Chapter 4

WORK AND ENERGY

LEARNING OBJECTIVES

At the end of this chapter the students will be able to:

- Understand the concept of work in terms of the product of a force and displacement in the direction of the force.
- Understand and derive the formula Work = wd = mgh for work done in a gravitational field near Earth's surface.
- Relate power to work done.
- Define power as the product of force and velocity.
- Quote examples of power from everyday life.
- Explain the two types of mechanical energy.
- Understand the work-energy principle.
- Derive an expression for absolute potential energy.
- Define escape velocity.
- Give examples of conservation of energies from everyday life.
- Describe some non-conventional sources of energy.

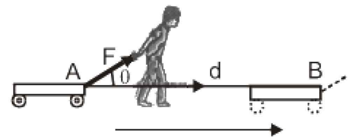
Q.1 Define the term work done. Describe the special cases when the work done is positive, negative and zero.

Ans. **WORK DONE BY A CONSTANT FORCE**

Work done on a body by a constant force is defined as;

"The product of the magnitude of the displacement and the component of the force in the direction of the displacement."

Consider, an object which is being pulled by a constant force F, at an angle θ to the direction of motion. The force moves the object from A to B through a displacement $\vec{d}$, as shown in figure.



Since

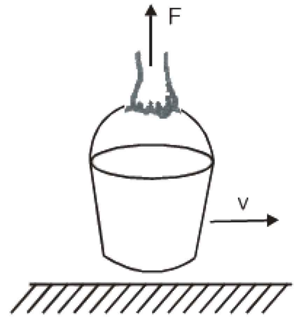
$$W = Fd$$

$$W = (F \cos\theta)\, d$$

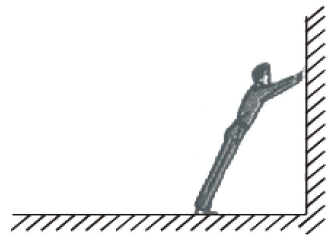
$$W = F\, d \cos\theta$$

Where $F \cos\theta$ is the component of the force in the direction of $\vec{d}$. So,

$$W = \vec{F} \cdot \vec{d}$$



(a)

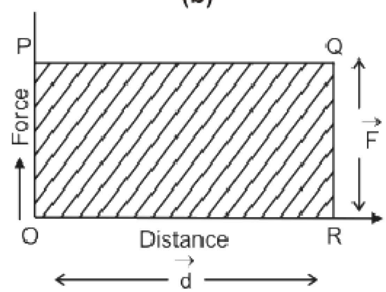


(b)

So work is also defined as; "The dot product of force $\vec{F}$ and displacement $\vec{d}$." When a constant force acts through a distance $\vec{d}$, the event can be plotted on a simple graph as show in figure. The displacement is plotted along the x-axis and the force along y-axis.

As the force is constant so the graph will be a horizontal straight line. Clearly, the shaded area in figure is also Fd.

Hence area under a force-displacement graph can be taken to represent the work done by the force. If $\vec{F}$ is not in the direction of the displacement the graph is plotted between F cos θ and d.



From the definition of work we find the following important results

(i) Work is a scalar quantity.

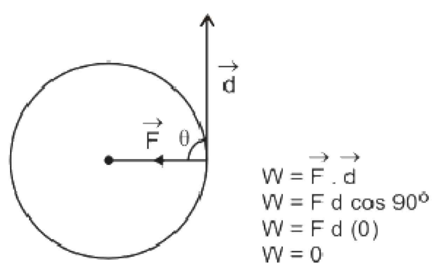
(ii) If $\theta < 90^o$, work is done and it is said to be positive work e.g., work is maximum when F is parallel to the direction of motion of the body i.e., $\theta = 0^o$.

$$W = F\, d \cos 0^o$$

$$W = Fd \times 1$$

$$W = F\, d$$

(iii) If $\theta = 90^o$, no work is done



Example

Work done by the centripetal force is 0 and when a person holding a pail by force F is moving forward then $\theta = 90^o$.

$$W = 0$$

(iv) If $\theta > 90^o$, the work done is said to be negative.

Example

When a body moves against the force of friction on a horizontal plane i.e., $\theta = 180^o$.

$$\therefore \quad W = F\, d \cos 180^o$$

$$W = F\, d\,(-1)$$

$$W = -F\, d$$

(v) SI unit of work is Nm known as joule (J).

$$\boxed{1J = 1N \times 1m}$$

Joule

The SI unit of work is Joule.

The amount of work done is 1 J when a force of 1N is acting on a body displaces it, through a distance of 1m.

$$\therefore \quad 1J = Nm$$

Dimensions

$$[W] = J$$

$$= Nm$$

$$= kg\, m / s^2 \times m$$

$$= kg\, m^2 / s^2 \qquad (\because \; F = ma, \quad N = kg\, m/s^2)$$

$$= [ML^2\, T^{-2}]$$

Q.2 Explain how can you calculate the work done by a variable force?

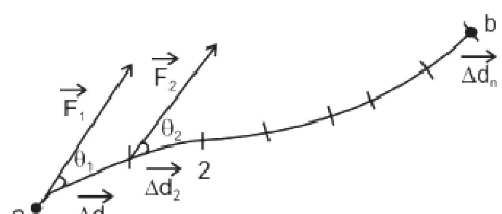
Ans. WORK DONE BY A VARIABLE FORCE

Variable Force

"If magnitude or direction of force changes during the process of the doing work, is called variable force."

Example

(i) As a rocket moves away from the earth, work is done against the force of gravity, which varies as the inverse square of the distance from Earth's centre $\left(F = \dfrac{G\,Mm}{r^2}\right)$



(ii) Force exerted by a spring increases with the amount of stretch $(F \propto x)$.

Consider the path of a particle in the xy-plane as it intervals.

During each small interval, the force is supposed to be approximately constant. So the work done by the first interval can then be written as;

$$\Delta W_1 = F_1 . \Delta d_1 = F_1 \cos\theta_1 \Delta d_1$$

and in the second interval;

$$\Delta W_2 = F_2 . \Delta d_2 = F_2 \cos\theta_2 \Delta d_2$$

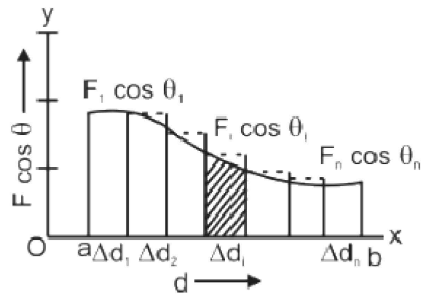
and so on. The total work done in moving the object can be calculated by adding all these terms

$$W_{total} = \Delta W_1 + \Delta W_2 + \ldots\ldots\ldots + \Delta W_n \qquad \ldots\ldots\ldots (i)$$

$$= F_1 \cos\theta_1 \Delta d_1 + F_2 \cos\theta_2 \Delta d_2 + \ldots\ldots\ldots + F_n \cos\theta_n d_n$$

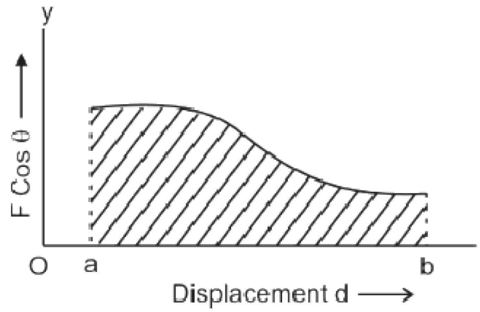
$$W_{total} = \sum_{i=1}^{n} F_i \cos\theta_i \Delta d_i \qquad \ldots\ldots\ldots (ii)$$

We can examine this graphically by plotting $F \cos\theta$ verses d as shown in Fig. The displacement d has been subdivided into n equal intervals. The value of $F \cos\theta$ at the beginning of each interval is indicated in the figure by horizontal lines.



Now the ith shaded rectangle has an area $F_i \cos\theta_i \Delta d_i$ which is the work done during the ith interval. Thus, the work done by given equation (ii) equals the sum of the areas of all the rectangles. If we subdivide the distance into a large number of intervals so that each Δd becomes very small, the work done given by equation (ii) becomes more accurate. If we let each Δd to approach zero then we obtain an exact result for the work done, such as

$$W_{total} = \lim_{\Delta d \to 0} \sum_{i=1}^{n} F_i \cos\theta_i \Delta d_i$$



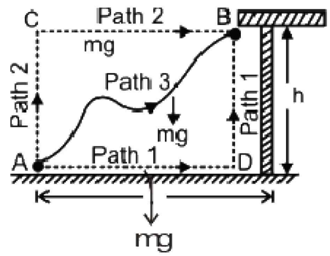
In this limit Δd approaches to zero, the total area of the rectangles approaches the area between the $F \cos\theta$ curve and d-axis from a to b as show in figure shaded.

Thus, the work done by a variable force in moving a particle between two points is equal to the area under the $F \cos\theta$ verses d curve between the two points a and b as shown in figure.

Q.3 Prove that the work done by gravitational field is independent of the path followed by the body.

Ans. WORK DONE BY GRAVITATIONAL FIELD

"The space around the Earth in which its gravitational force acts on a body, is called the gravitational field."



When an object is moved in the gravitational field, the work is done by the gravitational force. if displacement is in the direction of the gravitational force, the work is positive. If the displacement is against the gravitational force, the work is negative.

Consider an object of mass "m" being displaced with constant velocity from point A to B along various paths in the presence of a gravitational force, as shown in figure.

In this case gravitational force is equal to the weight "mg" of the object.

Work Done Along Path ADB

The work done by the gravitational field along the path ADB can be divided into two steps i.e., the 1st step from A to D, the work done along this path is zero because the weight mg is perpendicular to this path i.e.,

$$\begin{aligned} W_{AD} &= \vec{F} \cdot \vec{d} \\ &= Fd\cos 90^\circ \\ &= mgd\,(0) \\ &= 0 \end{aligned}$$

and the 2nd step is from D to B, the work done along this path is

$$\begin{aligned} W_{DB} &= \vec{F} \cdot \vec{d} \\ &= Fd\cos 180^\circ \\ &= mgh\,(-1) \\ &= -mgh \end{aligned}$$

The total work done along ADB is

$$\begin{aligned} W_{ADB} &= W_{AD} - W_{DB} \\ &= 0 + (-mgh) \\ &= -mgh \qquad \ldots\ldots \text{(i)} \end{aligned}$$

Work Done Along Path ACB

The work done by the gravitational field along the path ACB can be divided into two steps i.e., one from A to C, the work done along AC is

$$\begin{aligned} W_{AC} &= \vec{F} \cdot \vec{d} \\ &= Fd\cos 180^\circ \\ &= mgh\,(-1) \\ &= -mgh \end{aligned}$$

and the second step from C to A the work done along CA is zero because

$$W_{CA} = \vec{F} \cdot \vec{d}$$

Thus the total work done along ACB is

$$W_{ACB} = W_{AC} + W_{CB}$$
$$= -mgh + 0$$
$$W_{ACD} = -mgh \qquad \ldots\ldots \text{(ii)}$$

Work Done Along the Curved Path

Imagine the curved path, to be divided into a series of horizontal and vertical steps as shown in figure. There is no work done along the horizontal steps, because mg is perpendicular to the displacement for these steps. Work is done by the force of gravity only along the vertical displacement.

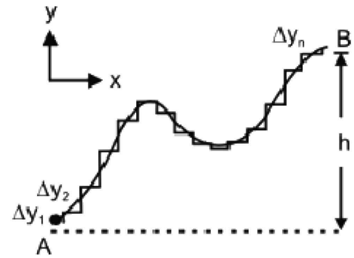
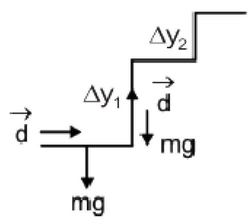


Fig. A smooth path may be replaced by a series of infinitesimal x and y displacements. Work is done only during they displacements.

$$W_1 = \vec{F} \cdot \vec{d}$$
$$= mg\,\Delta y_1 \cos 180^\circ$$
$$= -mg\,\Delta y_1$$

Similarly

$$W_2 = -mg\,\Delta y_2$$
$$\vdots \qquad \vdots \quad \vdots$$
$$\vdots \qquad \vdots \quad \vdots$$
$$W_n = -mg\,\Delta y_n$$
$$W_{AB} = W_1 + W_2 + \ldots\ldots + W_n$$
$$= -\,mg\,\Delta y_1 - mg\,\Delta y_2 \ldots\ldots mg\,\Delta y_n$$
$$= -\,mg\,(\Delta y_1 + \Delta y_2 + \ldots\ldots + \Delta y_n)$$



As $\Delta y_1 + \Delta y_2 + \ldots\ldots + \Delta y_n = h$

$\therefore$ $W_{AB} = -mg\,h \qquad \ldots\ldots \text{(iii)}$

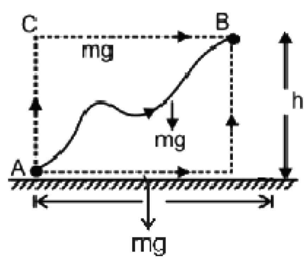
We conclude that

From eq. (i), (ii) and (iii)

"Work done in the Earth's gravitational field is independent of the path followed by the body".

Q.4 Prove that work done along the closed path in a gravitational field is zero.

Ans. Consider a closed path ABCA in a gravitational field. In order to calculate the work done along a closed path, we proceed as.



The total work done along this path is

$$W_{total} = W_{A \to B} + W_{B \to C} + W_{C \to A}$$

Since $W_{A \to B} = -mgh$

and $W_{B \to C} = \vec{F} \cdot \vec{d}$

$$= Fd \cos 90^\circ$$

and $W_{C\to A} = \vec{F} \cdot \vec{d}$

$$= Fd\cos 0°$$

$$= mgh\,(0)$$

$$= mgh$$

Thus $W_{total} = -mgh + 0 + mgh$

$$= 0$$

Thus the work done along a closed path is zero.

Conservative Field

"The field in which the work done is independent of the path followed and work done in a closed path be zero, is called a conservative field."

For Example:

Gravitational field and electrostatic field.

Note: *The frictional force is a non-conservative force, because if an object is moved over a rough surface between two points along different paths, the work done against the frictional force certainly depends on the path followed.*

Q.5 Define power and give the values of average power and instantaneous power. Also give the unit of power.

Ans. POWER

"The rate of doing work is known as power." **(OR)** Power is the measure of the rate at which work is being done. If work ΔW is done in the time interval Δt then average power P_{ave} during the interval Δt is defined as

$$P_{ave} = \frac{\Delta W}{\Delta t}$$

Instantaneous Power

If work is expressed as a function of time, the instantaneous power P at any instant is defined as:

$$P_{inst} = \lim_{\Delta t\to 0} \frac{\Delta W}{\Delta t}$$

Average power is equal to instantaneous power if work is done at constant rate.

Power and Velocity

For Your Information

Conservative Forces

Gravitational force

Elastic spring force

Electric force

Non-Conservative Forces

Frictional force

Air resistance

Tension in a string

Normal force

Propulsion force of a rocket

Propulsion force of a motor

For Your Information

Approximate Powers

Device	Power (W)
Jumbo Jet Aircraft	1.3×10^{8}

Colour TV	120
Flash light (two cells)	1.5
Pocket calculator	7.5×10^{-4}

exert, a constant force $\vec{F}$ on the boat, it moves with constant velocity $\vec{V}$. The power delivered by the motor at any instant is given by;

$$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t}$$

Since, $$\Delta W = \vec{F} \cdot \Delta \vec{d}$$

$$\therefore \quad P = \lim_{\Delta t \to 0} \frac{\vec{F} \cdot \Delta \vec{d}}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \vec{F} \cdot \frac{\Delta \vec{d}}{\Delta t}$$

$$= \vec{F} \cdot \lim_{\Delta t \to 0} \frac{\Delta \vec{d}}{\Delta t} \quad \left(\lim_{\Delta t \to 0} \frac{\Delta \vec{d}}{\Delta t} = \vec{V} \right)$$

$$P = \vec{F} \cdot \vec{V}$$

Hence power is also defined as: "The dot product of force and velocity."

Power is a scalar quantity.

Do You Know?

It takes about 9×10^9 J to make a car and the car then uses about 1×10^{12} J of energy from petrol in its lifetime

Unit

SI unit of power is watt.

Watt

"Rate of doing 1J of work in one second is called watt."

$$1\ W = 1\ J/1\ sec$$

Dimensions

$$P = \text{Watt}$$

As $$P = J/s$$

$$[P] = \frac{Nm}{Sec} \quad (\because\ 1J = Nm)$$

$$= \frac{kg\ m/s^2 \times m}{s}$$

$$= kg\ m^2/s^3$$

$$= [ML^2/T^3]$$

$$= [ML^2 T^{-3}]$$

In electrical measurements, the unit of work is watt x second

Commercial unit of electrical energy is kilowatt hour (k w h).

Kilowatt Hour

One kilowatt hour is the work done in one hour by an agency whose power is one kilowatt.

$$\therefore \quad 1\text{ k w h} = 1000\text{ W} \times 3600\text{ sec}$$
$$= 36 \times 10^5\text{ W sec}$$
$$= 36 \times 10^5\text{ J} \qquad (\because\ \text{W sec} = \text{J})$$
$$= 3.6 \times 10^6\text{ J}$$
$$= 3.6\text{ M J}$$
$$1\text{ h p} = 746\text{ watt}$$

Q.6 Define energy. What do you mean by K.E and P.E with its formula?

Ans. **ENERGY**

"Energy of a body is its capacity to do work."

For example; mechanical, heat, sound energies etc.

There are two basic forms of mechanical energy

1. Kinetic energy
2. Potential energy

Kinetic Energy

The energy possessed by a body due to its motion is called kinetic energy.

Mathematically:

$$\text{K.E.} = \frac{1}{2} m v^2$$

Potential Energy

The energy possessed by a body due to its position is called potential energy. There are two types of potential energy:

(i) Gravitational potential energy.

(ii) Elastic potential energy.

Gravitational Potential Energy

The energy possessed by a body because of its position in a force field. e.g. gravitational field. The potential energy due to gravitational field near the surface of the Earth at a height h is given by the formula.

$$\text{P.E.} = mg\,h$$

arbitrary position which is assigned the value of zero P.E. In the present case, this reference level is the surface of the Earth as position of zero P.E. In some cases a point at infinity from the Earth can also be chosen as zero reference level.

Source	Energy (J)
Burning 1 ton coal	30×10^{9}
Burning 1 litre petrol	5×10^{7}
K.E. of car at 90 km h^{-1}	1×10^{6}
Running person at 10 km h^{-1}	3×10^{2}
Fission of one atom of uranium	1.8×10^{-11}
K.E. of a molecule of air	6×10^{-21}

Elastic Potential Energy

The energy stored in a compressed spring is the potential energy possessed by the spring due to its compressed or stretched state. This form of energy is called the elastic potential energy.

Q.7 Derive work energy relation or principle.

Ans. WORK-ENERGY PRINCIPLE

This principle states that, work done on the body equals change in its K.E.

Consider a body of mass "m" moving with velocity "V_i". A force F acting through a distance d increases the velocity to V_f, then by using third equation of motion.

Tid-bits

[illegible]

$$2ad = V_f^2 - V_i^2$$

$$d = \frac{1}{2a}\left(V_f^2 - V_i^2\right) \quad \text{......... (1)}$$

As $$F = m\,a \quad \text{......... (2)}$$

Multiply equation (1) with equation (2)

$$Fd = \frac{m\,a}{2a}\left(V_f^2 - V_i^2\right)$$

$$W = \frac{1}{2}m\left(V_f^2 - V_i^2\right) \qquad (\because Fd = W)$$

$$= \frac{1}{2}m\,V_f^2 - \frac{1}{2}m\,V_i^2$$

$$= K.E_f - K.E_i$$

$$\therefore \quad W = \text{Change in K.E.}$$

This is known as work–energy principle.

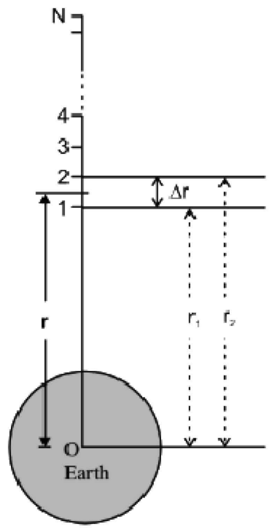
Note: *If a body is raised up from the Earth's surface, the work done changes the gravitational potential energy and if a spring is compressed, the work done on it equals the increase in its elastic potential energy.*

Q.8 Define absolute potential energy. Calculate the value of absolute potential in the gravitational field.

Ans. ABSOLUTE POTENTIAL ENERGY

Work done by the gravitational force is true only near the surface of the earth where the gravitational force is nearly constant. But if the body is displaced through a large distance from point 1 to N as shown in figure. Then the gravitational force will not remain same, since it varies inversely to the square of the distance $\left(F_g = \frac{G\,M\,m}{r^2}\right)\left(\therefore \quad F_g \propto \frac{1}{r^2}\right)$.

Thus we divide the distance between point 1 and N into small steps each of length Δr so that the force remains constant for each small step. Hence, the total work done can be calculated by adding the work done during all these steps.



If r_1 and r_2 are distances of points 1 and 2 respectively from the centre O of the Earth as shown in Fig.

The distance, between the centre of 1st step and the centre of Earth will be

$$r = \frac{r_1 + r_2}{2} \quad \ldots\ldots \ (1) \text{ (average distance)}$$

As $\quad \Delta r = r_2 - r_1$

$$r_2 = r_1 + \Delta r$$

Putting this value of r_2 in equation (1)

$$\therefore \quad r = \frac{r_1 + r_1 + \Delta r}{2}$$

$$r = \frac{2r_1 + \Delta r}{2}$$

$$r = \frac{2r_1}{2} + \frac{\Delta r}{2}$$

$$\therefore \quad r = r_1 + \frac{\Delta r}{2}$$

Squaring both sides

$$r^2 = r_1^2 + \left(\frac{\Delta r}{2}\right)^2 + 2r_1\frac{\Delta r}{2}$$

$$r^2 = r_1^2 + \frac{(\Delta r)^2}{4} + r_1\,\Delta r$$

Since Δr is very small so $\frac{\Delta r^2}{4}$ is neglected.

$$\therefore \quad r^2 = r_1^2 + r_1 \Delta r$$

Putting value of Δr

$$\therefore \quad r^2 = r_1^2 + r_1 (r_2 - r_1)$$

$$\therefore \quad r^2 = r_1^2 + r_1 r_2 - r_1^2$$

$$r^2 = r_1 r_2$$

The gravitational force F at the centre of this step is

$$F = \frac{G\,Mm}{r^2}$$

Putting value of r^2

$$\therefore \quad F = \frac{G\,Mm}{r_1 r_2}$$

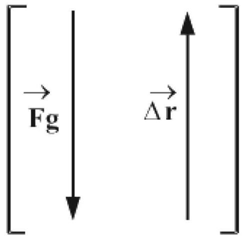
As this force is assumed to be constant during the interval Δr, so the work done is

$$W_{1 \to 2} = \vec{F} \cdot \Delta \vec{r}$$

$$= F \Delta r \cos 180^\circ$$

$$= F \Delta r (-1)$$

$$= -F \Delta r$$



Putting the value of F and Δr

$$\therefore \quad W_{1 \to 2} = -\frac{G\,Mm}{r_1 r_2}(r_2 - r_1)$$

$$= -G\,Mm\left(\frac{r_2}{r_1 r_2} - \frac{r_1}{r_1 r_2}\right)$$

$$= -G\,Mm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Similarly,

$$W_{2 \to 3} = -G\,Mm\left(\frac{1}{r_2} - \frac{1}{r_3}\right)$$

$$\vdots \qquad \vdots \qquad \vdots$$

Do You Know?

There is more energy reaching Earth in 10 days of sunlight than in all the fossil fuels on the Earth.

Tid-bits

[illegible]

Hence, $W_{total} = W_{1\to2} + W_{2\to3} + \dots\dots\dots + W_{N-1\to N}$

$$\therefore \quad W_{total} = -G\,Mm\left(\frac{1}{r_1}-\frac{1}{r_2}\right) - G\,Mm\left(\frac{1}{r_2}-\frac{1}{r_3}\right)\dots\dots\dots - G\,Mm\left(\frac{1}{r_{N-1}}-\frac{1}{r_N}\right)$$

$$W_{total} = -G\,Mm\left[\left(\frac{1}{r_1}-\frac{1}{r_2}\right)+\left(\frac{1}{r_2}-\frac{1}{r_3}\right)+\dots\dots\dots+\left(\frac{1}{r_{N-1}}-\frac{1}{r_N}\right)\right]$$

$$= -G\,Mm\left[\frac{1}{r_1}-\frac{1}{r_2}+\frac{1}{r_2}-\frac{1}{r_3}+\dots\dots\dots+\frac{1}{r_{N-1}}-\frac{1}{r_N}\right]$$

$$= -G\,Mm\left(\frac{1}{r_1}-\frac{1}{r_N}\right)$$

If the point N is situated at an infinity distance from the centre of the earth then

$$r_N = \infty$$

$$\frac{1}{r_N} = \frac{1}{\infty} = 0$$

$$\therefore \quad W_{total} = -G\,Mm\left(\frac{1}{r_1}-0\right)$$

$$W_{total} = -\frac{G\,Mm}{r_1}$$

Therefore, gravitational potential energy of a body at a distance 'r' from the center of Earth is;

$$U = -\frac{G\,Mm}{r}$$

This is also known as the absolute value of potential energy.

(i) When r increases, the gravitational force does negative work and U increases i.e., becomes less negative.

(ii) When r decreases, the body falls towards the Earth, the work, is positive and potential energy decreases i.e., becomes more negative.

(iii) The absolute P.E. on the surface of Earth is found by putting r = R (Radius of Earth).

$$\therefore \quad U = -\frac{G\,Mm}{R}$$

Negative sign shows that Earth's gravitational field for mass 'm' is attractive.

Q.9 Define escape velocity. Also derive the relation for the escape velocity.

Ans. ESCAPE VELOCITY

"The minimum initial velocity of an object with which it goes out of the Earth's gravitational field, is known as escape velocity."

The escape velocity corresponds to initial K.E. gained by the body which carries it to an infinite

We know that work done in lifting a body from Earth's surface to an infinite distance is equal to increase (change) in its potential energy. i.e.,

$$\therefore \quad \text{Increase in P.E.} = 0 - \left(-\frac{G\,Mm}{R}\right)$$

$$= G\frac{Mm}{R}$$

The body will escape out of the gravitational field if the initial K.E. of the body is equal to absolute P.E.

$$\frac{1}{2} m V_{esc}^2 = \frac{G\,Mm}{R}$$

$$V_{esc}^2 = \frac{2\,G\,M}{R}$$

Taking square root

$$V_{esc} = \sqrt{\frac{2\,G\,M}{R}} \quad \ldots\ldots\ldots (1)$$

As mass of earth is

$$M = \frac{g\,R^2}{G}$$

$$\therefore \quad G\,M = g\,R^2$$

Putting this value in equation (1)

$$\therefore \quad V_{esc} = \sqrt{\frac{2\,g\,R^2}{R}}$$

$$V_{esc} = \sqrt{2\,g\,R}$$

As $R = 6.4 \times 10^6$ m

$g = 9.8\ m/s^2$

$$\therefore \quad V_{esc} = \sqrt{2 \times 9.8 \times 6.4 \times 10^6}$$

$$V_{esc} = 11.2 \times 10^3\ m/s$$

or $V_{esc} = 11.2\ km/s$

The value of V_{esc} comes at to be approximately 11 km s^{-1}.

Note: *Escape velocity does not depend on mass of the object.*

For Your Information

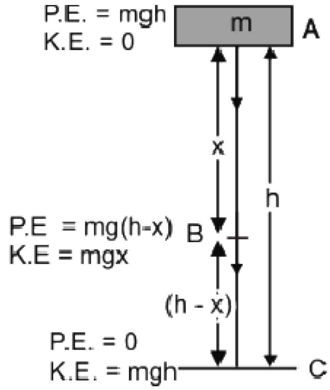
Some Escape Speeds (kms^{-1})

Heavenly body	Escape speed
Moon	2.4
Mercury	4.3
Mars	5.0
Venus	10.4
Earth	11.2
Uranus	22.4
Naptuna	25.4
Satum	37.0
Jupiter	61

Q.10 Explain the phenomena inter conversion of potential energy and kinetic energy.

Ans. INTER CONVERSION OF POTENTIAL ENERGY AND KINETIC ENERGY

Consider a body of mass 'm' at rest, at a height 'h' above the surface of the Earth as shown in figure. At the position A, the body has P.E. = mg h with respect to the earth and K.E. = 0. We release the body and as it falls, we can examine how K.E. and P.E. associated with it interchange.



$$P.E. = mg\,(h - x)$$

$$K.E. = \frac{1}{2} m V_B^2$$

V_B can be calculated by

Using $V_f^2 = V_i^2 + 2\,g\,S$ .... (1)

Here $V_f = V_B$

$V_i = 0$

$S = x$

Putting these values in equation (1)

$\therefore \quad V_B^2 = 0 + 2\,g\,x$

$= 2\,g\,x$

$\therefore$ K.E. at point B is

$\therefore \quad K.E. = \frac{1}{2} m V_B^2$

$K.E. = \frac{1}{2} m\,(2\,g\,x)$

$= mg\,x$

Total energy at B $= P.E. + K.E.$

$= mg\,(h - x) + mg\,x$

$= mg\,h - mg\,x + mg\,x$

$= mgh$

At position 'C', just before the body strikes the Earth, $P.E. = 0$ and $K.E. = \frac{1}{2} m V_C^2$ where V_C can be found out by the following expression.

$V_C^2 = V_i^2 + 2\,g\,h = 2\,g\,h$

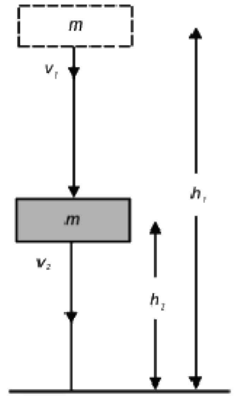
As $V_i = 0$

i.e., $K.E. = \frac{1}{2} m V_C^2$

$= \frac{1}{2} m \times 2\,g\,h$

$= mg\,h$

Thus at point C, K.E. is equal to the original value of the P.E. of the body. Actually when a body falls, its velocity increases i.e., the body is being accelerated under the action of gravity. The increase in velocity results in the increase in its K.E. On the other hand, as the body falls, its height decreases and hence, its potential energy also decreases. Thus, from figure



Loss in P.E. = Gain in K.E.

$$mg\,(h_1 - h_2) = \frac{1}{2}\,m\left(V_2^2 - V_1^2\right)$$

Where V_1 and V_2 are velocities of the body at the height h_1 and h_2 respectively. This result is truly only when frictional force is not considered.

If we assume that a frictional force 'f' is present during the downward motion, then a part of P.E. is used in doing work against friction equal to f h. The remaining P.E. = mg h – f h is converted into K.E.

Hence, $$mg\,h - f\,h = \frac{1}{2}\,m\,V^2$$

or $$mg\,h = \frac{1}{2}\,m\,V^2 + f\,h$$

Thus, Loss in P.E. = Gain in K.E. + Work done against friction.

CONSERVATION OF ENERGY

The kinetic and potential energies are both different forms of the same basic quantity, i.e., mechanical energy. Total mechanical energy of a body is the sum of the kinetic energy and potential energy. In our previous discussion of a falling body, potential energy may change into kinetic energy and vice versa, but the total energy remains constant. Mathematically,

Total energy = P.E. + K.E. = Constant

This is a special case of the law of conservation of energy which states that:

"Energy cannot be destroyed. It can be transformed from one kind into another, but the total amount of energy remains constant".

This is one of the basic laws of physics. We daily observe many energy transformations from one form to another. Some forms, such as electrical and chemical energy, are more easily transferred than others, such as heat. Ultimately all energy transfers result in heating of the environment and energy is wasted. For example, the P.E. of the falling object changes to K.E., but on striking the ground, the K.E. changes into heat and sound. If it

For Your Information

Source of energy	Original source
Solar	Sun
Bio mass	Sun
Fossil fuels	Sun
Wind	Sun
Waves	Sun
Hydro electric	Sun
Tides	Moon
Geothermal	Earth

Energy Sources

Renewable	Non-renewable
Hydroelectric	Coal
Wind	Natural gas
Tides	Oil
Geothermal*	Uranium
Biomass	Oil shale
Sunlight	Tar sands
Ethanol/Methanol**	

* Individual fields may run off

Q.11 Describe briefly the non-conventional energy sources.

Ans. **NON-CONVENTIONAL ENERGY SOURCES**

These are the energy sources not very common these days. However, it is expected, that these sources will contribute substantially to the world energy demand of the future. Some of these are introduced briefly here.

Energy From Tides

One very novel example of obtaining energy from gravitational field is the energy obtained from tides. Gravitational force of the moon gives rise to tides in the sea. The tides raise the water in the sea roughly twice a day. If the water at the high tide is trapped in a basin by constructing a dam, then it is possible to use this as a source of energy. The dam is filled at high tide and water is released in a controlled way at low tide to drive the turbines. At the next high tide the dam is filled again and the in rushing water also drives turbines and generates electricity as shown systematically in the figure.

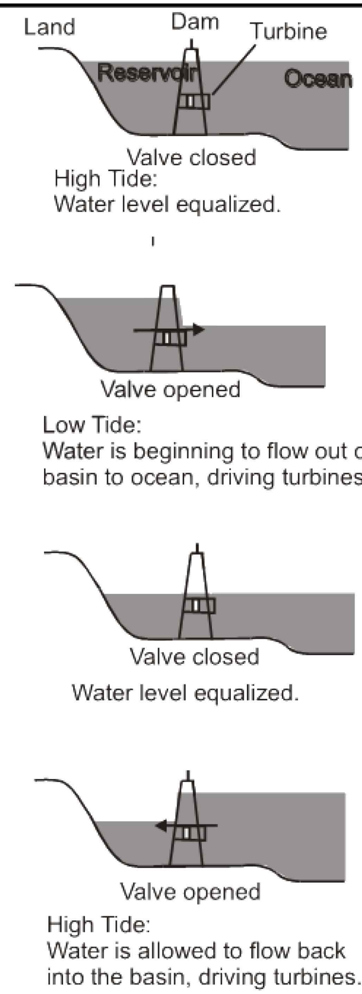
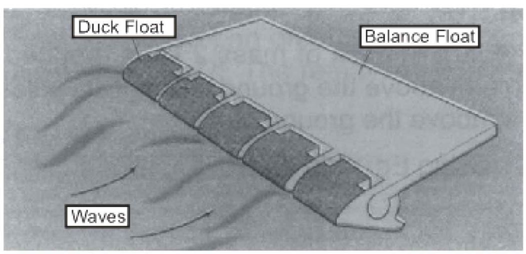


Fig. Tidal power plant. Turbine are located inside the dam.

Energy From Waves

The tidal movement and the winds blowing across the surface of the ocean produce strong water waves. Their energy can be utilized to generate electricity. A method of harnessing wave energy is to use large floats which move up and down with the waves. One such device invented by Professor Salter is known Salter's duck (figure). It consists of two parts (i) Duck float (ii) Balance float.



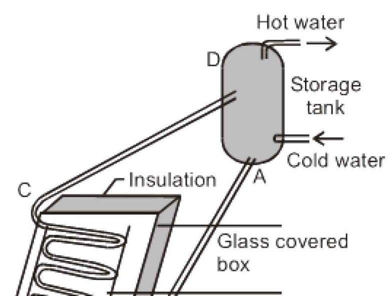
The wave energy makes duck float move relative to the balance float. The relative motion of the duck float is then used to run electricity generators.

Do You Know?

The pull of the Moon does not only pull the sea up and down. This tidal effect can also distort the continents pulling land up and down by as much as 25cm.

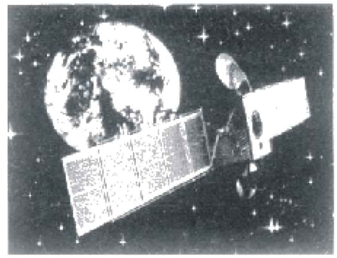
Solar Energy

The Earth receives huge amount of energy directly from the Sun each day. Solar energy at normal incidence outside the Earth's atmosphere is about 1.4 kWm^{-2} which is referred as solar constant. While passing through the atmosphere, the total energy is reduced due to reflection, scattering and absorption by dust particles, water vapours and other gases. On a clear day at noon, the intensity of the solar energy reaching the Earth's surface is about 1kWm^{-2}. This energy can be used directly to heat water using large solar reflectors and thermal absorbers or be converted to electricity. In one method



Much higher temperature can be achieved by concentrating solar radiation on to a small surface area by using huge reflectors (mirrors) or lenses to produced steam for running a turbine.

The other method is the direct conversion of sunlight into electricity through the use of semi conductor devices called solar cells also known as photo voltaic cells. Solar cells are thin wafers made from silicon. Electrons in the silicon gain energy from sunlight to create a voltage. The voltage produced by a single voltaic cell is very low. In order to get sufficient high voltage for practical use, a large number of such cells are connected in series forming a solar cell panel.



(b)

For cloudy days or nights, electric energy can be stored during the Sun light in Nickle cadmium batteries by connecting them to solar panels. These batteries can then provide power to electrical appliances at nights or on cloudy days.

Solar cells, although, are expensive but last a long time and have low running cost. Solar cells are used to power satellites having large solar panels which are kept facing the Sun figure (b). Other examples of the use of solar cells are remote ground based weather stations and rain forecast communication systems. Solar calculators are also in use now a days.

Q.12 Explain the energy from biomass.

Ans. ENERGY FROM BIOMASS

Biomass is a potential source of renewable energy. This includes all the organic materials such as crop residue, natural vegetation, trees, animal dung and sewage. Biomass energy or bio conversion refers to the use of this material as fuel or its conversion into fuels.

There are many methods used for the conversion of biomass into fuels. But the most common are:

(1) Direct combustion.

(2) Fermentation.

Direct combustion method is usually applied to get energy from waste products commonly known as solid waste. It will be discussed in the next section.

Biofuel such as ethanol (alcohol) is a replacement of gasoline. It is obtained by fermentation of biomass using enzymes and by decomposition through bacterial action in the absence of air (oxygen).

The rotting of biomass in a closed tank called a digester produces Biogas which can be piped out to use for cooking and heating (figure).

For Your Information

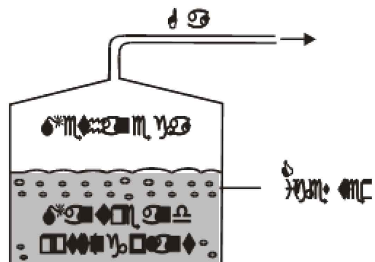
The rapid growth of human population has put a strain on our natural resources. A sustainable society minimizes waste and maximizes the benefit from each resource. Minimizing the use of energy is an other method of conservation we can save energy by,

(i) turning off lights and electrical appliances when not in use

(ii) using fluorescent bulbs instead of incandescent bulbs

(iii) using sunlight in offices, commercial centers and houses during daylight hours

(iv) taking short hot shawers.



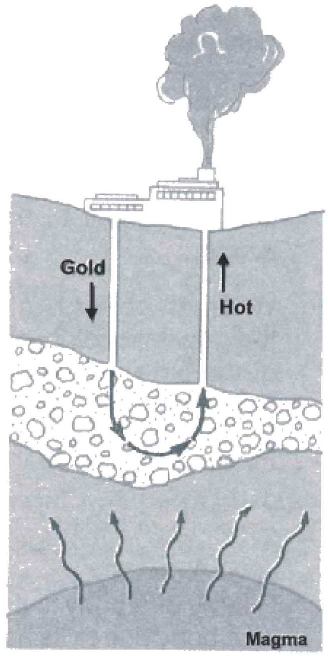
Energy from Waste Products

Waste products like wood waste, crop residue, and particularly municipal solid waste can be used to get energy by direct conversion. It is probably the most commonly used conversion process in which waste material is burnt in a confined container. Heat produced in this way is directly utilized in the boiler to produce steam that can run turbine generator.

Q.13 Explain geothermal energy.

Ans. ## GEOTHERMAL ENERGY

This is the heat energy extracted from inside the Earth in the form of hot water or steam. Heat within the Earth is generated by the following processes.



1. Radioactive Decay

The energy, heating the rocks, is constantly being released by the decay of radioactive elements.

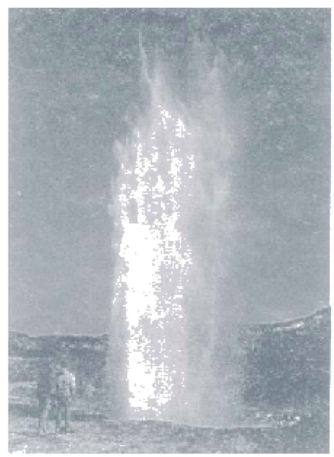
2. Residual Heat of the Earth

At some places hot igneous rocks, usually within 10 km of the Earth's surface, are in a molten and partly molten state. They conduct heat energy from the Earth's interior which is still very hot. The temperature of these rocks is about 200°C or more.

3. Compression of Material

The compression of material deep inside the Earth also causes generation of heat energy.

In some place water beneath the ground is in contact with hot rocks and is raised to high temperature and pressure. It comes to the surface as hot springs, geysers, or steam vents. The steam can be directed to turn turbines of electric generators.



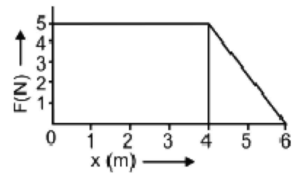
Where water is not present and hot rocks are not very deep, the water is pumped down through them which returns as steam (figure). The steam then can be used to drive turbines or for direct heating.

An interesting phenomenon of geothermal energy is a geyser. It is a hot spring that discharges steam and hot water, intermittently releasing an explosive column into the air (figure). Most geysers erupt at irregular intervals. They usually occur in volcanic regions. Extraction of geothermal heat energy often occurs closer to geyser sights. This extraction seriously disturbs geyser system by reducing heat flow and aquifer pressure. Aquifer is a layer of rock holding water that allows water to percolate through it with pressure.

SOLVED EXAMPLES

EXAMPLE 4.1

A force F acting on an object varies with distance x as shown in the figure. Calculate the work done by the force as the object moves from x = 0 to x = 6 m.



Data

Force = F

Distance = x

To Find

Work done = W = ?

SOLUTION

[Total area under the graph] = (Area of rectangle) + (Area of triangle)

$$= (4 \times 5) + \frac{1}{2}(6-4)(5)$$

$$= 20 + \frac{1}{2}(2)(5)$$

$$= 20 + 5 = 25 \text{ J}$$

Since area under F–d graph is equal to work done

Hence, W = 25 J

Result

Work done by the force = W = 25 J

EXAMPLE 4.2

A 70 kg man runs up a long flight of stairs in 4.05 s. The vertical height to the stairs is 4.5m calculate his power out put in watts.

Data

Mass of man = m = 70 kg

= t = 4 sec

Height of stairs = h = 4.5 m

To Find

Power output = P = ?

SOLUTION

Since $W = P.E. = mgh$

$\therefore$ $P = \dfrac{mgh}{t}$

$= \dfrac{70 \times 9.8 \times 4.5}{4}$

$= 7.7 \times 10^2$ watt

Result

Power output $= P = 7.7 \times 10^2$ watt

EXAMPLE 4.3

A brick of mass 2.0 kg is dropped from a rest position 5.0 m above the ground. What is its velocity at a height of 3.0 m above the ground?

Data

Mass of brick $= m = 2$ kg

rest position $= h_1 = 5$ m

At a height $= h_2 = 3$ m

To Find

Velocity $= V = ?$

SOLUTION

Using Loss of P.E. = Gain in K.E.

$$mg(h_1 - h_2) = \frac{1}{2} m (V_2^2 - V_1^2)$$

$\therefore$ $V_1 = 0$ and $V_2 = V$

$$\therefore \quad mg(h_1 - h_2) = \frac{1}{2} m (V^2 - 0^2)$$

$$g(h_1 - h_2) = \frac{1}{2} V^2$$

$$V^2 = 2g(h_1 - h_2)$$

Taking square root

$$V = \sqrt{2g(h_1 - h_2)}$$

Putting values

$$V = \sqrt{2 \times 9.8 (5 - 3)}$$

$$= \sqrt{2 \times 9.8 \times 2}$$

$$= \sqrt{39.2}$$

$$V = 6.26 \text{ m/s}$$