Chapter 6

FLUID DYNAMICS

LEARNING OBJECTIVES

At the end of this chapter the students will be able to:

- Understand that viscous forces in a fluid cause a retarding force on an object moving through it.
- Use Stokes' law to derive an expression for terminal velocity of a spherical body falling through a viscous fluid under laminar conditions.
- Understand the terms steady (laminar, streamline) flow, incompressible flow, non-viscous flow as applied to the motion of an ideal fluid.
- Appreciate the equation of continuity Av = Constant for the flow of an ideal and incompressible fluid.
- Appreciate that the equation of continuity is a form of the principle of conservation of mass.
- Derive Bernoulli's equation in form $P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$.
- Explain how Bernoulli effect is applied in the filter pump, atomizers, in the flow of air over an aerofoil, venturimeter and in blood physics.
- Give qualitative explanations for the swing of a spinning ball.

INTRODUCTION

The study of fluids in motion is relatively complicated, but analysis can be simplified by making a few assumptions. The analysis is further simplified by the use of two important conservation principles; the conservation of mass and the conservation of energy. The law of conservation of mass gives us the equation of continuity while the law of conservation of energy is the basis of Bernoulli's equation. The equation of continuity and the Bernoulli's equation along with their applications in aeroplane and blood circulation are discussed in this chapter.

Q.1 Define fluid dynamics.

Ans FLUID DYNAMICS

Fluid

"Those substances which can flow from one point to the other are called fluid."

For examples; liquids and gases.

Q.2 Explain what do you understand by viscosity.

Ans. VISCOSITY

"The frictional effect between different layers of the flowing fluid is described in terms of viscosity of fluid." **(OR)** "The property of fluid due to which they resist their flow is also known as viscosity." **(OR)** Fraction in fluids is known as viscosity.

Viscosity measures how much force is required to slide one layer of liquid over another layer. Substances that do not flow easily, such as thick tar and honey etc; have large co–efficient of viscosities, (η). Substances which flow easily like water have small coefficient of viscosities. Since, liquids and gases have non zero viscosity, a force is required if an object is to be moved through them.

Unit

As, $F = 6\pi \eta r v$

$$\eta = \frac{F}{6\pi r v}$$

As $= \frac{N}{m\ m/s}$

$$= \frac{Ns}{m^2}$$

$$= Nm^{-2}\ s$$

or $\eta = kg\ m / s^2\ m^{-2}\ s$

$$= kg\ m^{-1}\ s^{-1}$$

Dimensions

$$[\eta] = [ML^{-1}\ T^{-1}]$$

Q.3 Define drag force and Stoke's law.

Ans. DRAG FORCE (VISCOUS DRAG)

"An object moving through a fluid experiences a retarding force is called a drag force." The drag force

For Your Information

Viscosities of Liquids and Gases at 30°C	
Material	**Viscosity 10^{-3} (Nsm^{-2})**

increases as the speed of the object increases.

Stoke's Law

The drag force 'F' on a sphere of radius 'r' moving slowly with speed 'v' through a fluid of viscosity 'η' is given by Stoke's law as under

$$F_d = 6\pi\eta r v_t$$

At high speeds the force is no longer proportional to the speed.

Note: *With rise in temperature, viscosity of liquid decreases and viscosity of gases increases.*

Air	0.019
Acetone	0.295
Methanol	0.510
Benzene	0.564
Water	0.801
Ethanol	1.000
Plasma	1.6
Glycerin	6.29

Q.4 Define terminal velocity. Also derive the expression for terminal velocity. (OR) Define terminal velocity. Prove that terminal velocity is directly proportional to the square of the radius.

Ans. TERMINAL VELOCITY

Maximum constant velocity of an object falling vertically downward when the weight of the object is equal to drag force is called terminal velocity.

Explanation

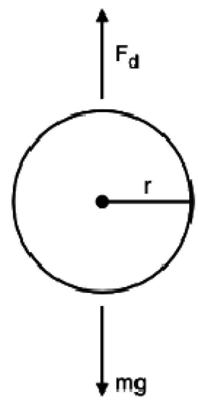
Consider a water droplet such as that of fog falling vertically, the air drag on the water droplet increases with speed. The droplet accelerates rapidly under the force of gravity which pulls the droplet downward. However, the upward drag force on it increases as the speed of the droplet increases.

Since, the droplet is moving downward.

$$\therefore \quad mg > F_d$$

The net force on droplet is

$$F_{net} = mg - F_d$$



As the speed of droplet continues to increase, the drag force eventually approaches the weight in the magnitude. Finally when the magnitude of drag force becomes equal to the weight, the net force acting on the droplet is zero. Then the droplet will fall with Constant speed (Maximum speed) is called Terminal velocity.

$$\therefore \quad F_{net} = 0$$

$$\therefore \quad 0 = mg - F_d$$

$$mg = F_d$$

$$mg = 6\pi\eta r V_t$$

$$V_t = \frac{mg}{6\pi \eta r} \quad \text{......... (1)}$$

or $V_t \propto m$

Where r, g, η are constant then

The more massive an object, faster it falls through a fluid.

As, $\rho \text{ (density)} = m / V$

$$m = \rho V$$

As droplet is of spherical shape; so

Volume of sphere is

$$V = \frac{4}{3}\pi r^3$$

Then $m = \rho \times \frac{4}{3}\pi r^3$

Putting this value in equation (1);

$$\therefore \quad V_t = \frac{\rho\, 4/3\, \pi r^3 g}{6\pi \eta r}$$

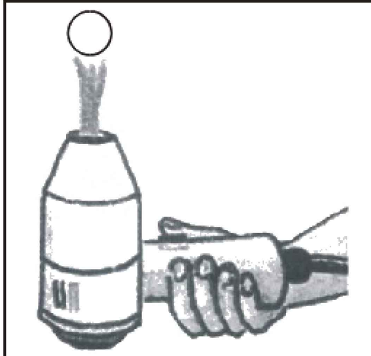
$$= \frac{4r^2 \rho g}{18 \eta}$$

$$V_t = \frac{2\rho g r^2}{9\eta}$$ which is the expression for the terminal velocity

or $V_t \propto r^2$ where $\frac{2\rho g}{9\eta}$ is constant

Hence, terminal velocity is directly proportional to the square of radius of droplet.

Can You Do That?



A table tennis ball can be made suspended in the stream of air coming from the nozzle of chair dryer.

Q.5 What is fluid flow? What is the difference between stream line and turbulent flow?

Ans. FLUID FLOW

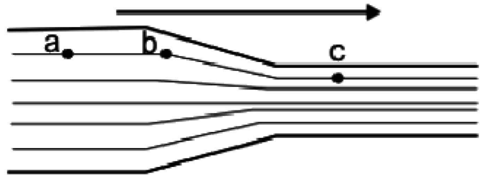
When a fluid is in motion, its flow can be:

(i) Streamline flow

(ii) Turbulent flow

(i) Streamline Flow

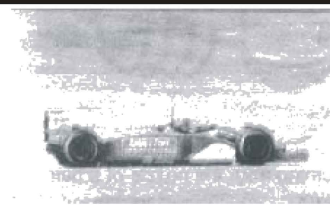
"The flow is said to be streamline or laminar if every particle that passes a particular point, moves along exactly the same path as followed by particles which passed that points earlier." **(OR)** "If velocity of the particles at different points does not change with time, the flow is called streamline flow."

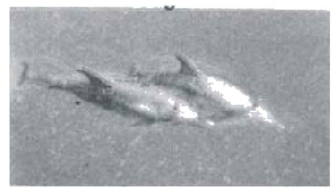


Stream Line

In this case each particle of fluid moves along a smooth path called a streamline as shown in figure. The different streamline cannot cross each other. This condition is called steady flow condition. If the streamlines cross each other the particle will go in one direction or other and flow will not be steady flow, it will be a turbulent flow.

Do You Know?


Formula one racing cars have a streamlined design.


Dolphins have streamlined bodies to assist their movement in water.

(ii) Turbulent Flow

"The irregular or unsteady flow of the fluid is called turbulent flow."

Ideal Fluid

Such a fluid which satisfy the following conditions is called an ideal fluid.

(i) The fluid is non–viscous i.e. there is no internal frictional force between adjacent layers of fluid.

(ii) The fluid is incompressible i.e. its density is constant.

(iii) The fluid motion is steady.

Q.6 State and explain equation of continuity.

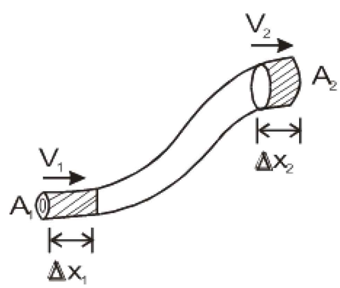
Ans. EQUATION OF CONTINUITY

Statement

The product of cross-sectional area of the pipe and the fluid speed at any point along the pipe is a constant. This constant equals the volume flow per second of the fluid or simply flow rate. **(OR)** If the fluid is incompressible and the flow is steady, the mass of the fluid is conserved i.e., the mass that flows into the bottom of the pipe through A_1 in a time Δt must be equal to mass of the liquid that flows out A_2 in the same time.

Explanation

Consider a fluid flowing through a pipe of non–uniform size. The particles in the fluid move along the streamlines in a steady state flow as shown in Fig.



In a small time Δt, the fluid at the lower end of tube moves a distance Δx_1, with a velocity V_1. If A_1 is area of cross-section of this end, then the mass of the fluid contained in the shaded region is

As, $\rho = m / V$

$$= \Delta x_1 \times A_1$$

$$\therefore \quad \Delta m_1 = \rho_1 A_1 \Delta x_1 \quad \text{......... (1)}$$

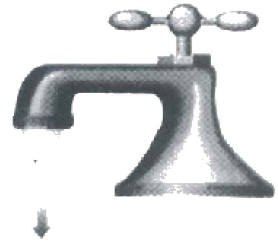
As, $\quad S = V t$

$$\therefore \quad \Delta x_1 = V_1 \Delta t$$

Putting in equation (1)

$$\therefore \quad \Delta m_1 = \rho_1 A_1 V_1 \Delta t \quad \text{......... (2)}$$

Tidbits



As the water falls, its speed increases and so its cross sectional area decreases as mandated by the continuity equation.

Where ρ_1 is the density of fluid. Similarly the fluid that moves with velocity V_2 through the upper end of pipe of area A_2 in the same time Δt has a mass

$$\Delta m_2 = \rho_2 A_2 V_2 \Delta t \quad \text{......... (3)}$$

If the fluid is incompressible and the flow is steady, the mass of the fluid is conserved.

$$\therefore \quad \Delta m_1 = \Delta m_2$$

Putting values from equation (2) and equation (3)

$$\rho_1 A_1 V_1 \Delta t = \rho_2 A_2 V_2 \Delta t$$

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

This equation is known as the equation of continuity. Since density is constant for the steady flow of incompressible fluid.

$$\therefore \quad \rho_1 = \rho_2 = \rho$$

$$\therefore \quad \rho A_1 V_1 = \rho A_2 V_2$$

$$A_1 V_1 = A_2 V_2$$

or $\quad AV = \text{Constant}$

$$\text{or} \quad V = \frac{1}{A}\text{Constant}$$

$$V \propto \frac{1}{A}$$

Note: *Equation of continuity obeys law of conservation of mass.*

Q.7 State and explain Bernoulli's equation.

Ans. BERNOULLI'S EQUATION (OR THEOREM)

Introduction

A fundamental equation in fluid dynamics which deals with the steady flow of an incompressible and non-viscous fluid is called Bernoulli's equation.

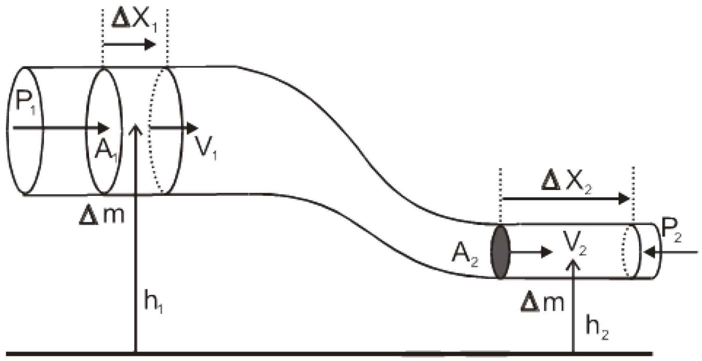
Statement

Bernoulli's theorem states that the sum of pressure, K.E. per unit volume and P.E. per unit volume, in a steady flow of an incompressible and non–viscous liquid has the same value."

Mathematically,

$$P + \frac{1}{2}\rho V^2 + \rho g h = \text{Constant}$$

Consider the flow of fluid which is incompressible, non-viscous and flows in a steady state manner through the pipe in time 't' as shown in figure.



The force on the upper end of the fluid is P_1A_1 (since $P = F/A \therefore F = PA$) where P_1 is the pressure and A_1 is the area of cross section of pipe at the upper end. The work done on the fluid, in moving it through a distance Δx_1 will be

$$W_1 = \vec{F} . \vec{\Delta x_1}$$
$$= F \Delta x_1 \cos 0^o$$
$$= P_1 A_1 \Delta x_1 \times 1$$
$$W_1 = P_1 A_1 \Delta x_1 \qquad \text{......... (1)}$$

If V_1 is velocity of fluid at this end then,

As $\qquad S = V t$

$$\Delta x_1 = V_1 t$$

$\therefore$ equation (1) becomes

$$W_1 = P_1 A_1 V_1 t$$

Similarly work done on the fluid at lower end is,

$$W_2 = \vec{F}_2 . \vec{\Delta x_2}$$
$$= F_2 \Delta x_2 \cos 180^o$$
$$= P_2 A_2 \Delta x_2 (-1) \qquad (\because F_2 = P_2 A_2)$$
$$= -P_2 A_2 \Delta x_2$$
$$W_2 = -P_2 A_2 V_2 t \qquad (\because \Delta x_2 = V_2 t)$$

Where V_2 is the velocity of the fluid at lower end. P_2 is the pressure, A_2 is the area of cross section of lower end and Δx_2 is the distance moved by the fluid in the same time interval 't'. The work W_2 is taken to be negative because this work is done against the fluid force.

Total work done

$$W = W_1 + W_2$$
$$\therefore \quad W = p_1 A_1 V_1 t + (-p_2 V_2 A_2 t)$$
$$W = p_1 A_1 V_1 t - p_2 A_2 V_2 t \qquad \text{......... (2)}$$

From equation of continuity

$$A_1 V_1 = A_2 V_2$$

$$\left[\begin{array}{l} \Delta m = \rho AV t \\ \frac{\Delta m}{\rho} = AV t \end{array} \right]$$

$\therefore$ equation (2) becomes

$$\therefore \quad W = P_1 V - P_2 V$$

$$W = (P_1 - P_2) V$$

Since $$\rho = \frac{m}{V}$$

$$\therefore \quad V = \frac{m}{\rho}$$

$$\therefore \quad W = \frac{m}{\rho}(P_1 - P_2) \qquad \ldots\ldots\ldots (3)$$

Part of this work is utilized by the fluid in changing its K.E. and a part of it is used in changing its gravitational P.E.

$$\text{Change in K.E.} = \Delta \text{K.E} = \frac{1}{2} m V_2^2 - \frac{1}{2} m V_1^2$$

$$\text{Change in P.E.} = \Delta \text{P.E.} = mg h_2 - mg h_1$$

Where h_1 and h_2 are the heights of upper and lower ends of pipe respectively.

Applying law of conservation of energy to this volume of the fluid.

$$W = \Delta\text{K.E.} + \Delta\text{P.E.}$$

$$\frac{m}{\rho}(P_1 - P_2) = m\left(\frac{1}{2} V_2^2 - \frac{1}{2} V_1^2 + g h_2 - g h_1\right)$$

$$P_1 - P_2 = \frac{1}{2}\rho V_2^2 - \frac{1}{2}\rho V_1^2 + \rho g h_2 - \rho g h_1$$

$$P_1 + \frac{1}{2}\rho V_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g h_2$$

This is Bernoulli's equation and is often expressed

As $$P + \frac{1}{2}\rho V^2 + \rho g h = \text{Constant}$$

Note: Law of conservation of energy is the basis of Bernoulli's theorem.

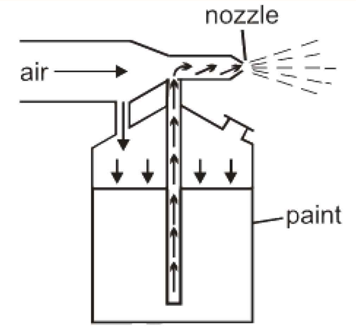
APPLICATIONS OF BERNOULLI'S EQUATION

There are two applications of Bernoulli's theorem:

Q.8 State and explain Torricelli's theorem.

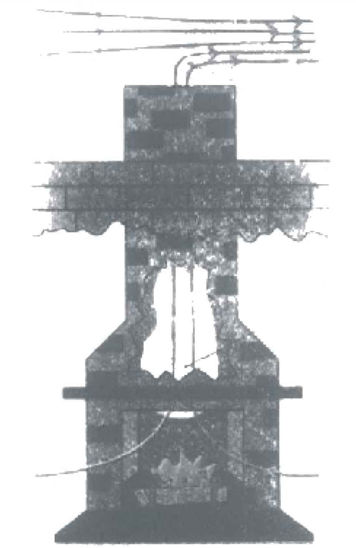
Ans. TORRICELLI'S THEOREM

Interesting Information



A stream of air passing over a tube dipped in a liquid will cause the liquid to rise in the tube as shown. This effect is used in perfume bottles and paint sprayers.

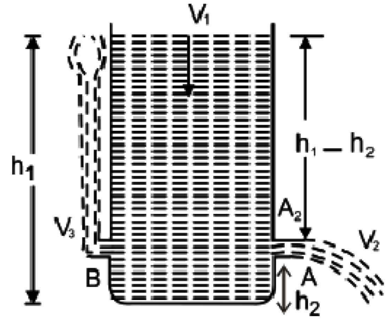
Do You Know?



A chimney works best when it is tall and exposed to air currents, which reduces the pressure at the top and force the upward flow of smoke.

Statement

"The speed of efflux (outward flow of gas or liquid) is equal to the velocity gained by the fluid in falling through the distance $(h_1 - h_2)$ under the action of gravity."



Explanation

Suppose a large tank of fluid has two small orifices A and B on it as shown in figure. Now we find speed with which the water flows from the orifice A.

Since the orifices are so small, the efflux speeds V_2 and V_3 will be much larger than the speed V_1 of the top of surface of water. Therefore, $V_1 \approx 0$.

As Bernoulli's equation is

$$P_1 + \frac{1}{2}\rho V_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g h_2$$

Putting $V_1 = 0$

$$\therefore \quad P_1 + \rho g h_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g h_2$$

Since $P_1 = P_2 = P$ (Atmospheric pressure)

$$\therefore \quad P + \rho g h_1 = P + \frac{1}{2}\rho V_2^2 + \rho g h_2$$

$$\rho g h_1 = \frac{1}{2}\rho V_2^2 + \rho g h_2$$

$$\frac{1}{2}\rho V_2^2 = \rho g h_1 - \rho g h_2$$

$$\frac{1}{2}\rho V_2^2 = \rho g (h_1 - h_2)$$

$$\frac{1}{2} V_2^2 = g (h_1 - h_2)$$

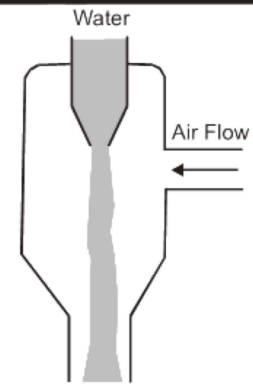
$$V_2^2 = 2g (h_1 - h_2)$$

Taking square root on both sides.

$$V_2 = \sqrt{2g (h_1 - h_2)}$$

Which is the Torricelli's theorem.

For Your Information



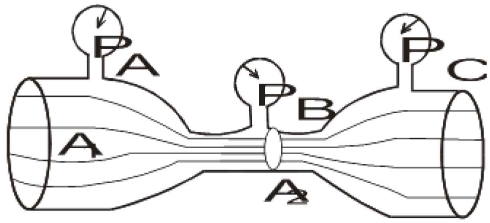
A filter pump has a constriction in the centre, so that a jet of water from the tap flows faster here. This cause a drop in pressure near it and air, therefore, flows in from the side tube. The air and water together are expelled through the lower part of the pump.

Note: Notice that the speed of the efflux of liquid is the same as the speed of a ball that falls through a height $(h_1 - h_2)$. The top level of the tank has moved down a little and the P.E. has been

Q.9 What is the relation between fluid speed and pressure? (OR) Prove that where the pressure is low, the speed will be high.

Ans. RELATION BETWEEN SPEED AND PRESSURE OF THE FLUID

Suppose that water flows through a pipe system as shown in figure.



Clearly, the water will flow faster at B than it does at A or C. $\left(\because V \propto \frac{1}{A}\right)$.

Now we compare the pressure at B with that at A. As the Bernoulli's equation is

$$P_A + \frac{1}{2}\rho V_A^2 + \rho g h_A = P_B + \frac{1}{2}\rho V_B^2 + \rho g h_B$$

Since the average height at both places is same.

$\therefore$ P.E. is same at both places.

$$\therefore \quad P_A + \frac{1}{2}\rho V_A^2 + \rho g h = P_B + \frac{1}{2}\rho V_B^2 + \rho g h$$

$$\therefore \quad P_A + \frac{1}{2}\rho V_A^2 = P_B + \frac{1}{2}\rho V_B^2$$

Let $V_A = 0.20$ m/s, $V_B = 2$m/s

For water;

$$\rho = 1000 \text{ kg/m}^3$$

$$\therefore \quad P_A + \frac{1}{2}(1000)(2)^2 = P_B + \frac{1}{2}(1000)(2)^2$$

$$P_A + 500\,(0.04) = P_B + 500 \times 4$$

$$P_A + 20 = P_B + 2000$$

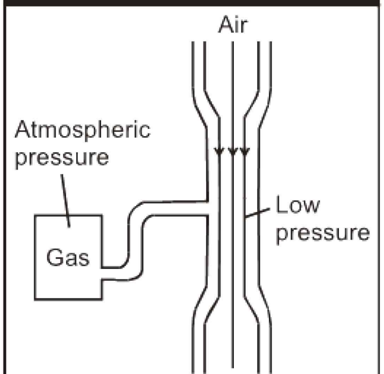
$$P_A - P_B = 2000 - 20$$

$$P_A - P_B = 1980 \text{ Nm}^{-2} \text{ (Pa)}$$

$$\therefore \quad P_A > P_B$$

This shows that the pressure in the narrow pipe where the streamlines are closer together is much smaller than in the wider pipe. Thus, where the speed is high, the pressure will be low.

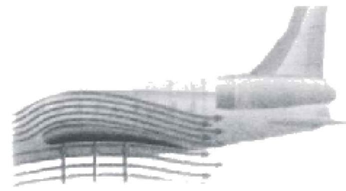
Interesting Information



The carburetor of a car engine uses a Venturi duct to feed the correct mix of air and petrol to the cylinders. Air is drawn through the duct and along a pipe to the cylinders. A tiny inlet at the side of duct is fed with petrol. The air through the duct moves very fast, creating low pressure in the duct, which draws petrol vapour into the air stream.

Example

The lift on an aeroplane is due to this effect. The flow of air around an aeroplane wing is illustrated in figure. The wing is designed to deflect the air so that streamlines are closer together above the wing than below it. We have seen in figure that where the streamlines are forced closer together, the speed is faster. Thus, air is travelling faster on the upper side of the wing than on the lower. The pressure will be lower at the top of the wing, and the wing will be forced upward.



Similarly, when a tennis ball is hit by a racket in such a way that spins as well as moves forward, the velocity of the air on one side of the ball increases (figure) due to spin and air speed in the same direction as at B and hence, the pressure decreases. This gives an extra curvature to the ball known as swing which deceives an opponent player.

Q.10 Explain venturi relation.

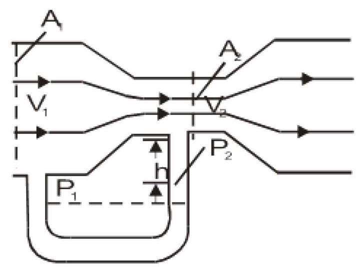
Ans. VENTURI RELATION

If one of the pipe has a much smaller diameter than the other, as shown in Fig. It is assumed that the pipes are horizontal so that $\rho g h$ terms become equal and can, therefore, be dropped then

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$

$$P_1 - P_2 = \frac{1}{2}\rho\left(V_2^2 - \frac{1}{2}\rho V_1^2\right)$$

$$= \frac{1}{2}\rho\left(V_2^2 - V_1^2\right)$$



As the cross sectional area A_2 is small as compared to the area A_1, then from equation of continuity $V_1 = (A_2/A_1)\, V_2$, will be small as compared to V_2. Thus for flow from a large pipe to a small pipe we can neglect V_1 on the right hand side of equation, hence,

$$P_1 - P_2 = \frac{1}{2}\rho V_2^2$$

This is known as Venturi relation, which is used in Venturi–meter, a device used to measure speed of liquid flow.

Q.11 Explain the flow of blood inside the human body. How the blood pressure is measured?

Ans. BLOOD FLOW

Blood is incompressible fluid with density nearly equal to that of water. High concentration of red blood cells (about 50%) increases its viscosity from three to five times that of water. Blood vessels are not rigid. They stretch like a rubber hose. Under normal circumstances the volume of blood is sufficient to keep the vessels inflated at all times, even in the relaxed state between heart beats. This means that there is tension in the walls of blood vessels and so

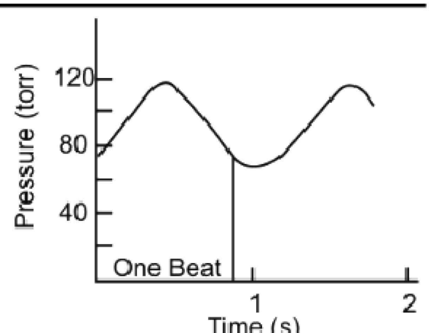
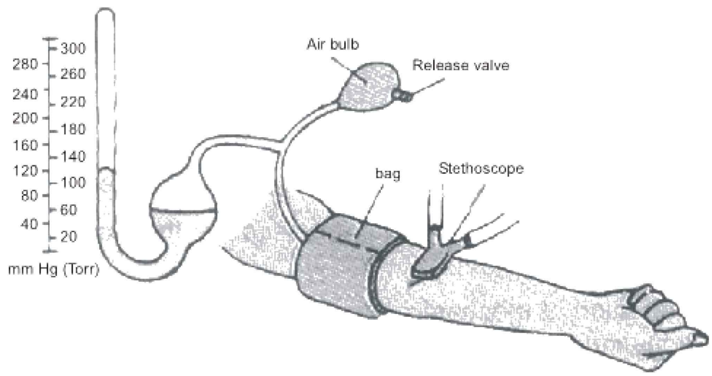


Figure shows the variation in blood pressure as the heart beats. The pressure varies from high (systolic) pressure of 120 torr to a low (diastolic) pressure of about 70 – 80 torr between beats in normal healthy person.

The instrument which is used to measure the blood pressure dynamically is called sphygmomanometer. It is shown in figure.



An inflatable bag is wound around the arm of a patient and external pressure on the arm is increased by inflating the bag. The effect is to squeeze the arm and compress the blood vessels inside. When the external pressure applied becomes larger than systolic pressure, the vessels collapse cutting off the flow of blood. Opening the release valve on the bag gradually decreases the external pressure.

A stethoscope detects the instant at which the external pressure becomes equal to the systolic pressure. At this point the first surges of blood flow through the narrow stricture which produces a high flow speed. As a result, the flow is initially turbulent.

As the pressure drops, the external pressure eventually equals the diastolic pressure. From this point, the vessel no longer collapse during any portion of the flow cycle. The flow changes from turbulent to laminar and the gurgle in stethoscope disappear. This is the signal to record diastolic pressure.

Note: (i) 1 Torr = 133.3 Nm^{-2}.

(ii) For blood pressure Torr or mm of Hg is used instead of SI unit of pressure i.e. Nm^{-2}.