Chapter 8

WAVES

LEARNING OBJECTIVE

At the end of this chapter the students will be able to:

Recall the generation and propagation of waves.

Describe the nature of the motions in transverse and longitudinal waves.

Understand and use the terms wavelength, frequency and speed of wave.

Understand and use the equation $v = f\lambda$.

Understand and describe Newton's formula of speed of sound.

Derive Laplace correction in Newton's formula of speed of sound.

Derive the formula $v = v_o + 0.61t$.

Explain and use the principle of superposition.

Understand the terms interference and beats.

Understand and describe reflection of waves.

Explain the formation of a stationary wave using graphical method.

Understand the terms node and anti-node.

Understand and describe modes of vibration of string.

Understand and describe Doppler's effect and its causes.

Waves transport energy without transporting matter. The energy transportation is carried by a disturbance, which spreads out from a source. We are well familiar with different types of waves such as water waves in the ocean, or gently formed ripples on a still pond due to rain drop. When a musician plucks a guitar-string, sound waves are generated which on reaching our ear, produce the sensation of music. Wave disturbances may also come in a concentrated bundle like the shock waves from an aeroplane flying at supersonic speed. Whatever may be the nature of waves, the mechanism by which it oscillations. The waves which propagate by the oscillation of material particles are known as mechanical waves.

There is another class of waves which, instead of material particles, propagate out in space due to oscillations of electric and magnetic fields. Such waves are known as electromagnetic waves. We will undertake the study of electromagnetic waves at a later stage. Here we will consider the mechanical waves only. The waves generated in ropes, strings, coil of springs, water and air are all mechanical waves.

So far we have been considering motion of individual particles but in case of mechanical waves, we study the collective motion of particles. An example will help us here. If you look at a black and white picture in a newspaper with a magnifying glass, you will discover that the picture is made up of many closely spaced dots. If you do not use the magnifier, you do not see the dots. What you see is the collective effect of dots in the form of a picture. Thus what we see as mechanical wave is actually the effect of oscillations of a very large number of particles of the medium through which the wave is passing.

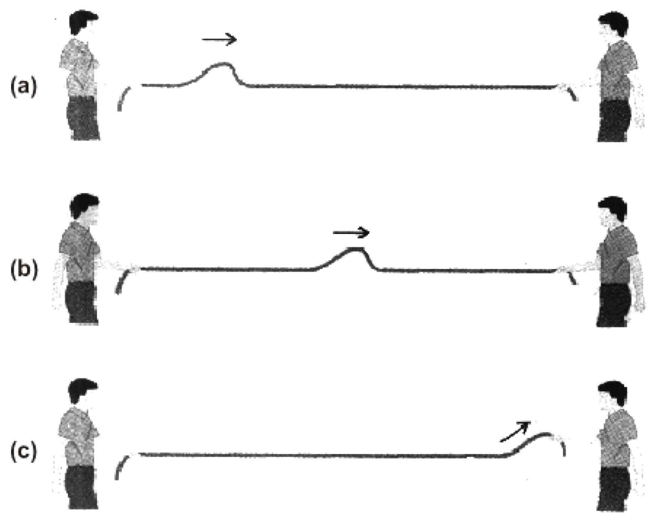
Do You Know?

Ultrasonic waves are particularly useful for undersea communication and detection systems. High frequency radio waves, used in radars travel just a few centimetres in water, whereas highly directional beams of ultrasonic waves can be made to travel many kilometres.

PROGRESSIVE WAVES

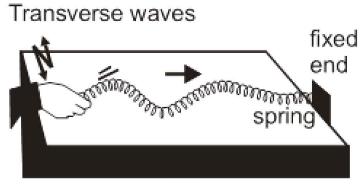
Drop a pebble into water. Ripples will be produced and spread out across the water. The ripples are the examples of progressive waves because they carry energy across the water surface. A wave, which transfers energy by moving away from the source of disturbance, is called a progressive or travelling wave. There are two kinds of progressive waves – transverse waves and longitudinal waves.

Transverse and Longitudinal Waves

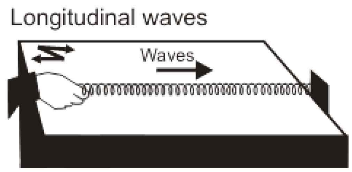


Consider two persons holding opposite ends of a rope or a hosepipe. Suddenly one person gives one up and down jerk to the rope. This disturbs the rope and creates a hump in it which travels along the rope towards the other person (Figure a and b).

When this hump reaches the other person, it causes his hand to move up (Figure c). Thus the energy and momentum imparted to the end of the rope by the first person has reached the other end of the rope by travelling through the rope i.e., a wave has been set up on the rope in the form of a moving hump. We call this type of wave a pulse. The forward motion of the pulse from one end of the rope to the other is an example of progressive wave. The hand jerking the end of the rope is the source of the wave. The rope is the medium in which the wave moves.



(a)



(b)

A large and loose spring coil (slinky spring) can be used to demonstrate the effect of the motion of the source in generating waves in a medium. It is better that the spring is laid on a smooth table with its one end fixed so that the spring does not sag under gravity.

If the free end of the spring is vibrated from side to side, a pulse of wave having a displacement pattern shown in figure (a) will be generated which will move along the spring.

If the end of the spring is moved back and forth, along the direction of the spring itself as shown in figure (b), a wave with back and forth displacement will travel along the spring. Waves like those in figure (a) in which displacement of the spring is perpendicular to the direction of the waves are called transverse waves. Waves like those in figure (b) in which displacements are in the direction of propagation of the waves are called longitudinal waves. In this example the coil of spring is the medium, so in general we can say that

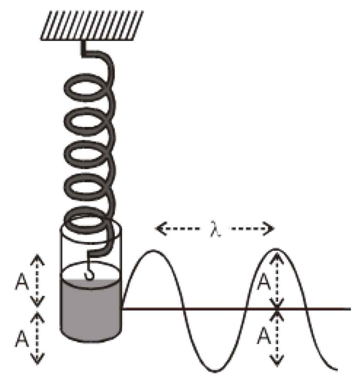
Transverse waves are those in which particles of the medium are displaced in a direction perpendicular to the direction of propagation of waves and longitudinal waves are those in which the particles of the medium have displacements along the direction of propagation of waves.

Both types of waves can be set up in solids. In fluids, however, transverse waves die out very quickly and usually cannot be produced at all. That is why, sound waves in air are longitudinal in nature.

PERIODIC WAVES

Upto now we have considered wave in the form of a pulse which is set up by a single disturbance in a medium like the snapping of one end of a rope or a coil spring. Continuous, regular and rhythmic disturbances in a medium result from periodic vibrations of a source which cause periodic waves in that medium. A good example of a periodic vibrator is an oscillating mass-spring system (figure a). We have already studied in the previous chapter that the mass of such a system executes SHM.

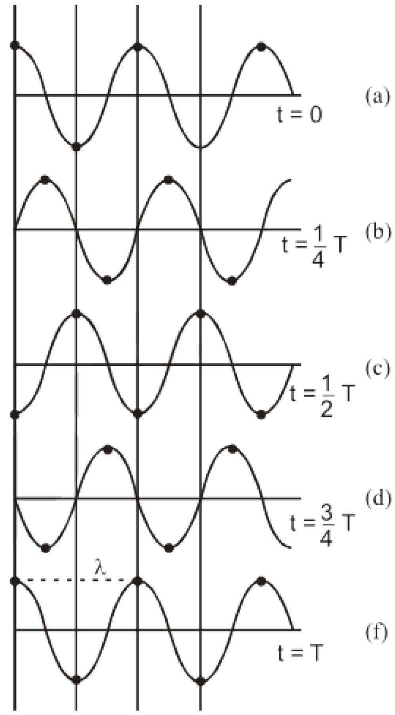
Transverse Periodic Waves



Imagine an experiment where one end of a rope is fastened to a mass spring vibrator. As the mass vibrates up and down, we observe a transverse periodic wave travelling along the length of rope (figure b). The wave consists of crests and troughs. The crest is a pattern in which the rope is displaced above its equilibrium position, and in troughs, it has a displacement below its equilibrium position.

As the source executes harmonic motion up and down with amplitude A and frequency f, ideally every point along the length of the rope executes SHM in turn, with the same amplitude and frequency. The wave travels towards right as crests and troughs in turn, replace one another, but the points on the rope simply oscillates up and down. The amplitude of the wave is the maximum value of the letter lambda λ (figure b).

In principle, the speed of the wave can be measured by timing the motion of a wave crest over a measured distance. But it is not always convenient to observe the motion of the crest. As discussed below, however, the speed of a periodic wave can be found indirectly from its frequency and wavelength.



As a wave progresses, each point in the medium oscillates periodically with the frequency and period of the source. Figure illustrates a periodic wave moving to the right, as it might look in photographic snapshots taken every 1/4 period. Follow the progress of the crest that started out from the extreme left at t = 0. The time that this crest takes to move a distance of one wavelength is equal to the time required for a point in the medium to go through one complete oscillation. That is the crest moves one wavelength λ in one period of oscillation T. The speed v of the crest is therefore,

$$v = \frac{\text{Distance moved}}{\text{Corresponding time interval}} = \frac{\lambda}{T}$$

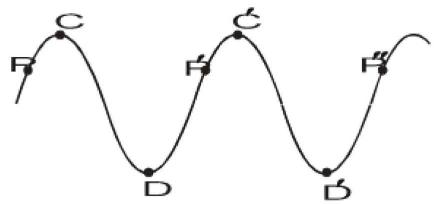
All parts of the wave pattern move with the same speed, so the speed of any one crest is just the speed of the wave. We can therefore, say that the speed v of the waves is

$$v = \frac{\lambda}{T} \qquad \text{...... (1)}$$

but $\frac{1}{T} = f$, where f is the frequency of the wave. It is the same as the frequency of the vibrator, generating the waves. Thus eq. (1) becomes

$$v = f\lambda \qquad \text{...... (2)}$$

Phase Relationship between Two Points on a Wave



The profile of periodic waves generated by a source executing SHM is represented by a sine curve. Figure shows the snapshot of a periodic wave passing through a medium. In this figure, set of points are shown which are moving in unison as the periodic wave passes. The points C and C′, as they move up and down, are always in the same state of vibration i.e., they always have identical displacements and velocities. Alternatively, we can say that as the wave passes, the points C and C′ move in phase. We may also say that C′ leads C by one time period of 2π radian. Any point at a distance x, C lags behind by phase angle.

$$\phi = \frac{2\pi x}{\lambda}$$

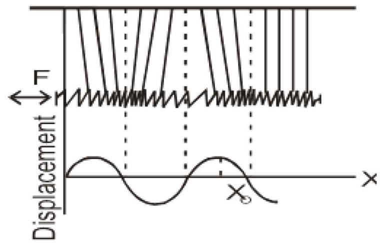
...... are all in phase with each other. These points can be anywhere along the wave and need not correspond with only the highest and lowest points. For example, points such as P, P′, P″, ... are all in phase. Each is separated from the next by a distance λ.

Some of the points are exactly out of step. For example, when point C reaches its maximum upward displacement, at the same time D reaches its maximum downward displacement. At the instant that C begins to go down, D begins to move up. Points such as these are called one half period out of phase. Any two points separated from one another by $\frac{\lambda}{2}$, $3\frac{\lambda}{2}$, $5\frac{\lambda}{2}$, ... are out of phase.

Longitudinal Periodic Waves

In the previous section we have considered the generation of transverse periodic waves. Now we will see how the longitudinal periodic waves can be generated.

Consider a coil of spring as shown in figure. It is suspended by threads so that it can vibrate horizontally. Suppose an oscillating force F is applied to its end as indicated. The force will alternately stretch and compress the spring, thereby sending a series of stretched regions (called rarefaction) and compressions down the spring. We will see the oscillating force causes a longitudinal wave to move down the spring. This type of wave generated in springs is also called a compressional wave. Clearly in a compressional wave, the particles in the path of wave move back and forth along the line of propagation of the wave.



Notice in figure, the supporting threads would be exactly vertical if the spring were undisturbed. The disturbance passing down the spring causes displacements of the elements of the spring from their equilibrium positions. In figure, the displacements of the threads from the vertical are a direct measure of the displacements of the spring elements. It is, therefore, an easy way to graph the displacements of the spring elements from their equilibrium positions and this is done in the lower part of the figure.

Q.1 Define progressive waves with its types.

Ans. **PROGRESSIVE WAVES (TRAVELLING WAVE)**

A wave, which transfers energy in moving away from the source of disturbance, is called a progressive or travelling waves

There are two kinds of traveling waves

(i) Transverse waves.

(ii) Longitudinal waves.

(i) Transverse Waves

"Transverse waves are those in which particles of the medium are displaced in a direction perpendicular to the direction of propagation of waves."

e.g., water waves, light waves.

(ii) Longitudinal Waves (Compressional Waves)

Both types of waves can be setup in solids in liquids however, transverse waves die out very quickly and usually cannot be produced at all that is why sound waves in air one longitudinal in nature.

Periodic Waves

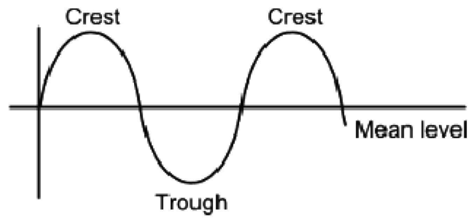
"Continuous, regular and rhythmic disturbances in a medium result from periodic vibrations of a source which cause periodic waves in that medium."

e.g., Oscillating mass–spring system.

Transverse Periodic Waves

Crest

"The portion of transverse wave above its mean position, is called crest."



Trough

"The potion of transverse wave below its mean position, is called trough."

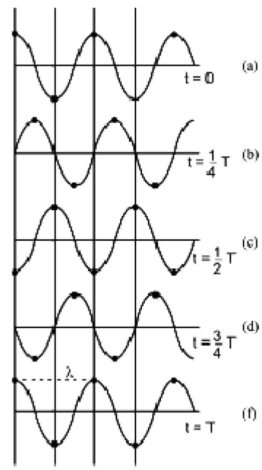
Wavelength

"The distance between any two consecutive crests or troughs, is called wave length."

It is denoted by a Greek letter Lambda (λ).

Q.2 Show that $V = \lambda f$.

Ans. The time that the crest takes to move a distance of one wave length is equal to the time required for a point in the medium to complete one oscillation i.e., crest moves one wave length 'λ' in one period of oscillation 'T', the speed 'V of the crest (wave) is



As, $V = \dfrac{\text{Distance moved}}{\text{Corresponding time interval}}$

$\therefore \quad V = \dfrac{\lambda}{T} \quad \left(\begin{array}{l} S = V\,t \\ V = \dfrac{S}{t} \end{array} \right)$

Since, $\dfrac{1}{T} = f$

$\therefore \quad V = \lambda f$

Which is the relation between speed, frequency and wavelength.

Longitudinal Periodic Waves

"The portion of longitudinal wave where particles of medium are very close to each other is called compression."

Rarefaction

"The portion of longitudinal wave where particles of medium are far apart from each other is called rarefaction."

Q.3 Explain Newton's formula for the speed of sound in air.

Ans. SPEED OF SOUND IN AIR

(i) Newton's Formula for Speed of Sound in Air

When one particle of the medium is disturbed, the disturbance in the form of wave travel in all directions in the medium. The velocity of disturbance depends upon the density and the elasticity of the medium. The lighter the density of the medium, more quickly the disturbance moves from point to point and similarly greater the elasticity of the medium, more quickly disturbance will be propogated from point to point in the medium. Newton proposed the following formula for the velocity of sound through the materials which is as follows:

$$V = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{\text{Elasticity}}{\text{Density}}}$$

Where $\in$ is the elasticity of the medium and ρ is the density.

Newton assumed that when a sound wave travels through air, the temperature of the air during compression remains constant and pressure changes from P to $(P + \Delta P)$ and therefore, the volume changes from V to $(V - \Delta V)$. According to Boyle's law

$$PV = (P + \Delta P)(V - \Delta V)$$

or

$$PV = PV - P\Delta V + V\Delta P - \Delta P\Delta V$$

The product $\Delta P\Delta V$ is very small and can be neglected. So, the above equation becomes:

$$0 = -P\Delta V + V\Delta P$$

$$P = \frac{V\Delta P}{\Delta V}$$

$$P = \frac{\Delta P}{\frac{\Delta V}{V}}$$

Speed of sound in different media	
Medium	**Speed ms^{-1}**
Solids at 20°C	
Lead	1320
Copper	3600
Aluminium	5100
Iron	5130
Glass	5500
Liquids at 20°C	
Methanol	1120
Water	1483
Gasses of S.T.P.	
Carbon dioxide	258
Oxygen	315
Air	332
Helium	972

$$\frac{\Delta V}{V} = \text{Volume strain}$$

$$P = \frac{\text{Stress}}{\text{Volume strain}} = \text{Elasticity}$$

The above equation becomes:

$$V = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{P}{\rho}}$$

On substituting the values of atmospheric pressure and density of air at S.T.P. in above equation, we find that the speed of sound waves in air comes out to be 280 ms^{-1}, whereas its experimental value is 332 ms^{-1}.

Q.4 How laplace correct the speed of sound in air?

Ans. LAPLACE CORRECTION FOR VELOCITY OF SOUND IN AIR

The sound waves travel in the form of compressions and rarefactions. The compressions and rarefactions are so rapid, the temperature of air does not remain constant. The temperature increases due to compressions and the temperature decreases due to the rarefactions. Therefore during compression the air does not lose heat due to conduction and during rarefaction it does not gain heat. Thus the temperature throughout the medium does not remain constant. The relation between volume and pressure (PV = Constant) is not true but it is given as

$$PV^{\gamma} = \text{Constant}$$

Where γ is constant and its value depends upon the nature of the gas where

$$\gamma = \frac{\text{Molar specific heat at constant pressure}}{\text{Molar specific heat at constant volume}}$$

If 'P' be the pressure then the change in pressure is very small which is $P + \Delta P$ therefore volume decreases from V to $V - \Delta V$ then

$$PV^{\gamma} = (P + \Delta P)(V - \Delta V)^{\gamma}$$

$$PV^{\gamma} = (P + \Delta P)V^{\gamma}\left(1 - \frac{\Delta V}{V}\right)^{\gamma}$$

Applying Binomial theorem:

$$P = (P + \Delta P)\left(1 - \gamma\frac{\Delta V}{V}\right)$$

or $$P = P - \gamma P\frac{\Delta V}{V} + \Delta P - \gamma \Delta P\frac{\Delta V}{V}$$

Where $\left(\gamma \Delta P \frac{\Delta V}{V}\right)$ is negligible. Hence, we have

For Your Information

Values of Constant

Types of gas	γ
Monoatomic	1.67
Diatomic	1.40
Polyatomic	1.29

For Your Information

Ranges of Hearing

Organisms	Frequencies (Hz)

$$\text{or} \quad \frac{\Delta P}{\frac{\Delta V}{V}} = \gamma P = E$$

Cat	60 – 70,000
Dog	15 – 50,000
Human	20 – 20,000

Therefore

$$V = \sqrt{\frac{E}{\rho}}$$

$$V = \sqrt{\frac{\gamma P}{\rho}}$$

On substituting the value of atmospheric pressure and γ then the speed of the sound is 333 m/s. This value of speed of sound is very close to the experimental value. Thus the laplace correction must therefore be correct.

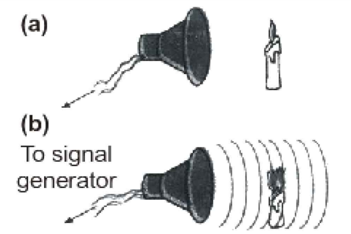
Q.5 What is effect of variation of pressure on the speed of sound?

Ans. **EFFECT OF PRESSURE ON SPEED OF SOUND**

$$\text{As,} \quad V = \sqrt{\frac{\gamma P}{\rho}} \quad \left(\begin{array}{l} \rho = \frac{m}{V} \\ P \rightarrow \text{increase} \\ V \rightarrow \text{decreases} \\ \rho \rightarrow \text{increase} \\ \therefore \; P \propto \rho \end{array} \right)$$

Since density is proportional to the pressure so the speed of sound is not affected by the variation in pressure of the gas.

Tidbits



Sound waves cause the candle flame to flicker.

Q.6 What is effect of density on the speed of sound?

Ans. **EFFECT OF DENSITY ON SPEED OF SOUND**

$$\text{As,} \quad V = \sqrt{\frac{\gamma P}{\rho}}$$

At the same temperature and pressure for the gases having the same value of γ, the velocity is inversely proportional to the square root of their density.

$$\text{i.e.,} \quad V \propto \frac{1}{\sqrt{\rho}}$$

Note: *Speed of sound in hydrogen is four times its speed in oxygen as density of the oxygen is sixteen times that of the hydrogen.*

Q.7 What is the effect of temperature on the speed of sound in air?

Ans. **EFFECT OF TEMPERATURE ON SPEED OF SOUND**

When a gas is heated at constant pressure, its volume is increased and hence, its density is decreased.

Let, V_0 = Speed of sound at 0^oC , ρ_0 = Density of gas at 0^oC

V_t = Speed of sound at t^oC , ρ_t = Density of gas at t^oC

then, $$V_0 = \sqrt{\frac{\gamma P}{\rho_0}} \quad \text{......... (1)}$$

and $$V_t = \sqrt{\frac{\gamma P}{\rho_t}} \quad \text{......... (2)}$$

Dividing equation (2) by (1)

$$\frac{V_t}{V_0} = \sqrt{\frac{\gamma P/\rho_t}{\gamma P/\rho_0}}$$

$$\frac{V_t}{V_0} = \frac{\sqrt{\gamma P}}{\sqrt{\rho_t}} \times \frac{\sqrt{\rho_0}}{\sqrt{\gamma P}}$$

$$\frac{V_t}{V_0} = \frac{\sqrt{\rho_0}}{\sqrt{\rho_t}}$$

$$\frac{V_t}{V_0} = \frac{\sqrt{\rho_0}}{\sqrt{\rho_t}} \quad \text{......... (3)}$$

If V_0 is the volume of a gas at temperature 0^oC and V_t is volume at t^oC, then by using volume expansion.

$$V_t = V_0 (1 + \beta t)$$

Where β is the coefficient of volume expansion of the gas. For all gases, its value is about $\frac{1}{273}$.

Hence, $$V_t = V_0\left(1 + \frac{t}{273}\right)$$

As $$\rho = \frac{m}{V}$$

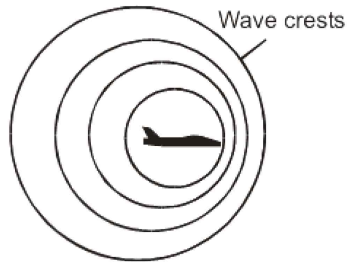
$$\text{Volume} = \frac{\text{Mass}}{\text{Density}}$$

$$V = \frac{m}{\rho}$$

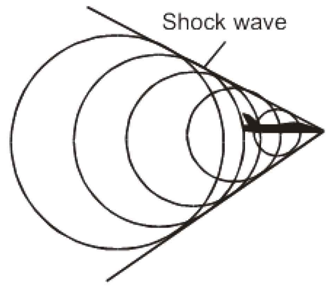
$$\frac{m}{\rho_t} = \frac{m}{\rho_0}\left(1 + \frac{t}{273}\right)$$

$$\frac{1}{\rho_t} = \frac{1}{\rho_0}\left(1 + \frac{t}{273}\right)$$

Do You Know?



Slower than the speed of sound.



Faster than the speed of sound.

What happens when a jet plane like Concorde flies faster than the speed of sound?
A conical surface of concentrated sound energy sweeps over the ground as a supersonic plane passes overhead. It is known as sonic boom.

$$V_t - V_o \propto V, t$$
$$V_t - V_o = \beta V_o t$$
$$V_t = V_o + \beta V_o t$$
$$= V_o(1 + \beta t)$$

$$\frac{V_t}{V_0} = \sqrt{\frac{\rho_t\left(1+\frac{t}{273}\right)}{\rho_t}}$$

$$\frac{V_t}{V_0} = \sqrt{1+\frac{t}{273}} \qquad \text{.........} \ (4)$$

$$\frac{V_t}{V_0} = \sqrt{\frac{273+t}{273}}$$

$$\frac{V_t}{V_0} = \sqrt{\frac{T}{T_0}}$$

Where T and T_0 are the absolute temperature. Corresponding to 5°C and 0°C respectively. Thus the speed of sound is directly proportional to the square root of the absolute temperature.

Now, using Binomial theorem and neglecting high power, we have, eq. (4) as:

$$\frac{V_t}{V_0} = \left(1+\frac{t}{273}\right)^{1/2}$$

$$\frac{V_t}{V_0} = 1+\frac{1}{2}\left(\frac{t}{273}\right)$$

$$\frac{V_t}{V_0} = \left(1+\frac{t}{546}\right)$$

$$V_t = V_0+\frac{V_0\, t}{546}$$

As, $V_0 = 332$ m/s

Putting the value in the 2nd factor,

$$V_t = V_0+\frac{332}{546}t$$

$$V_t = V_0 + 0.61\ t$$

This shows that one degree Celsius rise in temperature produces approximately 0.61 m/s (61 cm/s) increase in the speed of sound.

Q.8 State the principle of superposition.

Ans. PRINCIPLE OF SUPERPOSITION

So far, we have considered single waves. What happens when two waves encounter each other in the same medium? Suppose two waves approach each other on a coil of spring, one travelling towards the right and the other travelling towards left. Figure shows that you would see happening on the spring. The waves pass through each other without being modified. After the encounter, each wave shape looks just as it did before and is travelling along just as it was before.

This phenomenon of passing through each other unchanged can be observed with all types of waves. You can easily see that it is true for surface ripples.

But what is going on during the time when the two waves overlap? Figure (c) shows that the displacements they produce just add up. At each instant, the spring's displacement at any point in the overlap region is just the sum of the displacements that would be caused by each of the two waves separately.

Thus, if a particle of a medium is simultaneously acted upon by n waves such that its displacement due to each of the individual n waves be y_1, y_2, ……, y_n, then the resultant displacement of the particle, under the simultaneous action of these n waves is the algebraic sum of all the displacements i.e.,

$$Y = y_1 + y_2 + \ldots\ldots + y_n$$

This is called principle of superposition.

Again, if two waves which cross each other have opposite phase, their resultant displacement will be

$$Y = y_1 - y_2$$

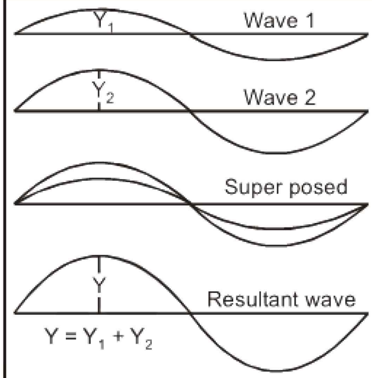
Particularly if $y_1 = y_2$ then result displacement $Y = 0$. Principle of superposition leads to many interesting phenomena with waves.

(i) Two waves having same frequency and travelling in the same direction (Interference).

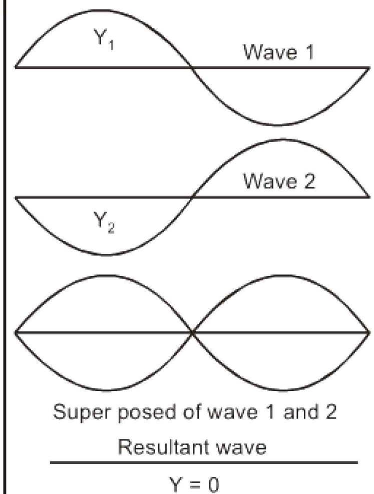
(ii) Two waves of slightly different frequencies and travelling in the same direction (Beats).

(iii) Two waves of equal frequency travelling in opposite direction (Stationary waves).

For Your Information



Superposition of two waves of the same frequency which are exactly in phase.



Superposition of two waves of the same frequency which are exactly out of phase.

Q.9 State and explain the phenomenon of interference of sound.

Ans. INTERFERENCE

"Superposition of two waves having the same frequency and traveling in the same direction results in a phenomenon, called interference." There are two types:

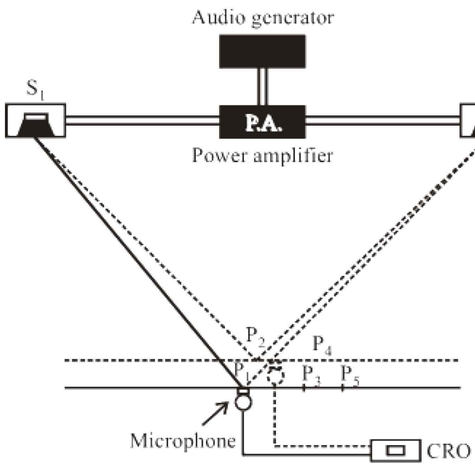


Fig. (a)

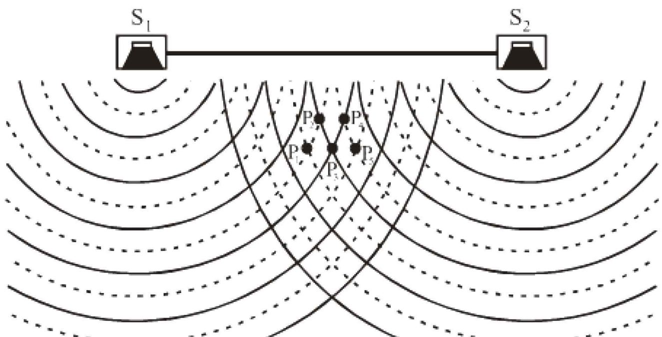


Fig. (b)

(i) Constructive Interference

"If two waves arrive at a point in phase i.e., compression of one wave falls on compression on other wave and rarefaction of one wave falls on the rarefaction of other wave, then resultant sound is loudest."

or Whenever path difference is an integral multiple of wavelength the two waves are added up. This effect is called constructive interference.

∴ Condition for constructive interference can be written as

$$\Delta S = n\lambda$$

where $n = \pm 1, \pm 2, \pm 3, \ldots\ldots$

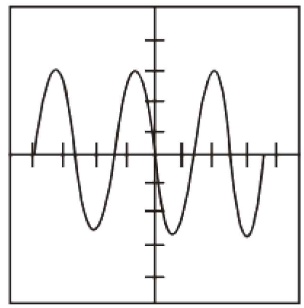


Fig. (c)
Constructive interference
Large displacement is displayed on the CRO screen

(ii) Destructive Interference

"If two waves reach a point out of phase i.e., compression of one wave falls on the rarefaction of other wave and rarefaction of one wave falls on the compression of other wave, then resultant sound will be minimum."

or At points where displacements of two waves cancel each other's effect, the path difference is an odd integral multiple of half the wavelength. This effect is called destructive interference.

Condition for destructive interference is

$$\Delta S = (2n + 1)\frac{\lambda}{2}$$

where $n = 0, \pm 1, \pm 2, \ldots\ldots$

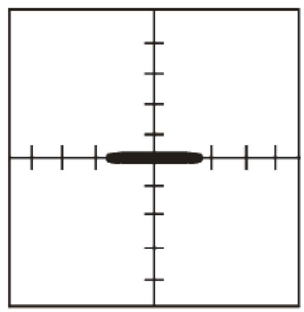


Fig. (d)
Destructive interference
Zero displacement is displayed on the CRO screen

Explanation

An experimental set up to observe interference effect in sound waves as shown in figure.

Two loud speakers S_1 and S_2 act as two sources of harmonic sound waves of a fixed frequency screen. The microphone is placed at various points, turn by turn, infront of the loud speakers as shown in the figure.

At points P_1, P_3 and P_5, we find that compressions met with compressions and rarefactions. So, the displacement of the two waves are added up at these points and large resultant displacement is produced. At points P_2 and P_4, compressions met with rarefactions so they cancel each other effect. The resultant displacement becomes zero. Now we have to find the path difference between the waves at point P_1 is

$$\Delta S = S_2P_1 - S_1P_1$$

$$\Delta S = 4\frac{1}{2}\lambda - 3\frac{1}{2}\lambda = \lambda$$

But $$\Delta S = n\lambda$$

For constructive interference.

Where n = 0, 1, 2, 3, ……

For destructive interference

$$\Delta S = S_2P_2 - S_1P_2$$

$$\Delta S = 4\lambda - 3\frac{1}{2}\lambda = \frac{1}{2}\lambda$$

So, $$\Delta S = \left(n + \frac{1}{2}\right)\lambda$$

Where n = 0, 1, 2, 3, ……

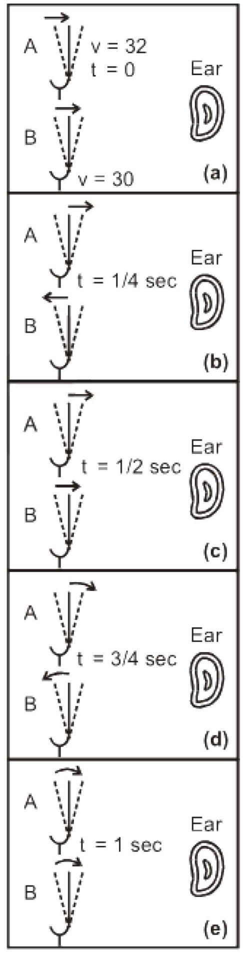
Q.10 What are beats? How they are produced? Show that the number of beats is equal to the difference between the frequencies of the tuning forks.

Ans. **BEATS**

Beat is the combined effect of two sound waves having frequencies slightly different from each other.

Consider two tuning forks each having frequency 32 cps. Slightly load (with wax or ring) one tuning fork so that frequency decreases a little. Let the frequency becomes 30 cps. The two tuning forks are sounded together and held at equal distance from the ear. Let at t = 0, the two forks are in phase. i.e., their right prongs moves towards right producing compressions at the same time and louder sound is heard by the listener.

With passage of time, the tuning fork B (30 cps) begins to fall behind 'A'.



At $t = \frac{1}{4}$ sec, 'A' has completed 8 vib and 'B' has completed $7\frac{1}{2}$ vib. The prongs are now out of phase and no sound is heard due to destructive interference.

At $t = \frac{1}{2}$ sec, 'A' has completed 16 vib and 'B' completes 15 vib. The prongs of forks become in phase and louder sound is heard. At $t = \frac{3}{4}$ sec, 'A' has completed 24 vib while 'B' completes $22\frac{1}{2}$ vib. The prongs are now in opposite phase and once again no sound is heard.

At t = 1 sec, both forks have completed 32 vib and 30 vib. The prongs become in phase and max or louder sound is heard.

It is observed that in 1 sec, the sound falls in intensity twice. The sudden fall of sound in intensity is known as beat. Thus two beats are produced/sec which is equal to the difference in frequencies of two forks (32 – 30 = 2).

Definition of Beats

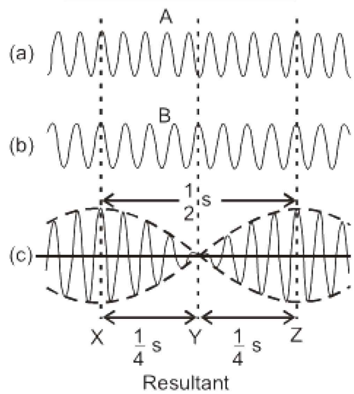
The periodic alteration of sound between maximum and minimum loudness as many times a second as the difference in frequencies is called phenomenon of beats.

No of beats/sec = Difference in frequencies

$$\pm n = f_A - f_B$$

Graphical Explanation of Beats

The displacements of the particles of the medium due to two waves are plotted separately as function of time. The resultant displacement of any particle will be the sum of the displacement due to each of the two waves. The resultant wave which is produced in shown in figure (c). It is seen that amplitude of resultant waves changes with time. The change in amplitude gives rise to production of beat.



Formation of Beats

Uses of Phenomenon of Beats

The phenomenon of beats is used to find out:

(i) Unknown frequency

(ii) To tune a musical instrument

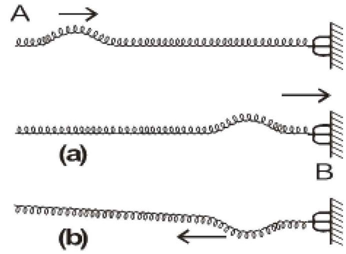
Q.11 Write a note on reflection of waves.

Ans. **REFLECTION OF WAVES**

In an extensivemedium, a wave travels in all direction from its source with a velocity depending upon the properties of the medium. However, when the wave comes across the boundary of two media, a part of it is reflected back. The reflected wave has the same wavelength and frequency but its phase may

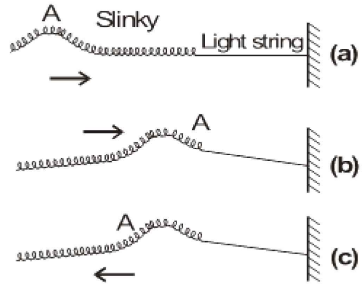
Now we will discuss two most common cases of reflection at the boundary. These cases will be explained with the help of waves travelling in slinky spring. (A slinky spring is a loose spring which has small initial length but a relatively large extended length).

One end of the slinky spring is tied to a rigid support on a smooth horizontal table. When a sharp jerk is given up to the free end of the slinky spring towards the side A, a displacement or a crest will travel from free end to the boundary (Figure a). It will exert a force on bound end towards the side A. Since this end is rigidly bound and acts as a denser medium, It will exert a reaction force on the spring in opposite direction. This force will produce displacement towards B and a trough will travel backwards along the spring (Figure b).



From the above discussion it can be concluded that whenever a transverse wave, travelling in a rarer medium, encounters a denser medium, it bounces back such that the direction of its displacement is reversed. An incident crest on reflection becomes a trough.

This experiment is repeated with a little variation by attaching one end of a light string to a slinky spring and the other end to the rigid support as shown in figure. If now the spring is given a sharp jerk towards A, a crest travels along the spring as shown in figure. When this crest reaches the spring-string boundary, it exert a force on the string towards the side it does not oppose the motion of the spring. The end of the spring, therefore, continues its displacement towards A. The spring behaves as if it has been plucked up. In other words a spring crest is again created at the boundary of the spring-string system, which travels backwards along the spring. From this it can be concluded that when a transverse wave travelling in a denser medium, is reflected from the **boundary of a rarer** medium, the direction of its displacement remains the same. An incident crest is reflected as a crest. We are already familiar with the fact that the direction of displacement is reversed when there is change of 180° in the phase of vibration. So, the above conclusion can be written as follows:



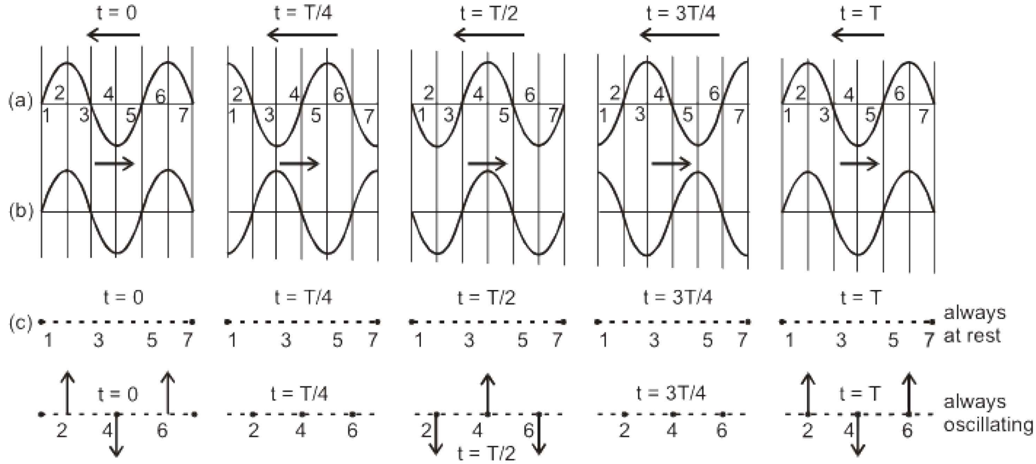
(i) If a transverse wave travelling in a rarer medium is incident on a denser medium, it is reflected such that it undergoes a phase change of 180°.

(ii) If a transverse wave travelling in a denser medium is incident on a rarer medium, it is reflected without any change in phase.

Q.12 What are stationary waves? How they one produced? Define node and anti-node.

Ans. **STATIONARY WAVES**

Now let us consider the superposition of two waves moving along a string in opposite directions. Figure (a, b) shows the profile of two such waves at instants = 0, T/4, 3/4 T and T, where T is the time period of the wave. We are interested in finding out the displacements of the points 1, 2, 3, 4, 5, 6 and 7 at these instants as the waves superpose. From the figure (a, b), it is obvious that the points 1, 2, 3, etc.,



are distant λ/4 apart, λ being the wavelength of the waves. We can determine the resultant displacement of these points by applying the principle of superposition. Figure (c) shows the resultant displacement of the points 1, 3, 5 and 7 at the instants t = 0, T/4, T/2, 3T/4 and T. It can be seen that the resultant displacement of these points is always zero. These points of the medium are known as nodes. Figure (c) shows that the distance between two consecutive nodes is λ/2. Figure (d) shows the resultant displacement of the points 2, 4 and 6 at the instant t = 0, T/4, T/2, 3T/4 and T. The figure shows that these points are moving with an amplitude which is the sum of the amplitudes of the component waves. These points are known as antinodes. They are situated midway between the nodes and are also λ/2 apart. The distance between a node and the next antinode is λ/4. Such a pattern of nodes and antinodes is known as a stationary or standing wave.

Energy in a wave moves because of the motion of the particles of the medium. The nodes always remain at rest, so energy cannot flow past these points. Hence energy remains "standing" in the medium between nodes, although it alternates between potential and kinetic forms. When the antinodes are all at their extreme displacements, the energy stored is wholly potential and when they are simultaneously passing through their equilibrium positions, the energy is wholly kinetic.

An easy way to generate a stationary wave is to superpose a wave travelling down a string with its reflection travelling in opposite direction as explained in the next section.

Q.13 Explain the stationary waves in a stretched string. Also calculate the frequencies.

Ans. STATIONARY WAVES IN A STRETCHED STRING

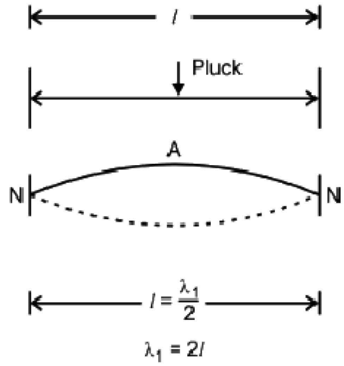
Consider a string of length '*l*' which is kept stretched by clamping its ends so that the tension in the string is F. If the string is plucked at its middle point, two transverse waves will originate from this point. One of them will move towards the left end of the string and the other towards the right end. When these waves reach the two clamped ends, they are reflected back, thus giving rise to stationary waves. The string will vibrate with such a frequency f_1, so that nodes are formed at two fixed ends (clamped ends) and anti-nodes between them. Thus the string vibrates in one loop as shown in Fig.

If λ_1 is the wavelength of this mode of vibration (1st mode of vibration) so, As $l = \frac{\lambda_1}{2}$

$$\lambda_1 = 2l \qquad \text{......... (i)}$$

$$V = \lambda f$$

$$f_1 = \frac{V}{\lambda_1}$$



Putting value of λ_1.

$$\therefore \quad f_1 = \frac{V}{2l} \qquad \text{......... (ii)}$$

The speed 'V' of the waves in the string depends upon the tension F of the string and mass per unit length of the string. It is given by

$$V = \sqrt{\frac{F}{m}}$$

$$\therefore \quad f_1 = \frac{\sqrt{F/m}}{2l}$$

$$f_1 = \frac{1}{2l}\sqrt{\frac{F}{m}} \qquad \text{......... (iii)}$$

If the string is plucked from $\frac{1}{4}$ of its length, then again stationary waves will be set up, but now the string vibrate in two loops with f_2. If λ_2 is wave length in this case then,

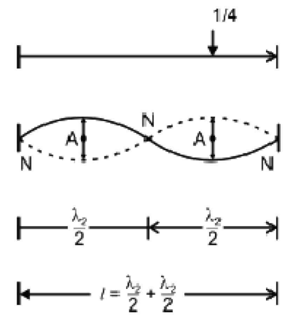
As, $l = \frac{\lambda_2}{2} + \frac{\lambda_2}{2}$

$$l = \frac{\lambda_2 + \lambda_2}{2}$$

$$l = \frac{2\lambda_2}{2}$$

$$l = \lambda_2$$

$$\therefore \quad f_2 = \frac{V}{\lambda_2}$$



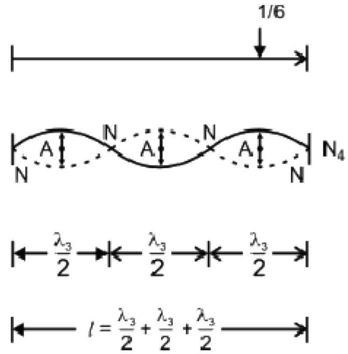
Putting value of λ_2

$$\therefore \quad f_2 = 2\left(\frac{V}{2l}\right)$$

$$f_2 = 2f_1$$

This shows that when string vibrates in two loops, its frequency is doubled and wave length becomes half than it vibrates in one loop.

Similarly if the string is plucked from $\frac{1}{6}$th of its length, it vibrates in three loops as shown in figure.



As $\quad l = \frac{\lambda_3}{2} + \frac{\lambda_3}{2} + \frac{\lambda_3}{2}$

$$l = 3\frac{\lambda_3}{2}$$

$$\lambda_3 = \frac{2l}{3}$$

$$\therefore \quad f_3 = \frac{V}{\lambda_3}$$

Putting value of λ_3

$$f_3 = \frac{V}{2l/3}$$

$$f_3 = 3\left(\frac{V}{2l}\right)$$

$$f_3 = 3f_1$$

It means that when string vibrates in three loops, its frequency is three times the frequency when it vibrates in one loop.

Thus we can generalize that if the string is made to vibrate in 'n' loop, then

$$f_n = nf_1$$

and wave length is;

$$\lambda_n = \frac{2l}{n}$$

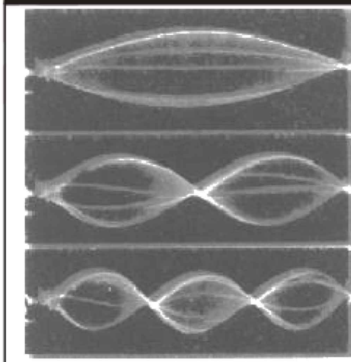
where n = 1, 2, 3, ……

It is clear that as the string vibrates in more and more loops, its frequency goes on increasing and the wave length gets shorter. However the product of frequency and wave length is always equal to V, (speed of the wave).

The above discussion clearly establishes that the stationary waves have a discrete set of frequencies f_1, $2f_1$, $3f_1$, ………, n f_1 which is known as harmonic series. The fundamental frequency 'f_1' is called first harmonic (over tone), 'f_2' is called second harmonic (over tone), and so on.

Note: *The frequency of a string on a musical instrument can be changed either by varying the tension or by changing the length. For example the tension in guitar and violin strings is*

Do You Know?



A standing-wave pattern is formed when the length of the string is an integral multiple of half wavelength; otherwise no standing wave is formed.

For Your Information

In an organ pipe, the primary driving mechanism is wavering,

Q.14 Describe the stationary waves in air column.

Ans. STATIONARY WAVES IN AIR COLUMNS

Stationary waves can be set in other media also, such as air column. A common example of vibrating air column is in the organ pipe. The relationship between the incident wave and the reflected wave depends on whether the reflecting end of the pipe is open or closed. If the reflecting end is open, the air molecules have complete freedom of motion and this behaves as an antinode. If the reflecting end is closed, then it behaves as a node because the movement of the molecules is restricted. The modes of vibration of an air column in a pipe open at both ends are shown in figure.

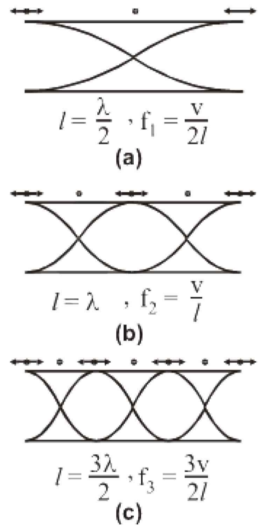


Fig. Stationary longitudinal waves in a pipe open at both ends.

In figure, the longitudinal waves set up in the pipe have been represented by transverse curved lines indicating the varying amplitude of vibration of the air particles at points along the axis of the pipe. However, it must be kept in mind that air vibrations are longitudinal along the length of the pipe. The wavelength 'λ_n' of nth harmonic and its frequency 'f_n' of any harmonic is given by

$$\lambda_n = \frac{2l}{n} \quad , \quad f_n = \frac{V}{\lambda_n} = \frac{nV}{2l} \quad \ldots\ldots (1)$$

$$n = 1, 2, 3, 4, \ldots\ldots$$

where 'V' is the speed of sound in air and 'l' is the length of the pipe. The equation (10 can also be written as

$$f_n = nf_1 \quad \ldots\ldots (2)$$

If a pipe is closed at one end and open at the other, the closed end is a node. The modes of vibration in this case are shown in figure.

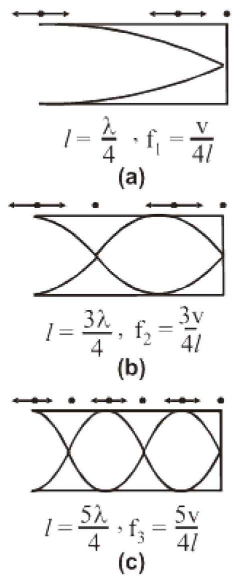


Fig. Stationary longitudinal waves in a pipe closed at one end. Only odd harmonics are present.

In case of fundamental note, the distance between a node and antinode is one fourth of the wavelength,

Hence, $l = \frac{\lambda_1}{4}$ or $\lambda_1 = 4l$

Since $V = f\lambda$

Hence, $f_1 = \frac{V}{\lambda_1} = \frac{V}{4l}$

It can be proved that in a pipe closed at one end, only odd harmonics are generated, which are given by the equation.

$$f_n = \frac{nV}{4l} \quad \ldots\ldots (3)$$

where $n = 1, 3, 5, \ldots\ldots$

This shows that the pipe, which is open at both ends, is richer in harmonics.

In 1^{st} mode

$$l = \frac{2\lambda_1}{4}$$

$$l = \frac{\lambda_1}{2}$$

$$\lambda_1 = 2l$$

As, $$V = \lambda f$$

$$f_1 = \frac{V}{\lambda_1}$$

Putting value of 'λ_1',

$$\therefore \quad f_1 = \frac{V}{2l}$$

In 2^{nd} mode

$$l = \frac{\lambda_2 + 2\lambda_2 + \lambda_2}{4}$$

$$l = \frac{4\lambda_2}{4}$$

$$l = \lambda_2$$

$$\therefore \quad f_2 = \frac{V}{l}$$

Multiply and divided by (2).

$$f_2 = 2\left(\frac{V}{2l}\right)$$

In 3^{rd} mode

$$l = \frac{\lambda_3 + 2\lambda_3 + 2\lambda_3 + \lambda_3}{4}$$

$$l = \frac{6\lambda_3}{4}$$

$$l = \frac{3\lambda_3}{2}$$

$$l = \frac{2l}{3}$$

As, $$f_3 = \frac{V}{\lambda_3}$$

Putting value of λ_3

$$\therefore \quad f_3 = \frac{V}{2l / 3}$$

$$f_3 = 3\left(\frac{V}{2l}\right)$$

Case II

When pipe is open at one end and closed at other end.

In 1st mode

$$\lambda_1 = 4l$$

$$f_1 = \frac{V}{\lambda_1}$$

Putting value of λ_1

$$f_1 = \frac{V}{4l}$$

In 2nd mode

$$l = \frac{\lambda_2}{4} + \frac{\lambda_2}{2}$$

$$l = \frac{\lambda_2 + 2\lambda_2}{4}$$

$$l = \frac{3\lambda_2}{4}$$

$$\lambda_2 = \frac{4l}{3}$$

$$f = \frac{V}{\lambda_2}$$

Putting value of λ_2

$$f_3 = \frac{V}{4l/3}$$

$$f_3 = \frac{3V}{4l}$$

$$f_3 = 3(f_1)$$

We can generalize,

$$\therefore \quad f_n = \frac{nV}{4l}$$

where $n = 1, 3, 5, \ldots\ldots$

Note: *The pipe which is open at both ends is richer in harmonics.*

Interesting Information

Echolocation allows dolphins to detect small differences in the shape, size and thickness of objects.

Q.15 What is Doppler effect? Describe expression for apparently changed frequency when the observer is at rest while the source is in motion. (OR) What is Doppler effect? Describe the expression for apparently changed frequency when the observer is in motion while source is at rest.

Ans. DOPPLER EFFECT

Introduction

This effect was observed by Johann Doppler while he was observing the frequency of light emitted from distant stars. In some cases the frequency of light emitted from a star was found to be

Definition

"The apparent change in the pitch OR frequency of a source of sound, when there is a relative motion between the source of sound and the observer, is called Doppler effect."

For example, when an observer is standing on a railway platform, the pitch of the whistle of a approaching train is heard to be higher. But when the same train moves away, the pitch of the whistle becomes lower.

Explanation

Suppose 'V' is the velocity of the sound in a medium and a source emits a sound of frequency 'f' and wave length 'λ'. If both the source and the observer are stationary, then the waves received by the observer in one second are;

$$f = \frac{V}{\lambda}$$

Case-A

When observer 'O' is moving towards a stationary source 'S'

If an observer ('O') moves towards, a source 'S' with velocity U_0, the relative velocity of the waves and the observer is increased to $(V + U_0)$, then number of waves received in one second apparent frequency increases

$$f_A = \frac{V}{\lambda}$$

Putting the value

$$f_A = \frac{V + U_0}{V/f} \quad \left(\because \lambda = \frac{V}{f}\right)$$

$$f_A = \left(\frac{V + U_0}{V}\right)f$$

Hence, $\frac{V + U_0}{V} > 1$

$\therefore$ $f_A > f$

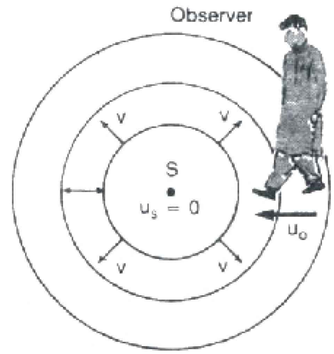


Fig. An observer moving with velocity u_o towards a stationary source hears a frequency f_A that is greater than the source frequency.

This means that when an observer 'O' is moving towards a stationary source S, the frequency of sound increases.

Case-B

When observer 'O' moves away from a stationary source S

When observer 'O' moves away from stationary source 'S' with velocity 'U_0,' then relative velocity of sound waves and the observer is, $(V - U_0)$, hence, number of waves received by the observer, per–second are reduced and is given by

$$f_B = \frac{V - U_0}{\lambda}$$

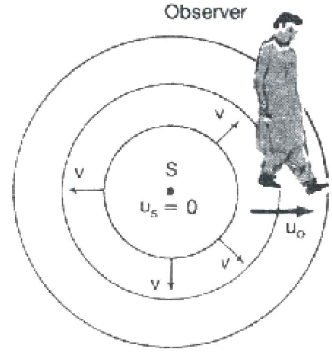


Fig. An observer moving with velocity u_o away from stationary source hears a frequency f_B that is smaller than the source frequency.

Putting value of

$$\lambda = \frac{V}{f}$$

$$\therefore \quad f_B = \frac{V - U_0}{V/f}$$

$$f_B = \left(\frac{V - U_0}{V}\right) f$$

As, $$\frac{V - U_0}{V} < 1$$

$$\therefore \quad f_B < f$$

This means that when observer 'O' moves away from stationary source 'S,' the apparent frequency decreases.

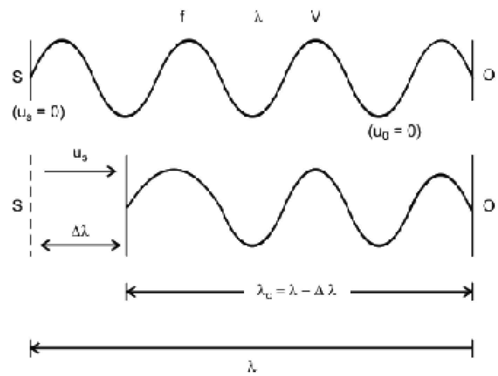
Case-C

When source 'S' is moving towards the stationary observer 'O'

If the source 'S' is moving towards the observer with velocity 'U_s' then in one second, the waves are compressed, (Wave length decreases), by an amount known as Doppler shift represented by $\Delta\lambda$.

As, $$V = f\lambda$$

$$\Delta\lambda = \frac{U_s}{f}$$



The compression of waves is due to the fact that same number of waves are contained in a shorter space depending upon the velocity of the source.

The wave length for observer 'C' is then, $\lambda_c = \lambda - \Delta\lambda$.

Putting the values of λ and $\Delta\lambda$.

$$\therefore \quad \lambda_c = \frac{V}{f} - \frac{U_s}{f}$$

$$\lambda_c = \frac{V - U_s}{f}$$

The modified frequency for observer 'C' is

$$f_c = \frac{V}{\lambda_c}$$

Putting value of λ_c

$$\therefore \quad f_c = \frac{V}{\frac{V - U_s}{f}}$$

$$f_c = \left(\frac{V}{V - U_s}\right) f$$

As, $$\frac{V}{V - U_s} > 1$$

$$\therefore \quad f_c > f$$

This means that the observed frequency increases when the source is moving towards the observer.

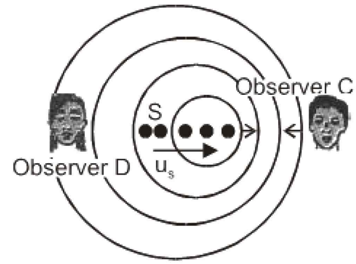
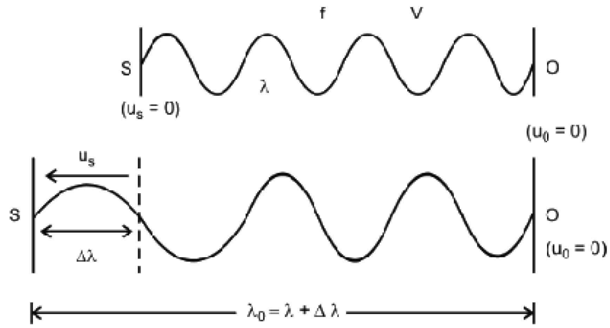


Fig. A source moving with velocity u_s towards a stationary observer C and away from stationary observer D. Observer C hears an increased and observer D hears a decreased frequency.

Case-D

When source 'S' is moving away from stationary observer 'O'

If the source is moving away from the observer 'O' with velocity 'U_s,' then in one second the waves are stretched (wave length increase) by an amount $\Delta\lambda$.



$$\Delta\lambda = \frac{U_s}{f}$$

The stretched of the waves is due to the fact that same number of wave are contained in larger (longer) space, depending upon the velocity of the source.

The wave length for observer 'D' is, $\lambda_D = \lambda + \Delta\lambda$.

Putting values of λ and $\Delta\lambda$.

$$\therefore \quad \lambda_D = \frac{V}{f} + \frac{U_s}{f}$$

$$\lambda_D = \frac{V + U_s}{f}$$

The modified frequency for observer 'D' will be

$$f_D = \frac{V}{\lambda_D}$$

Putting value of λ_D

$$\therefore \quad f_D = \frac{V}{\frac{V + U_s}{f}}$$

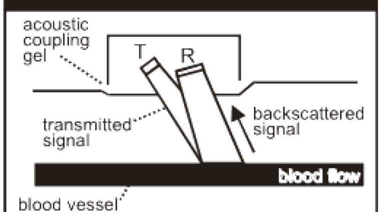
$$f_D = \left(\frac{V}{V + U_s}\right) f$$

As, $$\frac{V}{V + U_s} < 1$$

$$\therefore \quad f_D < f$$

This means that the observed frequency decreases, when the source is moving away from the observer.

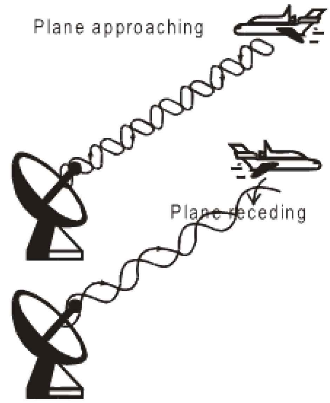
Do You Know?



The Doppler effect can be used to monitor blood flow through major arteries. Ultrasound waves of frequencies 5 MHz to 10 MHz are directed towards the artery and a receiver detects the back scattered signal. The apparent frequency depends on the velocity of flow of the blood.

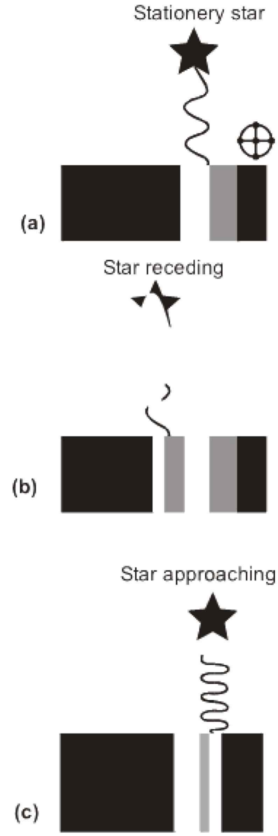
Ans. APPLICATIONS OF DOPPLER EFFECT

Doppler effect is applied in working of radar system. Radar uses radio waves to find the elevation and speed of an aeroplane. Radar is a device which transmits and receives radio waves. The radio waves transmitted from radar are reflected back from aeroplane and are received by radar. If the aeroplane is moving towards the radar then the wavelength of reflected wave is shorter. If the plane is moving away from the radar then the wavelength of reflected wave is longer as shown in figure.



A frequency shift is used in a radar to detect the motion of an aeroplane.

The difference of wavelength of transmitted and reflected waves is used to determine the speed of aeroplane. Term SONAR (is acronym) stands for sound navigation and ranging. Sonar is the name of the technique used for detecting the presence of objects under water by acoustical echo. In Sonar "Doppler detection" depends upon relative speed of the target and the detector. It employs. The Doppler effect in which an apparent change in frequency occurs when source and observer are in relative motion with respect to one another.



In military it is used for detection and location of submarine antisubmarine weapons and depth measurement of sea. Astronomers use Doppler effect to calculate the speeds of distant stars and galaxies. By comparing the line spectrum of light from the star with light from a laboratory source, the Doppler shift of stars light can be measured. Then speed of star can be calculated.

Doppler effect is used to determine whether a particular star or galaxy is approaching the earth or moving away from the earth. Light from the star is measured with the help of spectrometer. It has been found that stars moving towards the earth show a blue shift. Thus is because the emitted waves by the star have shorter wavelength than of the star had been at rest. So the spectrum is shifted towards shorter wavelength i.e., to the blue end of the spectrum.

It has been found that stars moving away from the earth show a red shift. This is because the emitted waves by the star have longer wavelength than of the star had been at rest. So the spectrum is shifted towards longer wavelength i.e., to the red end of the spectrum. Astronomers have discovered that all the distant galaxies are moving away from us. They have also measured their speed by measuring their red shift. Another important application of the Doppler shift using electromagnetic waves is radar speed trap. Microwaves are emitted from a transmitter in the form of short bursts. Each burst is reflected back by any moving car in the path of microwaves. The reflected microwaves are received back as Doppler's shift. By measuring Doppler shift the speed of the car can be calculated by computer programme.

Do You Know?