Chapter 7

OSCILLATIONS

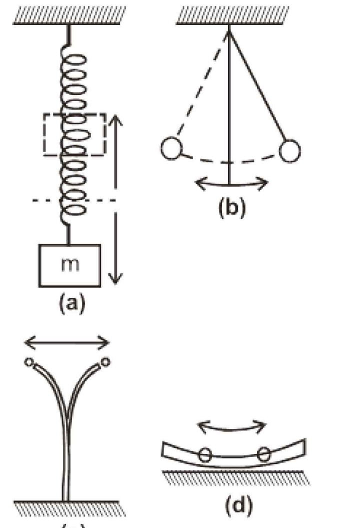
LEARNING OBJECTIVES

At the end of this chapter the students will be able to:

- Investigate the motion of an oscillator using experimental, analytical and graphical methods.
- Show that the motion of mass attached to a spring is simple harmonic.
- Understand that the motion of simple pendulum is simple harmonic and to calculate its time period.
- Understand and use the terms amplitude, time period, frequency, angular frequency and phase difference.
- Know and use of solutions in the form of $x = x_o \cos \omega t$ or $y = y_o \sin \omega t$.
- Describe the interchange between kinetic and potential energies during SHM.
- Describe practical examples of free and forced oscillations.

Q.1 Define oscillatory motion.

Ans. OSCILLATORY MOTION



(Vibrating objects)

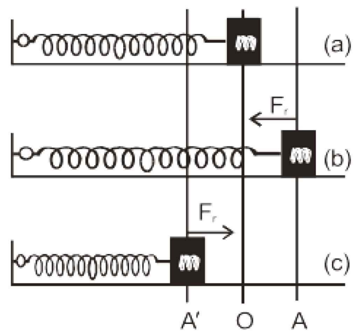
"Such a motion in which a body moves to and fro about a mean position, is called oscillatory or vibratory motion."

The oscillatory motion is called periodic motion when it repeats itself after regular intervals of time.

Examples

1. A mass, suspended from a spring, when pulled down and then released starts oscillating.
2. The bob of a simple pendulum, when displaced from its rest position and released vibrates.
3. A steel ruler clamped at one end to a bench oscillates when the free end is displaced sideways.
4. A steel ball rolling in a curved dish.

Thus to get oscillations, a body is pulled away from its rest or equilibrium position and then released. The body oscillates due to a restoring force. Under the action of this restoring force, the body accelerates and it overshoots the rest position due to inertia. The restoring force then pulls it back. The restoring force is always directed towards the rest position and so the acceleration is also directed towards the rest or mean position.

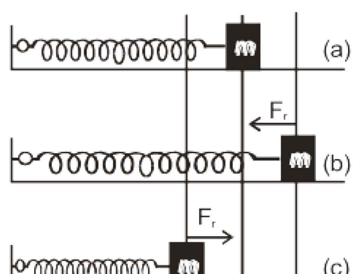


It is observed that the vibrating bodies produce waves. There are many phenomena in nature whose explanation requires the understanding of the concepts of vibrations and waves. Although many large structures, such as skyscrapers and bridges, appear to be rigid, they actually vibrate. The architects and the engineers who design and build them, take this fact into account.

Q.2 Explain simple harmonic motion. (OR) Derive an expression for the acceleration of a body vibrating under elastic restoring force.

Ans. SIMPLE HARMONIC MOTION

Consider a body of mass 'm' attached with a spring of spring constant (K). The other end of the spring attached with the fixed support and the spring is placed on the smooth horizontal surface. When the body is pulled towards right from the mean position through a displacement 'x', a deforming force F = Kx is needed. When the body is released, an elastic restoring force which is equal and opposite to the deforming force comes into play and restore the position of the body. But due to inertia, the body over shoot the mean position and



Thus under the action of restoring force and inertia, the body continue its vibratory motion for a long time between these two positions A and A′. The body speeds up while moving towards mean position and slows down while moving away from mean position. This means that the acceleration of the body is always directed towards the mean position. This acceleration can be calculated as follows:

$$F = -Kx \quad \text{(Elastic restoring force)}$$

$$F = ma \quad \text{(Newton's IInd law of motion)}$$

Comparing

$$ma = -Kx$$

$$a = -\frac{Kx}{m}$$

Where $\frac{K}{m}$ is constant

then, $a = \text{Constant}(-x)$

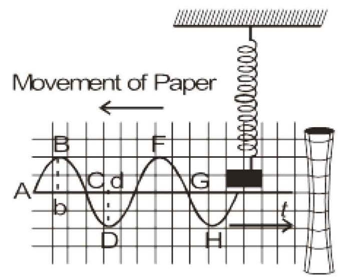
$$a \propto -x$$

Thus such a motion in which acceleration of the body is always directed towards the mean position and is directly proportional to the displacement called simple harmonic motion.

Q.3 Define instantaneous displacement and amplitude.

Ans. **INSTANTANEOUS DISPLACEMENT AND AMPLITUDE OF VIBRATION**

"When a body is vibrating, its displacement from the mean position changes with time. The value of its distance from the mean position at any time is known as its instantaneous displacement." It is zero at the instant when the body is at the mean position and is maximum at the extreme positions.



It is zero when the body is at mean position and is maximum at the extreme positions. It is denoted by x.

Amplitude

"The maximum value of displacement (where Hook's law is valid), on either side of mean position is called amplitude. It is denoted by x_0.

Vibrations

"A complete round trip of body in motion vibrating motion is called vibration."

Time Period

"It is the time required to complete one vibration. It is denoted by 'T'."

Frequency

"Number of vibrations completed in one second is called frequency. It is denoted by 'f'."

$$f = \frac{1}{T}$$

Angular Frequency

If 'T' is the time period of a body executing SHM, its angular frequency is

$$\omega = \frac{2\pi}{T}$$

Since $\frac{1}{T} = f$

$$\therefore \quad \omega = 2\pi f$$

ω = Angular frequency

$\theta = \omega t$

$s = Vt$

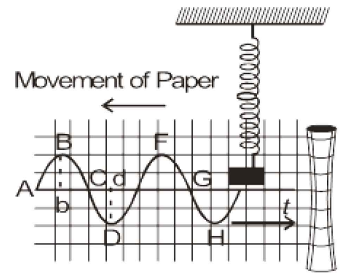
$2\pi r = r\omega T$

$2\pi = \omega T$

$\omega = \frac{2\pi}{T}$

Wave Form of SHM

The arrangement as shown in figure can be used to record the vibration in displacement with time for a mass-spring system. The strip of paper is moving at a constant speed from right to left, thus providing a time scale on the strip. A pen is attached with the vibrating mass, which records its displacement against time as shown in figure.



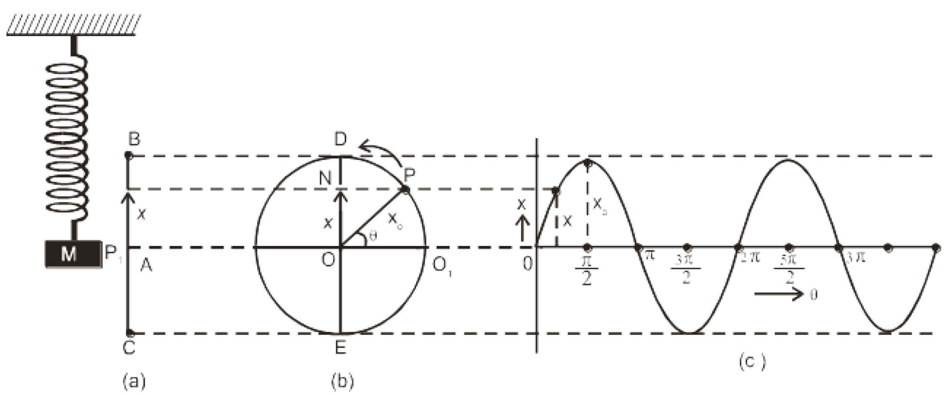
It can be seen that the curve showing the variation of displacement with time is a sine curve. It is usually known as wave-form of SHM.

Q.4 Discuss simple harmonic motion on the bases of uniform circular motion. Also calculate displacement and instantaneous velocity.

Ans. SIMPLE HARMONIC MOTION AND UNIFORM CIRCULAR MOTION

Consider a mass 'm', attached with the end of a vertically suspended spring, vibrate simple harmonically with period 'T', frequency 'f' and amplitude 'x_0'. The motion of mass is displayed by the 'P_1' on the line 'BC' with 'A' as mean position, and 'B', 'C' as extreme position, as shown in figure.

Assuming 'A' as the position of the pointer at $t = 0$, it will move so that it is at 'B', 'A', 'C' and back to 'A' at instants T/4, T/2, 3T/4 and T respectively. This will complete one cycle of vibration with amplitude of vibration being x_0 = AB = AC.



Consider another point 'P' moving on a circle of radius 'x_0' with a uniform angular frequency $\omega = 2\pi/T$, where 'T' is the time period of the vibration of the pointer. It may be noted that the radius of the circle is equal to the amplitude of the pointer's motion.

Consider the motion of the point 'N', the projection of 'P' on the diameter 'DE' drawn parallel to the line of vibration of the pointer as shown in figure. Note that the level of points 'D' and 'E' is the same as the points 'B' and 'C'. As 'P' describes uniform circular motion with a constant angular speed 'ω', 'N' oscillates to and fro on the diameter 'DE' with time period T. Assuming O_1, to be the position of P at t = 0, the position of the point N at the instants 0, T/4, T/2, 3T/4 and T will be at the points O, D, O, E and O respectively. A comparison of the motion of N with that of the pointer P_1 shows that it is a replica of the pointer's motion. Thus the expressions of the displacement, velocity and acceleration for the motion of N also hold good for the pointer P_1, executing SHM.

Displacement

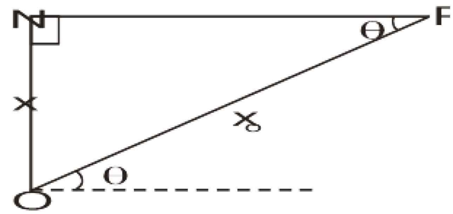
If we count the time t = 0 from the instant when P is passing through O_1, the angle, which the radius 'OP' sweeps out in time 't', is $\theta = \omega t$. The displacement x of the point N at instant will be

$$x = ON = OP \sin \angle QOP$$

Since $OP = x_o$

$$\angle O_1OP = \theta = \omega t$$

$$x = x_o \sin \omega t$$



This will be also the displacement of the pointer at the instant 't'.

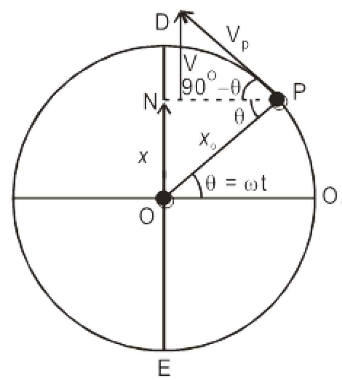
Instantaneous Velocity

The velocity of point 'P' at the instant 't' will be directed along the tangent to the circle at 'P' and its magnitude will be

$$V = r\,\omega$$

$$V_P = x_0\,\omega \quad (\because\ r = x_0) \qquad \text{........ (i)}$$

As the motion of 'N' on the diameter 'DE' is due to the motion of 'P' on the circle, the velocity of 'N' is actually the component of 'V_P' in a direction parallel to the diameter 'DE'.



From figure this component is

$$\frac{V}{V_P} = \sin(90^\circ - \theta)$$

$$V = V_P \cos\theta \quad \because \sin(90^\circ - \theta) = \cos\theta$$

Putting the value of V_P.

$$\therefore \quad V = x_0\,\omega \cos\theta \qquad \text{........ (ii)}$$

The direction of velocity of 'N' depends upon the value of phase angle θ. When 'θ' is between 0° to 90° the direction is from 'O' to 'D', for 'θ' between 90° to 270°, its direction is

$$\frac{\text{Base}}{\text{Hypt}} = \cos\theta$$

$$\frac{NP}{OP} = \cos\theta$$

$$\frac{NP}{x_0} = \cos\theta \qquad \text{.........} \quad \text{(iii)}$$

By using Pythagorean theorem,

$$(H)^2 = (P)^2 + (B)^2$$

$$x_0^2 = x^2 + (NP)^2$$

$$(NP)^2 = x_0^2 - x^2$$

$$NP = \sqrt{x_0^2 - x^2}$$

Putting this value in equation (iii).

$$\therefore \quad \cos\theta = \frac{\sqrt{x_0^2 - x^2}}{x_0}$$

Putting this value in equation (ii).

$$\therefore \quad V = x_0\,\omega\,\frac{\sqrt{x_0^2 - x^2}}{x_0}$$

$$\boxed{V = \omega\sqrt{x_0^2 - x^2}}$$

This equation shows that at mean position, where $x = 0$, the velocity is maximum and at the extreme positions where $x = x_0$, the velocity is zero.

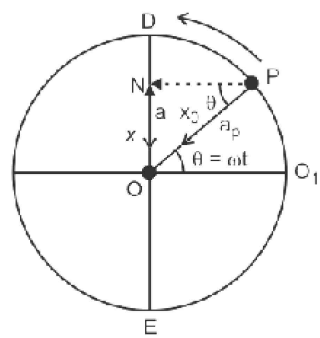
Q.5 Derive the relation for acceleration in terms of ω.

Ans. ACCELERATION IN TERMS OF ω

When the point P is moving on a circle, it has an acceleration

$$a_p = x_o\omega^2 \quad \text{because} \quad \boxed{a_p = \frac{v^2}{r} = \frac{r^2\omega^2}{r} = r\omega^2}$$

$$a_p = \frac{V_P^2}{r} \quad \dots \text{(i)} \quad (\because a_c = a_p)$$



At instant t, its direction will be along PO. The acceleration of the point N will be component of the acceleration a_p along the diameter DE on which N moves due to motion of P. The value of this component is

$$a_p \sin\theta = x_o\omega^2 \sin\theta$$

Thus the acceleration of N is

$$a = x_o\omega^2 \sin\theta$$

$$\frac{a}{a_p} = \sin\theta$$

$$a = a_p \sin\theta$$

Putting value of a_p

$$\therefore \quad a = x_0\,\omega^2 \sin\theta \quad \ldots\ldots\ldots \text{(ii)}$$

Considering ΔONP

$$\frac{ON}{OP} = \sin\theta$$

$$\therefore \quad \sin\theta = \frac{x}{x_0}$$

Putting this value in equation (ii)

$$\therefore \quad a = x_0\,\omega^2\left(\frac{x}{x_0}\right)$$

$$\vec{a} = \omega^2\,\vec{x}$$

Since acceleration of N is directed towards mean position. So

$$\therefore \quad \vec{a} = -\omega^2\,\vec{x}$$

$$\text{or} \quad \vec{a} \propto -\vec{x}$$

This shows that the acceleration is proportional to displacement and is directed towards the mean position, which is the characteristics of SHM. Thus N is executing SHM with same amplitude, period and instantaneous displacement as the pointer P_1. This conforms that the motion of N is just a replica, of the pointers motion.

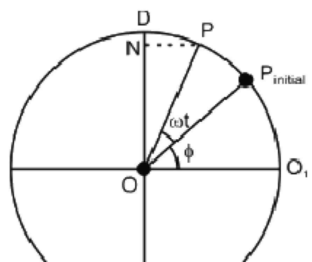
Q.6 What is meant by phase angle?

Ans. **PHASE**

"The angle $\theta = \omega t$, which specifies the displacement as well as the direction of motion of the point, executing SHM is known as phase."

It is the angle, which the rotating radius 'OP' makes with reference direction 'OO_1' at any instant 't'.

At 't' = 0, the position of rotating radius 'OP' is along 'OO_1' so that 'N' is at its mean position and the displacement is 'O'. In general at t = 0, 'OP' can make any angle 'ϕ' with the reference 'OO_1' as shown in Fig. 1.



In time 't' the radius will rotate by 'ωt'. So, now 'OP' would make an angle $(\omega t + \phi)$ with 'OO_1' as the instant 't' and displacement

Now the phase angle is

$$\theta = \omega t + \phi$$

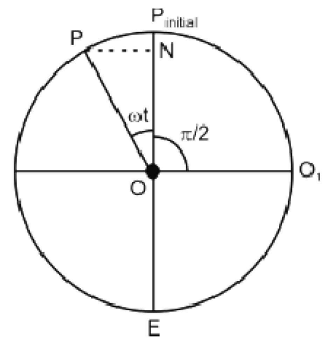
at $t = 0$

$$\therefore \quad \theta = \phi$$

So ϕ is the initial phase. If we take initial phase as 90^o then,

$$x = x_0 \sin(\omega t + 90^o)$$

$$x = x_0 \cos \omega t$$



This also gives the "displacement" of SHM, but in this case the 'N' is starting its motion from extreme position instead of mean position as shown in figure. Now shadow 'N' is moving along horizontal diameter.

Q.7 Discuss the motion of mass attach with one end of spring placed on horizontal surface. A horizontal mass spring system also derive the expressions for time period, instantaneous displacement and velocity.

Ans. A HORIZONTAL MASS-SPRING SYSTEM

Consider the vibrating mass attached to a spring as shown whose acceleration at any instant is given by

$$a = -\frac{k}{m}x \quad \text{......... (1)}$$

As k and m are constant. We see that the acceleration is directly proportional to the displacement x and its direction is towards the mean position. In case of circular motion, the acceleration is

$$a = -\omega^2 x \quad \text{......... (2)}$$

Comparing (1) and (2)

$$-\omega^2 x = -\frac{k}{m}x$$

$$\omega^2 = \frac{k}{m}$$

Taking square root

$$\omega = \sqrt{\frac{k}{m}}$$

As $T = \dfrac{2\pi}{\omega}$

Time Period

$$T = \frac{2\pi}{\sqrt{\frac{k}{m}}}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Instantaneous Displacement

The instantaneous displacement of mass is given by

$$x = x_0 \sin \omega t$$

Putting value of ω

$$x = x_0 \sin \sqrt{\frac{k}{m}}\ t$$

Instantaneous Velocity

The instantaneous velocity V of mass m is given by

$$V = \omega\sqrt{x_0^2 - x^2}$$

Putting value of ω

$$V = \sqrt{\frac{k}{m}}\sqrt{x_0^2 - x^2}$$

$$V = \sqrt{\frac{k}{m}}\sqrt{x_0^2\left(1 - \frac{x^2}{x_0^2}\right)}$$

$$V = x_0\sqrt{\frac{k}{m}}\ \sqrt{1 - \frac{x^2}{x_0^2}} \qquad \text{......... (3)}$$

This equation shows that velocity of mass gets maximum equal to V_0 when $x = 0$ i.e. at mean position.

$$V_0 = \sqrt{\frac{k}{m}}\sqrt{x_0^2(1-0)}$$

$$V_0 = \sqrt{\frac{k}{m}}\sqrt{x_0^2}$$

$$V_0 = \sqrt{\frac{k}{m}}\,(x_0)$$

$$V_0 = x_0\sqrt{\frac{k}{m}}$$

Putting $V_0 = x_0\sqrt{\frac{k}{m}}$ in equation (3)

Note: The formula derived for displacement and velocity are also valid for vertically suspended mass-spring system provided air friction is not considered.

Q.8 What is simple pendulum. Show that its motion is S.H.M. Also derive the expression for time period and frequency.

Ans. **SIMPLE PENDULUM**

It is an arrangement in which a heavy bob is suspended with light and inextensible string. "A simple pendulum consists of a small heavy mass 'm', suspended by a light inextensible string of length 'l', fixed at its upper end," by a frictionless support as shown in the figure.

Explanation

When such a pendulum is displaced from its mean position through a small angle 'θ' to position 'B' and released. It starts oscillating to and fro.

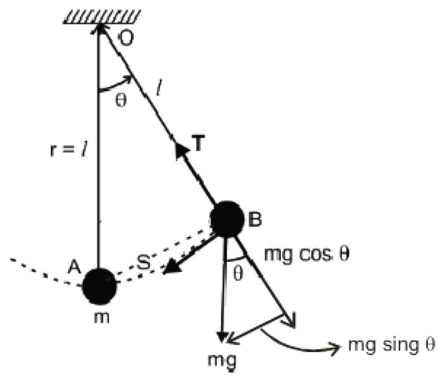
Two forces are acting on bob

(i) tension 'T' along the string.

(ii) Its weight 'mg' downwards. The weight 'mg' can be resolved into two components.

(i) $mg \sin\theta$ along the tangent at B.

(ii) $mg \cos\theta$ along the string to balance the tension in the string.



i.e. $T = mg \cos\theta$

Therefore '$mg \sin\theta$' is responsible for the motion of the bob. The restoring force at 'B' will be,

$$F = -mg \sin\theta$$

As $F = ma$

$\therefore$ $ma = -mg \sin\theta$

$$a = -g \sin\theta \qquad \text{......... (1)}$$

If θ is very small then $\sin\theta \approx \theta$ (in radian)

Equation (1) becomes

$\therefore$ $a = -g\theta$ .... (2)

As θ is very small.

$\therefore$ Arc x = chord x

As $s = r\theta$

Arc length = s

$\because$ $r = l$ and $s = x$

$\therefore$ $x = l\theta$

$$\theta = \frac{x}{l}$$

Putting this value in equation (2).

$$\therefore \quad a = -g\frac{x}{l}$$

$$a = -\frac{g}{l}x \quad \text{......... (3)}$$

Since, $\frac{g}{l}$ is constant

So, $a = -\text{Constant } x$

$\therefore$ $a \propto -x$

which shows that the acceleration of a simple pendulum is directly proportional towards the displacement and is always directed toward the mean position. So the motion of simple pendulum is a simple harmonic motion.

But for simple harmonic motion

$$a = -\omega^2 x \quad \text{......... (4)}$$

From (3) and (4)

$$-\omega^2 x = -\frac{g}{l} x$$

$$\omega^2 = \frac{g}{l}$$

Taking square root

$$\therefore \quad \omega = \sqrt{\frac{g}{l}}$$

Time Period

∴ time period of simple pendulum is

$$T = \frac{2\pi}{\omega}$$

$$T = \frac{2\pi}{\sqrt{g/l}}$$

$$\therefore \quad T = 2\pi\sqrt{\frac{l}{g}}$$

"This equation shows that the time period depends only on the length of the pendulum and the acceleration due to gravity. It is independent of the mass of the bob."

It is also independent of amplitude.

Q. What is the length of the second's pendulum?

Ans. **As,** $\mathbf{T = 2\ sec}$

$$\therefore \quad T = 2\pi\sqrt{\frac{l}{g}}$$

Squaring the both sides

$$T^2 = 4\pi^2\left(\sqrt{\frac{l}{g}}\right)$$

$$l = \frac{T^2 g}{4\pi^2}$$

$$l = \frac{(2)^2 \times 9.8}{4\,(3.14)^2}$$

$$l = \frac{4 \times 9.8}{4 \times 9.86}$$

$$l = \frac{9.8}{9.86}$$

$$= 0.9939 \text{ m}$$

$$l = 99.39 \text{ cm}$$

Second's Pendulum

"A second's pendulum is a pendulum which completes one vibration in two seconds."

Hence time period of such a pendulum is 2 second.

As, $T = 2$ sec

$$\therefore \quad f = \frac{1}{T}$$

$$f = \frac{1}{2}$$

$$f = 0.5 \text{ Hz}$$

Q.9 Explain the energy conservation in S.H.M.

Ans. ENERGY CONSERVATION IN SHM

When the mass 'm' is pulled slowly, the spring is stretched by an amount 'x_0' against the elastic restoring force 'F'. It is assumed that the stretching is done slowly, so that acceleration is zero.

According to Hook's law:

$$F = k x$$

$$F = \text{Average force}$$

$$= \frac{0 + k\,x_0}{2}$$

$$= \frac{1}{2}k\,x_0$$

Work done in moving the mass 'm' through 'x_0' is

$$W = F\,d$$

$$W = \frac{1}{2}k\,x_0 \times x_0$$

$$W = \frac{1}{2}k\,x_0^2$$

This work appears as elastic P.E. of spring.

Hence, Elastic P.E. $= \frac{1}{2}k\,x_0^2$

This gives maximum P.E. at the extreme position.

$$P.E_{max} = \frac{1}{2}k\,x_0^2$$

Energy at Extreme Position

At extreme position,

$$P.E. = \frac{1}{2}k\,x_0^2$$

$$K.E. = 0$$

$\therefore$ Total energy $= E = P.E. + K.E.$

$$E = \frac{1}{2}k\,x_0^2 + 0$$

So, $$E = \frac{1}{2}k\,x_0^2$$

Energy at any Instant 't'

At any instant 't', if the displacement is 'x', then P.E. at that instant is

$$P.E. = \frac{1}{2}k\,x^2$$

the velocity at that instant is;

K.E. at that instant is

$$\text{K.E.} = \frac{1}{2} m V^2$$

Putting the value of 'V'.

$$\text{K.E.} = \frac{1}{2} m x_0^2 \left(\frac{k}{m} \left(1 - \frac{x^2}{x_0^2} \right) \right)$$

$$\text{K.E.} = \frac{1}{2} k x_0^2 \left(1 - \frac{x^2}{x_0^2} \right)$$

$$\therefore \quad E = \text{P.E.} + \text{K.E.}$$

$$= \frac{1}{2} k x^2 + \frac{1}{2} k x_0^2 \left(1 - \frac{x^2}{x_0^2} \right)$$

$$= \frac{1}{2} k x^2 + \frac{1}{2} k x_0^2 - \frac{1}{2} k x_0^2 \left(\frac{x^2}{x_0^2} \right)$$

$$= \frac{1}{2} k x^2 + \frac{1}{2} k x_0^2 - \frac{1}{2} k x^2$$

$$E_{total} = \frac{1}{2} k x_0^2$$

Energy at Mean Position

K.E. is maximum at mean position.

i.e., $x = 0$

$$\text{K.E.}_{max} = \frac{1}{2} k x_0^2 \left(1 - \frac{(0)^2}{x_0^2} \right)$$

$$= \frac{1}{2} k x_0^2 \left(1 - \frac{0}{x_0^2} \right)$$

$$= \frac{1}{2} k x_0^2 (1 - 0)$$

$$\text{K.E.}_{max} = \frac{1}{2} k x_0^2$$

Also $\text{P.E.} = 0$

$$\therefore \quad E = \text{P.E.} + \text{K.E.}$$

$$E = 0 + \frac{1}{2} k x_0^2$$

"Hence the total energy of the vibrating mass and spring system is constant." When the K.E of the mass is maximum, the P.E of the spring is zero. On the other hand, when the P.E of the spring is maximum, the K.E of the mean is zero.

Q.10 Define free and forced oscillations.

Ans. FREE AND FORCED OSCILLATIONS

Free Oscillations

"A body is said to be executing free vibrations, when it oscillates without the interference of an external force. the frequency of these free vibrations is known as its natural frequency."

Example

A simple pendulum when slightly displaced from its mean position vibrates freely with its natural frequency that depends only upon the length of the pendulum.

Forced Oscillations

"If a freely oscillating system is subjected to an external force, then forced vibrations will take place."

Example

When the mass of a vibrating pendulum is struck repeatedly, then forced vibrations are produced.

The vibrations of a factory floor caused by running of heavy machinery.

DRIVEN HARMONIC OSCILLATOR

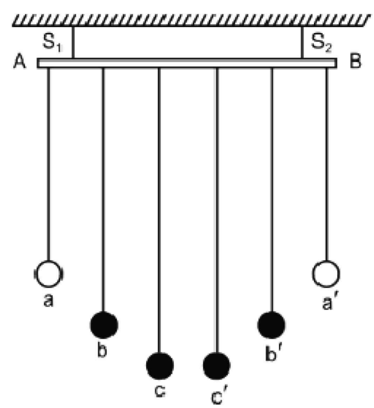
"A physical system under going forced vibrations is known as driven harmonic oscillator."

Q.11 Explain the phenomenon resonance with example.

Ans. RESONANCE

Definition

"The resonance occurs, when the frequency of the applied force is equal to one of the natural frequencies of vibration of the forced or driven harmonic oscillator."



To demonstrate the resonance effect, an apparatus is shown in the Fig. A horizontal rod 'AB' is supported by two strings S_1 and S_2. Three pairs of pendulums aa', bb' and cc' are suspended to this rod. The length of each pair is the same but is different for different pairs. If one of these pendulums, says c, is displaced in a direction perpendicular to the plane of the paper, then its resultant oscillatory motion causes in rod AB a very slight disturbing motion, whose period is the same as that of c'.

other pendulums remain small through out the subsequent motions of c and c′, because their natural periods are not the same as that of the disturbing force due to rod AB.

Example 1

A swing is a good example of mechanical resonance.

Do You Know?

All structures are likely to resonate at one or more frequencies. This can cause problem. It is especially important to test all the components in helicopters and aeroplanes; resonance in an aeroplane's wing or a helicopter rotor could be very dangerous.

Example 2

The column of soldiers, while marching on a bridge of long span are advised to break their steps. Their rhythmic march might set up oscillations of dangerously large amplitude in the bridge structure.

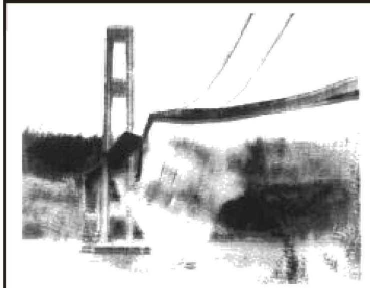
Example 3

Tuning a radio is the best example of electrical resonance.

Example 4

The heating and cooking of food very efficiently and evenly by microwave oven.

Interesting Information



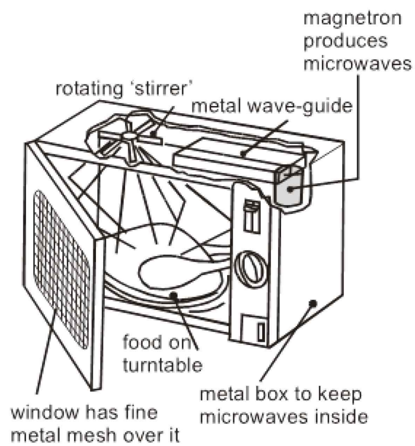
The collapse of Tacoma Narrow Bridge (USA) is suspected to be due to violent resonance oscillations.

Advantages and Disadvantages of Resonance

We come across many examples of resonance in every day life. A swing is a good example of mechanical resonance. It is like a pendulum with a single natural frequency depending on its length. If a series of regular pushes are given to the swing, its motion can be built up enormously. If pushes are given irregularly, the swing will hardly vibrate.

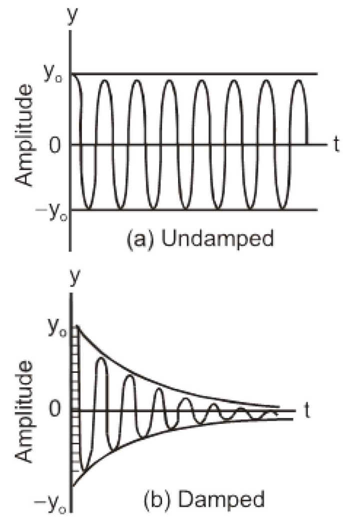
The column of soldiers, while marching on a bridge of long span are advised to break their steps. Their rhythmic march might set up oscillations of dangerously large amplitude in the bridge structure.

Tuning a radio is the best example of electrical resonance. When we turn the knob of a radio, to tune a station, we are changing the natural frequency of the electric circuit of the receiver, to make it equal to the transmission frequency of the radio station. When the two frequencies match, energy absorption is maximum and this is the only station we hear.



Another good example of resonance is the heating and cooking of food very efficiently and evenly by microwave oven figure. The waves produced in this type of oven have a wavelength of 12 cm at a frequency of 2450 MHz. At this frequency, the waves are absorbed due to resonance by water and fat molecules in the food, heating them up and so cooking the food.

This is a common observation that the amplitude of an oscillating simple pendulum decreases gradually with time till it becomes zero. Such oscillations, in which the amplitude decreases steadily with time, are called damped oscillations.



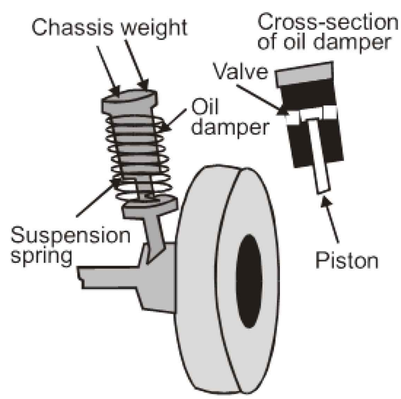
We know from our everyday experience that the motion of any macroscopic system is accompanied by frictional effects. While describing the motion of a simple pendulum, this effects was completely ignored. As the bob of the pendulum moves to and fro, then in addition to the weight of the bob and the tension in the string, bob experiences viscous drag due to its motion through the air. Thus simple harmonic motion is an idealization (Fig. a). In practice, the amplitude of this motion gradually becomes smaller and smaller because of friction and air resistance because the energy of the oscillator is used up in doing work against the resistive forces. Fig. b shows how the amplitude of a damped simple harmonic wave changes, with time as compared with an ideal un-damped harmonic wave. Thus we see that

Damping is the process whereby energy is dissipated from the oscillating system.

An application of damped oscillations is the shock absorber of a car which provides a damping force to prevent excessive oscillation.

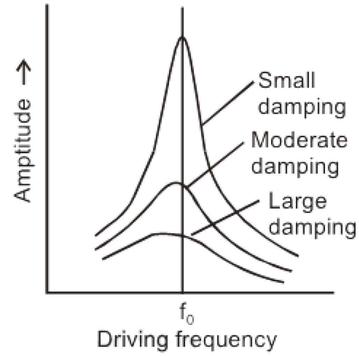
Q.13 What is sharpness of resonance?

Ans. SHARPNESS OF RESONANCE



At resonance, the amplitude of vibration becomes very large when damping is small. Thus, damping prevents the amplitude from becoming excessively large. The amplitude decreases rapidly at a frequency slightly different from the resonant frequency. Where as a heavily damped system has a fairly flat resonance curve as is shown in an amplitude frequency graph in figure.

The effect of damping can be observed by attaching a very light mass such as pith ball, and another of the same length carrying a heavy mass such as lead bob of equal size, to a rod as shown in figure (1). They are set into motion (vibrations) by a third pendulum of equal length attached to the same rod. It is observed that the amplitude of lead bob is much greater than that of pith ball. The


Small damping
Moderate damping
Large damping
Amptitude →
f_0
Driving frequency