Chapter 5

CIRCULAR MOTION

LEARNING OBJECTIVES

At the end of this chapter the students will be able to:

- Describe angular motion.
- Define angular displacement, angular velocity and angular acceleration.
- Define radian and convert an angle from radian measure to degree and vice versa.
- Derive the equation $a_c = r\omega^2 = v^2/r$ and $F_c = m\omega^2\ r = mv^2/r$.
- Understand and describe moment of inertia of a body.
- Understand the concept of angular momentum.
- Describe examples of conservation of angular momentum.
- Describe the motion of artificial satellites.
- Understand that the objects in satellites appear to be weightless.
- Understand that how and why artificial gravity is produced.
- Calculate the radius of geo-stationary orbits and orbital velocity of satellites.
- Describe Newton's and Einstein's views of gravitation.

Q.1 Define circular motion.

Ans. CIRCULAR MOTION

When a body moves in such a way that its distance from the fixed point remains constant then such a motion is called circular motion.

For Example:

(i) Motion of earth around the sun.

(ii) Motion of moon around the earth.

Q.2 Explain angular displacement with its units.

Ans. ANGULAR DISPLACEMENT

The angle through which a particle moves in an interval of time while moving along a circle is called angular displacement.

Consider the motion of a single particle P of mass m in a circular path of radius r. Suppose this motion is taking place by attaching the particle P at the end of a mass less rigid rod of length r whose other end is pivoted at the centre O of the circular path, as shown in Fig. (i). As the particle is moving on the circular path, the rod OP rotates in the plane of the circle. The axis of rotation passes through the pivot O and is normal to the plane of rotation. Consider a system of axes as shown in Fig. (ii). The z–axis is taken along the axis of rotation with the pivot O as origin of coordinates. Axes x and y are taken in the plane of rotation. While OP is rotating, suppose at any instant t, its position is OP_1, making angle θ with x-axis. At later time $t + \Delta t$, let its position be OP_2 making angle $\theta + \Delta\theta$ with x-axis Fig. (iii).

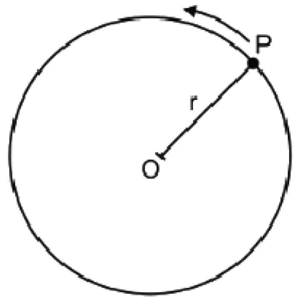


Fig. (i)

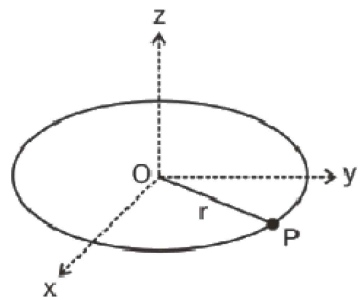


Fig. (ii)

Angle $\Delta\theta$ defines the angular displacement of OP during the time interval Δt.

The angular displacement $\Delta\theta$ is assigned a positive sign when the sense of rotation of OP is counter clockwise.

For very small values of $\Delta\theta$, the angular displacement is a vector quantity.

The direction associated with $\Delta\theta$ is along the axis rotation and is given by right hand rule which states that

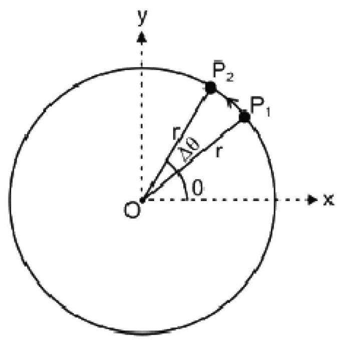


Fig. (iii)

Right Hand Rule

Grasp the axis of rotation in right hand with fingers curling in the direction of rotation; the thumb points in the direction of angular displacement, as shown in Fig. (iv).

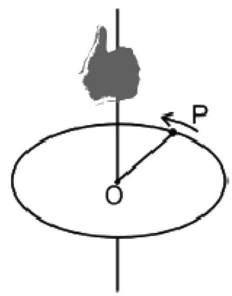


Fig. (iv)

Unit

There are three units of angular displacement

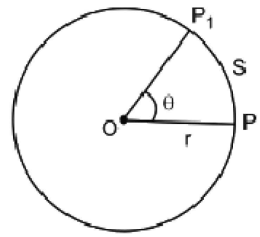
(i) degree

(ii) revolution

(iii) radian

But radian is SI unit. The angular displacement is one radian if the angle between two radii of a circle which cut off on the circumference an arc equal in length to the radius.

Q.3 Show that $S = r\theta$



Ans. Consider an arc of length S of a circle of radius r which subtends an angle θ at the centre of the circle. Its value in radian is given by

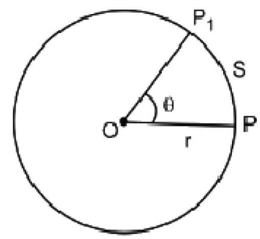
$$\theta = \frac{\text{Arc length}}{\text{Radius}} \text{ (rad)}$$

$$\theta = \frac{S}{r}$$

$$S = r\theta$$

Q.4 Show that 1 radian = 57.3°.

Ans. If 'OP' is rotating, the point 'P' covers a distance S = 2πr in one revolution of P. In radian, it is



$$\frac{S}{r} = \frac{2\pi r}{r} = 2\pi \text{ radian}$$

$$1 \text{ revolution} = 2\pi \text{ rad.} = 360°$$

$$1 \text{ rad.} = \frac{360°}{2\pi}$$

$$1 \text{ rad.} = 57.3°$$

$$\therefore \quad 2\pi \text{ radian} = 360°$$

$$1 \text{ radian} = \frac{360°}{2\pi}$$

$$= \frac{180°}{3.14}$$

$$1 \text{ radian} = 57.3°$$

Q.5 Define angular velocity and instantaneous angular velocity with its units.

Ans. **ANGULAR VELOCITY**

"The rate of change of angular displacement is called angular velocity". **(OR)** The angular velocity is also defined as the rate at which the angular displacement is changing with time. It is denoted by ω. If Δθ is the angular displacement during the time interval Δt, the average angular velocity during the interval is

$$\omega_{ave} = \frac{\Delta\theta}{\Delta t}$$

It is a vector quantity. Its direction is along the axis of rotation and given by right hand rule.

Dimensions

$$[\omega] = [T^{-1}]$$

The instantaneous angular velocity ω is the limit of the ratio $\frac{\Delta\theta}{\Delta t}$ as Δt following instant t, approaches to zero. Thus

$$\omega_{ins} = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$$

Units

Angular velocity measured in radians per second which is SI units. Sometime it is also measures in terms of **revolution per minute**.

Q.6 Define angular acceleration and instantaneous angular acceleration with its units.

Ans. **ANGULAR ACCELERATION**

The rate of change of angular velocity is called angular acceleration.

It is denoted by α.

It is a vector quantity. Its direction is along axis of rotation and given by right hand rule. If ω_i and ω_f are the value of instantaneous velocity of a rotating body at time t_i and t_f then, the average angular acceleration during the intervals $t_f - t_i$ is given by

$$\alpha_{ave} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t}$$

$$\therefore \quad \alpha = \frac{\Delta\omega}{\Delta t}$$

Unit

Its SI unit is radian per sec^2.

Dimensions

$$[\alpha] = [T^{-2}]$$

Instantaneous Angular Acceleration

The instantaneous angular acceleration is the limit of the ratio $\frac{\Delta\omega}{\Delta t}$ as Δt approaches to zero then mathematically

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t}$$

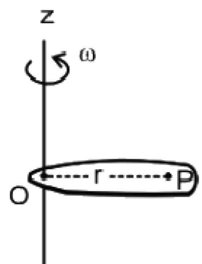
Explanation

Till now we have been considering the motion of a particle P on a circular path. The point P was fixed at the end of a rotating massless rigid rod. Now we consider the rotation of a rigid body as shown in figure. Imagine a point P on the rigid body. Line OP is the perpendicular dropped from P on the axis of rotation. It is usually referred as reference line. As the body rotates, line OP also rotates with it with the same angular velocity and angular acceleration. Thus the rotation of a rigid body can be described by

Q.7 Show that $V = r\omega$ (relation between linear and angular velocity).

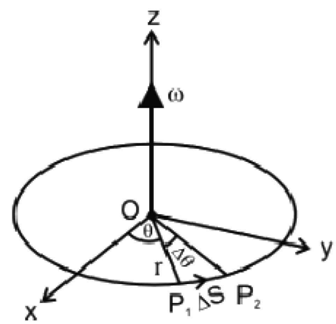
Ans. Consider a rigid body rotating about z-axis with an angular velocity ω as shown in Fig. (a).

Imagine a point P in the rigid body at a perpendicular distance r from the axis of rotation. OP represents the reference line of the rigid body. As the body rotates, the point P moves along a circle of radius r with a linear velocity v whereas the line OP rotates with angular velocity ω as shown in Fig. (b). We are interested in finding out the relation between ω and v. As the axis of rotation is fixed, so the direction of ω always remains the same and ω can be manipulated as a scalar. As regards the linear velocity of the point P, we consider its magnitude only which can also be treated as a scalar.



(a)

Suppose during the course of its motion, the point P moves through a distance $P_1P_2 = \Delta S$ in a time interval Δt during which reference line OP has an angular displacement $\Delta\theta$ radian during this interval. ΔS and $\Delta\theta$ are related by equation.



(b)

$$\Delta S = r\Delta\theta$$

Dividing both sides by Δt

$$\frac{\Delta S}{\Delta t} = r\frac{\Delta\theta}{\Delta t} \qquad \ldots\ldots (1)$$

In the limit when $\Delta t \to 0$ the ratio $\frac{\Delta S}{\Delta t}$ represents v, the magnitude of the velocity with which point P is moving on the circumference of the circle. Similarly $\Delta\theta/\Delta t$ represents the angular velocity ω of the reference line OP. So Eq. (1) becomes

$$v = r\omega \qquad \ldots\ldots (2)$$

In Fig. (b), it can be seen that the point P is moving along the arc P_1P_2. In the limit when $\Delta t \to 0$, the length of arc P_1P_2 becomes very small and its direction represents the direction of tangent to the circle at point P_1. Thus the velocity with which point P is moving on the circumference of the circle has a magnitude v and its direction is always along the tangent to the circle at that point. That is why the linear velocity of the point P is also known as tangential velocity.

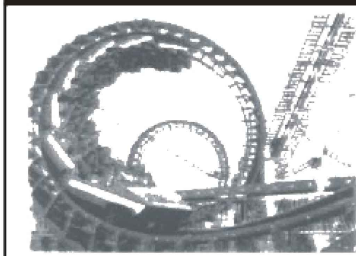
Q.8 Show that $a_t = r\alpha$ (relation between linear and angular acceleration).

Ans. If the reference line OP is rotating with an angular acceleration α, the point P will also have a linear or tangential acceleration a_t. Using Eq. (1) it can be shown that the two accelerations are related by

$$a_t = r\alpha \qquad \ldots\ldots (1)$$

Eqs. (1) and (2) show that on a rotating body, points that are at different distances from the axis do not have the same speed or acceleration, but all points on a rigid body displacement, angular speed and angular acceleration at any instant. Thus by the use of angular variables we can describe the motion of the entire body in a simple way.

Point to Ponder

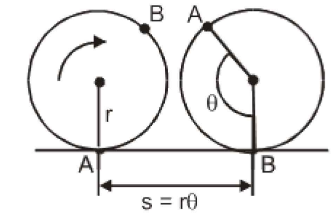


You may feel scared at the top of

in linear motion except that θ, ω and α have replaced S, v and a, respectively. As the other equations of linear motion were obtained by algebraic manipulation of these equations, it follows that analogous equations will also apply to angular motion. Given below are angular equations together with their linear counterparts.

Linear	Angular	
$v_f = v_i + at$	$\omega_f = \omega_i + \alpha t$	...... (1)
$2aS = v_f^2 - v_i^2$	$2\alpha\theta = \omega_f^2 - \omega_i^2$	...... (2)
$S = v_i t + \frac{1}{2} at^2$	$\theta = \omega_i t + \frac{1}{2} \alpha t^2$	...... (3)

Do You Know?



As the wheel turns through an angle, it lays out a tangential distance S = rθ.

The angular equations (1) to (3) hold true only in the case when the axis of rotation is fixed, so that all the angular vectors have the same direction. Hence they can be manipulated as scalars.

Q.9 What is centripetal force? Also derive the expression for centripetal force.

Ans. CENTRIPETAL FORCE

The force needed to bend the normally straight path of the particle into a circular path is called the centripetal force.

It is denoted by F_C and mathematically

$$F_C = \frac{m V^2}{r}$$

Explanation

If the particle moves from A to B with uniform speed V as shown in Fig. (i). The velocity of the particle changes its direction but not its magnitude. The change in velocity is shown in Fig. (ii). Hence acceleration of the particle is

$$a = \frac{\Delta V}{\Delta t} \quad .... (1)$$

Where Δt is the time taken by the particle to travel from A to B. Let the velocities at A and B are $\vec{V}_1$ and $\vec{V}_2$. Since speed of the particle is V, so the time taken to travel a distance S, as shown in Fig. (i) is

$$\Delta t = \frac{S}{V} \qquad (\because S = V t)$$

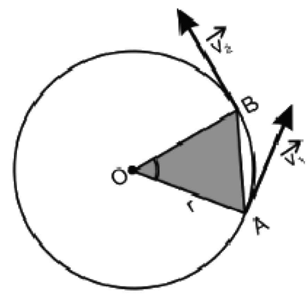


Fig. (i)

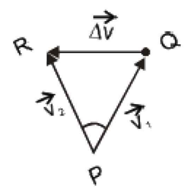


Fig. (ii)

Put in equation (1)

$$\text{So} \quad a = V\frac{\Delta V}{S} \quad .... (2)$$

Let us now draw a triangle PQR such that PQ is parallel and equal to $\vec{V}_1$ and PR is parallel and equal to $\vec{V}_2$, as shown in Fig. (ii). We know that the radius of a circle is perpendicular to its tangent, so OA is perpendicular to $\vec{V}_1$ and OB is perpendicular to $\vec{V}_2$ Fig. (i). Therefore, angle AOB between their equal arms are equal. Hence, the triangle OAB Fig. (i) is similar to triangle PQR Fig. (ii). Hence, we can write

$$\frac{\Delta V}{AB} = \frac{V}{r}$$

$$\frac{\Delta V}{V} = \frac{AB}{r}$$

If the point B is close to the point A on the circle, as will be the case when $\Delta t \rightarrow 0$, the arc AB is of nearly the same length as the line AB. To that approximation,

$$AB = S$$

$$\therefore \quad \frac{\Delta V}{V} = \frac{S}{r}$$

$$\therefore \quad \Delta V = S\frac{V}{r}$$

Putting this value in the equation (2), we get

$$a = \frac{V}{S}\, S\, \frac{V}{r}$$

$$\therefore \quad a = \frac{V^2}{r} \qquad \ldots\ldots\ldots (3)$$

Where a is the instantaneous acceleration. As this acceleration is caused by the centripetal force, it is called **centripetal acceleration** denoted by a_c. This acceleration is directed along the radius towards the centre of the circle. In Fig. (i) and (ii), since PQ is perpendicular to OA and PR is perpendicular to OB, so QR is perpendicular to AB. It may be noted that QR is parallel to the perpendicular bisector of AB. As the acceleration of the object moving in the circle parallel to ΔV when $AB \rightarrow 0$, so centripetal acceleration is directed along radius towards the centre of the circle. It can, concluded that

The instantaneous acceleration of an object traveling with uniform speed in a circle is directed towards the centre of the circle and is called centripetal acceleration.

The centripetal force has the same direction as the centripetal acceleration and its value is given by

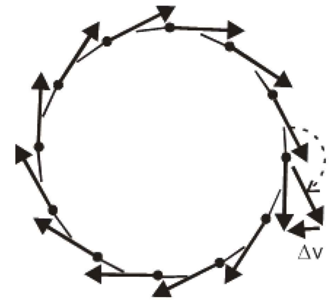
$$F_C = m\, a_c = \frac{m V^2}{r}$$

In angular motion.

Put $\quad V = r\,\omega$

then $\quad F_C = \frac{mr\omega^2}{r}$

Do You Know?

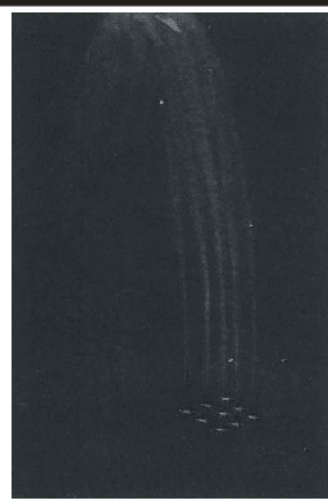


Direction of motion changes continuously in circular motion.

Tid-bits

Banked tracks are needed for turns that are taken so quickly that friction alone cannot provide energy for centripetal force.

Do You Know?



Curved flight at high speed requires a large centripetal force that makes the stunt dangerous even if the air planes are not so close.

Ans. MOMENT OF INERTIA

Consider a mass m attached to a massless rod at O as shown in Fig. (i). Let us assume that the bearing at the pivot O is frictionless and that the mass of the rod is negligible. Let the system be in a horizontal plane. A force F is acting on the mass perpendicular to the rod and hence this will accelerate the mass according to

$$F = ma$$

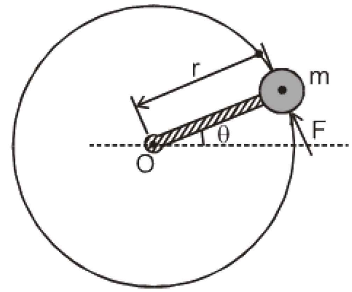


Fig. The force F causes a torque about the axis O and gives the mass m an angular acceleration about the pivot point.

In doing so the force will cause the mass to rotate about O. Since tangential acceleration a_T is related to angular acceleration α by the equation.

$$a_T = r\alpha$$

So, $F = m r \alpha$

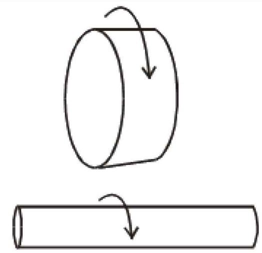
Multiply both sides by r

$\therefore \quad rF = \tau = \text{torque} = m r^2 \alpha$

which is rotational analogue of the Newton's second law of motion, F = ma.

Here F is replaced by τ, a by α and m by mr^2. The quantity $m r^2$ is known as the moment of inertia and is represented by I. It may be defined as the product of mass and square of distance from the point of rotation. The moment of inertia plays the same role in angular motion as the mass in linear motion. It may be noted that moment of inertia depends not only on mass m but also on r^2.

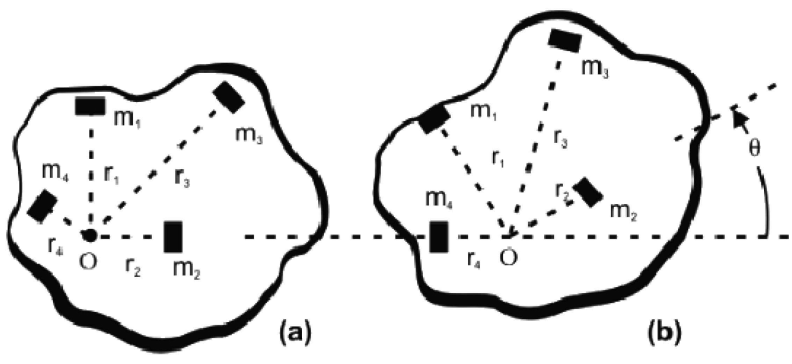
Do You Know?



Two cylinders of equal mass. The one with the larger diameter has the greater rotational inertia.

Moment of Inertia of a Rigid Body

Most rigid bodies have different mass concentration at different distances from the axis of rotation, which means mass distribution is not uniform. As shown in figure, rigid body is made up of n small pieces of masses.



For Your Information

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$m_1, m_2, \ldots\ldots, m_n$ at distance $r_1, r_2, \ldots\ldots, r_n$ from the axis of rotation O. Let the body be rotating with the angular acceleration α, so the magnitude of the torque acting on m_1 is

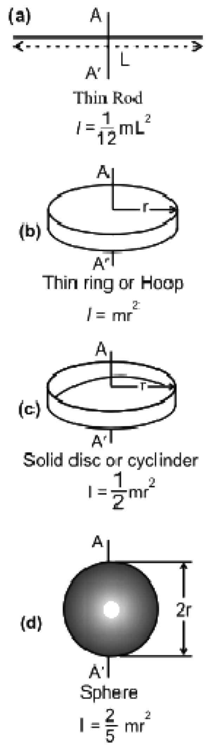
$$\tau_2 = m_2 r_2^2 \alpha_2$$

and so on.

Hence the body is rigid, so all the masses are rotating with same angular acceleration α,

Total torque τ_{total} is then given by

$$\tau_{total} = (m_1 r_1^2 + m_2 r_2^2 + \ldots\ldots + m_n r_n^2)\,\alpha$$



$$= \left(\sum_{i=1}^{n} m_i r_i^2\right)\alpha$$

$$\tau = I\alpha$$

where $$I = \sum_{i=1}^{n} m_i r_i^2$$

Q.11 Explain angular momentum. Derive a relation between angular momentum and moment of inertia.

Ans. ANGULAR MOMENTUM

Angular momentum is the product of linear momentum and moment arm for momentum. **(OR)** A particle is said to possess an angular momentum about a reference axis if it so moves that its angular position changes relative to that reference axis.

The angular momentum L of a particle of mass m moving with velocity V and momentum p relative to origin O is defined as

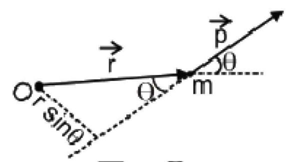


Fig (i)

$$\vec{L} = \vec{r} \times \vec{P}$$

where $\vec{r}$ is the position vector of the particle at that instant relative to the origin O. Angular momentum is a vector quantity. Its magnitude

$$L = rp\sin\theta = m\,r\,V\sin\theta \quad (\because \; P = mV)$$

If the particle is moving in a circle of radius r with uniform angular velocity ω, then angle between r and tangent velocity is 90^{o}. Hence

$$L = m\,r\,V \sin 90^{o} = m\,r\,V$$

But $V = r\,\omega$

Hence $L = m\,r^2\,\omega$

$$L = I\omega \qquad (\because I = mr^2)$$

Now consider a symmetric rigid rotating about a fixed axis through the centre of mass as shown in Fig. (ii). Each particle of the rigid body rotates about the same axis in a circle with an angular velocity ω. The magnitude of the angular momentum of the particle of mass m_i is $m_i V_i r_i$ about the origin O. The direction of L_i is the same as that of ω. Since $V_i = r_i\,\omega$, the angular momentum of the ith particle is $m_i r_i^2 \omega$. Summing this over all particles gives the total angular momentum of the rigid body.

Q. Show that $Kgm^2/s = Js$.

Ans. Taking:

$L.H.S = Kg\,m^2/s$

Multiply and divide by s

$$= Kgm^2/s \times \frac{s}{s}$$

$$= Kgm^2/s^2 \times s$$

$$= Kgm/s^2\; ms$$

$$= Nms \quad (\because Nm = J)$$

$$= Js$$

$$= R.H.S$$

$$L = \left(\sum_{i=1}^{n} m_i\, r_i^2\right) = I\,\omega$$

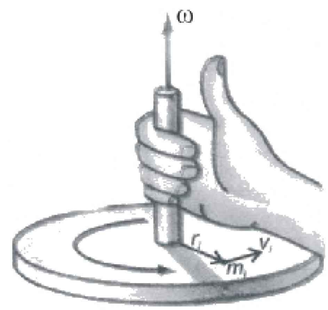


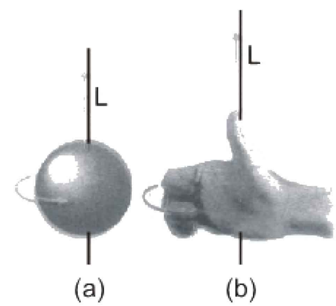
Fig. (ii)

Where I is the moment of inertia of the rigid body about the axis of rotation.

Physicists usually make a distinction between spin angular momentum (L_s) and orbital angular momentum (L_c). The spin angular momentum is the angular momentum of spinning body, while orbital angular momentum is associated with the motion of a body along a circular path.

The difference is illustrated in Fig. (iii). In the usual circumstances concerning orbital angular momentum, the orbital radius is large as compared to the size of the body, hence, the body may be considered to be a point object.

For Your Information



The sphere in (a) is rotating in the sense given by the gold arrow. Its angular velocity and angular momentum are taken to be upward along the rotational axis, as shown by the right-hand rule in (b).

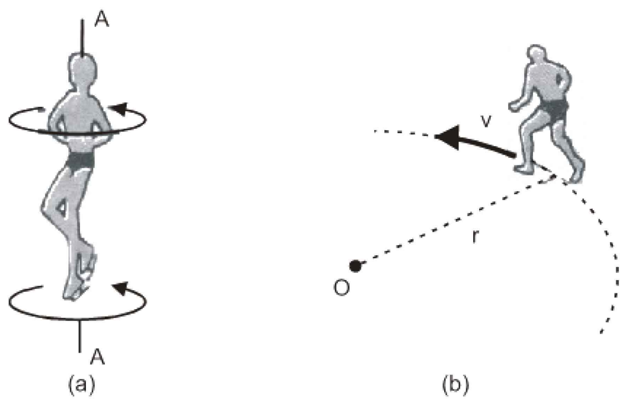


Fig. (iii)

Ans. LAW OF CONSERVATION OF ANGULAR MOMENTUM

The law of conservation of angular momentum states that if no external torque acts on a system, the total angular momentum of the system remains constant.

$$L_{total} = L_1 + L_2 + \ldots\ldots = \text{constant}$$

or

$$I\,\omega = \text{Constant}$$

This is illustrated by the diver in Fig. (i). The diver pushes off the board with a small angular velocity about a horizontal axis through his centre gravity G. Upon lifting off from the board, the diver's legs and arms are fully extended which means that the diver has a large moment of inertia I_1 about this axis. The moment of inertia is considerably reduced to a new value I_2, when the legs and arms are drawn into the closed position. As the angular momentum is conserved, so

$$I_1\,\omega_1 = I_2\,\omega_2$$

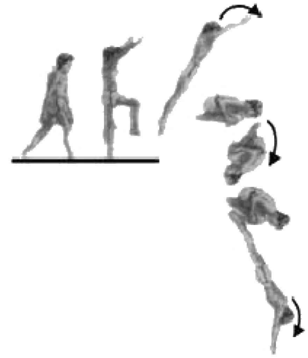
Hence, the diver must spin faster when moment of inertia becomes smaller to conserve angular momentum. This enables the diver to take extra somersaults.

The angular momentum is a vector quantity with direction along axis of rotation.

The direction of angular momentum along axis of rotation also remain fixed. This is illustrated by the fact given below.

The axis of rotation of an object will not change its orientation unless an external torque causes it to do so.

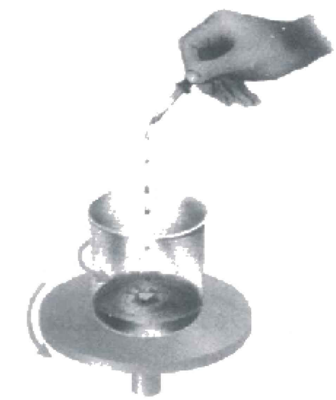
This fact is of great importance for the Earth as it moves around the Sun. no other sizeable torque is experienced by the Earth, because the major force acting on it is the pull of the Sun. The Earth's axis of rotation, therefore, remains fixed in one direction with reference to the universe around us.



A man diving from a diving board.

Fig. (i)

Point to Ponder



Why does the coasting rotating system slow down as water drips into the beaker?

Do You Know?

The law of conservation of angular momentum is important in many sports, particularly in diving gymnastics and ice skating

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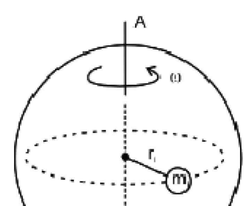
Q.13 Define rotational kinetic energy. Also calculate the expression for rotational K.E.

Ans. ROTATIONAL KINETIC ENERGY

If a body is spinning about an axis with constant angular velocity ω, each point of the body is moving in a circular path and, therefore, has some K.E. To determine the total K.E. of a spinning body, we imagine it to be composed of tiny pieces of mass m_1, m_2, …… If a piece of mass m_i is at a distance r_i from the axis of rotation,



Thus the K.E. of this piece is

$$K.E_i = \frac{1}{2} m_i V_i^2 = \frac{1}{2} m_i (r_i \omega)^2$$

$$K.E_i = \frac{1}{2} m_i r_i^2 \omega^2$$

The rotational K.E. of the whole body is the sum of the kinetic energies of all the parts. So we have

$$K.E_{rot} = \frac{1}{2}(m_1 r_1^2 \omega^2 + m_2 r_2^2 \omega^2 + \ldots\ldots)$$

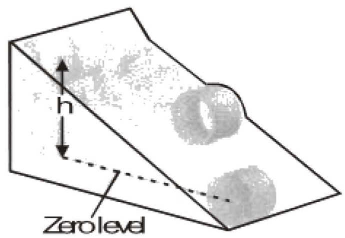
$$K.E_{rot} = \frac{1}{2}(m_1 r_1^2 + m_2 r_2^2 + \ldots\ldots)\,\omega^2$$

$$\therefore \quad K.E_{rot} = \frac{1}{2} I \omega^2$$

Q.14 Calculate the rotational K.E of a disc and a hoop with its velocities.

Ans. ROTATIONAL KINETIC ENERGY OF A DISC AND A HOOP

As, $$K.E_{rot} = \frac{1}{2} I \omega^2 \quad \ldots\ldots\ldots (1)$$



For disc

$$I = \frac{1}{2} m r^2$$

Putting in eq (1)

$$\therefore \quad K.E_{rot} = \frac{1}{2}\left(\frac{1}{2} m r^2\right)\omega^2$$

$$K.E_{rot} = \frac{1}{4} m r^2 \omega^2$$

As, $$V = r\omega$$

Squaring

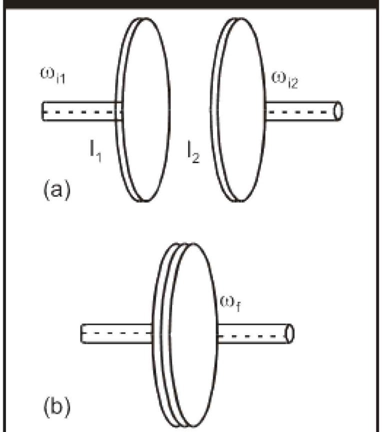
$$V^2 = r^2 \omega^2$$

$$\therefore \quad K.E_{rot} = \frac{1}{4} m V^2$$

For Hoop

$$I = m r^2$$

Interesting Information



$$\therefore \quad K.E_{rot} = \frac{1}{2} m r^2 \omega^2$$

$$K.E_{rot} = \frac{1}{2} m V^2$$

When both starts moving down an inclined plane of height h, their motion consists of both rotational and transnational motions. If no energy is lost against friction, the total K.E. of disc or hoop on reaching the bottom of incline must be equal to its P.E. at top.

$$P.E = K.E_{rot} + K.E_{trans}$$

Velocity of a Disc

$$mgh = \frac{1}{4} m V^2 + \frac{1}{2} m V^2$$

$$mgh = m V^2 \left(\frac{1}{4} + \frac{1}{2}\right)$$

$$mgh = m V^2 \left(\frac{1+2}{4}\right)$$

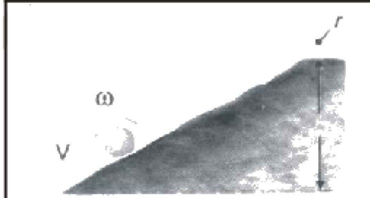
$$mgh = \frac{3}{4} m V^2$$

$$gh = \frac{3}{4} V^2$$

$$V^2 = \frac{4}{3} g h$$

$$V = \sqrt{\frac{4gh}{3}}$$

For Your Information



As the sphere rolls to the bottom of the incline, its gravitational potential energy is changed to kinetic energy of rotation and translation.

Velocity of a Hoop

$$P.E. = K.E_{rot} + K.E_{trans}$$

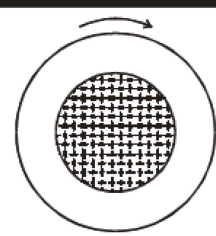
$$mgh = \frac{1}{2} m V^2 + \frac{1}{2} m V^2$$

$$mgh = m V^2 \left(\frac{1}{2} + \frac{1}{2}\right)$$

$$mgh = m V^2 \left(\frac{1+1}{2}\right)$$

$$mgh = m V^2$$

Do You Know?



As the wheel rolls, it has both rotational and translational kinetic energy.

Q.15 What are artificial satellite? Find the expression for minimum velocity and period to put a satellite into the orbit.

Ans. ARTIFICIAL SATELLITES

"Satellites are objects that orbit around the Earth." They are put into orbit by rockets and are held in orbit by the gravitational pull of the Earth. The low flying Earth satellites have acceleration 9.8 ms^{-2} towards the centre of the Earth. If they do not, they would fly off in a straight line tangent to the Earth.

When the satellite is moving in a circle, it has an acceleration

$$a_c = \frac{V^2}{r}$$

In a circular orbit around the Earth, the centripetal acceleration is supplied by gravity and we have;

$$g = \frac{V^2}{R} \qquad \ldots\ldots\ldots (1)$$

where V is the orbital velocity and R is the radius of earth, which is 6400 km.

From equation (1)

$$V^2 = g\,h$$

$$V = \sqrt{g\,R}$$

$$= \sqrt{9.8 \times 6.4 \times 10^6}$$

$$= \sqrt{62.72 \times 10^6}$$

$$= 7.9 \times 10^3 \text{ m/s}$$

$$V = 7.9 \text{ km/s}$$

This is the minimum velocity necessary to put a satellite into orbit and called critical velocity.

Time Period of a Satellite

The period T is given by

$$T = \frac{2\pi R}{V} \qquad \left(\because \begin{array}{l} S = V\,T \\ S = 2\pi R \end{array} \right)$$

$$T = \frac{2\,(3.14)\,6400}{7.9}$$

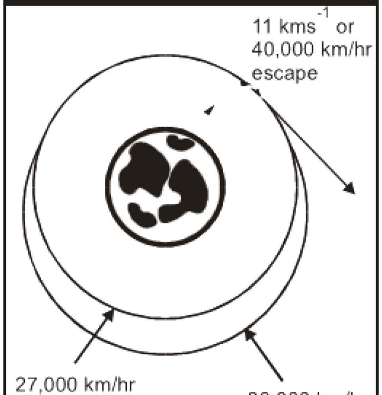
$$= 5060 \text{ sec}$$

$$= \frac{5060}{60} \text{ min}$$

$$T = 84 \text{ min}$$

GMm

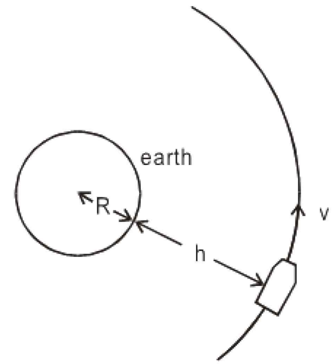
For Your Information



$$a \propto \frac{1}{r^2}$$

Higher the satellite the slower will the required speed and longer it will take to complete one revolution around the earth.

Close orbiting satellites orbit the Earth at a height of about 400 km. Twenty four such satellites form the Global positioning system. An airline pilot, sailor or any other person can now use a pocket size instrument or mobile phone to find his position on the Earth's surface to within 10 m accuracy.



Q.16 What do you understand by real and apparent weight?

Ans. REAL AND APPARENT WEIGHT

On the Earth the weight of an object is the gravitational pull of the Earth on the object. The weight of an object is measured by a spring balance. The force exerted by the object on the scale is equal to the pull due to gravity on the object i.e., weight of the object.

Consider an object of mass 'm' suspended by a string and spring balance in a lift as shown in fig. The reading of spring balance indicates tension in string and it shows apparent weight of object. Its value depends upon acceleration of lift.

Tidbits

The moment you switch on your mobile phone, your location can be tracked immediately by global positioning system.

Case I

When lift is at rest or moving with uniform velocity.

In this case

$$a = 0$$

As $F = ma$

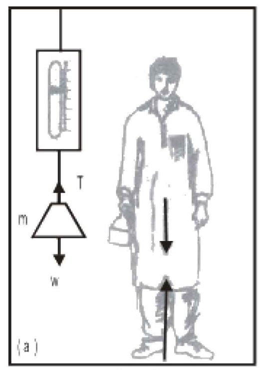
$\therefore$ $F_{net} = m(0) = 0$

As $F_{net} = T - w$

$$0 = T - w$$

$$T = w$$

The scale shows real weight of the object.



at rest
$a = 0$
$T = w$

Case II

When lift is moving upward with acceleration 'a'.

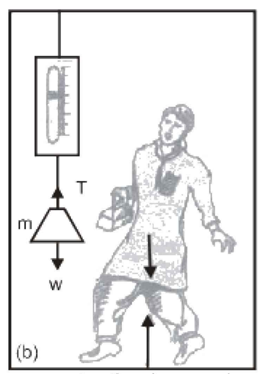
In this case

$$F_{net} = T - w$$

$$T - w = ma \qquad (\because F_{net} = ma)$$

$$T = w + ma$$

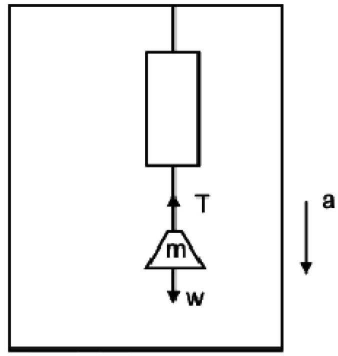
$$T = mg + ma$$



acceleration downward
$w - T = ma$

amount ma.

Case III



When lift is moving downward with acceleration 'a'.

In this case

$$F_{net} = w - T$$

$$ma = w - T$$

$$T = w - ma \quad \ldots\ldots\ldots (1)$$

The tension is less than w by an amount ma. To a person in the accelerated lift the object appears to weight less than w.

Do You Know?

Your apparent weight differs from your true weight when the velocity of the elevator changes at the start and end of a ride, not during the rest of the ridge when that velocity is constant.

Case IV

When lift is falling freely under gravity

i.e., $a = g$

From equation (1)

$$T = mg - ma$$

$$T = m(g - a)$$

$$\because \quad a = g$$

$$\therefore \quad T = m(g - g)$$

$$T = m(0)$$

$$T = 0$$

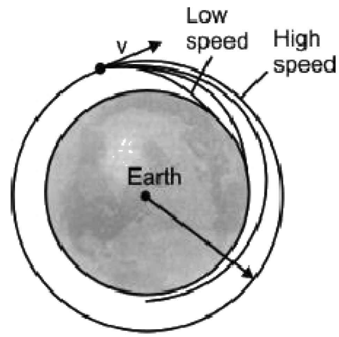
The apparent weight of the object will be shown by the scale to be zero. It is understood from these considerations that apparent weight of the object is not equal to its true weight in an accelerating system. It is equal and opposite to the force required to stop it from falling in that frame of reference.

Q.17 Explain the phenomena of weightless in a satellite.

Ans. **WEIGHTLESSNESS IN A SATELLITE AND GRAVITY FREE SYSTEM**

When a satellite is falling freely in space, everything within this freely falling system will appear to be weightless. It does not matter where the object is whether it is falling under force of attraction of the Earth, the Sun, or some distant star.

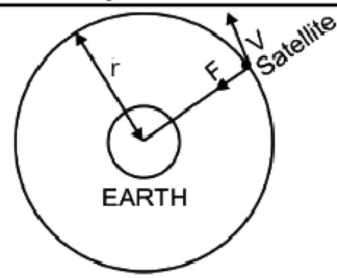
An Earth's satellite is freely falling object. The statement may be surprising at first, but it is easily seen to be correct. Consider the behaviour of a projectile shot parallel to horizontal surface of the Earth in the absence of air friction. If the projectile is thrown at successively larger speed then during its free fall to the Earth, the curvature of path decreases with increasing horizontal speeds. If object is thrown fast enough parallel to the Earth, curvature of its path will match the curvature of the Earth as shown in figure. In this case the space ship will simply circle the Earth.



The space ship is accelerating towards the centre of the Earth at all times since it circles round the Earth. Its radial acceleration is simply g, the free fall acceleration. In fact the space ship is falling towards the centre of the Earth at all the times, but the curvature of the Earth prevents space ship from hitting. Since the space ship is in free fall all the objects within it appear to be weightless. Thus no force is required to hold an object falling in the frame of reference of the space craft or satellites. Such a system is called gravity free system.

Q.18 What is orbital velocity? Define an expression for orbital velocity.

Ans. **ORBITAL VELOCITY**



The earth and some other planets revolve round the Sun nearly circular paths. The artificial satellites launched by men also adopt nearly circular course around the Earth. This type of motion is called orbital motion.

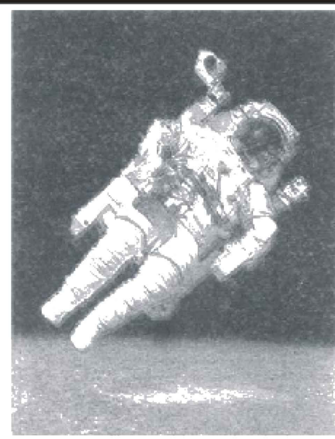
Figure shows a satellite going round the Earth in a circular path. The mass of the satellite is m_s and V is its orbital speed. The mass of the Earth is M and r represents radius of the orbit. This force $\frac{m_s V^2}{r}$ is required to hold the satellite in orbit. This force is provided by the gravitational force of attraction between the Earth and the satellite. Equation the gravitational force to the required centripetal force, gives

$$\frac{G m_s M}{r^2} = \frac{m_s V^2}{r}$$

$$V = \sqrt{\frac{GM}{r}} \qquad \text{......... (1)}$$

This shows that the mass of the satellite is unimportant in describing the satellite's orbit. Thus any satellite orbiting at distance r from Earth's centre must have the orbital speed given by equation (1). Any speed less than this will bring the satellite tumbling back to the Earth.

Tid-bits

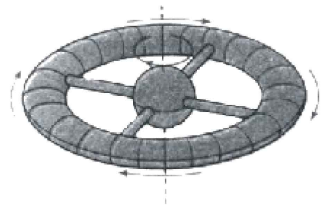


In 1984, at a height of 100 km above Hawaii island with a speed of 29000 kmh^{-1} Bruce McCandless stepped into space from a space shuttle and became the first human satellite of the Earth.

Q.19 What do you understand by artificial gravity? Derive an expression for

In a gravity free space satellite there will be no force that will force any body to any side of the spacecraft. If this satellite is to stay in orbit over an extended period of time this weightlessness may affect the performance of astronauts present in that spacecraft. To over come this difficulty, an artificial gravity is created in the spacecraft. This could enable the crew of the spaceships to function in an almost normal manner. For

this the spaceship is set into rotation around its own axis. The astronaut then is pressed towards the outer rim and exerts a force on the 'floor' of the spaceship in much the same way as on the Earth.



Consider a spacecraft of the shape as shown in Fig. The outer radius of the spaceship is R and it rotates around its own central axis with angular speed ω.

$$\therefore \quad a_c = \frac{V^2}{R}$$

$$V = R\omega$$

$$\therefore \quad a_c = \frac{V^2}{R}$$

$$a_c = R\omega^2$$

Then its angular acceleration is given by

$$a_c = R\,\omega^2$$

But

$$\omega = \frac{2\pi}{t}$$

Where t is period of revolution of spaceship.

Hence,

$$a_c = R\left(\frac{2\pi}{t}\right)^2 = \frac{4\pi^2 R}{t^2}$$

As frequency

$$f = \frac{1}{t}$$

$$a_c = R\,4\pi^2 f^2$$

or

$$f^2 = \frac{a_c}{4\pi^2 R}$$

or

$$f = \frac{1}{2\pi}\sqrt{\frac{a_c}{R}}$$

Do You Know?

The surface of the rotating space ship pushes on an object with which it is in contact and thereby provides the centripetal force needed to keep the object moving on a circular path.

Here the force of gravity provides required centripetal acceleration, therefore,

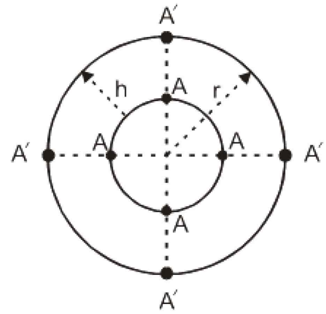
$$a_c = g$$

and

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{R}}$$

When the spaceship rotates with this frequency, artificial gravity like Earth is provided to the inhabitants of the spaceship.

Q.20 What are geo stationary satellites? Derive an expression for the orbital radius of



"The satellite whose orbital motion is synchronized with the rotation of Earth is called geo stationary satellite and the orbit in which geo stationary satellite revolves around the Earth is called geo stationary orbit."

In this way the synchronous satellite remains always over same point on equator as Earth spins on its axis.

Such a satellite is very useful for worldwide communication, weather observations, navigation and other military uses.

Radius of Geo Stationary Orbit

As, $$V = \sqrt{\frac{GM}{r}} \quad \ldots\ldots\ldots (1)$$

But this speed must be equal to the average speed of the satellite in one day.

i.e. $$V = \frac{S}{T}$$

$$V = \frac{2\pi r}{T} \quad \ldots\ldots\ldots (2)$$

Where T is the revolution of satellite, that is equal to one day. This means that the satellite must move in one complete orbit in a time of exactly one day. As the Earth rotates in one day and the satellite will revolve around. Earth is one day, the satellite at A will always stay over same point A on Earth as shown in figure.

Equating above equation (1) and (2),

$$\frac{2\pi r}{T} = \sqrt{\frac{GM}{r}}$$

Squaring both sides;

$$\frac{4\pi^2 r^2}{T^2} = \frac{GM}{r}$$

or $$r^3 = \frac{GM\,T^2}{4\pi^2}$$

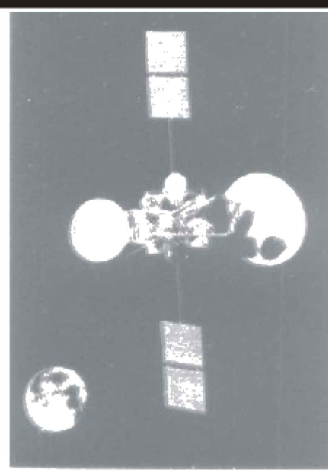
or $$r = \left(\frac{GM\,T^2}{4\pi^2}\right)^{1/3}$$

Putting the value we get

$$r = 4.23 \times 10^4 \text{ km}$$

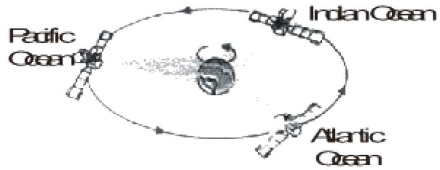
which is orbital radius measured from centre of the Earth, for a geostationary satellite. A satellite at this height will always stay directly above a particular point on the surface of the Earth. The height of such satellite above equator is 36000 km.

Do You Know?



A geostationary satellite orbits the Earth once per day over the equator so it appears to be stationery. It is used now for international communications.

Ans. COMMUNICATION SATELLITES



The whole Earth can be covered by just three geostationary satellite.

A satellite communication system can be set up by placing several geo stationary satellites in orbit over different point on the surface of the Earth. One such satellite covers 120^{o} of longitude, so that whole of the populated Earth's surface can be covered by three correctly positioned satellites shown in figure.

Since these geo stationary satellites seem to hover over one place on the Earth, continuous communication with any place on the surface of the Earth can be made. Microwaves are used because they travel in a narrow beam, in a straight line and pass easily through the atmosphere of the Earth. The energy needed to amplify and retransmit the signals is provided by large solar cell panels fitted on the satellites. There are over 200 Earth stations which transmit signals to satellites and receive signals via satellites from other countries can also pick up the signals from the satellite using a dish antenna on your house.

The largest satellite system is managed by 126 countries, international Telecommunication satellite organization (INTELSAT). An INTELSAT VI satellite is shown in Fig. It operates at microwave frequency of 4, 6, 11 and 14 GHz and has capacity of 30,000 two way telephone circuits plus three T.V. channels.

Do You Know?

[illegible]

Q.22 Discuss Newton's and Einstein's views of gravitation.

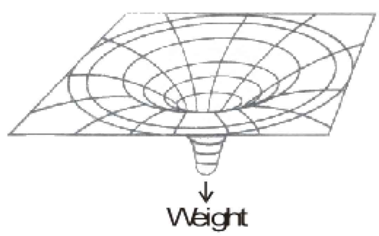
Ans. NEWTON'S AND EINSTEIN'S VIEWS OF GRAVITATION

According to Newton, the gravitation is the intrinsic property of matter that every particle of matter attracts every other particle with a force that is directly proportional to the product of their masses and is inversely proportional to the square of the distance between them.

Do You Know?

[illegible]

According to Einstein's theory, space time is curved, especially locally near massive bodies. To visualize this, we might think of space as a thin rubber sheet; if a heavy weight is hung from it, curves as shown in Fig. The weight corresponds to a huge mass that causes space itself to curve. Thus, in Einstein's theory we do not speak of the force of gravity acting on bodies; instead we say that bodies and light rays move along geodesics (equivalent to straight lines in plane geometry) in curved space time. Thus a body at rest or moving slowly near the great mass would follow a geodesic towards that body.

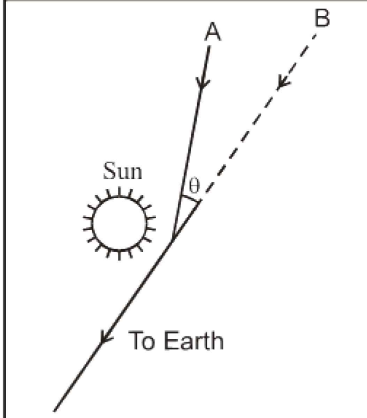


Rubber sheet analogy for curved space-time.

Einstein's theory gives us a physical picture of how gravity works. Newton discovered the inverse square law of gravity; but explicitly said that he offered no explanation of why gravity should follow an inverse square law. Einstein's theory also says that gravity follow inverse square law (Except in strong gravitational fields), but it tells us why this should be so that is why Einstein's theory is better

Einstein inferred that if gravity and acceleration are precisely equivalent, gravity must bend light, by a precisely amount that could be calculated. This was not entirely startling suggestions Newton's theory, based on the idea light beam would be deflected by gravity. But in Einstein's theory, the deflection of light is predicted to be exact twice as great as it is according to Newton's theory. Why the bending to straight caused by the gravity of the Sun was measured during a solar eclipse in 1919, and found, match Einstein's prediction rather than Newton's, then Einstein's theory was hailed as a scientific triumph.

Interesting Information



Bending of starlight by the Sun. Light from the star A is deflected as it passes close to the Sun on its way to Earth. We see the star in the apparent direction B, shifted by the angle ϕ = 1.745 seconds of angle which was found to be the same during the solar eclipse of 1919.

SOLVED EXAMPLES

EXAMPLE 5.1

An electric fan rotating at 3 rev s^{-1} is switched off. It comes to rest in 18.0 s. Assuming declaration to be uniform, find its value. How many revolutions did it turn before coming to rest?

Data

Initial angular velocity $= \omega_i = 3$ rev / s

Final angular velocity $= \omega_f = 0$

Time $= \Delta t = 18$ sec

To Find

Angular declaration $= \alpha = ?$

Number of revolutions $= \theta = ?$

SOLUTION

Using $\alpha = \dfrac{\Delta\omega}{\Delta t}$

$$\alpha = \frac{\omega_f - \omega_i}{\Delta t}$$

$$\alpha = \frac{0 - 3}{18}$$

$$\alpha = -0.167 \text{ rev/s}^2$$

For angular motion

$$\theta = \omega_i\, t + \frac{1}{2}\alpha\, t^2$$

$$\theta = 3 \times 18 + \frac{1}{2}(-0.167)\,(18)^2$$

$$\theta = 27 \text{ rev}$$

Result

Angular declaration $= \alpha = 0.167$ rev/s^2

Number of revolutions $= \theta = 27$ rev

EXAMPLE 5.2

A 1000 kg car is turning round a corner at 10 ms^{-1} as it travels along an arc of a circle. If the radius of circular path is 10 m, how large a force must be exerted by the pavement on the tyres to hold the car in the circular path?

Data

Mass of car = m = 1000 kg

Radius of circular path = r = 10 m

Speed of car = v = 10 ms^{-1}

To Find

Force required = F_C = ?

SOLUTION

Using $F_C = \frac{m V^2}{r}$

$$F_C = \frac{1000\,(10)^2}{10}$$

$$F_C = 100 \times 100$$

$$F_C = 10^4 \text{ N}$$

Result

Force required = $F_C = 1.0 \times 10^4$ N

This force must be supplied by the frictional force of the pavement on the wheels.

EXAMPLE 5.3

A ball tied to the end of a string, is swung in a vertical circle of radius r under the action of gravity as shown in Fig. What will be the tension in the spring when the ball is at the point A of the path and its speed is v at this point?

SOLUTION

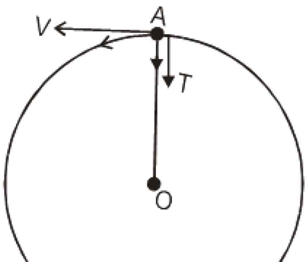
At point A two forces are acting on the ball.

(1) Tension in string.

(2) Weight of ball w.

As these force act along the radius at point A, therefore,

$$T + w = \frac{m V^2}{r} \quad \ldots\ldots\ldots (1)$$



$$T = \frac{m V^2}{r} - w$$

$$T = \frac{m V^2}{r} - mg$$

If $\frac{V^2}{r} = g$

Then $T = m(g - g) = 0$

$T = 0$ and the centripetal force just equal to weight.

EXAMPLE 5.4

The mass of Earth is 6.00 x 10^{24} kg. The distance r from Earth to the Sun is 1.50 x 10^{11} m. As seen from the direction of the North Star, the Earth revolves counter-clockwise around the Sun. Determine the orbital angular momentum of the Earth about the Sun, assuming that it traverses a circular orbit about the Sun once a year (3.16 x 10^7 s).

Data

Mass of earth = M_e = 6×10^{24} kg

Distance between Earth and Sun = r = 1.50×10^{11} m

Time period = T = 3.16×10^7 sec

To Find

Orbital angular momentum = L_o = ?

SOLUTION

Using $L_o = I\omega$ .... (1)

Here $I = Mer^2$

As $T = \frac{2\pi}{T}$

$\therefore \omega = \frac{2\pi}{T}$

$\therefore$ equation (1) becomes

$$L_o = M_e r^2 \frac{2\pi}{T}$$

Putting values

$$L_o = 6 \times 10^{24} (1.50 \times 10^{11})^2 \frac{2(3.14)}{3.16 \times 10^7}$$

$$= \frac{37.68 \times 2.25 \times 10^{24} \times 10^{22}}{3.16 \times 10^7}$$

$$= 26.83 \times 10^{39}$$

$$L_o = 2.68 \times 10^{40} \text{ Js}$$

Result

Orbital angular momentum $= L_o = 2.68 \times 10^{40}$ Js

EXAMPLE 5.5

A disc without slipping rolls down a hill of height 10.0 m. If the disc stars from rest at the top of the hill, what is its speed at the bottom?

Data

Height of hill $= h = 10$ m

To Find

Speed of disc at bottom $= V = ?$

SOLUTION

Using $$V = \sqrt{\frac{4\,gh}{3}}$$

$$V = \sqrt{\frac{4 \times 9.8 \times 10}{3}}$$

$$V = \sqrt{\frac{392}{3}}$$

$$V = \sqrt{130.66}$$

$$V = 11.4 \text{ m/s}$$

Result

Speed of disc at bottom $= V = 11.4$ m/s

EXAMPLE 5.6

An Earth satellite is in circular orbit at a distance of 384,000 km from the Earth's surface. What is its period of one revolution in days? Take mass of the Earth $M = 6.0 \times 10^{24}$ kg and its radius $R = 6400$ km.

Data

Distance of satellite from surface of Earth $= h = 384000$ km

Mass of earth $= M_e = 6 \times 10^{24}$ kg

Radius $= R = 6400$ km

To Find

As distance of satellite from centre of Earth is

$$r = h + R$$

$$\therefore \quad \text{Using} \quad V = \sqrt{\frac{GM}{r}}$$

$$= \sqrt{\frac{40.02 \times 10^{13}}{390400000}}$$

$$= \sqrt{0.000001025 \times 10^{12}}$$

$$= 0.00101 \times 10^{6}$$

$$= 1.01 \times 10^{3} \text{ m/s}$$

$$= 1.01 \text{ km/s}$$

Now using:

$$T = \frac{2 \times 3.14 \times 290400000}{1.01 \times 10^{3}}$$

$$= \frac{1 \text{ day}}{60 \times 60 \times 24}$$

$$T = 27.5 \text{ day}$$

Result

Period of one revolution = T = 27.5 days

EXAMPLE 5.7

Radio and T.V. signals bounce from a synchronous satellite. This satellite circles the Earth once in 24 hours. So if the satellite circles eastward above the equator, it stays over the same spot on the Earth because the Earth is rotating at the same rate.

(a) What is the orbital radius for a synchronous satellite?

(b) What is its speed?

Data

$$T = 24 \text{ h}$$

$$T = 24 \times 60 \times 60 \text{ sec}$$

$$T = 86400 \text{ sec}$$

To Find

(a) Orbital radius = r = ?

(b) Speed of satellite = V = ?

SOLUTION

Putting values

$$r = \left[\frac{6.67 \times 10^{-11} \times 6 \times 10^{24} (86400)^2}{4 (3.14)^2}\right]^{1/3}$$

$$= \left[\frac{40.02 \times 10^{13} \times 7.46 \times 10^{9}}{4 \times 9.86}\right]^{1/3}$$

$$= \left[\frac{298.55 \times 10^{22}}{39.44}\right]^{1/3}$$

$$= [7.57 \times 10^{22}]^{1/3}$$

$$= (75.7 \times 10^{21})^{1/3}$$

$$= (75.7)^{1/3} \; 10^{21 \times 1/3}$$

$$= [(4.23)^3]^{1/3} \; 10^7$$

$$r = 4.23 \times 10^7 \text{ m}$$

(b) Now using

$$V = \frac{2\pi r}{T}$$

Putting values

$$= \frac{2 (3.14) (4.23 \times 10^7)}{86400}$$

$$= \frac{26.56 \times 10^7}{86400}$$

$$= 3.07 \times 10^{-4} \times 10^7$$

$$= 3.07 \times 10^3 \text{ m/s}$$

$$V = 3.1 \text{ km/s}$$

Result

(a) Orbital radius = r = 4.23×10^7 m

(b) Speed of satellite = V = 3.1 km/s