}=\frac{1}{5}$$

$$\Rightarrow x=-\frac{12}{5} \quad \text{and} \quad y=-\frac{21}{5}$$

$\Rightarrow \left(-\frac{12}{5},-\frac{21}{5}\right)$ is the point of intersection of l_1 and l_2.

For point of intersection of l_2 and l_3.

$$\frac{x}{4-3}=\frac{-y}{-12-6}=\frac{1}{3+2}$$

$$\Rightarrow \frac{x}{1}=\frac{-y}{-18}=\frac{1}{5}$$

$$\Rightarrow \frac{x}{1}=\frac{1}{5} \quad \text{and} \quad \frac{-y}{-18}=\frac{1}{5}$$

$$\Rightarrow x=\frac{1}{5} \quad \text{and} \quad y=\frac{18}{5}$$

$\Rightarrow \left(\frac{1}{5},\frac{18}{5}\right)$ is the point of intersection of l_2 and l_3.

Now for point of intersection of l_1 and l_3

$$\frac{x}{8+6}=\frac{-y}{-4+12}=\frac{1}{1+4}$$

$$\Rightarrow \frac{x}{14}=\frac{-y}{8}=\frac{1}{5}$$

$$\Rightarrow \frac{x}{14}=\frac{1}{5} \quad \text{and} \quad \frac{-y}{8}=\frac{1}{5}$$

$$\Rightarrow x=\frac{14}{5} \quad \text{and} \quad y=-\frac{8}{5}$$

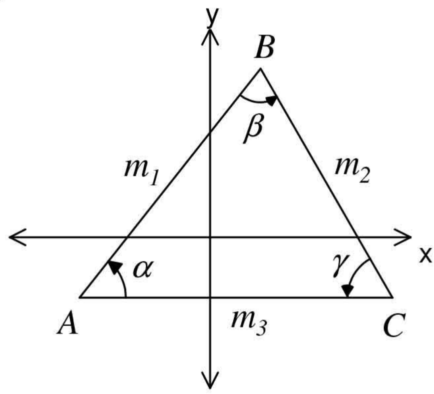
$\Rightarrow \left(\frac{14}{5},-\frac{8}{5}\right)$ is the point of intersection of l_1 and l_3.

Hence $\left(-\frac{12}{5},-\frac{21}{5}\right)$, $\left(\frac{1}{5},\frac{18}{5}\right)$ and $\left(\frac{14}{5},-\frac{8}{5}\right)$ are vertices of triangle made by l_1, l_2 and l_3. We say these vertices as A, B and C respectively.

Slope of side $AB = m_1 = \dfrac{18/5+21/5}{1/5+12/5}=\dfrac{39/5}{13/5}=\dfrac{39}{13}=3$

Slope of side $BC = m_2 = \dfrac{-8/5-18/5}{14/5-1/5}=\dfrac{-26/5}{13/5}=-\dfrac{26}{13}=-2$

Slope of side $CA = m_3 = \dfrac{-21/5+8/5}{-12/5-14/5}=\dfrac{-13/5}{-26/5}=\dfrac{13}{26}=\dfrac{1}{2}$



Let α, β and γ denotes angles of triangle at vertices A, B and C respectively. Then

$$\tan\alpha = \frac{m_1 - m_3}{1 + m_1 m_3} = \frac{3 - \frac{1}{2}}{1 + (3)\left(\frac{1}{2}\right)} = \frac{\frac{5}{2}}{\frac{5}{2}} = 1$$

$$\Rightarrow \alpha = \tan^{-1}(1) \quad \Rightarrow \boxed{\alpha = 45^\circ}$$

Now $$\tan\beta = \frac{m_2 - m_1}{1 + m_2 m_1} = \frac{-2-3}{1 + (-3)(3)} = \frac{-5}{-5} = 1$$

$$\Rightarrow \beta = \tan^{-1}(1) \quad \Rightarrow \boxed{\beta = 45^\circ}$$

Now $$\tan\gamma = \frac{m_3 - m_2}{1 + m_3 m_2} = \frac{\frac{1}{2} + 2}{1 + \left(\frac{1}{2}\right)(-2)} = \frac{\frac{5}{2}}{0} = \infty$$

$$\Rightarrow \gamma = \tan^{-1}(\infty) \quad \Rightarrow \boxed{\gamma = 90^\circ}.$$

Question # 10

Find the angle measured from the line l_1 to the line l_2 where

(a) l_1: joining $(2,7)$ and $(7,10)$
  l_2: joining $(1,1)$ and $(-5,3)$

(b) l_1: joining $(3,-1)$ and $(5,7)$
  l_2: joining $(2,4)$ and $(-8,2)$

(c) l_1: joining $(1,-7)$ and $(6,-4)$
  l_2: joining $(-1,2)$ and $(-6,-1)$

(d) l_1: joining $(-9,-1)$ and $(3,-5)$
  l_2: joining $(2,7)$ and $(-6,-7)$

Solution

(a) Since l_1: joining $(2,7)$ and $(7,10)$

Therefore slope of $l_1 = m_1 = \dfrac{10-7}{7-2} = \dfrac{3}{5}$

Also l_2: joining $(1,1)$ and $(-5,3)$

Therefore slope of $l_2 = m_2 = \dfrac{3-1}{-5-1} = \dfrac{2}{-6} = -\dfrac{1}{3}$

Let θ be a angle from l_1 to l_2 then

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2} = \frac{-\frac{1}{3} - \frac{3}{5}}{1 + \left(\frac{3}{5}\right)\left(-\frac{1}{3}\right)}$$

$$= \frac{-\frac{14}{15}}{1 - \frac{1}{5}} = \frac{-\frac{14}{15}}{\frac{4}{5}} = -\frac{14}{15} \times \frac{5}{4} = -\frac{7}{6}$$

$$\Rightarrow -\tan\theta = \frac{7}{6} \quad \Rightarrow \tan(180 - \theta) = \frac{7}{6}$$

$$\because \tan(180 - \theta) = -\tan\theta$$

$$\Rightarrow\ 180-\theta=\tan^{-1}\left(\frac{7}{6}\right)=49.4$$

$$\Rightarrow\ \theta=180-49.4 \quad \Rightarrow \boxed{\theta=130.6^\circ}$$

Now acute angle between lines $= 180-130.6=49.4^\circ$

(b) *Do yourself as above.*

(c) Since l_1: joining $(1,-7)$ and $(6,-4)$

Therefore slope of $l_1=m_1=\dfrac{-4+7}{6-1}=\dfrac{3}{5}$

Also l_2: joining $(-1,2)$ and $(-6,-1)$

Therefore slope of $l_2=m_2=\dfrac{-1-2}{-6+1}=\dfrac{-3}{-5}=\dfrac{3}{5}$

Let θ be a angle from l_1 to l_2 then

$$\tan\theta=\frac{m_2-m_1}{1+m_1m_2}=\frac{3/5-3/5}{1+\left(3/5\right)\left(3/5\right)}=\frac{0}{1+9/25}=0$$

$$\Rightarrow\ \theta=\tan^{-1}(0) \quad \Rightarrow \boxed{\theta=0^\circ}$$

Also acute angle between lines $= 0^\circ$

(d) Since l_1: joining $(-9,-1)$ and $(3,-5)$

Therefore slope of $l_1=m_1=\dfrac{-5+1}{3+9}=\dfrac{-4}{12}=-\dfrac{1}{3}$

Also l_2: joining $(2,7)$ and $(-6,-7)$

Therefore slope of $l_2=m_2=\dfrac{-7-7}{-6-2}=\dfrac{-14}{-8}=\dfrac{7}{4}$

Let θ be a angle from l_1 to l_2 then

$$\tan\theta=\frac{m_2-m_1}{1+m_1m_2}=\frac{7/4-\left(-1/3\right)}{1+\left(7/4\right)\left(-1/3\right)}$$

$$=\frac{7/4+1/3}{1-7/12}=\frac{25/12}{5/12}=\frac{25}{12}\times\frac{12}{5}=5$$

$$\Rightarrow\ \theta=\tan^{-1}(5) \quad \Rightarrow \boxed{\theta=78.69^\circ}$$

Also acute angle between lines $=78.69^\circ$

Question # 11

Find the interior angle of the triangle whose vertices are

(a) $A(-2,11)$, $B(-6,-3)$, $C(4,-9)$

(b) $A(6,1)$, $B(2,7)$, $C(-6,-7)$

(c) $A(2,-5)$, $B(-4,-3)$,$C(-1,5)$

(d) $A(2,8)$, $B(-5,4)$ and $C(4,-9)$

Solution

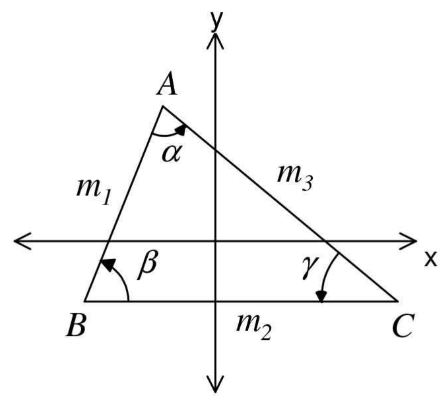
(a) Given vertices $A(-2,11)$, $B(-6,-3)$ and $C(4,-9)$

Let m_1, m_2 and m_3 denotes the slopes of side AB, BC and CA respectively. Then

$$m_1 = \frac{-3-11}{-6+2} = \frac{-14}{-4} = \frac{7}{2}$$

$$m_2 = \frac{-9+3}{4+6} = \frac{-6}{10} = -\frac{3}{5}$$

$$m_3 = \frac{11+9}{-2-4} = \frac{20}{-6} = -\frac{10}{3}$$



Let α, β and γ denotes angles of triangle at vertex A, B and C respectively. Then

$$\tan\alpha = \frac{m_3 - m_1}{1 + m_3 m_1} = \frac{-10/3 - 7/2}{1 + (-10/3)(7/2)}$$

$$= \frac{-41/6}{1 - 35/3} = \frac{-41/6}{-32/3} = \frac{41}{6} \times \frac{3}{32} = \frac{41}{64}$$

$$\Rightarrow \alpha = \tan^{-1}\left(\frac{41}{64}\right) \qquad \Rightarrow \boxed{\alpha = 32.64^\circ}$$

$$\tan\beta = \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{7/2 - (-3/5)}{1 + (7/2)(-3/5)}$$

$$= \frac{7/2 + 3/5}{1 - 21/10} = \frac{41/10}{-11/10} = -\frac{41}{10} \times \frac{10}{11} = -\frac{41}{11}$$

$$\Rightarrow -\tan\beta = \frac{41}{11} \qquad \Rightarrow \tan(180 - \beta) = \frac{41}{11} \qquad \because \tan(180-\theta) = -\tan\theta$$

$$\Rightarrow 180 - \beta = \tan^{-1}\left(\frac{41}{11}\right) = 74.98$$

$$\Rightarrow \beta = 180 - 74.98 \qquad \Rightarrow \boxed{\beta = 105.02}$$

$$\tan\gamma = \frac{m_2 - m_3}{1 + m_2 m_3} = \frac{-3/5 - (-10/3)}{1 + (-3/5)(-10/3)}$$

$$= \frac{-3/5 + 10/3}{1 + 2} = \frac{41/15}{3} = \frac{41}{15 \times 3} = \frac{41}{45}$$

$$\Rightarrow\ \gamma=\tan^{-1}\left(\frac{41}{45}\right)\qquad\Rightarrow\ \boxed{\gamma=42.34^\circ}$$

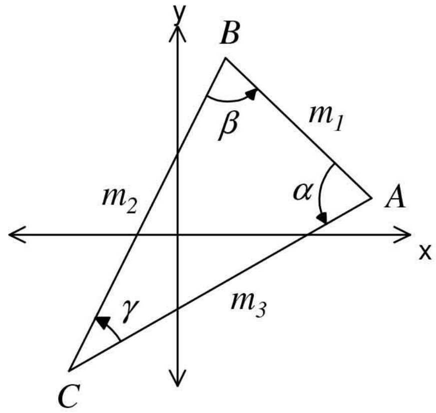
(b) Given vertices $A(6,1)$, $B(2,7)$ and $C(-6,-7)$

Let m_1, m_2 and m_3 denotes the slopes of side AB, BC and CA respectively. Then

$$m_1=\frac{7-1}{2-6}=\frac{6}{-4}=-\frac{3}{2}$$

$$m_2=\frac{-7-7}{-6-2}=\frac{-14}{-8}=\frac{7}{4}$$

$$m_3=\frac{1+7}{6+6}=\frac{8}{12}=\frac{2}{3}$$



Let α, β and γ denotes angles of triangle at vertex A, B and C respectively. Then

$$\tan\alpha=\frac{m_3-m_1}{1+m_3m_1}=\frac{\frac{2}{3}-\left(-\frac{3}{2}\right)}{1+\left(\frac{2}{3}\right)\left(-\frac{3}{2}\right)}$$

$$=\frac{\frac{2}{3}+\frac{3}{2}}{1-1}=\frac{\frac{13}{6}}{0}=\infty$$

$$\Rightarrow\ \alpha=\tan^{-1}(\infty)\qquad\Rightarrow\ \boxed{\alpha=90^\circ}$$

$$\tan\beta=\frac{m_1-m_2}{1+m_1m_2}=\frac{-\frac{3}{2}-\frac{7}{4}}{1+\left(-\frac{3}{2}\right)\left(\frac{7}{4}\right)}$$

$$=\frac{-\frac{13}{4}}{1-\frac{21}{8}}=\frac{-\frac{13}{4}}{-\frac{13}{8}}=\frac{13}{4}\times\frac{8}{13}=2$$

$$\Rightarrow\ \beta=\tan^{-1}(2)\qquad\Rightarrow\ \boxed{\beta=63.43^\circ}$$

$$\tan\gamma=\frac{m_2-m_3}{1+m_2m_3}=\frac{\frac{7}{4}-\frac{2}{3}}{1+\left(\frac{7}{4}\right)\left(\frac{2}{3}\right)}$$

$$=\frac{\frac{13}{12}}{1+\frac{7}{6}}=\frac{\frac{13}{12}}{\frac{13}{6}}=\frac{13}{12}\times\frac{6}{13}=\frac{1}{2}$$

$$\Rightarrow\ \gamma=\tan^{-1}\left(\frac{1}{2}\right)\qquad\Rightarrow\ \boxed{\gamma=26.57^\circ}$$

(c) *Do yourself as above.*

(d) *Do yourself as above.*

Question # 12
Find the interior angles of the quadrilateral whose vertices are $A(5,2)$, $B(-2,3)$, $C(-3,-4)$ and $D(4,-5)$

Solution Given vertices are $A(5,2)$, $B(-2,3)$, $C(-3,-4)$ and $D(4,-5)$

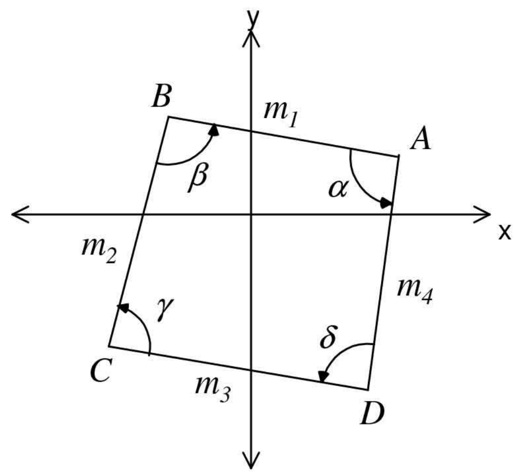
Let m_1, m_2, m_3 and m_4 be slopes of side AB, BC, CD and DA. Then

$$m_1 = \frac{3-2}{-2-5} = \frac{1}{-7}$$

$$m_2 = \frac{-4-3}{-3+2} = \frac{-7}{-1} = 7$$

$$m_3 = \frac{-5+4}{4+3} = \frac{-1}{7}$$

$$m_4 = \frac{2+5}{5-4} = \frac{7}{1} = 7$$



Now suppose α, β, γ and δ are angels of quadrilateral at vertices A, B, C and D respectively. Then

$$\tan\alpha = \frac{m_4 - m_1}{1 + m_4 m_1} = \frac{7-\left(-\frac{1}{7}\right)}{1+(7)\left(-\frac{1}{7}\right)} = \frac{7+\frac{1}{7}}{1-1} = \frac{\frac{50}{7}}{0} = \infty$$

$$\Rightarrow \alpha = \tan^{-1}(\infty) \qquad \Rightarrow \boxed{\alpha = 90^\circ}$$

$$\tan\beta = \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{-\frac{1}{7}-7}{1+\left(-\frac{1}{7}\right)(7)} = \frac{-\frac{50}{7}}{0} = \infty$$

$$\Rightarrow \beta = \tan^{-1}(\infty) \qquad \Rightarrow \boxed{\beta = 90^\circ}$$

$$\tan\gamma = \frac{m_2 - m_3}{1 + m_2 m_3} = \frac{7-\left(-\frac{1}{7}\right)}{1+(7)\left(-\frac{1}{7}\right)} = \frac{7+\frac{1}{7}}{1-1} = \frac{\frac{50}{7}}{0} = \infty$$

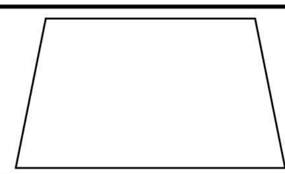
$$\Rightarrow \gamma = \tan^{-1}(\infty) \qquad \Rightarrow \boxed{\gamma = 90^\circ}$$

$$\tan\delta = \frac{m_3 - m_4}{1 + m_3 m_4} = \frac{-\frac{1}{7}-7}{1+\left(-\frac{1}{7}\right)(7)} = \frac{-\frac{50}{7}}{0} = \infty$$

$$\Rightarrow \delta = \tan^{-1}(\infty) \qquad \Rightarrow \boxed{\delta = 90^\circ}$$

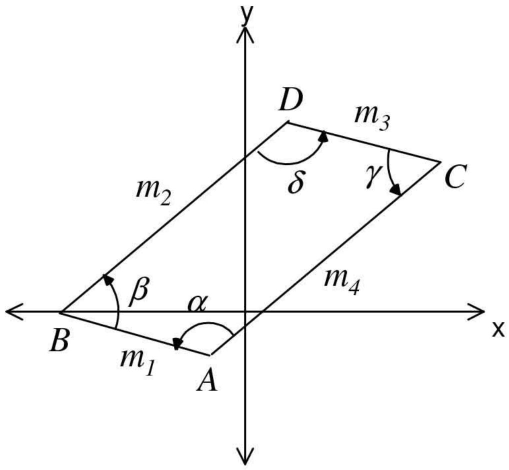
Trapezium

If any two opposite sides of the quadrilateral are parallel then it is called *trapezium.*



Question # 13
Show that the points $A(-1,-1)$, $B(-3,0)$, $C(3,7)$ and $D(1,8)$ are the vertices of the rhombus and find its interior angle.
Solution Given vertices are $A(-1,-1)$, $B(-3,0)$, $C(3,7)$ and $D(1,8)$
Let m_1, m_2, m_3 and m_4 be slopes of side $\overline{AB}, \overline{BD}, \overline{DC}$ and $\overline{CA}$. Then

$$m_1 = \frac{0+1}{-3+1} = \frac{1}{-2}$$
$$m_2 = \frac{8-0}{1+3} = \frac{8}{4} = 2$$
$$m_3 = \frac{7-8}{3-1} = \frac{-1}{2}$$
$$m_4 = \frac{-1-7}{-1-3} = \frac{-8}{-4} = 2$$



Since $m_2 = m_4$ or $m_1 = m_3$
Hence A, B, C and D are vertices of trapezium.
Now suppose α, β, γ and δ are angels of quadrilateral at vertices A, B, C and D respectively. Then

Now do yourself as above in Question # 12

Question # 14
Find the area of the region bounded by the triangle, whose sides are
$$7x - y - 10 = 0;\ 10x + y - 41 = 0;\ 3x + 2y + 3 = 0$$
Solution
Let $l_1:\ 7x - y - 10 = 0$
$l_2:\ 10x + y - 41 = 0$
$l_3:\ 3x + 2y + 3 = 0$

For intersection of l_1 and l_2
$$\frac{x}{41+10} = \frac{-y}{-287+100} = \frac{1}{7+10}$$
$$\Rightarrow \frac{x}{51} = \frac{-y}{-187} = \frac{1}{17}$$
$$\Rightarrow \frac{x}{51} = \frac{1}{17} \quad \text{and} \quad \frac{y}{187} = \frac{1}{17}$$
$$\Rightarrow x = \frac{51}{17} = 3 \quad \text{and} \quad y = \frac{187}{17} = 11$$
$\Rightarrow (3,11)$ is the point of intersection of l_1 and l_2.
Now for point of intersection of l_2 and l_3
$$\frac{x}{3+82} = \frac{-y}{30+123} = \frac{1}{20-3}$$
$$\Rightarrow \frac{x}{85} = \frac{-y}{153} = \frac{1}{17}$$

$$\Rightarrow \frac{x}{85}=\frac{-y}{153}=\frac{1}{17}$$

$$\Rightarrow \frac{x}{85}=\frac{1}{17} \quad \text{and} \quad \frac{-y}{153}=\frac{1}{17}$$

$$\Rightarrow x=\frac{85}{17}=5 \quad \text{and} \quad y=-\frac{153}{17}=-9$$

$\Rightarrow (5,-9)$ is the point of intersection of l_2 and l_3.

For point of intersection of l_1 and l_3

$$\frac{x}{-3+20}=\frac{-y}{21+30}=\frac{1}{14+3}$$

$$\Rightarrow \frac{x}{17}=\frac{-y}{51}=\frac{1}{17}$$

$$\Rightarrow \frac{x}{17}=\frac{1}{17} \quad \text{and} \quad \frac{-y}{51}=\frac{1}{17}$$

$$\Rightarrow x=\frac{17}{17}=1 \quad \text{and} \quad y=-\frac{51}{17}=3$$

$\Rightarrow (1,-3)$ is the point of intersection of l_1 and l_3.

Now area of triangle having vertices $(3,11)$, $(5,-9)$ and $(1,-3)$ is given by:

$$\frac{1}{2}\begin{vmatrix} 3 & 11 & 1 \\ 5 & -9 & 1 \\ 1 & -3 & 1 \end{vmatrix}$$

$$=\frac{1}{2}\left|3(-9+3)-11(5-1)+1(-15+9)\right|$$

$$=\frac{1}{2}\left|3(-6)-11(4)+1(-6)\right| =\frac{1}{2}\left|-18-44-6\right|$$

$$=\frac{1}{2}\left|-68\right| =\frac{1}{2}(68) \quad =34 \text{ sq. unit}$$

Question # 15

The vertices of a triangle are $A(-2,3)$, $B(-4,1)$ and $C(3,5)$. Find the centre of the circum centre of the triangle?

Solution *Same Question # 7(c)*

Question # 16

Express the given system of equations in matrix form. Find in each case whether in lines are concurrent.

(a) $x+3y-2=0$; $2x-y+4=0$; $x-11y+14=0$

(b) $2x+3y+4=0$; $x-2y-3=0$; $3x+y-8=0$

(c) $3x-4y-2=0$; $x+2y-4=0$; $3x-2y+5=0$

Solution

(a)

$$x+3y-2=0$$
$$2x-y+4=0$$
$$x-11y+14=0$$

In matrix form

$$\begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Coefficient matrix of the system is

$$A=\begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix}$$

$$\Rightarrow |A|=1(-14+44)-3(28-4)-2(-22+1)$$
$$=1(30)-3(24)-2(-21)$$
$$=30-72+42=0$$

Hence given lines are concurrent.

(b)

$$2x+3y+4=0$$
$$x-2y-3=0$$
$$3x+y-8=0$$

In matrix form

$$\begin{bmatrix} 2 & 3 & 4 \\ 1 & -2 & -3 \\ 3 & 1 & -8 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Coefficient matrix of the system is

$$A=\begin{bmatrix} 2 & 3 & 4 \\ 1 & -2 & -3 \\ 3 & 1 & -8 \end{bmatrix}$$

$$\Rightarrow |A|=2(16+3)-3(-8+9)+4(1+6)$$
$$=2(19)-3(1)+4(7) \quad =38-3+28 \quad =63\neq 0$$

Hence given lines are not concurrent.

(c) *Do yourself as above*

Question # 17

Find a system of linear equations corresponding to the given matrix form .Check whether the lines responded by the system are concurrent.

$$(a)\begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & 1 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad (b)\begin{bmatrix} 1 & 1 & 2 \\ 2 & 4 & -3 \\ 3 & 6 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Solution

(a)

$$\begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & 1 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x+0-1 \\ 2x+0+1 \\ 0-y+2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \Rightarrow \begin{bmatrix} x-1 \\ 2x+1 \\ -y+2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Equating the elements

$$x-1=0$$
$$2x+1=0$$
$$-y+2=0$$

are the required equation of lines.

Coefficients matrix of the system

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & 1 \\ 0 & -1 & 2 \end{bmatrix}$$

$$\Rightarrow \det A = 1(0+1) - 0 - 1(-2-0)$$
$$= 1+2 = 3 \neq 0$$

Hence system is not concurrent.

(b) *Do yourself as above.*